Research Article

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# A new result for entire functions and their shifts with two shared values

https://doi.org/10.1515/math-2025-0138
received September 19, 2024; accepted February 5, 2025

**Abstract:** Let $f$ be a transcendental entire function of hyperorder strictly less than 1, and let $c$ be a nonzero finite complex number. We prove that if $f(z)$ and $f(z+c)$ partially share 0, 1 ignoring multiplicity (i.e., $\overline{E}(0,f(z)) \subseteq \overline{E}(0,f(z+c))$ and $\overline{E}(1,f(z)) \subseteq \overline{E}(1,f(z+c))$), then $f(z) \equiv f(z+c)$. This result is a generalization and improvement of the previous theorem due to Li and Yi.

**Keywords:** entire function, shift, uniqueness, shared value

**MSC 2020:** 30D35, 39A10

## 1 Introduction and main results

We assume the reader is familiar with the fundamental concepts and standard notations of the classical Nevanlinna theory [1,2], such as $T(r,f)$, $N(r,f)$, and $m(r,f)$. In addition, we denote by $S(r,f)$ a quantity of $o(T(r,f))$ as $r$ tends to infinity outside of a possible exceptional set of finite logarithmic measure.

For a given value $a \in \mathbb{C} \cup \{\infty\}$ and two meromorphic functions $f$ and $g$, we denote by $\overline{E}(a,f)$ (or $E(a,f)$) the set of all zeros of $f-a$ ignoring multiplicity (or counting multiplicity, respectively), and let us abbreviate this with IM (or CM). If $\overline{E}(a,f) = \overline{E}(a,g)$, then we say that $f$ and $g$ share $a$ IM; if $E(a,f) = E(a,g)$, then we say that $f$ and $g$ share $a$ CM; if $\overline{E}(a,f) \subseteq \overline{E}(a,g)$, then we say that $f$ and $g$ partially share $a$ IM; and if $E(a,f) \subseteq E(a,g)$, then we say that $f$ and $g$ partially share $a$ CM.

In recent decades, by applying the difference analog of classical Nevanlinna theory [3–6] for meromorphic functions $f(z)$ and their shifts $f(z+c)$ or difference operators $\Delta_c f = f(z+c) - f(z)$, many uniqueness theorems between functions satisfying some certain shared conditions were obtained. Here, we recall some previous results as follows.

**Theorem 1.1.** [7] *Let f be a nonconstant meromorphic function of finite order, and let $a_1$, $a_2$, and $a_3$ be three distinct values in the extended complex plane. If $f(z)$ and $f(z+c)$ share $a_1$, $a_2$, and $a_3$* CM, *where c is a nonzero complex number, then $f(z) \equiv f(z+c)$.*

**Corollary 1.2.** [7] *Let f be a nonconstant entire function of finite order, and let $a_1$ and $a_2$ be two distinct finite complex values. If $f(z)$ and $f(z+c)$ share $a_1$ and $a_2$ CM, where c is a nonzero complex number, then $f(z) \equiv f(z+c)$.*

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The main idea of the proof of Theorem 1.1 is based on the difference analog of classical Nevanlinna's theory for meromorphic functions of finite order, which is established by Halburd and Korhonen [3,4], Chiang and Feng [5], independently, and developed by Halburd et al. [6] to hyperorder strictly less than 1. Correspondingly, the assumption "finite order" in Theorem 1.1 and Corollary 1.2 can naturally be replaced by "hyperorder $\rho_2(f) < 1$."

It is a natural question whether the shared conditions can be weakened or not. In 2017, one of the authors [8] of this article improved the shared conditions of Theorem 1.1 from "3CM" to "2CM + 1IM." In 2016, Li and Yi [9] improved Corollary 1.2 from "2CM" to "2IM." We recall those theorems as follows.

**Theorem 1.3.** [8] *Let f be a nonconstant meromorphic function of hyperorder $\rho_2(f) < 1$, and let $a_1$, $a_2$, and $a_3$ be three distinct values in the extended complex plane. If $f(z)$ and $f(z+c)$ share $a_1$ and $a_2$ CM and $a_3$ IM, where c is a nonzero complex number, then $f(z) \equiv f(z+c)$.*

**Theorem 1.4.** [9] *Let f be a nonconstant entire function of hyperorder $\rho_2(f) < 1$, and let $a_1$ and $a_2$ be two distinct finite complex values. If $f(z)$ and $f(z+c)$ share $a_1$ and $a_2$ IM, where c is a nonzero complex number, then $f(z) \equiv f(z+c)$.*

In this article, we shall replace the shared value conditions of Theorem 1.4 by two partially shared values.

**Theorem 1.5.** *Suppose that f is a transcendental entire function of hyperorder $\rho_2(f) < 1$, and let $a_1$ and $a_2$ be two distinct finite complex values. If $\overline{E}(a_1, f(z)) \subseteq \overline{E}(a_1, f(z+c))$ and $\overline{E}(a_2, f(z)) \subseteq \overline{E}(a_2, f(z+c))$, where c is a nonzero complex number, then $f(z) \equiv f(z+c)$.*

The following example shows that the word "entire function" cannot be replaced by "meromorphic function."

**Example 1.** Let $f(z) = \frac{e^z}{e^z-1}$ and $c$ be a given finite complex number except for $2ki\pi$, where $k$ is an integer. Then, $f(z+c) = \frac{e^{z+c}}{e^{z+c}-1}$, which implies that $E(0, f(z)) = \varnothing$ and $E(0, f(z+c)) = \varnothing$, i.e., $f(z)$ and $f(z+c)$ share 0 CM. Similarly, one can see that $f(z)$ and $f(z+c)$ share 1 CM. But $f(z) \not\equiv f(z+c)$.

## 2 Some lemmas

Next, we shall recall the following lemmas that are needed in the sequel.

**Lemma 2.1.** [6] *Let f be a nonconstant meromorphic function of hyperorder $\rho_2(f) < 1$ and $c \in \mathbb{C}\backslash\{0\}$. Then,*

$$m\left(r, \frac{f(z+c)}{f(z)}\right) + m\left(r, \frac{f(z)}{f(z+c)}\right) = S(r, f),$$

*for all $z \in \mathbb{C}$.*

**Lemma 2.2.** [6] *Let $T : [0, +\infty) \to [0, +\infty)$ be a non-decreasing continuous function, and let $s \in (0, \infty)$. If the hyperorder of T is strictly less than one, i.e.,*

$$\rho_2 = \rho_2(T) = \limsup_{r\to\infty} \frac{\log^+ \log^+ T(r)}{\log r} < 1,$$

*and $\delta \in (0, 1 - \rho_2)$, then*

$$T(r+s) = T(r) + o\left(\frac{T(r)}{r^\delta}\right),$$

*where r runs to infinity outside of a set of finite logarithmic measures.*

**Remark 2.3.** The following properties are immediate consequences of Lemma 2.2 for a nonconstant meromorphic function $f(z)$ of hyperorder $\rho_2(f) < 1$, i.e., we have $T(r, f(z \pm c)) = T(r, f) + S(r, f)$, $\overline{N}\left(r, \frac{1}{f(z \pm c) - a}\right) = \overline{N}\left(r, \frac{1}{f(z) - a}\right) + S(r, f)$, and so on.

## 3 Proof of Theorem 1.5

The proof is based on the idea of Lin and Ishizaki [10]. Without loss of generality, we can suppose that $a_1 = 0$ and $a_2 = 1$. Assume, on the contrary, that $f(z) \not\equiv f(z + c)$.

Set

$$\varphi(z) = \frac{f'(z)[f(z) - f(z + c)]}{f(z)(f(z) - 1)}. \tag{3.1}$$

According to the shared value conditions, we know that any zero of $f(f - 1)$ is a regular point of $\varphi$. And since $f$ is an entire function, one can see that $\varphi$ is an entire function. In addition, using the lemma on logarithmic derivatives and Lemma 2.1, we have

$$m(r, \varphi) \le m\left(r, \frac{f'(z)}{f(z) - 1}\right) + m\left(r, \frac{f(z) - f(z + c)}{f(z)}\right) + O(1) \le S(r, f),$$

which implies that

$$T(r, \varphi) = S(r, f). \tag{3.2}$$

Next, we denote by $N_0\left(r, \frac{1}{h'}\right)$ the counting function of those zeros of $h'(z)$, which are not the zeros of $h(z)(h(z) - 1)$. Noting that $\varphi \not\equiv 0$ by the assumptions, from (3.1) and (3.2), we have

$$N_0\left(r, \frac{1}{f'(z)}\right) \le N\left(r, \frac{1}{\varphi}\right) \le T\left(r, \frac{1}{\varphi}\right) \le T(r, \varphi) = S(r, f). \tag{3.3}$$

Rewrite (3.1) as

$$\varphi(z + c) = \frac{f'(z + c)[f(z + c) - f(z + c + c)]}{f(z + c)(f(z + c) - 1)}. \tag{3.4}$$

Then, we can deduce from (3.2), (3.4), and Lemma 2.2 that

$$N_0\left(r, \frac{1}{f'(z + c)}\right) \le N\left(r, \frac{1}{\varphi(z + c)}\right) \le T\left(r, \frac{1}{\varphi(z + c)}\right) \le T(r, \varphi(z + c)) + O(1) = S(r, f). \tag{3.5}$$

Set

$$F(z) = \frac{f'(z)\{f(z + c)[(f(z + c) - 1)]\}}{f'(z + c)[f(z)(f(z) - 1)]}. \tag{3.6}$$

Immediately, we obtain

$$\begin{aligned} m(r, F) &\le m\left(r, \frac{f'(z)}{f'(z + c)}\right) + m\left(r, \frac{f(z + c)}{f(z)}\right) + m\left(r, \frac{f(z + c) - 1}{f(z) - 1}\right) \\ &\le S(r, f') + S(r, f) + S(r, f - 1) = S(r, f). \end{aligned}$$

Moreover, we **claim** that $N(r, F) = S(r, f)$. Since $f$ is an entire function, we know that all the poles of $F(z)$ in (3.6) could only come from the zeros of $f'(z + c)[f(z)(f(z) - 1)]$. We consider the following two cases.

**Case 1.** Let $z_0$ be a zero of $f(z)$ (or $f(z)-1$, respectively) with multiplicity $m$, and of $f(z+c)$ (or $f(z+c)-1$, respectively) with multiplicity $n$. Then, $z_0$ is a zero of $f'(z+c)[f(z)(f(z)-1)]$ in (3.6) with multiplicity $n-1+m$, and a zero of $f'(z)[f(z+c)(f(z+c)-1)]$ in (3.6) with multiplicity $m-1+n$ too. Thus, $z_0$ is not a pole of $F(z)$.
**Case 2.** Let $z_1$ be a zero of $f'(z+c)$. We consider the following subcases.
**Subcase 2.1.** If $z_1$ is a common zero of $f(z+c)(f(z+c)-1)$ and $f(z)(f(z)-1)$, then $z_1$ is not a pole of $F(z)$ by the same argument as in Case 1.
**Subcase 2.2.** If $z_1$ is a zero of $f(z+c)(f(z+c)-1)$ but not of $f(z)(f(z)-1)$, then since the multiplicity at $z_1$ of $f'(z+c)$ is less than the multiplicity of $f(z+c)(f(z+c)-1)$, it is obvious that $z_1$ is also not a pole of $F(z)$.
**Subcase 2.3.** If $z_1$ is not a zero of $f(z+c)(f(z+c)-1)$, then it follows from (3.3) and the aforementioned two subcases that $N(r,F)=S(r,f)$.

Therefore, we have

$$T(r,F)=S(r,f). \tag{3.7}$$

In what follows, we rewrite (3.6) as

$$\frac{F(z)f'(z+c)}{f(z+c)[(f(z+c)-1)]}=\frac{f'(z)}{f(z)(f(z)-1)}. \tag{3.8}$$

Denote by $\overline{N}_D\left(r,\frac{1}{f(z+c)(f(z+c)-1)}\right)$ the reduced counting function of those zeros of $f(z+c)(f(z+c)-1)$ but not of $f(z)(f(z)-1)$. It follows from (3.7), (3.8), and $F\not\equiv 0$ that

$$\overline{N}_D\left(r,\frac{1}{f(z+c)(f(z+c)-1)}\right)\le \overline{N}\left(r,\frac{1}{F(z)}\right)=S(r,f).$$

Now, we denote by $\overline{N}_{(m,n)}(r,a;f(z),f(z+c))=\overline{N}_{(m,n)}(r,a)$ the reduced counting function of those common zeros of $f(z)-a$ with multiplicity $m$, and of $f(z+c)-a$ with multiplicity $n$, and **claim** that

$$\overline{N}_{(m,n)}(r,a)=S(r,f) \tag{3.9}$$

holds for any fixed pair $(m,n)$ and $a\in\{0,1\}$. Otherwise, without loss of generality, we can assume there exists some pair $(m,n)$ such that

$$\overline{N}_{(m,n)}(r,0)\ne S(r,f). \tag{3.10}$$

Next, we discuss the following two cases. In the following, let $z_0$ be a common zero of $f(z)$ and $f(z+c)$ with multiplicity $m$ and multiplicity $n$, respectively, and then, $F(z_0)\ne 0,\infty$ by (3.8), where we ignore those points in $\overline{N}_D\left(r,\frac{1}{f(z+c)(f(z+c)-1)}\right)$.

Because the coefficients of the term $(z-z_0)^{-1}$ of the Laurent expansions of the both sides of (3.8) are identical, we have

$$-F(z_0)n=-m. \tag{3.11}$$

**Case 1.** Suppose that $m=n$. Then, we have $F(z_0)=1$ by (3.11). If $F(z)\not\equiv 1$, then it follows from (3.7) that

$$\overline{N}_{(m,n)}(r,0)\le N\left(r,\frac{1}{F(z)-1}\right)=S(r,f),$$

which is a contradiction to (3.10). Hence, $F(z)\equiv 1$, and it follows from (3.8) that

$$\frac{f'(z+c)}{f(z+c)[(f(z+c)-1)]}\equiv\frac{f'(z)}{f(z)(f(z)-1)},$$

which implies $f(z)$ and $f(z+c)$ share 0, 1 CM by the assumptions of Theorem 1.5. In fact, from the aforementioned identical equation (for convenience, notate this equation as (*)), we can derive $\overline{E}(0,f(z))\supseteq\overline{E}(0,f(z+c))$. Otherwise, if there exists a point $\xi$ such that $f(\xi+c)=0$ but $f(\xi)\ne 0$, then we also have

$f(\xi)-1\neq 0$. Moreover, $\xi$ is a pole of the left side of (*) but is not a pole of the right of (*), which is a contradiction. Therefore, we have $\overline{E}(0, f(z))=\overline{E}(0, f(z+c))$. Similarly, we can obtain $\overline{E}(1, f(z))=\overline{E}(1, f(z+c))$. Next, by observing the Laurent expansions on both sides of (*), we can further deduce that $f(z)$ and $f(z+c)$ share 0, 1 CM.

Now, by applying Theorem 1.4 to this case, we know that $f(z)\equiv f(z+c)$, but this is a contradiction.

**Case 2.** Suppose that $m\neq n$. Then, we have $F(z_0)=\frac{m}{n}$ by (3.11). If $F(z)\not\equiv\frac{m}{n}$, then it follows from (3.7) that

$$\overline{N}_{(m,n)}(r,0)\leq N\left(r,\frac{1}{F(z)-\frac{m}{n}}\right)=S(r,f),$$

which is a contradiction to (3.10). Hence, $F(z)\equiv\frac{m}{n}$, and it follows from (3.8) that

$$\frac{mf'(z+c)}{f(z+c)[(f(z+c)-1)]}\equiv\frac{nf'(z)}{f(z)(f(z)-1)},$$

which implies

$$\left(\frac{f(z+c)-1}{f(z+c)}\right)^m\equiv A_0\left(\frac{f(z)-1}{f(z)}\right)^n,$$

for some nonzero constant $A_0$. This together with the Valiron-Mohon'ko theorem yields

$$m\cdot T(r, f(z+c))=n\cdot T(r,f)+S(r,f).$$

It follows from Lemma 2.2 immediately that $m=n$, which is a contradiction in this case.

Therefore, the claim of (3.9) is true.

Finally, we will derive a contradiction in the following, and complete our proof.

Since

$$\begin{aligned}\overline{N}\left(r,\frac{1}{f(z)}\right)&=\sum_{\substack{m\geq1\\n\geq1}}\overline{N}_{(m,n)}(r,0)\\&=\sum_{\substack{11\geq m\geq1\\11\geq n\geq1}}\overline{N}_{(m,n)}(r,0)+\sum_{\substack{12\leq m\\11\geq n\geq1}}\overline{N}_{(m,n)}(r,0)+\sum_{\substack{11\geq m\geq1\\12\leq n}}\overline{N}_{(m,n)}(r,0)+\sum_{\substack{12\leq m\\12\leq n}}\overline{N}_{(m,n)}(r,0),\end{aligned}$$

what is more, we can deduce that

$$\sum_{\substack{12\leq m\\11\geq n\geq1}}\overline{N}_{(m,n)}(r,0)\leq\frac{1}{12}N\left(r,\frac{1}{f(z)}\right)\leq\frac{1}{12}T(r,f)+S(r,f),$$

$$\sum_{\substack{11\geq m\geq1\\12\leq n}}\overline{N}_{(m,n)}(r,0)\leq\frac{1}{12}N\left(r,\frac{1}{f(z+c)}\right)\leq\frac{1}{12}T(r,f)+S(r,f),$$

$$\sum_{\substack{12\leq m\\12\leq n}}\overline{N}_{(m,n)}(r,0)\leq\frac{1}{12}N\left(r,\frac{1}{f(z)}\right)\leq\frac{1}{12}T(r,f)+S(r,f),$$

and from (3.9) that

$$\sum_{\substack{11\geq m\geq1\\11\geq n\geq1}}\overline{N}_{(m,n)}(r,0)=S(r,f).$$

Hence, we can obtain from the aforementioned formulas that

$$\overline{N}\left(r,\frac{1}{f(z)}\right)\leq\frac{1}{4}T(r,f)+S(r,f).\tag{3.12}$$

Similarly, we can also obtain

$$\overline{N}\left(r, \frac{1}{f(z)-1}\right) \leq \frac{1}{4}T(r,f) + S(r,f). \tag{3.13}$$

By (3.12) and (3.13), we have

$$\begin{aligned} T(r,f) &\leq \overline{N}(r,f) + \overline{N}\left(r, \frac{1}{f(z)}\right) + \overline{N}\left(r, \frac{1}{f(z)-1}\right) + S(r,f) \\ &\leq \frac{1}{4}T(r,f) + \frac{1}{4}T(r,f) + S(r,f) = \frac{1}{2}T(r,f) + S(r,f), \end{aligned}$$

which is a contradiction.

Therefore, this completes the proof of Theorem 1.5.

## 4 Discussion

In 1989, Brosch [11] proved the following sufficient conditions for periodicity of meromorphic functions in his doctoral dissertation.

**Theorem 4.1.** ([11] or [2], Theorem 5.14) *Let $f(z)$ and $g(z)$ be two nonconstant meromorphic functions sharing three values $a_1, a_2, a_3 \in \mathbb{C} \cup \{\infty\}$ CM. If $f(z)$ is a periodic function with period $c(\neq 0)$, then $g(z)$ is also a periodic function with period $c$.*

As we know, one can apply the uniqueness theorems on meromorphic functions and their shifts to study the periodicity of meromorphic functions. For example, the third author of this article improved Theorem 4.1 as follows.

**Theorem 4.2.** [12] *Let $f(z)$ and $g(z)$ be two nonconstant meromorphic functions with $f(z)$ of finite order, and let $c \in \mathbb{C}\backslash\{0\}$, and let $a_1, a_2, a_3 \in \mathbb{C} \cup \{\infty\}$ be three distinct values. Suppose that $f(z)$ and $g(z)$ share $a_1$ and $a_2$* CM *and $a_3$ IM. If $f(z) = f(z+c)$ for all $z \in \mathbb{C}$, then $g(z) = g(z+c)$ for all $z \in \mathbb{C}$.*

An immediate consequence of Theorem 4.2 is the following result.

**Corollary 4.3.** *Let $f(z)$ and $g(z)$ be two nonconstant entire functions with $f(z)$ of finite order, and let $c \in \mathbb{C}\backslash\{0\}$. Suppose that $f(z)$ and $g(z)$ share 0 CM and 1 IM. If $f(z) = f(z+c)$ for all $z \in \mathbb{C}$, then $g(z) = g(z+c)$ for all $z \in \mathbb{C}$.*

Now, using Theorem 1.4, we can obtain the following result.

**Theorem 4.4.** *Let $f(z)$ and $g(z)$ be two nonconstant entire functions of hyperorder strictly less than 1, and let $c \in \mathbb{C}\backslash\{0\}$. Suppose that $f(z)$ and $g(z)$ share 0, 1 IM. If $f(z) = f(z+c)$ for all $z \in \mathbb{C}$, then $g(z) = g(z+c)$ for all $z \in \mathbb{C}$.*

**Proof.** By the assumptions of Theorem 4.4, it follows that

$$g(z) = 0 \leftrightarrow f(z) = 0 \leftrightarrow f(z+c) = 0 \leftrightarrow g(z+c) = 0$$

and that

$$g(z) = 1 \leftrightarrow f(z) = 1 \leftrightarrow f(z+c) = 1 \leftrightarrow g(z+c) = 1.$$

This implies $g(z)$ and $g(z+c)$ share 0, 1 IM, and hence, $g(z) \equiv g(z+c)$ by Theorem 1.4. □

Therefore, regarding Corollary 4.3, Theorem 4.4, and the main result Theorem 1.5 of this article, we give the following question.

**Question** If the condition "$f(z)$ and $g(z)$ share 0, 1 IM" of Theorem 4.4 is replaced with "$\overline{E}(0, f(z)) \subseteq \overline{E}(0, g(z))$ and $\overline{E}(1, f(z)) \subseteq \overline{E}(1, g(z))$", does the conclusion of Theorem 4.4 still hold?

**Acknowledgement:** We thank the referees for their time and comments.

**Funding information**: This work was supported by the Natural Science Foundation of Fujian (No. 2022J011212), the Natural Science Foundation of China (No. 11801291), and the projects of Ningde Normal University (2020Y03, 2019T01, 2022FZ28).

**Author contributions**: All authors have accepted responsibility for the entire content of this manuscript and consented to its submission to the journal, reviewed all the results, and approved the final version of the manuscript.

**Conflict of interest**: The authors state no conflict of interest.

**Data availability statement**: Data sharing is not applicable to this article as no datasets were generated or analysed during the current study.

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