Research Article

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On multiplicative degree based topological indices for planar octahedron networks

Abstract: Chemical graph theory is a branch of graph theory in which a chemical compound is presented with a simple graph called a molecular graph. There are atomic bonds in the chemistry of the chemical atomic graph and edges. The graph is connected when there is at least one connection between its vertices. The number that describes the topology of the graph is called the topological index. Cheminformatics is a new subject which is a combination of chemistry, mathematics and information science. It studies quantitative structure-activity (QSAR) and structure-property (QSPR) relationships that are used to predict the biological activities and properties of chemical compounds. We evaluated the second multiplicative Zagreb index, first and second universal Zagreb indices, first and second hyper Zagreb indices, sum and product connectivity indices for the planar octahedron network, triangular prism network, hex planar octahedron network, and give these indices closed analytical formulas.

1 Introduction

Topological indices, provided by graph theory, are a valuable tool. Cheminformatics is a modern academic area that combines the subjects of chemistry, mathematics, and information science. It studies the relationships between quantitative structure-activity (QSAR) and structure-property (QSPR) used to suggest biological activities and chemical compound properties. That is why scholars around the world are extremely interested in it.

The molecular structures are those in which atoms are connected by covalent bonds. In graph theory, atoms are considered as vertices and covalent bonds are as edges. Chem-informatics is a new area of research in which the subjects chemistry, mathematics, and information science are combined.

At present, in the field computational chemistry topological indices have a rising interest which is actually associated to their use in nonempirical quantitative structure-property relationship and quantitative-structure activity relationship. Topological descriptor Top(G) may also be defined with the property of isomorphism i.e for every graph H isomorphic to G, Top(G) = Top(H). The idea of topological indices was first introduced by Weiner (1947) during the lab work on boiling point of paraffin and named this result as Path number which was later named as Weiner Index.

Nowadays there is an extensive research activity on *ABC* and *GA* indices and their variants, for further study of topological indices of various graphs (Ali and Sajjad, 2019; Ali et al., 2019, 2020; Akhtar and Imran, 2016; Aslam et al., 2019; Babar et al., 2020; Baig et al., 2015, 2018; Gao et al., 2017; Huo et al., 2020; Imran et al., 2016, 2018; Simonraj and George, 2012; Shirdel et al., 2013; Smith, 2019; Vetrík, 2018; Wei et al., 2019; Zheng et al., 2019). For the basic notations and definitions, see (Diudea et al., 2001; Trinajstić, 1983).

In this article, we consider the silicate structure (Manuel and Rajasingh, 2011) derived from the POH network, TP network, and hex POH network (Simon Raj and George, 2015). The method of drawing planar octahedron networks with dimension *m* is as follows:

Step 1: Let take a silicate network with dimension *m*.
Step 2: At the middle of each triangular face, fix new vertices, and connect them to oxide vertices in the corresponding triangle face.

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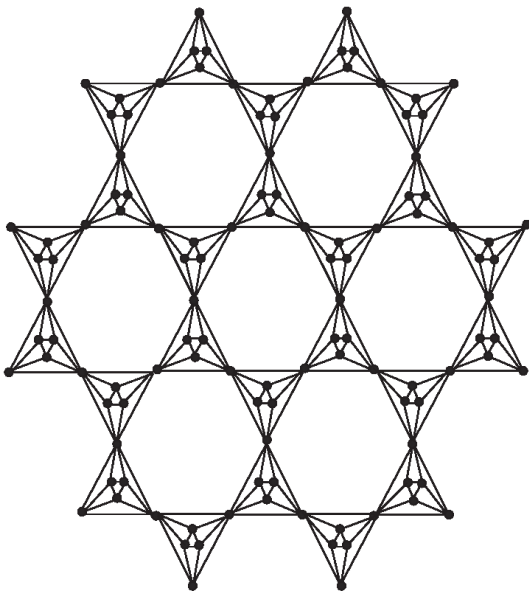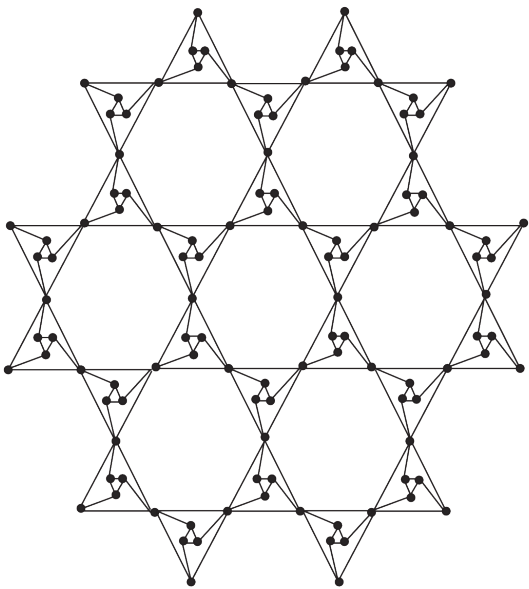
Figure 1: Planar octahedron network POH (2).

Figure 2: Triangular prism network TP (2).

Step 3: Link all these new vertices of the centre that lie in the same silicate cell.

Step 4: The resulting graph is called the planar octahedron network for the m dimension as shown in Figure 1. Delete all silicon vertices. We can also create the triangular prism network as shown in Figure 2 and the hex POH network as shown in Figure 3.

In the graph theory, the number of edges which is occurrence to the vertex that is the degree of vertex of the graph. Let φ be a graph. Then second
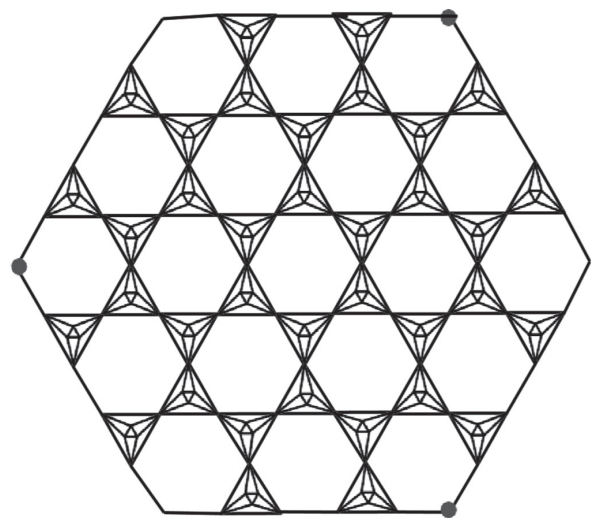

Figure 3: Hex planar octahedron network (2).

multiplicative Zagreb index (Gutman et al., 1975) can be defined as:

$$II_2(\varphi) = \prod_{ab \in E(\varphi)} (d_a \times d_b).$$

(1)

First and second indices (Kulli, 2016) of a graph φ are defined as:

$$HII_1(\varphi) = \prod_{ab \in E(\varphi)} (d_a + d_b)^2,$$

(2)

$$HII_2(\varphi) = \prod_{ab \in E(\varphi)} (d_a \times d_b)^2.$$

(3)

The first and second universal Zagreb index (Kulli, 2016) defined as:

$$MZ_1^\alpha(\varphi) = \prod_{ab \in E(\varphi)} (d_a + d_b)^\alpha,$$

(4)

$$MZ_2^\alpha(\varphi) = \prod_{ab \in E(\varphi)} (d_a \times d_b)^\alpha,$$

(5)

where: $\alpha \in R$.

The sum and product connectivity of Multiplicative indices (Kulli, 2016) defined as:

$$SCII(\varphi) = \prod_{ab \in E(\varphi)} \left(\frac{1}{\sqrt{d_a + d_b}} \right),$$

(6)

$$PCII(\varphi) = \prod_{ab \in E(\varphi)} \left(\frac{1}{\sqrt{d_a \times d_b}} \right).$$

(7)

2 Results

We research the Zagreb indices with its types, such as the second Zagreb multiplicative index, the first Zagreb hyper index, the second Zagreb hyper index, the first and second Zagreb universal indices and the multiplicative indices sum and product connectivity for the planar octahedron network, triangular prism network, hex planar octahedron network.

2.1 Results for planar octahedron network POH(m)

In this section, we calculate edge partition of topological indices of the dimension m for the planar octahedron network. In the coming theorems, we compute some important indices for planar octahedron network.

Theorem 2.1.1

Consider the planar octahedral network POH(m), then its second multiplicative Zagreb index is equal to:

$$II_2(\varphi_1) = 382205952m^6 - 169869312m^4.$$

Proof. Let $\varphi_1 \cong$ POH(m). From Eq. 1, we have:

$$II_2(\varphi_1) = \prod_{ab \in E(\varphi_1)} \left(d_a \times d_b\right).$$

Using Table 1, we have:

$$II_2(\varphi_1) = (4 \times 4) \,|\, E_1(POH(m))\,| \times (4 \times 8)\,|\, E_2(POH(m))\,|$$
$$\times (8 \times 8)\,|\, E_3(POH(m))\,|, = 16\,|\, E_1(POH(m))\,|$$
$$\times 32\,|\, E_2(POH(m))\,| \times 64\,|\, E_3(POH(m))\,|,$$
$$= 16(18m^2 + 12m) \times 32(36m^2) \times 64(18m^2 - 12m).$$

This value is what, we get after calculations:

$$\Rightarrow II_2(\varphi) = 382205952m^6 - 169869312m^4.$$

Theorem 2.1.2

Consider the POH(m) network, then its first and second hyper Zegreb indices are equal to:

$$HII_1(\varphi_1) = 27518828544m^6 - 12230590464m^4,$$
$$HII_2(\varphi_1) = 1.25 \times 10^{13} m^6 - 5.57 \times 10^{12} m^4.$$

Proof. Let $\varphi_1 \cong$ POH(m) network and by using Eq. 2, we have:

$$HII_1(\varphi_1) = \prod_{ab \in E(\varphi)} (d_a + d_b)^2.$$

Table 1: Edge partition of POH(m) network.

(d_a, d_b)	Number of edges
$E_1 = (4,4)$	$18m^2 + 12m$
$E_2 = (4,8)$	$36m^2$
$E_3 = (8,8)$	$18m^2 - 12m$

Using Table 1, we have:

$$HII_1(\varphi_1) = (4+4)^2 \,|\, E_1(POH(m))\,| \times (4+8)^2 \,|\, E_2(POH(m))\,|$$
$$\times (8+8)^2 \,|\, E_3(POH(m))\,|,$$
$$= 8^2(18m^2 + 12m) \times 12^2(36m^2) \times 16^2(18m^2 - 12m),$$
$$= 2359296(11664m^6 - 5184m^4).$$

This value is what, we get after calculations:

$$\Rightarrow HII_1(\varphi_1) = 27518828544m^6 - 12230590464m^4.$$

For second hyper Zegreb index and using Eq. 3, we have:

$$HII_2(\varphi_1) = \prod_{ab \in E(\varphi)} (d_a \times d_b)^2.$$

Using Table 1, we have:

$$HII_2(\varphi_1) = (4 \times 4)^2 \,|\, E_1(POH(m))\,| \times (4 \times 8)^2 \,|\, E_2(POH(m)\,|$$
$$\times (8 \times 8)^2 \,|\, E_3(POH(m))\,|,$$
$$= 16^2(18m^2 + 12m) \times 32^2(36m^2) \times 64^2(18m^2 - 12m),$$
$$= 1073741824(11664m^6 - 5184m^4).$$

This value is what, we get after calculations:

$$\Rightarrow HII_2(\varphi_1) = 1.25 \times 10^{13} m^6 - 5.57 \times 10^{12} m^4.$$

Theorem 2.1.3

Consider the POH(m) network, then its first and second universal Zagreb indices are equal to:

$$MZ_1^\alpha(\varphi_1) = 1536^\alpha (11664m^6 - 5184m^4),$$
$$MZ_2^\alpha(\varphi_1) = 32768^\alpha (11664m^6 - 5184m^4).$$

Proof. Let $\varphi_1 \cong$ POH(m) network. We have to prove first universal Zagreb index and using Eq. 4:

$$MZ_1^\alpha(\varphi_1) = \prod_{ab \in E(\varphi_1)} (d_a + d_b)^\alpha.$$

Using Table 1, we have:

$$MZ_1^\alpha(\varphi_1) = (4+4)^\alpha \,|\, E_1(POH(m))\,| \times (4+8)^\alpha \,|\, E_2(POH(m))\,|$$
$$\times (8+8)^\alpha \,|\, E_3(POH(m))\,|,$$
$$= 8^\alpha(18m^2 + 12m) \times 12^\alpha(36m^2) \times 16^\alpha(18m^2 - 12m),$$
$$= 8^\alpha \times 12^\alpha \times 16^\alpha (11664m^6 - 5184m^4).$$

This value is what, we get after calculations:

$$\Rightarrow MZ_1^\alpha(\varphi_1) = 1536^\alpha(11664m^6 - 5184m^4).$$

For second universal-Zegreb index and by using Eq. 5, we have:

$$MZ_2^\alpha(\varphi_1) = \prod_{ab\in E(\varphi_1)} (d_a \times d_b)^\alpha.$$

Using Table 1, we have:

$$MZ_2^\alpha(\varphi_1) = (4\times4)^\alpha \mid E_1(POH(m)) \mid \times(4\times8)^\alpha \mid E_2(POH(m)) \mid$$
$$\times(8\times8)^\alpha \mid E_3(POH(m)) \mid,$$
$$= 16^\alpha(18m^2 + 12m)\times32^\alpha(36m^2)\times64^\alpha(18m^2 - 12m),$$
$$= 16^\alpha \times32^\alpha \times64^\alpha(11664m^6 - 5184m^4).$$

This value is what, we get after calculations:

$$\Rightarrow MZ_2^\alpha(\varphi_1) = 32768^\alpha(11664m^6 - 5184m^4).$$

Theorem 2.1.4

Consider the POH(m) network, the sum and product connectivity of multiplicative indices described as:

$$SCII(\varphi_1) = \frac{\sqrt{6}}{96}(11664m^6 - 5184m^4),$$
$$PCII(\varphi_1) = \frac{1}{\sqrt{32768}}(11664m^6 - 5184m^4).$$

Proof. Let $\varphi_1 \cong$ POH(m) network. We have to prove the sum and product connectivity of multiplicative indices and by using Eq. 6, we have:

$$SCII(\varphi_1) = \prod_{ab\in E(\varphi_1)} \left(\frac{1}{\sqrt{d_a + d_b}}\right).$$

Using Table 1, we have:

$$SCII(\varphi_1) = \frac{1}{\sqrt{4+4}} \mid E_1(POH(m)) \mid \times\frac{1}{\sqrt{4+8}} \mid E_2(POH(m)) \mid$$
$$\times\frac{1}{\sqrt{8+8}} \mid E_3(POH(m)) \mid$$
$$= \frac{1}{\sqrt{4+4}}(18m^2 + 12m)\times\frac{1}{\sqrt{4+8}}(36m^2)$$
$$\times\frac{1}{\sqrt{8+8}}(18m^2 - 12m),$$
$$= \frac{1}{\sqrt{8}}\times\frac{1}{\sqrt{12}}\times\frac{1}{\sqrt{16}}(11664m^6 - 5184m^4),$$
$$= \frac{1}{\sqrt{8\times12\times16}}(11664m^6 - 5184m^4).$$

This value is what, we get after calculations:

$$\Rightarrow SCII(\varphi_1) = \frac{\sqrt{6}}{96}(11664m^6 - 5184m^4).$$

For product connectivity of multiplicative indices and by using Eq. 7, we have:

$$PCII(\varphi_1) = \prod_{ab\in E(\varphi_1)} \left(\frac{1}{\sqrt{d_a \times d_b}}\right).$$

Using Table 1, we have:

$$PCII(\varphi_1) = \frac{1}{\sqrt{4\times4}} \mid E_1(POH(m)) \mid \times\frac{1}{\sqrt{4\times8}} \mid E_2(POH(m)) \mid$$
$$\times\frac{1}{\sqrt{8\times8}} \mid E_3(POH(m)) \mid,$$
$$= \frac{1}{\sqrt{4\times4}}(18m^2 + 12m)\times\frac{1}{\sqrt{4\times8}}(36m^2)$$
$$\times\frac{1}{\sqrt{8\times8}}(18m^2 - 12m),$$
$$= \frac{1}{\sqrt{16}}\times\frac{1}{\sqrt{32}}\times\frac{1}{\sqrt{64}}(11664m^6 - 5184m^4),$$
$$= \frac{1}{\sqrt{16\times32\times64}}(11664m^6 - 5184m^4).$$

This value is what, we get after calculations:

$$\Rightarrow PCII(\varphi_1) = \frac{1}{\sqrt{32768}}(11664m^6 - 5184m^4).$$

2.2 Results for triangular prism network TP(m)

In this section, we calculate edge partition of topological indices of the dimension m for the Triangular prism network. In the coming theorems, we compute some important indices for triangular prism network.

Theorem 2.2.1

Consider the triangular prism network TP(m), then its second multiplicative Zagreb index is equal to:

$$II_2(\varphi_2) = 34012224m^6 - 11337408m^4 - 2519424m^3.$$

Proof. Let $\varphi_2 \cong$ TP network and by using Eq. 1, we have:

$$II_2(\varphi_2) = \prod_{ab\in E(\varphi_2)} (d_a \times d_b).$$

Table 2: Edge partition of TP network.

(d_a, d_b)	Number of edges
$E_1 = (3,3)$	$18m^2 + 6m$
$E_2 = (3,6)$	$18m^2 + 6m$
$E_3 = (6,6)$	$18m^2 - 12m$

Using Table 2, we have:

$$
\begin{aligned}
II_2(\varphi_2) &= (3\times3)\,|E_1(TP(m))|\times(3\times6)\,|E_2(TP(m))| \\
&\quad \times(6\times6)\,|E_3(TP(m))|, \\
&= 9\,|E_1(TP(m))|\times18\,|E_1(TP(m))|\times36\,|E_1(TP(m))|, \\
&= 9(18m^2+6m)\times18(18m^2+6m)\times36(18m^2-12m).
\end{aligned}
$$

This value is what, we get after calculations:

$$
\Rightarrow II_2(\varphi_2) = 34012224m^6 - 11337408m^4 - 2519424m^3.
$$

Theorem 2.2.2

Consider the TP(m) network, then its first and second hyper Zegreb indices are equal to:

$$
\begin{aligned}
HII_1(\varphi_2) &= 2448880128m^6 - 816293376m^4 - 181398528m^3, \\
HII_2(\varphi_2) &= 1.99\times10^{11}m^6 - 6.6\times10^{10}m^4 - 1.47\times10^{10}m^3.
\end{aligned}
$$

Proof. Let $\varphi_2 \cong$ TP Network and by using Eq. 2, we have:

$$
HII_2(\varphi_2) = \prod_{ab\in E(\varphi_2)} (d_a\times d_b)^2.
$$

Using Table 2, we have:

$$
\begin{aligned}
HII_1(\varphi_2) &= (3+3)^2\,|E_1(TP(m))|\times(3+6)^2\,|E_2(TP(m))| \\
&\quad \times(6+6)^2\,|E_3(TP(m))|, \\
&= 6^2(18m^2+6m)\times9^2(18m^2+6m)\times12^2(18m^2-12m), \\
&= 419904(5832m^6-1944m^4-432m^3).
\end{aligned}
$$

This value is what, we get after calculations:

$$
\Rightarrow HII_1(\varphi_2) = 2448880128m^6 - 816293376m^4 - 181398528m^3.
$$

For second hyper Zegreb index and by using Eq. 3, we have:

$$
HII_2(\varphi_2) = \prod_{ab\in E(\varphi_2)} (d_a\times d_b)^2.
$$

Using Table 2, we have:

$$
\begin{aligned}
HII_2(\varphi_2) &= (3\times3)^2\,|E_1(TP(m))|\times(3\times6)^2\,|E_2(TP(m))| \\
&\quad \times(6\times6)^2\,|E_3(TP(m))|, \\
&= 9^2(18m^2+6m)\times18^2(18m^2+6m)\times36^2(18m^2-12m), \\
&= 34012224(5832m^6-1944m^4-432m^3).
\end{aligned}
$$

This value is what, we get after calculations:

$$
\Rightarrow HII_2(\varphi_2) = 1.99\times10^{11}m^6 - 6.6\times10^{10}m^4 - 1.47\times10^{10}m^3.
$$

Theorem 2.2.3

Consider the TP network, then its first and second universal Zagreb indices are equal to:

$$
\begin{aligned}
MZ_1^\alpha(\varphi_2) &= 648^\alpha(5832m^6 - 1944m^4 - 432m^3), \\
MZ_2^\alpha(\varphi_2) &= 5832^\alpha(5832m^6 - 1944m^4 - 432m^3).
\end{aligned}
$$

Proof. Let $\varphi_2 \cong$ TP network. Using Eq. 4, we have:

$$
MZ_1^\alpha(\varphi_2) = \prod_{ab\in E(\varphi_2)} (d_a+d_b)^\alpha.
$$

Using Table 2, we have:

$$
\begin{aligned}
MZ_1^\alpha(\varphi_2) &= (3+3)^\alpha\,|E_1(TP(m))|\times(3+6)^\alpha\,|E_2(TP(m))| \\
&\quad \times(6+6)^\alpha\,|E_3(TP(m))| \\
&= 6^\alpha(18m^2+6m)\times9^\alpha(18m^2+6m)\times12^\alpha(18m^2-12m), \\
&= 6^\alpha\times9^\alpha\times12^\alpha(5832m^6-1944m^4-432m^3).
\end{aligned}
$$

This value is what, we get after calculations:

$$
\Rightarrow MZ_1^\alpha(\varphi_2) = 648^\alpha(5832m^6 - 1944m^4 - 432m^3).
$$

For second universal Zegreb index and by using Eq. 5:

$$
MZ_2^\alpha(\varphi_2) = \prod_{ab\in E(\varphi_2)} (d_a\times d_b)^\alpha.
$$

Using Table 2, we have:

$$
\begin{aligned}
MZ_2^\alpha(\varphi_2) &= (3\times3)^\alpha\,|E_1(TP(m))|\times(3\times6)^\alpha\,|E_2(TP(m))| \\
&\quad \times(6\times6)^\alpha\,|E_3(TP(m))|, \\
&= 9^\alpha(18m^2+6m)\times18^\alpha(18m^2+6m)\times36^\alpha(18m^2-12m), \\
&= 9^\alpha\times18^\alpha\times36^\alpha(5832m^6-1944m^4-432m^3).
\end{aligned}
$$

This value is what, we get after calculations:

$$
\Rightarrow MZ_2^\alpha(\varphi_2) = 5832^\alpha(5832m^6 - 1944m^4 - 432m^3).
$$

Theorem 2.2.4

Consider the TP(m) network, the sum and product connectivity of multiplicative indices described as:

$$
\begin{aligned}
SCII(\varphi_2) &= \frac{\sqrt{2}}{36}(5832m^6 - 1944m^4 - 432m^3), \\
PCII(\varphi_2) &= \frac{1}{\sqrt{5832}}(5832m^6 - 1944m^4 - 432m^3).
\end{aligned}
$$

Proof. Let $\varphi_2 \cong$ TP network. We have to prove the sum and product connectivity of multiplicative indices. First prove sum connectivity of multiplicative indices and by using Eq. 6:

$$SCII(\varphi_2) = \prod_{ab \in E(\varphi_2)} \left(\frac{1}{\sqrt{d_a + d_b}} \right).$$

Using Table 2, we have:

$$
\begin{aligned}
SCII(\varphi_2) &= \frac{1}{\sqrt{3+3}} \mid E_1(TP(m)) \mid \times \frac{1}{\sqrt{3+6}} \mid E_2(TP(m)) \mid \\
&\quad \times \frac{1}{\sqrt{6+6}} \mid E_3(TP(m)) \mid, \\
&= \frac{1}{\sqrt{3+3}} (18m^2 + 6m) \times \frac{1}{\sqrt{3+6}} (18m^2 + 6m) \\
&\quad \times \frac{1}{\sqrt{6+6}} (18m^2 - 12m), \\
&= \frac{1}{\sqrt{6} \times \sqrt{9} \times \sqrt{12}} (5832m^6 - 1944m^4 - 432m^3), \\
&= \frac{1}{\sqrt{6 \times 9 \times 12}} (5832m^6 - 1944m^4 - 432m^3).
\end{aligned}
$$

This value is what, we get after calculations:

$$\Rightarrow SCII(\varphi_2) = \frac{\sqrt{2}}{36} (5832m^6 - 1944m^4 - 432m^3).$$

For product connectivity of multiplicative indices and by using Eq. 7, we have:

$$PCII(\varphi_2) = \prod_{ab \in E(\varphi_2)} \left(\frac{1}{\sqrt{d_a \times d_b}} \right).$$

Using Table 2, we have:

$$
\begin{aligned}
PCII(\varphi_2) &= \frac{1}{\sqrt{3 \times 3}} \mid E_1(TP(m)) \mid \times \frac{1}{\sqrt{3 \times 6}} \mid E_2(TP(m)) \mid \\
&\quad \times \frac{1}{\sqrt{6 \times 6}} \mid E_3(TP(m)) \mid, \\
&= \frac{1}{\sqrt{3 \times 3}} (18m^2 + 6m) \times \frac{1}{\sqrt{3 \times 6}} (18m^2 + 6m) \\
&\quad \times \frac{1}{\sqrt{6 \times 6}} (18m^2 - 12m), \\
&= \frac{1}{\sqrt{9}} \times \frac{1}{\sqrt{18}} \times \frac{1}{\sqrt{36}} (5832m^6 - 1944m^4 - 432m^3), \\
&= \frac{1}{\sqrt{9 \times 18 \times 36}} (5832m^6 - 1944m^4 - 432m^3).
\end{aligned}
$$

This value is what, we get after calculations:

$$\Rightarrow PCII(\varphi_2) = \frac{1}{\sqrt{5832}} (5832m^6 - 1944m^4 - 432m^3).$$

2.3 Results for hex POH network (m)

In this section, we calculate edge partition of topological indices of the dimension m for the hex POH network (m). In the coming theorems, we compute some important indices for hex POH network (m).

Theorem 2.3.1

Consider the hex POH network (m), then its second multiplicative Zagreb index is equal to:

$$
\begin{aligned}
II_2(\varphi_3) = 32768(&11664m^6 - 27216m^5 - 11664m^4 \\
&+ 88128m^3 - 97200m^2 + 42768m - 6480).
\end{aligned}
$$

Proof. Let $\varphi_3 \cong$ hex POH(m) and by using Eq. 1, we have:

$$II_2(\varphi_3) = \prod_{ab \in E(\varphi_3)} (d_a \times d_b).$$

Using Table 3, we have:

$$
\begin{aligned}
II_2(\varphi_3) &= 16 \mid E_1(HexPOH(m)) \mid \times 32 \mid E_2(HexPOH(m)) \mid \\
&\quad \times 64 \mid E_3(HexPOH(m)) \mid, \\
&= (4 \times 4) \mid E_1 HexPOH(m) \mid \times (4 \times 8) \mid E_2 HexPOH(m) \mid \\
&\quad \times (8 \times 8) \mid E_3 HexPOH(m) \mid, \\
&= 16(18m^2 + 18m - 30) \times 32(36m^2 - 48m + 12) \\
&\quad \times 64(18m^2 - 36m + 18).
\end{aligned}
$$

This value is what, we get after calculations:

$$
\begin{aligned}
\Rightarrow II_2(\varphi_3) = 32768(&11664m^6 - 27216m^5 - 11664m^4 \\
&+ 88128m^3 - 97200m^2 + 42768m - 6480).
\end{aligned}
$$

Theorem 2.3.2

Consider the hex POH network, then its first and second hyper Zegreb indices are equal to:

$$
\begin{aligned}
HII_1(\varphi_3) = 2359296(&11664m^6 - 27216m^5 - 11664m^4 \\
&+ 88128m^3 - 97200m^2 + 42768m - 6480), \\
HII_2(\varphi_3) = 1073741824(&11664m^6 - 27216m^5 - 11664m^4 \\
&+ 88128m^3 - 97200m^2 + 42768m - 6480).
\end{aligned}
$$

Table 3: Edge partition of hex planar octahedron network.

(d_a, d_b)	Number of edges
$E_1 = (4,4)$	$18m^2 + 18m - 30$
$E_2 = (4,8)$	$36m^2 - 48m + 12$
$E_3 = (8,8)$	$18m^2 - 36m + 18$

Proof. Let $\varphi_3 \cong$ hex POH network and by using Eq. 2, we have:

$$HII_1(\varphi_3) = \prod_{ab \in E(\varphi_3)} (d_a + d_b)^2.$$

Using Table 3, we have:

$$\begin{aligned}
HII_1(\varphi_3) &= (4+4)^2 \, | E_1(HexPOH(m)) | \\
&\quad \times (4+8)^2 \, | E_2(HexPOH(m)) | \\
&\quad \times (8+8)^2 \, | E_3(HexPOH(m)) |, \\
&= 8^2 (18m^2 + 18m - 30) \times 12^2 (36m^2 - 48m + 12) \\
&\quad \times 16^2 (18m^2 - 36m + 18).
\end{aligned}$$

This value is what, we get after calculations:

$$\begin{aligned}
\Rightarrow HII_1(\varphi_3) &= 2359296(11664m^6 - 27216m^5 - 11664m^4 \\
&\quad + 88128m^3 - 97200m^2 + 42768m - 6480).
\end{aligned}$$

For second hyper Zegreb index and by using Eq. 3, we have:

$$HII_2(\varphi_3) = \prod_{ab \in E(\varphi_3)} (d_a \times d_b)^2.$$

Using Table 3, we have:

$$\begin{aligned}
HII_2(\varphi_3) &= (4 \times 4)^2 \, | E_1(HexPOH(m)) | \\
&\quad \times (4 \times 8)^2 \, | E_2(HexPOH(m)) | \\
&\quad \times (8 \times 8)^2 \, | E_3(HexPOH(m)) |, \\
&= 16^2 (18m^2 + 18m - 30) \times 32^2 (36m^2 - 48m + 12) \\
&\quad \times 64^2 (18m^2 - 36m + 18).
\end{aligned}$$

This value is what, we get after calculations:

$$\begin{aligned}
\Rightarrow HII_2(\varphi_3) &= 1073741824(11664m^6 - 27216m^5 - 11664m^4 \\
&\quad + 88128m^3 - 97200m^2 + 42768m - 6480).
\end{aligned}$$

Theorem 2.3.3

Consider the hex POH network, then its first and second universal Zagreb indices are equal to:

$$\begin{aligned}
MZ_1^\alpha(\varphi_3) &= 1536^\alpha(11664m^6 - 27216m^5 - 11664m^4 \\
&\quad + 88128m^3 - 97200m^2 + 42768m - 6480), \\
MZ_2^\alpha(\varphi_3) &= 32768^\alpha(11664m^6 - 27216m^5 - 11664m^4 \\
&\quad + 88128m^3 - 97200m^2 + 42768m - 6480).
\end{aligned}$$

Proof. Let $\varphi_3 \cong$ hex POH network (m). We have to prove first universal Zagreb index and by using Eq. 4, we have:

$$MZ_1^\alpha(\varphi_3) = \prod_{ab \in E(\varphi_3)} (d_a + d_b)^\alpha.$$

Using Table 3, we have:

$$\begin{aligned}
MZ_1^\alpha(\varphi_3) &= (4+4)^\alpha \, | E_1(HexPOH(m)) | \\
&\quad \times (4+8)^\alpha \, | E_2(HexPOH(m)) | \\
&\quad \times (8+8)^\alpha \, | E_3(HexPOH(m)) |, \\
&= 8^\alpha (18m^2 + 18m - 30) \times 12^\alpha (36m^2 - 48m + 12) \\
&\quad \times 18^\alpha (18m^2 - 36m + 18), \\
&= 8^\alpha \times 12^\alpha \times 16^\alpha (11664m^6 - 27216m^5 - 11664m^4 \\
&\quad + 88128m^3 - 97200m^2 + 42768m - 6480).
\end{aligned}$$

This value is what, we get after calculations:

$$\begin{aligned}
\Rightarrow MZ_1^\alpha(\varphi_3) &= 1536^\alpha(11664m^6 - 27216m^5 - 11664m^4 \\
&\quad + 88128m^3 - 97200m^2 + 42768m - 6480).
\end{aligned}$$

For second universal Zegreb index and by using Eq. 5, we have:

$$MZ_2^\alpha(\varphi_3) = \prod_{ab \in E(\varphi_3)} (d_a \times d_b)^\alpha.$$

Using Table 3, we have:

$$\begin{aligned}
MZ_2^\alpha(\varphi_2) &= (4 \times 4)^\alpha \, | E_1(HexPOH(m)) | \\
&\quad \times (4 \times 8)^\alpha \, | E_2(HexPOH(m)) | \\
&\quad \times (8 \times 8)^\alpha \, | E_3(HexPOH(m)) |, \\
&= 16^\alpha (18m^2 + 18m - 30) \times 32^\alpha (36m^2 - 48m + 12) \\
&\quad \times 64^\alpha (18m^2 - 36m + 18), \\
&= 16^\alpha \times 32^\alpha \times 64^\alpha (11664m^6 - 27216m^5 - 11664m^4 \\
&\quad + 88128m^3 - 97200m^2 + 42768m - 6480).
\end{aligned}$$

This value is what, we get after calculations:

$$\begin{aligned}
\Rightarrow MZ_2^\alpha(\varphi_3) &= 32768^\alpha(11664m^6 - 27216m^5 - 11664m^4 \\
&\quad + 88128m^3 - 97200m^2 + 42768m - 6480).
\end{aligned}$$

Theorem 2.3.4

Consider the hex POH(m) network, the sum and product connectivity of multiplicative indices described as:

$$SCII(\varphi_3) = \frac{1}{\sqrt{16}}(11664m^6 - 27216m^5 - 11664m^4$$
$$+ 88128m^3 - 97200m^2 + 42768m - 6480),$$

$$PCII(\varphi_3) = \frac{1}{\sqrt{32768}}(11664m^6 - 27216m^5 - 11664m^4$$
$$+ 88128m^3 - 97200m^2 + 42768m - 6480).$$

Proof. Let $\varphi_3 \cong$ hex POH(m). We have to prove the sum and product connectivity of multiplicative indices. First we prove sum connectivity of multiplicative indices and by using Eq. 6, we have:

$$SCII(\varphi_3) = \prod_{ab \in E(\varphi_3)} \left(\frac{1}{\sqrt{(d_a + d_b)}} \right).$$

Using Table 3, we have:

$$SCII(\varphi_3) = \frac{1}{\sqrt{4+4}} |E_1(HexPOH(m))|$$
$$\times \frac{1}{\sqrt{4+8}} |E_2(HexPOH(m))|$$
$$\times \frac{1}{\sqrt{8+8}} |E_3(HexPOH(m))|,$$
$$= \frac{1}{\sqrt{4+4}}(18m^2 + 18m - 30)$$
$$\times \frac{1}{\sqrt{4+8}}(36m^2 - 48m + 12)$$
$$\times \frac{1}{\sqrt{8+8}}(18m^2 - 36m + 18),$$
$$= \frac{1}{\sqrt{8}} \times \frac{1}{\sqrt{12}} \times \frac{1}{\sqrt{16}}(11664m^6 - 27216m^5 - 11664m^4$$
$$+ 88128m^3 + 97200m^2 + 42768m - 6480),$$
$$= \frac{1}{\sqrt{8 \times 12 \times 16}}(11664m^6 - 27216m^5 - 11664m^4$$
$$+ 88128m^3 - 97200m^2 + 42768m - 6480).$$

This value is what, we get after calculations:

$$\Rightarrow SCII(\varphi_3) = \frac{1}{\sqrt{16}}(11664m^6 - 27216m^5 - 11664m^4$$
$$+ 88128m^3 - 97200m^2 + 42768m - 6480).$$

For product connectivity of multiplicative indices and by using Eq. 7:

$$PCII(\varphi_3) = \prod_{ab \in E(\varphi_3)} \left(\frac{1}{\sqrt{(d_a \times d_b)}} \right).$$

Using Table 3, we have:

$$PCII(\varphi_3) = \frac{1}{\sqrt{4 \times 4}} |E_1(HexPOH(m))|$$
$$\times \frac{1}{\sqrt{4 \times 8}} |E_2(HexPOH(m))|$$
$$\times \frac{1}{\sqrt{8 \times 8}} |E_3(HexPOH(m))|,$$
$$= \frac{1}{\sqrt{4 \times 4}}(18m^2 + 18m - 30)$$
$$\times \frac{1}{\sqrt{4 \times 8}}(36m^2 - 48m + 12)$$
$$\times \frac{1}{\sqrt{8 \times 8}}(18m^2 - 36m + 18),$$
$$= \frac{1}{\sqrt{16}} \times \frac{1}{\sqrt{32}} \times \frac{1}{\sqrt{64}}(11664m^6 - 27216m^5 - 11664m^4$$
$$+ 88128m^3 - 97200m^2 + 42768m - 6480),$$
$$= \frac{1}{\sqrt{16 \times 32 \times 64}}(11664m^6 - 27216m^5 - 11664m^4$$
$$+ 88128m^3 - 97200m^2 + 42768m - 6480).$$

This value is what, we get after calculations:

$$\Rightarrow PCII(\varphi_3) = \frac{1}{\sqrt{32768}}(11664m^6 - 27216m^5 - 11664m^4$$
$$+ 88128m^3 - 97200m^2 + 42768m - 6480).$$

3 Discussion

The comparison of second kind multiplicative Zagrab, first and second kind of hyper Zagreb, first and second universal Zagreb, sum and product connectivity of multiplicative indices for the planar octahedron structure, triangular prism structure and hex planar octahedron structures have been computed for the different numerical values as shown in Figures 4-9. The graphical representations show the correctness of the results.
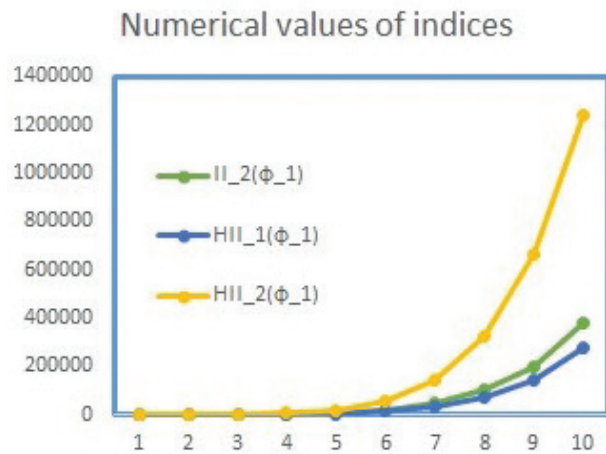
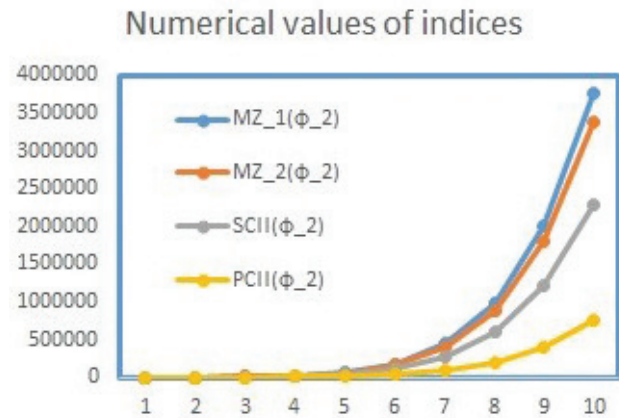**Figure 4:** Comparison of Indices for planar octahedron network.

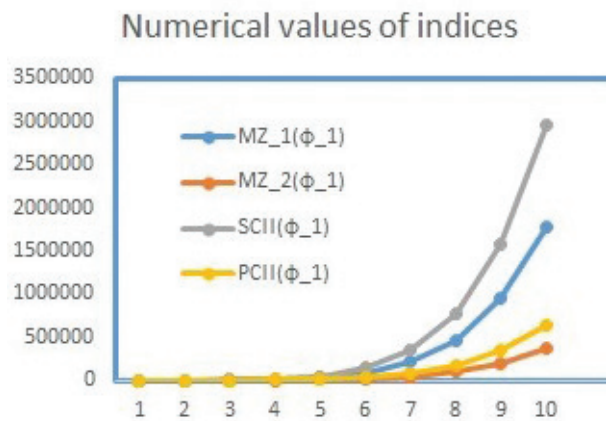**Figure 7:** Comparison of indices for triangular prism network.

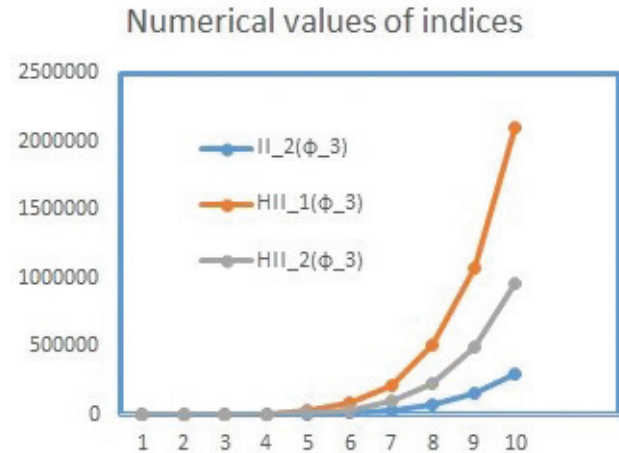**Figure 5:** Comparison of indices for planar octahedron network.

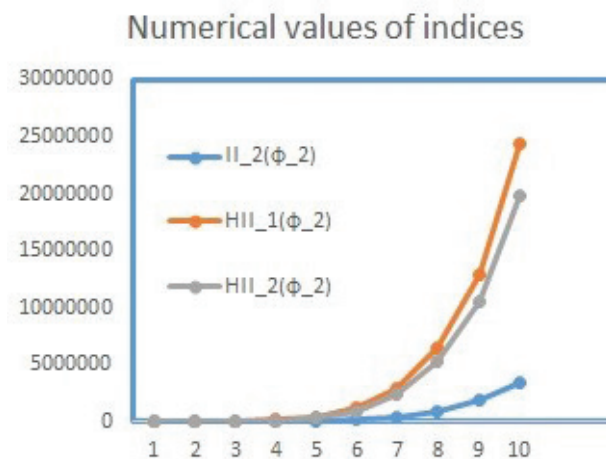**Figure 8:** Comparison of indices for hex POH.

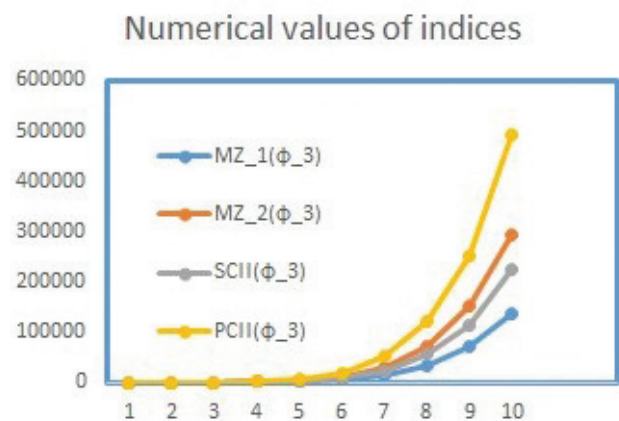**Figure 6:** Comparison of indices for triangular prism network.

Figure 9: Comparison of indices for hex POH.

4 Conclusions

In this paper, second multiplicative Zagreb index, first hyper Zagreb and second hyper Zagreb, first universal Zagreb and second hyper Zagreb, sum and product connectivity of multiplicative indices computed for the planar octahedron network, triangular prism network and hex planar octahedron network. Chemical point of view these results may be helpful for people working in computer science and chemistry who encounter hex-derived networks. There exist many open problems for calculating the expressions of similar derived networks we are hoping to compute the other multiplicative degree based indices.

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References

Ali H., Sajjad A., On further results of hex derived networks. Open J. Discret. Appl. Math., 2019, 2(1), 32-40.

Ali H., Babar U., Asghar S.S., Kausar F., On Some Topological Polynomials of Dominating David Derived Graphs. Punjab University Journal of Mathematics, 2020, 52(11), 19-44.

Ali H., Binyamin M.A., Shafiq M.K., Gao W., On the Degree-Based Topological Indices of Some Derived Networks. Mathematics, 2019, 7, 612.

Akhtar S., Imran M., On molecular topological properties of benzenoid structures. Can. J. Chem., 2016, 94, 687-698.

Aslam A., Ahmad S., Binyamin M. A., Gao W., Calculating topological indices of certain OTIS interconnection networks. Open Chem., 2019, 17(1), 220-228.

Babar U., Ali H., Arshad S.H., Sheikh U., Multiplicative topological properties of graphs derived from honeycomb structure. AIMS Mathematics, 2020, 5(2), 1562.

Baig A.Q., Imran M., Ali H., On topological indices of poly oxide, poly silicate, DOX, and DSL networks. Can. J. Chem., 2015, 93, 730-739.

Baig A.Q., Naeem, M., Gao W., Revan and hyper-Revan indices of Octahedral and icosahedral networks. Appl. Math. Nonlinear Sci., 2018, 3(1), 33-40.

Diudea M.V., Gutman I., Lorentz J., Molecular Topology Babes-Bolyai University: Cluj-Napoca, Romania, 2001.

Gao W., Baig A. Q., Khalid W., Farahani M.R., Molecular description of copper(II) oxide. Maced. J. Chem. Chem. Eng., 2017, 36, 93-99.

Gutman I., Rusić B., Trinajstić N., Wilcox Jr C.F., Graph theory and molecular orbitals. XII. Acyclic polyenes. J. Chem. Phys., 1975 , 62(9), 3399-3405.

Huo Y., Ali H., Binyamin M.A., Asghar S.S., Babar U., Liu J.B., On Topological Indices of mth Chain Hex-Derived Network of Third Type. Front. Phys., 2020, 8, 593275.

Imran M., Baig A.Q., Ali H., On topological properties of dominating David derived networks. Can. J. Chem., 2016, 94, 137-148.

Imran S., Siddiqui M.K., Imran M., Nadeem M.F., Computing topological indices and polynomials for line graphs. Mathematics, 2018, 6(8), 137.

Kulli V.R., Multiplicative hyper-Zagreb indices and coindices of graphs: Computing these indices of some nanostructures. I.R.J.P.A., 2016, 6(7), 342-347.

Manuel P.D., Rajasingh I., Minimum Metric Dimension of Silicate Networks. Ars Combinatoria, 2011, 98, 501-510.

Simon Raj F., George A., Network embedding on Planar Octahedron networks. In: 2015 IEEE International Conference on Electrical, Computer and Communication Technologies (ICECCT) (2015, March). 2015, 1-6.

Simonraj F., George A., Embedding of poly honeycomb networks and the metric dimension of star of david network. GRAPH-HOC., 2012, 4, 11, 28.

Shirdel G.H., Rezapour H., Sayadi A.M., The hyper Zagreb index of graph operations. Iran. J. Math. Chem., 2013, 4, 213-220.

Smith K.M., On neighbourhood degree sequences of complex networks. Sci. Rep.-UK, 2019, 9(1), 8340.

Trinajstić N., Chemical Graph Theory. CRC Press, Boca Raton, FL, USA, 1983.

Vetrík T., Degree-based topological indices of hexagonal nanotubes. J. Appl. Math. Comput., 2018, 58(1-2), 111-124.

Wei C.C., Ali H., Binyamin M.A., Naeem M.N., Liu J.B., Computing Degree Based Topological Properties of Third Type of Hex-Derived Networks. Mathematics, 2019, 7, 368.

Wiener H., Structural determination of the paraffin boiling points. J. Am. Chem. Soc., 1947, 69, 17-20.

Zheng J., Iqbal Z., Fahad A., Zafar A., Aslam A., Qureshi M.I., et al., Some eccentricity-based topological indices and polynomials of poly (EThyleneAmidoAmine)(PETAA) dendrimers. Processes, 2019, 7(7), 433.