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Supplementary Information — Observation of intensity flattened phase shifting enabled by unidirectional guided resonance

1 Complex background reflectivity and transmissivity

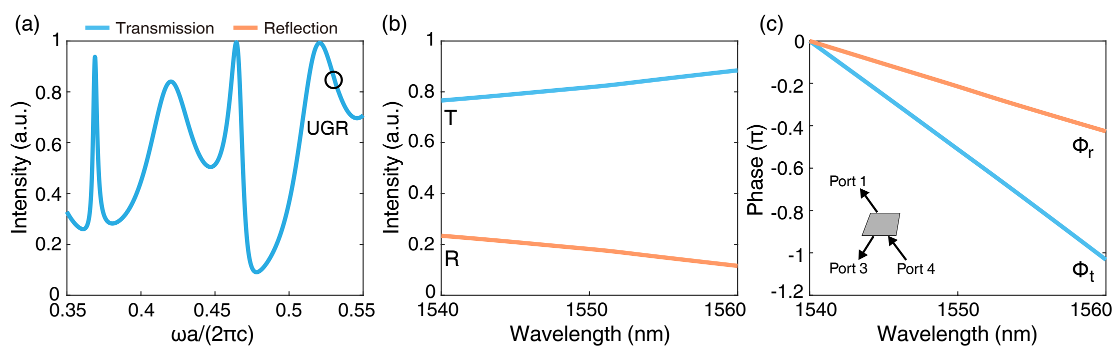

Fig. 1: Numerical simulation of the reflection and transmission: (a) transmittance over a wide frequency range, multiple resonances appear on the spectrum; (b) the background reflectance R (orange) and transmittance T (light blue) when the incident inputs from Port 4 and then measured from Port 1 and 3 (inset); (c) the phase response of reflection ϕ_r (orange) and transmission ϕ_t (light blue).

Temporal coupled-mode theory (TCMT)[1, 2] is a useful and accurate tool to model the resonances interacting with excitation, in which a scattering matrix C is used to depict the direct (nonresonant) transmission and reflection through the resonance, given by:

$$C = e^{j\phi} \begin{pmatrix} 0 & r & 0 & jt \\ r & 0 & jt & 0 \\ 0 & jt & 0 & r \\ jt & 0 & r & 0 \end{pmatrix} \tag{1}$$

in which r and t characterize the background reflectivity and transmissivity, and satisfies $r^2 + t^2 = 1$ because the law of energy-conservation. In many previous works, the r and t are assumed as Fabry–Perot background, and they are real numbers. This assumption works well for single-resonance TCMT models. While noticed from SFig.1(a), there exists multiple resonances near the UGR, and some of them are low-Q resonances that would influence a wide-range of the resonances nearby. If we keep using single-resonance TCMT to model our system, we find it is necessary to assume complex background r and t.

As illustrated in Fig.1 in the main text, the resonance radiation towards Port 4 is closed in our UGR design. Under time-reversal, it implies that an incident from Port 4 has no chance to excite the resonance. So that the transmission (Port 4 to 1) to reflection (Port 4 to 2) are purely attributed to the background of

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the photonic crystal slab. The transmittance T, reflectance R and phase related response ϕ_r and ϕ_t are presented in SFig. 1(b) and (c). Accordingly, the complex background reflectivity and transmissivity are obtained as:

$$\hat{r} = \sqrt{R}e^{-j\phi_r} \quad \hat{t} = \sqrt{T}e^{-j\phi_t} \tag{2}$$

2 Spatial correlation for phase extraction

When the signal and reference light arrive at the CCD camera, they create an equal inclination interference, which are concentric-ring fringes. To accurately extract the phase from fringe patterns, we take a cut-line that passes through the center of the rings, and obtain the intensity at each pixel from the CCD images, as shown in SFig.2.

Next, we generate a reference interference pattern by choosing a given reference wavelength at 1540 nm. From analytical theory of equal inclination interference, the radius of the bright circle is

$$r = f \cdot tan\theta \tag{3}$$

where f represents the focal length of the lens, and θ is the angle of incidence. In general, the Michelson interferometer configuration in our setup creates an air gap with a thickness of h, then the phase difference between the reference and signal light is written as:

$$\phi = \frac{2\pi}{\lambda} \cdot 2hcos\theta + \phi_{\text{res}} \tag{4}$$

Note that the phase different ϕ is wavelength dependent if h is not zero. After fine-tuning the arms' equality, we get $h = 0$, so we have $\phi = \phi_{\text{res}}$. Further, the interference intensity is

$$I = |1 + e^{-j\phi}|^2 \tag{5}$$

Combing the Gaussian envelope due to diffraction, we get the reference pattern as:

$$I_{\text{ref}} = Ie^{-\frac{(r-r_0)^2}{2w_0^2}} \tag{6}$$

where coefficients r_0 and w_0 is the center and waist width of the interference pattern, and they are fitted according to the experiment data at 1540 nm as shown in Fig. S2(a).

We use spatial correlation to accurately calculate the phase shifting for other wavelengths. Specifically, for each wavelength, we scan the phase $\phi = \phi_{\text{res}}$ in Eq.5 and calculate the correlation with respect to the experimental observed line-cut data. Further, we find out the ϕ that maximize the correlation, which is exactly the phase shifting given by the resonance.

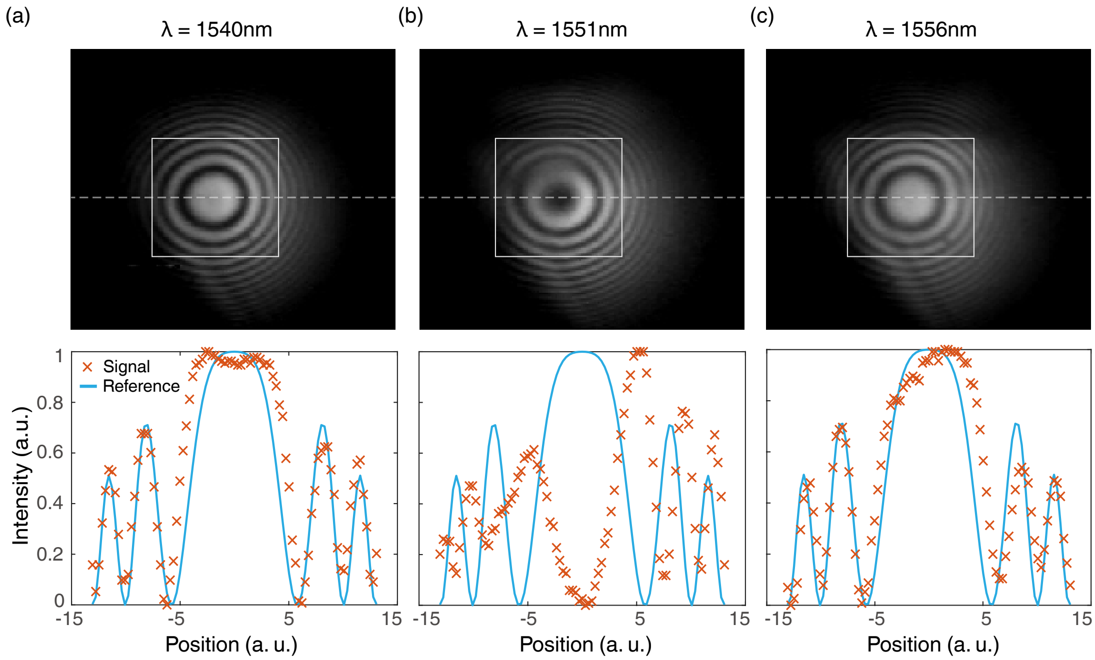

Fig. 2: Spatial correlation method to extract phase shifting. Top panel: the interference patterns and the dash-line shows the line-cut; Bottom panel: the reference profiles (lines) and line-cut of CCD image data (markers) for (a) 1540 nm; (b) 1551 nm; and (c) 1556 nm, respectively.

References

[1] S. Fan, W. Suh, and J. D. Joannopoulos, "Temporal coupled-mode theory for the Fano resonance in optical resonators," *J. Opt. Soc. Am. A* **20**, 569–572 (2003).
[2] H. Zhou, B. Zhen, C. W. Hsu et al., "Perfect single-sided radiation and absorption without mirrors," *Optica* **3**, 1079–1086 (2016).