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Supplemental Information for Research Article: Beyond dipolar Huygens' metasurfaces for full phase coverage and unity transmittance

1 Dipolar-quadrupolar Holograms

In this section, we investigate the direct consequences of phase-coverage deficiency in a dipolar metasurface by performing a case study on hologram images.

In particular, we study transmission holograms originating from the modulation of a plane wave by a pure phase mask, i.e. the mask (the metasurface) is assumed to have unity transmittance and a spatially varying phase-angle. Due to practical reasons, the metasurface is realized as a rectangular array of pixels, where each pixel is assumed to have unity transmittance and a spatially constant phase-angle that can be set by adjusting the lattice constant of the particle array associated with the pixel.

The algorithm used for the hologram generation is based on the inverse Fourier transform algorithm [1], and the band-limited angular spectrum method [2].

We employed the zero-normalized cross-correlation (ZNCC) as a measure for the similarity between the target intensity image I_{target} and the hologram intensity image

I_{hologram} :

$$\text{ZNCC} = \frac{\sum_{i,j} \tilde{I}_{\text{target}}[i,j] \tilde{I}_{\text{hologram}}[i,j]}{\sqrt{\sum_{i,j} \tilde{I}_{\text{target}}^2[i,j]} \sqrt{\sum_{i,j} \tilde{I}_{\text{hologram}}^2[i,j]}}, \quad (1a)$$

$$\tilde{I}_{(\cdot)}[i,j] = I_{(\cdot)}[i,j] - \frac{1}{IJ} \sum_{i=1}^I \sum_{j=1}^J I_{(\cdot)}[i,j], \quad (1b)$$

Here, (i,j) and (I,J) are the coordinates and numbers of image pixels per axis, respectively.

The inverse Fourier transform algorithm terminated as soon as the ZNCC changed by less than 10^{-5} between successive iterations. For the results presented here, this condition was usually met after 500 to 800 iterations, and since the algorithm requires an initial guess for the phase mask, each simulation was started with a different set of phase-angles taken from a uniform random distribution.

The target image is shown in Fig. 1a. We consider a monochromatic hologram image at an optical wavelength of 1500 nm with a total size of $10 \times 10 \text{ mm}^2$ in a distance of 70 mm behind the metasurface. The metasurface has a total size of $2 \times 2 \text{ mm}^2$ and consists of 200×200 pixels, which corresponds to a pixel size of $10 \times 10 \mu\text{m}^2$.

We investigated two scenarios: The first with pixels consisting of dipolar-quadrupolar particles providing 100% phase-coverage and the second with pixels consisting of dipolar particles providing only 75% phase-coverage. The achieved hologram images are shown in Fig. 1b and Fig. 1c, respectively. As expected, the dipolar-quadrupolar metasurface performs better (ZNCC = 0.772) than the dipolar metasurface (ZNCC = 0.707).

In order to quantify the maximum possible ZNCC in dependence on the spatial frequencies present in the target intensity image, we defined a parametrized test intensity image, where Λ is the spatial wavelength of an intensity modulation:

$$I_{\text{test}}(x,y;\Lambda) = \frac{1}{4} \left[2 - \cos\left(\frac{2\pi}{\Lambda}x\right) - \cos\left(\frac{2\pi}{\Lambda}y\right) \right]. \quad (2)$$

We considered a range of $\Lambda \in [0.05, 10] \text{ mm}$, where the lower limit corresponds to the Abbe resolution limit and the upper to the image size.

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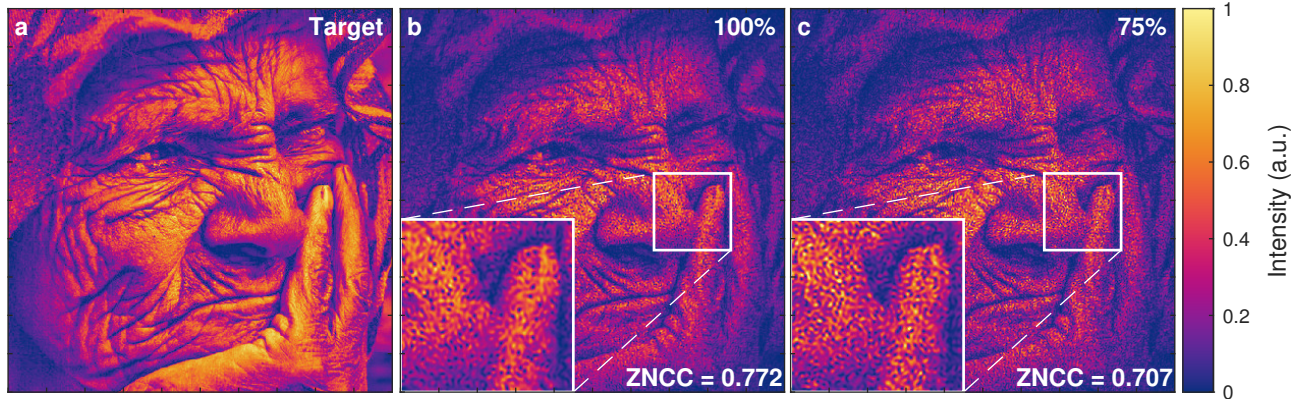


Fig. 1: Intensity images of the a) target and the holograms for b) 75% and c) 100% coverage of the full phase-shift range. All images have a physical size of $10 \times 10 \text{ mm}^2$. The insets show a magnification of the region enclosed by the white squares. Photo courtesy of Ebrahim Alipoor, taken with permission.

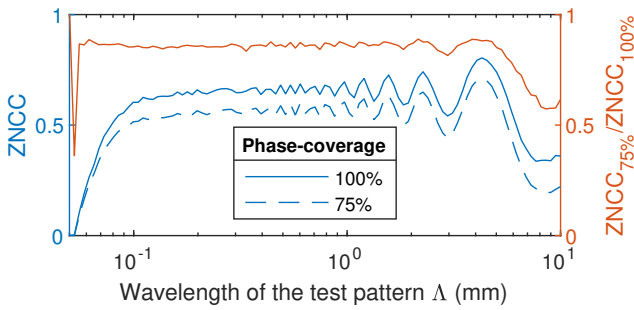


Fig. 2: The zero-normalized cross-correlation as a function of the feature size in the test intensity image I_{test} .

References

- [1] Antonie D. Verhoeven, Frank Wyrowski, and Jari Turunen. "Iterative Design of Diffractive Elements Made of Lossy Materials." In: *J. Opt. Soc. Am. A* 35.1 (Dec. 2017), p. 45. DOI: 10.1364/josaa.35.000045.
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At each Λ we recorded the ZNCC of the test pattern for both scenarios, i.e. dipolar and dipolar-quadrupolar metasurfaces. The results are shown in Fig. 2 and indicate that in such scenarios, the ZNCC of the generated hologram image can be consistently increased for all spatial frequencies by about 16% when instead of dipolar, dipolar-quadrupolar metasurfaces are used.

As a short note on the finiteness of the pixels, we have taken the response of the infinitely periodic arrangement as the response of the finite-size pixels. In that assumption, we have neglected the effect of the pixel finiteness and also inter-pixel coupling to avoid complications and to focus the study on the comparison of full and partial phase coverage. For larger pixels, the effect of finiteness and inter-pixel coupling diminishes as far as a minimum distance between pixels are considered. A pixel size of $10 \times 10 \mu\text{m}^2$ has been considered as a proper quantity for the study. Our future works will cover this topic in more detail.