Supplementary Material

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Arbitrary polarization conversion for pure vortex generation with a single metasurface

1 Extinction conditions

In the main text we solved the general problem corresponding to the determination of the pillar parameters that provide complete attenuation of an input field of arbitrary polarization passing through the pillar-polarizer system. The expression for the pillar orientation angle α_0 for the extinction condition was given in Eq. (3) of the main text. The analytical expression for the phase retardance $\Delta\phi_0$ is less compact and is given here:

$$\begin{aligned}\Delta\phi_0 =2 \Bigg\{ & - \cot^{-1}\Bigg[\csc\delta\Bigg(\cos\delta + \cot\alpha_0 \cot\chi + \csc\alpha_0 \csc\chi \\ & \sqrt{\sin^2\alpha_0\Big(\Big(\cos\delta + \cot\alpha_0 \cot\chi\Big)^2 + \sin^2\delta\Big)\sin^2\chi}\;\Bigg)\Bigg] \\ & + \cot^{-1}\Bigg[\csc\delta\Bigg(\cos\delta - \cot\chi\tan\alpha_0 + \csc\chi\sec\alpha_0 \\ & \sqrt{\cos^2\alpha_0 \sin^2\chi\Big(\sin^2\delta + \Big(\cos\delta - \cot\chi\tan\alpha_0\Big)^2\Big)}\;\Bigg)\Bigg]\Bigg\} \end{aligned} \tag{S1}$$

where $|\chi,\ \delta\rangle$ characterize the incident polarization state (Eq. (2) of the main text).

The dependence of α_0 and $\Delta\phi_0$ were plotted in the main text on the (χ, δ) plane. In Fig. S1 we represent them on the Poincaré sphere, together with the distribution of maximum field transmission for the case in which the phase retardance is fixed to $\Delta\phi_0$, as in Eq. (S1), and the orientation angle is allowed to vary to maximize the transmission. Only a limited fraction of input states results in a small transmission. Such states are oriented close to the x-axis, almost perpendicularly to the y-oriented polarizer and therefore nearly no amplitude can be transmitted, regardless of the angle of the pillar. The maximum transmitted amplitude on the Poincaré sphere

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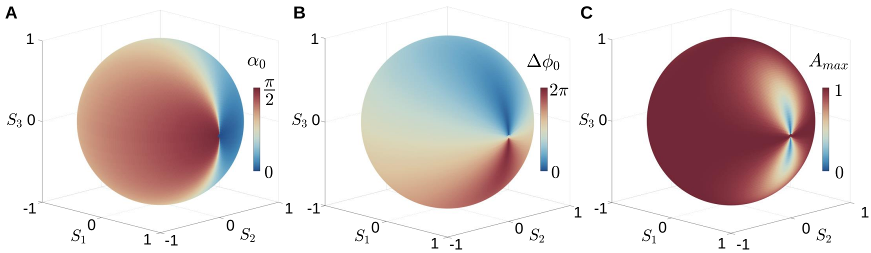

Fig. S1: Representation on the Poincaré sphere of the design parameters required for the extinction condition through the pillar-polarizer system, corresponding to the pillar angle (**A**) and phase retardance (**B**), for any input polarization state. **C**, Maximum field transmission through the system when the phase retardance is fixed to $\Delta\phi_0$ and the orientation angle is allowed to vary. These plots correspond to the planar representations shown in Fig. 2 D-F of the main text. The axis are the three Stokes parameters, S_1, S_2 and S_3.

shows a distribution that is reminiscent of that of a dipole emission pattern (Fig. S1C). This can be understood as in our system, similarly to a dipole, there is a preferential direction (in our case determined by the polarizer axis) from which no amplitude can be emitted. However we note that, as already mentioned in the main text, even for such polarization states it is possible to obtain unity transmission through the pillar-polarizer system if both the orientation angle and the phase retardance of the pillar are allowed to vary. In fact it can be shown that for any input polarization state a combination of these parameters always exist such to provide unity transmission (cf. Fig. 1B of the main text). The only disadvantage of the approach where both the phase retardance and the angle can vary is that the parameter combination for an arbitrary amplitude modulation needs to be determined numerically, rather than analytically.

2 Purity efficiency

The efficiency of the PA metasurface in converting an input beam into a pure Laguerre-Gauss (LG) mode depends on the radial distribution of both the source and LG fields, which are given by

$$E_S = e^{-(r/\omega_S)^2} \tag{S2}$$

and

$$E_{LG} = e^{-(r/\omega_0)^2} \left(\frac{r\sqrt{2}}{\omega_0}\right)^{|\ell|}, \tag{S3}$$

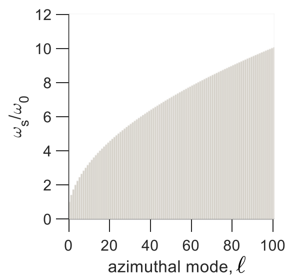

Fig. S2: Ratio of the beam waist of a Gaussian source and beam waist of the embedded Gaussian of a Laguerre-Gauss mode generated by the PA metasurface, such that the purity conversion efficiency is maximized. The ratio is plotted for different orbital angular momentum charges of the generated Laguerre-Gauss mode.

respectively, where r is the radial coordinate, ω_S is the beam waist of the Gaussian source and ω_0 is the beam waist of the embedded Gaussian in the LG mde. In Eq. (S2) and (S3) we omitted any constant or azimuthally-varying factor. The purity efficiency of the PA metasurface in terms of power conversion is then defined as

$$\eta = \frac{\int_0^\infty E_{LG}^2 \mathrm{d}r}{\mathcal{N}^2 \int_0^\infty E_S^2 \mathrm{d}r} \tag{S4}$$

where $\mathcal{N}$ is a normalization constant corresponding to the maximum of E_{LG}/E_S. By maximizing Eq. (S4) vs. ω_S it is calculated that the maximum efficiency for a given $|\ell|$ is obtained when $\omega_S/\omega_0 = \sqrt{|\ell|+1}$ (Fig. S2). In these conditions the purity efficiency can be written as

$$\eta = e^{|\ell|} \left[\frac{\omega_S}{\omega_0} \sqrt{\frac{|\ell|}{(\omega_S/\omega_0)^2 - 1}} \right]^{-2|\ell|} \Gamma\left(\frac{1}{2} + |\ell|\right) \frac{\omega_0}{\sqrt{\pi}\,\omega_S} \tag{S5}$$

where $\Gamma(...)$ is the gamma function.

3 Device design

In Fig. 2 of the main text only the central regions of the designed metasurfaces are shown due to space constraints. In Fig. S3A,B we show a comparison between the PO and PA metasurface designs on a larger scale. In both cases the target LG mode corresponds to an azimuthal mode with $|\ell|= 5$. It can be appreciated as the angle degree of freedom is used in the PA metasurface for the purpose of

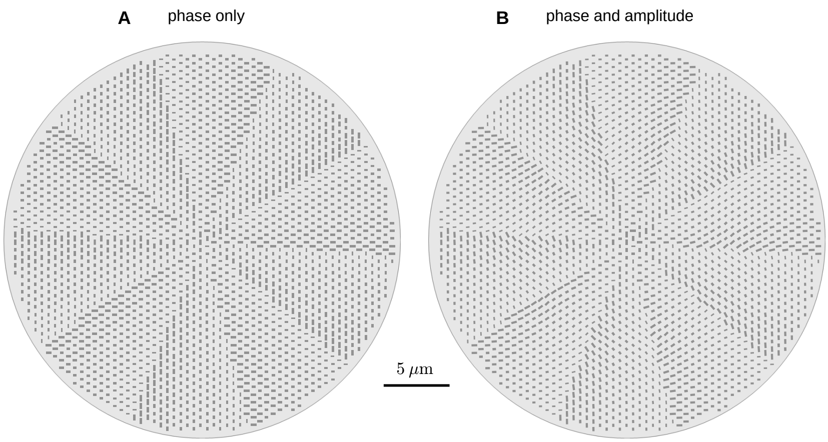

Fig. S3: Comparison of the device designs for a PO and PA metasurface imparting an orbital angular momentum charge of $|\ell|= 5$. Only the PA metasurface incorporates an amplitude transmission mask, which is embedded in the pillars rotation.

implementing the amplitude mask, which is essential for the generation of pure LG modes.