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Research Article

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Research of cooperation strategy of government-enterprise digital transformation based on differential game

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Abstract: To cope with the development of digital economy, it is particularly important to discuss the digital transformation strategy of government-enterprise cooperation. Taking government and enterprise as game subjects, this paper constructs a game model of government subsidy behavior and enterprise digital transformation effort in the process of digital transformation based on differential game theory and studies different game strategies in noncooperative game, Stackelberg game, and cooperative game. The results show that (1) as an effective regulation mechanism, the government subsidy coefficient can significantly improve the optimal effort of the digital transformation of enterprises; (2) the optimal strategy, the optimal benefit, and the total benefit of the digital ecosystem formed by the government and enterprises in the cooperative game are better than those in the non-cooperative game. Pareto optimality is achieved; (3) the results of theoretical derivation are verified by numerical simulation. This study provides a theoretical basis for the digital transformation cooperation between government and enterprises.

Keywords: digital transformation, digital economy, government subsidy, differential game

MSC 2020: 49L12, 49N70, 49N90

1 Introduction

According to the latest White Paper on the Development of China's Digital Economy (2021) released by the Chinese Academy of Information and Communication Technology, China's digital economy continued to develop vigorously in 2020, with a scale of 39.2 trillion yuan, an increase of 3.3 trillion yuan over last year, accounting for 38.6 percent of GDP, and an increase of 2.4 percentage points over the same period last year, effectively supporting epidemic prevention and control and economic and social development. Under the combined influence of the impact of the epidemic and the global economic downturn, China's digital economy still maintains a high growth rate of 9.7%, which is more than 3.2 times the nominal growth rate of GDP in the same period [1]. Thus, it can be seen that the digital economy has become the key driving force for stable economic growth. The Chinese government attaches great importance to the development of the digital economy and has successively issued policy documents such as the "implementation Plan of the National Digital Economy Innovation and Development Experimental Zone" to vigorously develop the digital economy industry. Leading the development of traditional industries with the digital economy

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and promoting the development speed and efficiency of the digital economy with traditional industries will have an important impact on economic development and social transformation [2]. Government policy is an important external factor to guide enterprises to promote digital transformation [3]. For the government, helping enterprises with digital transformation is conducive to increasing social benefits and tax revenue. Enterprises respond to the call of the government to actively achieve their own strategic coordination, achieve cooperative development, seek multiple innovative knowledge, and promote their own digital transformation [4]. These signs show that the government and enterprises attach great importance to the digital transformation of enterprises. However, for the specific national conditions of many kinds of enterprises, unbalanced regional economic development, and unequal distribution of material resources, it is difficult for the government and enterprises to make their own best decisions on the issue of digital transformation under the condition of maximizing their respective benefits. As the respective benefits of the government and enterprises are closely related to their own efforts to develop the level of digitization. Therefore, there is an urgent need to optimize the investment of the government and enterprises in the digital economy and the Nash equilibrium strategy to help enterprises realize the digital transformation, so as to make our traditional enterprises complete the digital transformation in line with the development of the times as soon as possible. This has important practical significance to meet the needs of enterprise development and give full play to the guiding role of the government.

The existing research has made a lot of discussions on the efforts invested by the government and enterprises in the process of digital transformation. On the one hand, it is the research on the relevant measures taken by the government in the process of digital transformation of enterprises. For example, Henderson [5] through the case study of Wales in the past ten years, and this paper obtained the impact of policy on enterprise digital transformation and confirmed that subsidy is an effective measure in the process of enterprise digital transformation. Pittaway and Montazemi [6] took the cases of 11 local governments in Canada as the research object, and this paper puts forward the specific measures for the government to guide the digital transformation of enterprises. It includes strengthening the training of the concept of digital transformation of enterprises, formulating incentives for each stage of the life cycle of digital transformation, and providing necessary financial subsidies. Troshani et al. [7] systematically expound the digital transformation work guided by the government from the aspects of pattern and path. The study pointed out that government investment in guiding digital transformation includes monitoring perception, detection feedback, decision-making, and implementation. Pérez-Morote et al. [8] draw lessons from the case of the British government to promote the digital transformation and analyze the content that the Chinese government needs to invest in promoting the digital transformation, that is, the national universal digital service platform and the basic network software and hardware facilities built by the government. Conversely, it aims at the related research on the government subsidy mechanism in the process of digital transformation of enterprises [9,10], for example, cooperating with enterprises, providing the necessary material conditions [11], the government calls on the public to participate [12,13], and so on. In terms of enterprise investment, Gurbaxani and Dunkle [14] pointed out that enterprises should integrate emerging digital technologies such as big data, cloud computing, Internet of things, blockchain, digital twin, and artificial intelligence into their own digital transformation according to their actual needs, and increase investment in digital technology to promote their own digital transformation. Through the interview with eight small- and medium-sized enterprises, Anim-Yeboah et al. [15] pointed out the problems existing in the digital transformation of small- and medium-sized enterprises, such as the skill gap of employees, the limited technical level, the defects at the strategic level, and so on, and puts forward the relevant solutions – team building and investment in business model innovation should be strengthened in the digital transformation of enterprises. Castagna et al. [16] through the survey of 73 enterprises concluded that enterprises should improve their digital cognition to the level of organizational strategy and adapt to the changes in the internal and external environment in time. The development of dynamic capability optimization adjusts the company's business model to promote its own digital transformation. These studies show that although the academic circles have done relevant research on the input of the efforts made by the government and enterprises in the process of digital transformation, there is no in-depth exploration of the problem that the government and enterprises make the best strategy for the digital transformation of enterprises under the condition of maximizing their respective benefits or the total benefits of cooperation. Therefore, it is necessary to conduct an in-depth study on the cooperation strategy of government-enterprise digital transformation.

Although scholars have different perspectives on the enterprise digital transformation of the relevant issues, involving a wide range, these studies still have limitations. First, there is much research on the digital transformation of enterprises by summary or qualitative analysis, but only a few studies on the related problems of the digital transformation of enterprises. Second, most of the existing studies focus on the investment efforts of the government or individual enterprises in the process of enterprise digital transformation, and only a small number of studies focus on the cooperation between government and enterprises in promoting enterprise digital transformation; however, questionnaire survey or interview is often used, which leads to a wide range of conclusions and recommendations and the lack of reliable theoretical support for cooperation strategies. In addition, in terms of research methods, most scholars have analyzed the equilibrium problem between the input and the benefit of the government's promotion of traditional enterprises' digital transformation through the static game or evolutionary game models; however, they do not realize that due to the asymmetry of information, the cooperation between government and enterprises to promote the digital transformation of enterprises is a dynamic process of change. Differential game is an important academic tool to deal with the confrontation or cooperation among the participants during the duration. Therefore, this paper, based on the differential game theory, discusses the digital transformation cooperation between government and enterprises, which is helpful to promote the digital transformation cooperation strategy of government and enterprises.

Taking the digital transformation of enterprises as the main line, this paper constructs a linear quadratic differential game model and quantitatively discusses the dynamic decision-making process between the government and enterprises and the influence of government subsidies on the digital transformation of enterprises under the three cases of Nash non-cooperative game, Stackelberg game, and cooperative game by using Hamilton-Jacobi-Bellman equation (HJB equation). To explore the coordination mechanism and optimal strategy between the two under the dynamic framework. This study uses game theory to deeply analyze the game relationship between stakeholders from the perspective of economics and management, and designs the optimal benefit strategy of the government and enterprises. On the one hand, it enriches the related research on the digital transformation of enterprises. It expands the application depth of game theory in the field of the digital economy. On the other hand, the theoretical model derivation and role positioning analysis of the participants in the digital transformation of enterprises can provide a theoretical basis for the cooperation between government and enterprises in digital transformation.

2 Model hypothesis

2.1 Hypothesis 1

The local digital economy and the effort cost of enterprises to use digital technology are related to their respective efforts. Digital technology has the characteristics of extensiveness and complexity, and this paper will no longer subdivide the specific digital technology. The subsequent digital technology refers to the content and considers the convex characteristics of the effort cost and the hypothesis of the effort cost in [17]. At the moment t, the effort costs of the government and local digital economy and the efforts of enterprises to use digital technology are, respectively:

$$C_1(t) = \frac{1}{2}k_1E_1^2(t),\ C_2(t) = \frac{1}{2}k_2E_2^2(t).$$

where $E_1(t)$ and $E_2(t) \geq 0$, respectively, represent the respective efforts of the government to develop the local digital economy and enterprises to use digital technology at the moment t. $C_1(t)$ and $C_2(t)$, respectively, indicate that the cost of the government's respective efforts to develop the local digital economy and the enterprises' use of digital technology increases with the increase of their respective efforts at the moment t. The extent of the increase shows an upward trend using the quadratic function to describe input or the cost is a common form in the literature [18]. $k_1(t)$ and $k_2(t) > 0$ represent the cost coefficient invested by the government and enterprises in the development of the digital economy, respectively.

2.2 Hypothesis 2

To encourage enterprises to use digital technology, the government subsidizes the cost of using digital technology (such as the current preferential tax policy), with a subsidy ratio of $S(t)$ and $0 \le S(t) < 1$. That is, the government only bears part of the cost of using digital technology for enterprises.

2.3 Hypothesis 3

The digitization level of an enterprise is represented by variable $E(t)$, and derivative $\frac{\mathrm{d}E(t)}{\mathrm{d}t}$ describes its change in time $t \in (0, +\infty)$, which is determined by the level of efforts of enterprises and governments. In addition, the digital transformation of enterprises is a dynamic process with the passage of time. With the advance of time, the digital technology will gradually upgrade so that the existing technology will be gradually eliminated, and the digital transformation policy of enterprises has certain timeliness. Therefore, the growth rate of enterprise digitization level will gradually decline. According to the idea of the dynamic model of data governance proposed in reference [2], the stochastic differential equation is used to express the changing law of enterprise digitization level with time:

$$\dot{E}(t) = \frac{\mathrm{d}E(t)}{\mathrm{d}t} = \lambda_1 E_1(t) + \lambda_2 E_2(t) - \delta E(t). \tag{1}$$

Among them, λ_1 and $\lambda_2 > 0$ represent the influence degree of the efforts of enterprises and governments to promote collaborative development on the level of enterprise digitization, that is, the influence coefficient of the digital level. $\delta > 0$ indicates the decline of the enterprise digitization level, that is, the growth of $E(t)$ will gradually slow down with the improvement of the efforts of the participants, that is, the marginal utility decreases.

2.4 Hypothesis 4

The government and enterprises actively participate in the digital construction, which is the main way to improve the digital level of enterprises. Their respective efforts in cooperative development will directly affect the benefits of the whole digital ecosystem. Therefore, the interaction between business and government needs to be considered [2]. The total benefits of government–enterprise cooperation in the development of digital ecosystem $\pi(t)$ in time t can be expressed as follows:

$$\pi(t) = \theta_1 E_1(t) + \theta_2 E_2(t) + \beta E(t), \tag{2}$$

where θ_1 and θ_2 represent the marginal income coefficient of the enterprise and the government, respectively. $\beta > 0$ represents the influence of enterprise digitization level on the total income of the digital ecosystem.

2.5 Hypothesis 5

The total income of the whole digital ecosystem is distributed between the enterprise and the government according to the pre-agreed distribution ratio. If the government gets the proportion of ω, the enterprise gets the proportion of $1 - \omega$, of which $\omega \in (0, 1)$ is the income distribution coefficient, which is determined according to the importance and contribution to the process of cooperative development between the enterprise and the government. Both the enterprise and the government have a positive discount rate of p, and their goal is to seek cooperative development and maximize their own interests in an unlimited time frame.

In this paper, a differential game is defined, which includes one player of the government and a single enterprise using digital technology. There are three control variables $E_1(t) > 0$, $E_2(t) > 0$ and $0 \leq S(t) < 1$ and one state variable $E(t) \geq 0$. Under the noncooperative contract, only two control variables are $E_1(t)$ and $E_2(t)$, and $S(t) = 0$. Because of the existence of dynamic parameters, the solution will become very difficult. In this paper, according to reference [19], the parameters in the model are assumed to be time-independent constants. In addition, to facilitate writing, the time unit t will be omitted later.

3 Game strategies under cooperation and noncooperation

Based on the description and hypothesis presented in Section 2, this section further analyzes whether the cost-sharing contract can make the optimal decision-making of the government and enterprises reach the optimal level of cooperation; if it cannot reach the optimal level of cooperation, then analyze whether cost sharing can make the optimal decision-making of the government and enterprises achieve Pareto improvement and the degree of improvement, to provide a decision-making basis for government–enterprise cooperation to realize the digital transformation of enterprises.

3.1 Noncooperative game between government and enterprises

In the actual situation, the decision-making order of the government and enterprises is as follows: the government first formulates its own local digital economy development strategy. Enterprises formulate their own strategies on the basis of the strategies formulated by the government. If the optimal level of a cooperative game is the upper limit of game coordination, then the optimal level of a noncooperative game without cost sharing can be regarded as the lower limit of the benefits of all parties in game coordination.

Superscript N is used to represent the noncooperative game without cost sharing. At this time, the government does not share the efforts of enterprises to use digital technology, that is, $S(t) = 0$, and in this case, the objective functions of the government and the enterprise are, respectively

$$\max_{E_1} V_1^N = \int_0^\infty e^{-\rho t}\left[\omega(\theta_1 E_1 + \theta_2 E_2 + \beta E) - \frac{1}{2}k_1 E_1^2\right]\mathrm{d}t, \tag{3}$$

$$\max_{E_2} V_2^N = \int_0^\infty e^{-\rho t}[(1-\omega)(\theta_1 E_1 + \theta_2 E_2 + \beta E) - \frac{1}{2}k_2 E_2^2 \mathrm{d}t. \tag{4}$$

Theorem 1. *In the noncooperative game, the static feedback equilibrium strategies of the government and the enterprise are, respectively*

$$E_1^{N*} = \frac{\omega[\theta_1(\rho + \delta) + \lambda_1\beta]}{k_1(\rho + \delta)}, \tag{5}$$

$$E_2^{N*} = \frac{(1-\omega)[\theta_2(\rho + \delta) + \beta\lambda_2]}{k_2(\rho + \delta)}. \tag{6}$$

Proof. To obtain the Nash equilibrium state in this case, we first assume that both the government and the enterprise have optimal policy return functions $V_1(E)$ and $V_2(E)$; that they are continuously bounded and differentiable; and that all $E \geq 0$ must satisfy the HJB equation, that is, □

$$\rho V_1^N = \max_{E_1}\left\{\omega(\theta_1E_1 + \theta_2E_2 + \beta E) - \frac{1}{2}k_1E_1^2 + \frac{\partial V_1^N}{\partial E}(\lambda_1E_1 + \lambda_2E_2 - \delta E)\right\}, \tag{7}$$

$$\rho V_2^N = \max_{E_2}\left\{(1-\omega)(\theta_1E_1 + \theta_2E_2 + \beta E) - \frac{1}{2}k_2E_2^2 + \frac{\partial V_2^N}{\partial E}(\lambda_1E_1 + \lambda_2E_2 - \delta E)\right\}. \tag{8}$$

To solve the right-hand side of the HJB equation, the condition of maximizing it is that formula (7) is used to find the first-order partial derivative for E_1, and formula (8) for E_2 to find the first-order partial derivative, so that all of them are zero, and the solution can be obtained.

$$E_1 = \frac{\omega\theta_1 + \frac{\partial V_1^N}{\partial E}\lambda_1}{k_1}, \tag{9}$$

$$E_2 = \frac{(1-\omega)\theta_2 + \frac{\partial V_2^N}{\partial E}\lambda_2}{k_2}. \tag{10}$$

It can be obtained by substituting formulas (9) and (10) into formulas (7) and (8), respectively.

$$\rho V_1^N = \left(\omega\beta - \frac{\partial V_1^N}{\partial E}\delta\right)E + \frac{\left(\omega\theta_1 + \frac{\partial V_1^N}{\partial E}\lambda_1\right)^2}{2k_1} + \frac{\left(\omega\theta_2 + \frac{\partial V_1^N}{\partial E}\lambda_2\right)\left[(1-\omega)\theta_2 + \frac{\partial V_2^N}{\partial E}\lambda_2\right]}{k_2}, \tag{11}$$

$$\rho V_2^N = \left[(1-\omega)\beta - \frac{\partial V_2^N}{\partial E}\delta\right]E + \frac{\left[(1-\omega)\theta_1 + \frac{\partial V_2^N}{\partial E}\lambda_1\right]\left(\omega\theta_1 + \frac{\partial V_1^N}{\partial E}\lambda_1\right)}{k_1} + \frac{\left[(1-\omega)\theta_2 + \frac{\partial V_2^N}{\partial E}\lambda_2\right]^2}{2k_2}. \tag{12}$$

It can be seen from the structure of formulas (11) and (12) that the univariate first-order function with E as an independent variable is the solution of the HJB equation.

$$V_1^N = m_1E + m_2, \tag{13}$$

$$V_2^N = n_1E + n_2. \tag{14}$$

Among them, m_1, m_2, n_1, and n_2 are constants to be solved. Formulas (11) and (12) are substituted in formulas (13) and (14), respectively.

$$\rho V_1^N = (\omega\beta - m_1\delta)E + \frac{(\omega\theta_1 + m_1\lambda_1)^2}{2k_1} + \frac{(\omega\theta_2 + m_1\lambda_2)[(1-\omega)\theta_2 + n_1\lambda_2]}{k_2}, \tag{15}$$

$$\rho V_2^N = [(1-\omega)\beta - n_1\delta]E + \frac{[(1-\omega)\theta_1 + n_1\lambda_1](\omega\theta_1 + m_1\lambda_1)}{k_1} + \frac{[(1-\omega)\theta_2 + n_1\lambda_2]^2}{2k_2}. \tag{16}$$

From the previous hypothesis, formulas (15) and (16) should satisfy all $E \geq 0$. Therefore, we can obtain the value of m_1, m_2, n_1, and n_2, and the values are expressed as follows:

$$\begin{cases} m_1 = \dfrac{\omega\beta}{\rho + \delta}, \\ m_2 = \dfrac{\omega^2[(\rho+\delta)\theta_1 + \beta\lambda_1]^2}{2\rho k_1(\rho+\delta)^2} + \dfrac{\omega(1-\omega)[\theta_2(\rho+\delta) + \beta\lambda_2][(\theta_2(\rho+\delta) + \beta\lambda_2]}{\rho k_2(\rho+\delta)^2}, \end{cases} \tag{17}$$

$$\begin{cases} n_1 = \dfrac{(1-\omega)\beta}{\rho + \delta}, \\ n_2 = \dfrac{\omega(1-\omega)[\theta_1(\rho+\delta) + \beta\lambda_1][\theta_1(\rho+\delta) + \beta\lambda_2]}{\rho k_1(\rho+\delta)^2} + \dfrac{(1-\omega)^2[\theta_2(\rho+\delta) + \lambda_2\beta]^2}{2\rho k_2(\rho+\delta)^2}. \end{cases} \tag{18}$$

By substituting the values of m_1, m_2, n_1, and n_2 for type (11) and formula (12), the explicit expressions of the optimal effort strategy $E_1^\star$ and $E_2^\star$ of government and enterprise are obtained, respectively:

$$E_1^{N*} = \frac{\omega[\theta_1(\rho + \delta) + \lambda_1\beta]}{k_1(\rho + \delta)},$$

$$E_2^{N*} = \frac{(1 - \omega)[\theta_2(\rho + \delta) + \beta\lambda_2]}{k_2(\rho + \delta)}.$$

At the same time, the optimal benefit functions of the government and the enterprise are, respectively:

$$V_1^{N*} = \frac{\omega\beta}{\rho + \delta}E + \frac{\omega[(\rho + \delta)\theta_1 + \beta\lambda_1]^2}{2\rho k_1(\rho + \delta)^2} + \frac{\omega(1 - \omega)[\theta_2(\rho + \delta) + \beta\lambda_2][\theta_2(\rho + \delta) + \beta\lambda_2]}{\rho k_2(\rho + \delta)^2}, \tag{19}$$

$$V_2^{N*} = \frac{(1 - \omega)\beta}{\rho + \delta}E + \frac{\omega(1 - \omega)[\theta_1(\rho + \delta) + \beta\lambda_1][\theta_1(\rho + \delta) + \beta\lambda_2]}{\rho k_1(\rho + \delta)^2} + \frac{(1 - \omega)^2[\theta_2(\rho + \delta) + \lambda_2\beta]^2}{2\rho k_2(\rho + \delta)^2}. \tag{20}$$

Corollary 1. *In the noncooperative game, the optimal effort level of the government to develop the local digital economy E_1^{N*} and the income ratio of the government in the digital ecosystem ω, the marginal revenue coefficient of the government θ_1, the influence coefficient of digitization level on the government λ_1, the influence degree of enterprise digitization level on the total income of digital ecosystem β are positively correlated. These are negatively related to the effort cost of the government k_1.*

Corollary 2. *In the noncooperative game, the optimal effort level of enterprises using digital technology E_2^N, proportion to the income of enterprises in the digital ecosystem $1 - \omega$, marginal income coefficient of an enterprise θ_2, the influence of enterprise digitization level on the total benefits of digital ecosystem β, and the influence coefficient λ_2 of digitalization level on enterprises are positively correlated. These are negatively correlated with the effort cost of the enterprise.*

This conclusion is consistent with reality. For example, when the market economy is more developed, the government pays more attention to the development level of the local digital economy, and the degree to which enterprises use digital technology becomes more important. The government's investment in the development of the local digital economy will increase with the improvement of the degree of digitalization of enterprises, that is, E_1^{N*} will increase. To increase revenue and successfully complete the digital transformation, enterprises will take the initiative to increase their efforts to use digital technology, which to a certain extent explains why enterprises should strive to actively invest in the use of digital technology for digital transformation, for example, the unmanned supermarket of Ali Group, the uninhabited farm of country Garden, and so on.

It can be seen from Theorem 1 that the optimal effort level of the government and the optimal effort level of the enterprise are positively related to their own marginal income, which shows that both sides make decisions from the point of view of maximizing their own interests. The overall benefit of the whole digital ecosystem is not taken into account. When both parties in the system formulate their own strategies of the benefit maximization of the system as a whole, the benefits obtained by the government and enterprises will be improved. It can be seen from Corollary 2 that the optimal effort level of enterprises using digital technology is negatively related to the effort cost of enterprises, and the reduction of the cost of enterprises using digital technology can improve their efforts to use digital technology. The government can consider giving enterprises a certain proportion of subsidies or preferential policies to help enterprises complete digital transformation and improve the overall efficiency of the digital ecosystem.

3.2 The situation of Stackelberg game between government and enterprise

As the government plays a leading role in the digital transformation of enterprises, to encourage enterprises to use digital technology for digital transformation, the government will provide a certain proportion of subsidies or preferential policies for enterprises to use digital technology. From a long-term and dynamic point of view, the decision on digital transformation between the government and enterprises constitutes a government-guided Stackelberg differential game model, which is represented by superscript *ST*.

The government and enterprises make their own decisions to maximize their own benefits, and the decision-making process is divided into two stages. In the first stage, the government determines the proportion of subsidies to enterprises according to the standard of the development level of the local digital economy; in the second stage, enterprises make corresponding decisions after obtaining government decision-making information to ensure the maximization of their own interests. The government can effectively predict the follow-up decisions of enterprises before making decisions. In this case, the objective functions of the government and the enterprise are, respectively:

$$\max_{E_1,S} V_1^{ST} = \int_0^{\infty} e^{-\rho t}\left[\omega(\theta_1E_1 + \theta_2E_2 + \beta E) - \frac{1}{2}k_1E_1{}^2 - S\frac{1}{2}k_2E_2{}^2\right]\mathrm{d}t, \tag{21}$$

$$\max_{E_2,s} V_2{}^{ST} = \int_0^{\infty} e^{-\rho t}\left[(1-\omega)(\theta_1E_1 + \theta_2E_2 + \beta E) - \frac{1}{2}k_2E_2{}^2 + S\frac{1}{2}k_2E_2^2\right]\mathrm{d}t. \tag{22}$$

Theorem 2. *When the government provides a certain proportion of subsidies or preferential policies for enterprises to use digital technology, the static feedback equilibrium strategy between the government and enterprises is expressed as follows:*

$$E_1^{ST\star} = \frac{\omega\theta(\rho+\delta) + \omega\beta\lambda_1}{(\rho+\delta)k_1}, \tag{23}$$

$$E_2^{ST\star} = \frac{(1-\omega)[\theta 2(\rho+\delta) + \beta\lambda_2]}{(1-S^\star)(\rho+\delta)k_2}, \tag{24}$$

$$S^{ST\star} = \frac{[(2-\theta_2)\omega + \theta_2](\rho+\delta) - (1-3\omega)\beta\lambda_2}{[(2-\theta_2)\omega + \theta_2](\rho+\delta) + (1+\omega)\beta\lambda_2}. \tag{25}$$

Proof. To obtain the Stackelberg equilibrium in this case, we should first assume that both the government and enterprises have optimal benefits. Shared function $V_1(E)$ and $V_2(E)$, and continuous bounded differentiable, for all $E \geq 0$, must satisfy the HJB equation. By using the method of backward induction, this paper first constructs the HJB equation of the unilateral optimal effort level of enterprises using digital technology. □

$$\rho V_2{}^{ST} = \max_{E_2,S}\left\{(1-\omega)(\theta_1E_1 + \theta_2E_2 + \beta E) - \frac{1}{2}k_2E_2{}^2 + S\frac{1}{2}k_2E_2{}^2 + \frac{\partial V_2{}^{ST}}{\partial E}(\lambda_1E_1 + \lambda_2E_2 - \delta E)\right\}. \tag{26}$$

The condition for solving the right-hand side of the HJB equation to maximize it is that equation (26) needs to find the first-order partial derivative of E_2 and make it zero, so that the following solution can be obtained.

$$E_2 = \frac{(1-\omega)\theta_2 + \frac{\partial V_2{}^{ST}}{\partial E}\lambda^2}{(1-S)k_2}. \tag{27}$$

Enterprises will determine the level of their efforts to use digital technology according to formula (27) E_2, and so the government should determine its own optimal subsidy according to the level of efforts of enterprises to use digital technology to maximize benefits. At this time, the HJB equation for the government to develop the local digital economy is expressed as follows:

$$\rho V_1{}^{ST} = \max_{E_1,S}\left\{\omega(\theta_1E_1 + \theta_2E_2 + \beta E) - \frac{1}{2}k_1E_1{}^2 - S\frac{1}{2}k_2E_2{}^2 + \frac{\partial V_1{}^{ST}}{\partial E}(\lambda_1E_1 + \lambda_2E_2 - \delta E)\right\}. \tag{28}$$

Formula (27) is substituted for (28), and the right-hand side is solved, so that the condition for its maximization is that formula (28) calculates the first-order partial derivatives for E_1 and S, respectively, and makes them all zero, so that the solution can be obtained.

$$E_1 = \frac{\omega\theta 1 + \frac{\partial V_1^{ST}}{\partial E}\lambda_1}{k1}, \tag{29}$$

$$S = \frac{2\omega\theta_2 - (1-\omega)\theta_2 - \frac{\partial V_2^{ST}}{\partial E}\lambda_2 + 2\frac{\partial V_1^{ST}}{\partial E}\lambda_2}{2\omega\theta_2 + (1-\omega)\theta_2 + \frac{\partial V_2^{ST}}{\partial E}\lambda_2 + 2\frac{\partial V_1^{ST}}{\partial E}\lambda_2}. \tag{30}$$

Substitute the expressions (27), (29), and (30) into (26) and (28):

$$\begin{aligned}\rho V_1^{ST} &= \left(\omega\beta - \frac{\partial V_1^{ST}}{\partial E}\delta\right)E + \frac{\left(\omega\theta_1 + \frac{\partial V_1^{ST}}{\partial E}\right)^2}{2k_1} \\ &+ \frac{\left[2S(1-S)(\omega\theta_2 + \frac{\partial V_1^{ST}}{\partial E}\lambda_2 - 1\right]\left[(1-\omega)\theta_2 + \frac{\partial V_2^{ST}}{\partial E}\lambda_2\right]}{2(1-S)^2 k_2},\end{aligned} \tag{31}$$

$$\begin{aligned}\rho V_2^{ST} &= \left[(1-\omega)\beta - \frac{\partial V_2^{ST}}{\partial E}\delta\right]E + \frac{\left(\omega\theta_1 + \frac{\partial V_1^{ST}}{\partial E}\lambda_1\right)\left[(1-\omega)\theta_1 + \frac{\partial V_2^{ST}}{\partial E}\lambda_1\right]}{k_1} \\ &+ \frac{[2(1-S) + Sk_2 - 1]\left[(1-\omega)\theta_2 + \frac{\partial V_2^{ST}}{\partial E}\lambda_2\right]^2}{2(1-S_2)^2 k_2^{\,2}}.\end{aligned} \tag{32}$$

From the structure of formulas (31) and (32), it can be seen that the univariate first-order function with E as an independent variable is the solution of the HJB equation.

$$V_1^{ST} = a_1 E + a_2, \tag{33}$$

$$V_2^{ST} = b_1 E + b_2, \tag{34}$$

where a_1, a_2, b_1, and b_2 are constants, and the derivative of E in formulas (33) and (34) is substituted in formulas (31) and (32), respectively. The parameter expression of the optimal benefit function can be obtained as follows:

$$\begin{cases} a_1 = \dfrac{\omega\beta}{\rho + \alpha}, \\ a_2 = \dfrac{[\omega\theta_1(\rho+\delta) + \omega\beta]^2}{2k_1\rho(\rho+\delta)^2} + \dfrac{\{2\omega S(1-S)[\theta_2(\rho+\delta) + \beta\lambda_2] - \rho - \delta\}\{(1-\omega)[\theta_2(\rho+\delta) + \beta\lambda_2]\}}{2k^2\rho(1-S)^2(\rho+\delta)^2}, \end{cases} \tag{35}$$

$$\begin{cases} b_1 = \dfrac{(1-\omega)\beta}{\rho+\delta}, \\ b_2 = \dfrac{\{\omega[\theta_1(\rho+\delta) + \beta\lambda_1]\}\{(1-\omega)[\theta_1(\rho+\delta) + \beta\lambda 1]\}}{k_1\rho(\rho+\delta)^2} + \dfrac{(1-\omega)^2[2(1-S) + Sk_2 - 1][\theta_2(\rho+\delta) + \beta\lambda_2]^2}{2(1-S)^2 k_2^{\,2}\rho(\rho+\delta)^2}, \end{cases} \tag{36}$$

a_1, b_1 substitution (27), formula (29) and formula (30) can be used to obtain the optimal sharing effort strategies $E_1^\star$ and $E_2^\star$ for the government and enterprises, respectively, and the government provides a certain proportion of subsidies or preferential policies for enterprises to use digital technology $S^\star$.

$$E_1^{ST\star} = \frac{\omega\theta_1(\rho+\delta) + \omega\beta\lambda_1}{(\rho+\delta)k_1},$$

$$E_2^{ST\star} = \frac{(1-\omega)[\theta_2(\rho+\delta) + \beta\lambda_2]}{(1-S^\star)(\rho+\delta)k_2},$$

and

$$S^{ST\star} = \frac{[(2-\theta_2)\omega + \theta_2](\rho+\delta) - (1-3\omega)\beta\lambda_2}{[(2-\theta_2)\omega + \theta_2](\rho+\delta) + (1+\omega)\beta\lambda_2}.$$

The optimal benefit functions of the government and the enterprise are obtained by using a_1, a_2 and b_1, b_2, and generation formula (28) and formula (29). The optimal benefit functions of the government and the enterprise are expressed as follows:

$$V_1^{ST\star} = \frac{\omega\beta}{\rho+\delta}E + \frac{[\omega\theta_1(\rho+\delta)+\omega\beta]^2}{2k_1\rho(\rho+\delta)^2} + \frac{\{2\omega S^{\star}(1-S^{\star})[\theta_2(\rho+\delta)+\beta\lambda_2]-\rho-\delta\}\{(1-\omega)[\theta_2(\rho+\delta)+\beta\lambda_2]\}}{2k_2\rho(1-S^{\star})(\rho+\delta)^2},$$

$$V_2^{ST\star} = \frac{(1-\omega)\beta}{\rho+\delta}E + \frac{\omega(1-\omega)[\theta_1(\rho+\delta)+\beta\lambda_1]^2}{k_1\rho(\rho+\delta)^2} + \frac{(1-\omega)^2[1-2S^{\star}+k_2S^{\star}][\theta_2(\rho+\delta)+\beta\lambda_2]^2}{2k_2{}^2\rho(1-S^{\star})(\rho+\delta)^2}.$$

Corollary 4. *When the government gives enterprises a certain proportion of subsidies or preferential policies, the optimal level of efforts of enterprises to use digital technology is negatively correlated with their own effort costs k_2 and positively correlated with the proportion of government subsidies S^*.*

Corollary 5. *When the government gives enterprises a certain proportion of subsidies or preferential policies, the optimal effort level of enterprises using digital technology is greater than that without subsidies.*

As shown in Corollary 5, the government has shared the proportion of the cost of enterprises using digital technology, reduced the cost pressure of enterprises, moved their optimal level of efforts to the right, and raised their level of efforts to use digital technology, which is consistent with the previous expectations.

Similar to Corollary 1, when the government gives enterprises a certain proportion of subsidies or preferential policies, $E_1^{ST\star}$ of the government's optimal efforts to develop the local digital economy is positively correlated with the proportion of the government's income in the digital ecosystem ω, the government's marginal income coefficient θ_1, the influence coefficient of the digitization level on the government λ_1, and the influence degree of the enterprise digitization level on the total income of the digital ecosystem β. It is negatively correlated with the effort cost of the government k_1.

3.3 Cooperative game between government and enterprises

To maintain the prosperity of the local economy and enhance the digital level of enterprises, so that enterprises not only promote their own digital transformation but also improve their brand influence, from any point of view, the government and enterprises hope to strive to improve the level of digitization. This part discusses the coordination and cooperation between the government and enterprises, considering the whole digital ecosystem composed by both government and enterprises, and assumes that there is a central decision maker who aims to maximize the overall benefit of the system, and the decision maker aims to maximize the overall benefit of the system. By determining the optimal level of use of digital technology, the whole system achieves the optimal state. Superscript C is used to indicate the situation at this time.

When the government and enterprises cooperate, the two sides will take the benefit maximization of the digital ecosystem composed of both as the goal and jointly determine the optimal level of each effort, and its objective function is expressed as follows:

$$\max_{E_1,E_2} V_{12}^{C} = \int_0^{\infty} e^{-\rho t}\left[2(\theta_1E_1 + \theta_2E_2 + \beta E) - \frac{1}{2}k_1E_1{}^2 - \frac{1}{2}k_2E_2{}^2\right]\mathrm{d}t. \tag{37}$$

Theorem 3. *In the case of the cooperative game, the static feedback equilibrium strategies of government and enterprise are, respectively, expressed as follows:*

$$E_1^{C\star} = \frac{2\theta(\rho + \delta) + 2\beta\lambda_1}{(\rho + \delta)k_1}, \tag{38}$$

$$E_2^{C\star} = \frac{2\theta(\rho + \delta) + 2\beta\lambda_2}{(\rho + \delta)k_2}. \tag{39}$$

Proof. To obtain the cooperative equilibrium state in this case, we should first assume that there is an optimal return function V_{12} in the digital ecosystem of government–enterprise cooperation, and it is continuously bounded and differentiable, and all $E \geq 0$ must satisfy the HJB equation. □

$$\rho V_{12}^C = \max_{E_1, E_2}\left\{2(\theta_1 E_1 + \theta_2 E_2 + \beta E) - \frac{1}{2}k_1 E_1{}^2 - \frac{1}{2}k_2 E_2{}^2 + 2\beta E + \frac{\partial V_{12}{}^C}{\partial E}(\lambda_1 E_1 + \lambda_2 E_2 - \delta E)\right\}. \tag{40}$$

The first-order conditions of E_1 and E_2 in the aforementioned formula can be obtained:

$$E_1 = \frac{2\theta + \frac{\partial V_{12}^C}{\partial E}\lambda_1}{k1}, \tag{41}$$

$$E_2 = \frac{2\theta + \frac{\partial V_{12}^C}{\partial E}\lambda_2}{k_2}. \tag{42}$$

Formulas (41) and (42) are available in the belt type (40).

$$\rho V_{12}^C = \left(2\beta - \frac{\partial V_{12}^C}{\partial E}\delta\right)E + \frac{\left(2\theta + \lambda_1 \frac{\partial V_{12}^C}{\partial E}\right)^2}{2k_1} + \frac{\left(2\theta + \lambda_2 \frac{\partial V_{12}^C}{\partial E}\right)^2}{2k_2}. \tag{43}$$

Similarly, the linear optimal benefit function with respect to E is the solution of the HJB equation.

$$V_{12}{}^C = l_1 E + l_2, \tag{44}$$

where l_1 and l_2 are constants, the derivative of formula (44) with respect to E is substituted (43), and the parameter value of the optimal benefit function is expressed as follows:

$$l_1 = \frac{2\beta}{\rho + \delta}, \tag{45}$$

$$l_2 = \frac{[2\theta(\rho + \delta) + 2\beta\lambda_1]^2}{2k_1\rho(\rho + \delta)} + \frac{[2\theta_2(\rho + \delta) + 2\beta\lambda_2]}{2k_2\rho(\rho + \delta)}. \tag{46}$$

Formulas (45) and (46) can be obtained by replacing E_1 and E_2 with (43) and the derivative equals 0, respectively, from which the optimal effort level of local governments and traditional enterprises in the cooperative game can be obtained as follows:

$$E_1^{C\star} = \frac{2\theta_1(\rho + \delta) + 2\beta\lambda_1}{(\rho + \delta)k_1},$$

$$E_2^{C\star} = \frac{2\theta_2(\rho + \delta) + 2\beta\lambda_2}{(\rho + \delta)k_2}.$$

The optimal benefit function of the system can be obtained by inserting l_1 and l_2 generations into (28).

$$V_{12}^{C\star} = \frac{2\beta}{\rho + \delta}E + \frac{[2\theta_1(\rho + \delta) + 2\beta\lambda_1]^2}{2k_1\rho(\rho + \delta)^2} + \frac{[2\theta_2(\rho + \delta) + 2\beta\lambda_2]}{2k_2\rho(\rho + \delta)}. \tag{47}$$

Corollary 6. *In the cooperative game, the optimal effort level of the government $E_1^\star$, the optimal effort level of the enterprise $E_2^\star$, and the overall benefit of the system $V_{12}^{C\star}$ are all related to the marginal benefit of the government and the enterprise θ_1 and θ_2, respectively. These are positively related to the influence of enterprise digitization level on the total benefits of digital ecosystem β.*

It can be seen from Corollary 6 that in the case of cooperative game, the higher the marginal income of the government or the higher the digitization level of the enterprise, the higher the effort level of the government and the enterprise as well as the overall benefit of the digital ecosystem. That is, the increase in the marginal benefit of the government will increase the investment of enterprises in the use of digital technology. As far as the government is concerned, a reasonable proportion of subsidies, preferential policies for enterprises to use digital technology, and cooperation with enterprises to develop digital technology for the benefit of society can well reduce the cost of enterprises' efforts to use digital technology alone. When the cost of the efforts of the government and enterprises in the whole digital ecosystem is low, the overall benefit of the digital ecosystem can be better improved.

4 Comparative analysis of equilibrium results

By comparing the optimal effort strategies and benefits of the government and enterprises in the three cases of noncooperative game, Stackelberg game, and cooperative game, we can get some relevant conclusions as follows.

Theorem 4. *In the noncooperative game, the optimal effort level of the government and enterprises is the lowest. In the cooperative game, the optimal effort level of the government is the highest, and the optimal effort level of enterprises is closely related to the optimal subsidy of the government.*

Proof. As far as the government is concerned, we can know from formulas (7), (20), and (37) that □

$$E_1^{N\star} = E_1^{ST\star} = \frac{\omega[\theta_1(\rho + \delta) + \lambda_1\beta]}{k_1(\rho + \delta)},\ E_1^{C\star} = \frac{2\theta_1(\rho + \delta) + 2\beta\lambda_1}{(\rho + \delta)k_1}.$$

Therefore, $E_1^{N\star} = E_1^{ST\star} < E_1^{C\star}$.

As far as the enterprise is concerned, you can know from formulas (7), (20), and (37).

$$E_2^{N\star} - E_2^{ST\star} = -\frac{S^\star(1 - \omega)[\theta_2(\rho + \delta) + \beta\lambda_2]}{(1 - S^\star)k_2(\rho + \delta)} < 0.$$

Therefore, $E_2^{N\star} = E_2^{ST\star}$.

$$E_2^{N\star} - E_2^{C\star} = -\frac{(\omega + 1)[\theta_2(\rho + \delta) + \beta\lambda_2]}{k_2(\rho + \delta)} < 0.$$

Therefore, $E_2^{N\star} = E_2^{C\star}$.

$$E_2^{C\star} - E_2^{ST\star} = \frac{(1 - 2S^\star + \omega)[\theta_2(\rho + \delta) + \beta\lambda_2]}{(1 - S^\star)k_2(\rho + \delta)}.$$

Therefore, $\begin{cases} S^\star < \frac{1+\omega}{2}, & E_2^{C\star} > E_2^{ST\star}, \\ S^\star \geq \frac{1+\omega}{2}, & E_2^{C\star} \leq E_2^{ST\star}. \end{cases}$

To sum up, $\begin{cases} S^\star < \frac{1+\omega}{2}, & E_2^{C\star} > E_2^{ST\star} > E_2^{N\star}, \\ S^\star \geq \frac{1+\omega}{2}, & E_2^{ST\star} \geq E_2^{C\star} > E_2^{N\star}. \end{cases}$

It can be seen from Theorem 4 that when the government's optimal subsidy is $S^\star < \frac{1+\omega}{2}$, the cooperation between the government and enterprises will maximize their respective efforts. When the government's optimal subsidy is $S^\star \geq \frac{1+\omega}{2}$, the cooperation between government and enterprises reduces the optimal effort level of enterprises. Therefore, the government's optimal subsidy is not the more the better, but should be properly subsidized.

From the aforementioned analysis, we can see that, on the one hand, from the perspective of the government, regardless of the intensity of subsidies, the optimal level of government effort is the largest in the cooperative game. On the other hand, from the point of view of the enterprise, when $S^* < \frac{1+\omega}{2}$, the optimal effort level of the enterprise is the largest in the cooperative game. When $S^* \geq \frac{1+\omega}{2}$, the optimal effort level of the enterprise is the largest in the Steinberg game. Therefore, the condition for the government and enterprises to cooperate to complete the digital transformation of enterprises is that the government subsidy to enterprises is less than $\frac{1+\omega}{2}$.

Theorem 5. *Compared with the noncooperative game, in the case of partial subsidies or preferential policies given by the government, both the government and the enterprise have achieved Pareto improvement.*

Proof.

$$V_1^{ST*} - V_1^{N*} = \frac{\omega[\omega(1+\beta)^2 + (A + B\lambda_1)^2]}{2\rho k_1(\rho+\delta)^2} + \frac{B(1-\omega)[2B\omega(1-S^*)^2 + \rho + \delta]}{2\rho k_2(\rho+\delta)^2(1-S^*)} > 0,$$

$$V_2^{ST*} - V_2^{N*} = \frac{\beta\omega(1-\omega)[A + \beta\lambda_1](\lambda_1 + \lambda_2)}{\rho k_1(\rho+\delta)^2} + \frac{B^2(1-\omega)^2(1-k_2)}{2\rho k_2{}^2(1-S^*)(\rho+\delta)^2} > 0.$$

Among them $A = \theta_1(\rho+\delta)$, $B = \theta_2(\rho+\delta) + \beta\lambda_2$.

Therefore, it is proved that $V_1^{ST*} - V_1^{N*}$, $V_2^{ST*} - V_2^{N*}$. □

It can be seen from Theorem 5 that in cost sharing, the benefits of the government and enterprises are greater than those without cost sharing, which shows that in cost sharing, through the government to give enterprises a certain proportion of subsidies or preferential policies to achieve the Pareto improvement of the digital ecosystem, and the benefits of both governments and enterprises have been improved.

Theorem 6. *In the case of cooperation, the optimal benefit of the digital ecosystem is greater than that of the other two cases.*

Proof.

$$\begin{aligned} & V_{12}^{C*} - (V_1^{ST*} + V_2^{ST*}) \\ &= \frac{\beta}{\rho+\delta}E + \frac{(A+\beta\lambda_1)^2[4 - 2\omega(1-\omega)] + \omega^2(A+\beta)^2}{2\rho k_1(\rho+\delta)^2} \\ &\quad + \frac{B\{2k_2(1-S^*)(\rho+\delta) + k_2[2\omega S^*(1-S^*)(A+\beta\lambda_2) - (\rho+\delta)] + (1-\omega)^2(1 - 2S^* + k_2 S^*)\}}{2\rho k_2{}^2(1-S^*)(\rho+\delta)^2} > 0. \end{aligned}$$

Therefore, $V_{12}^{C*} - V_1^{ST*} + V_2^{ST*} > V_1^{N*} + V_2^{N*}$. □

According to Theorems 5 and 6, compared with the noncooperative contract, the cost-sharing contract realizes the Pareto improvement of both members in the digital ecosystem, and the optimal benefit of the whole digital ecosystem in the case of the cost-sharing contract is greater than that of the noncooperative contract. Conversely, the optimal benefit of the digital ecosystem in the case of cooperation is greater than that of cost sharing. However, it is worth noting that only the optimal strategy that makes the respective benefits of the cooperative cooperation of the government and enterprises greater than that of the noncooperative cooperation can be accepted by both the government and the enterprise. As for the incremental share of the benefits of the digital ecosystem, it depends on the negotiation ability of both sides. That is, if the final income distribution scheme is reasonable and feasible, then for the government and enterprises, the cooperative contract is Pareto optimal.

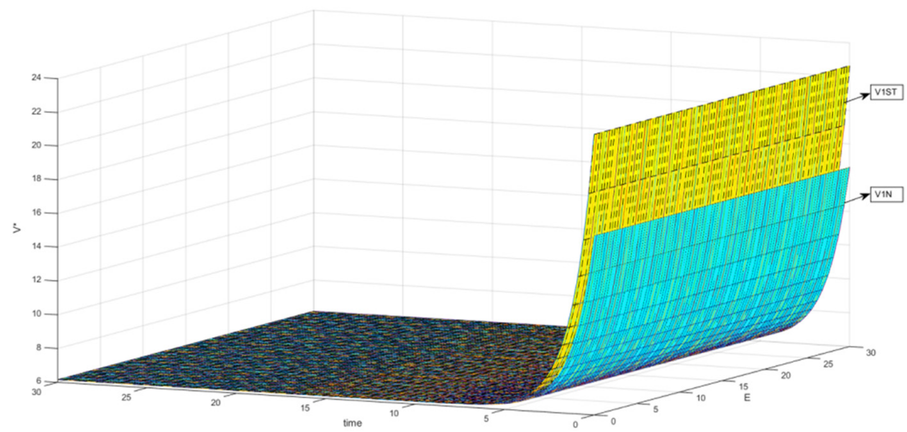

Figure 1: The change of government's optimal benefit under noncooperative game and Stackelberg game.

5 Numerical simulation

In the differential game between the government and the enterprise in the case of cooperation and non-cooperation, the optimal effort and benefit of the government and the enterprise and the benefit of the whole digital ecosystem depend on the parameters in the model. According to the research results of references [2] and considering the actual situation, the benchmark parameters are set as follows: $\alpha = 0.8$, $k_1 = 2$, $k_2 = 2$, $\lambda_2 = 2$, $\delta = 0.2$, $\theta_1 = 3$, $\theta_2 = 3$, $\beta = 2$, $\omega = 0.6$. The numerical simulation is carried out with Matlab7.0 software.

First, according to benchmark parameters, the improvement effect of government subsidies is studied, and the comparative relationship between the benefits of government and enterprises before and after government subsidies is drawn (Figures 1 and 2). The change in the total income in the digital ecosystem under three game modes is observed (Figure 3); second, the sensitivity analysis of the government optimal subsidy coefficient is carried out (Figures 4 and 5).

As shown in Figures 1 and 2, the optimal benefits of the government and enterprises increase with the increase of the digitization level of enterprises and gradually decrease with the passage of time and finally stabilize. In addition, the optimal benefit of the government and enterprises after the government subsidy is higher than that of the government and enterprises before the subsidy. This shows that when enterprises carry out digital transformation, the government can carry out a certain proportion of subsidies, which can

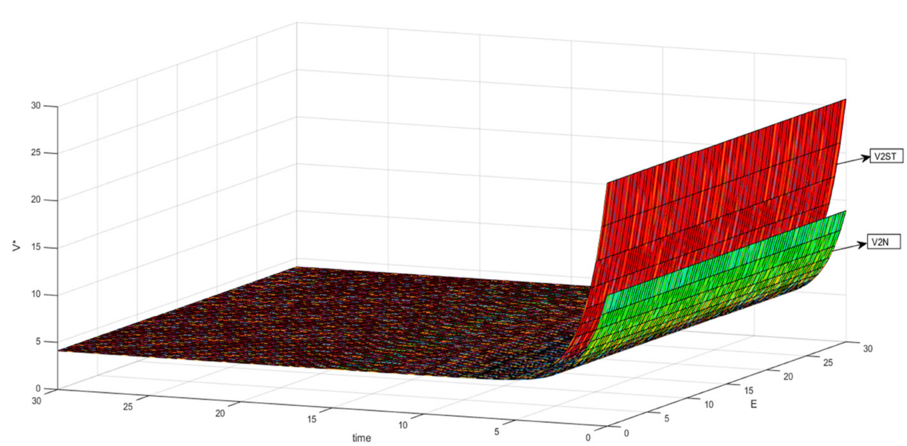

Figure 2: The change of enterprise's optimal benefit under noncooperative game and Stackelberg game.

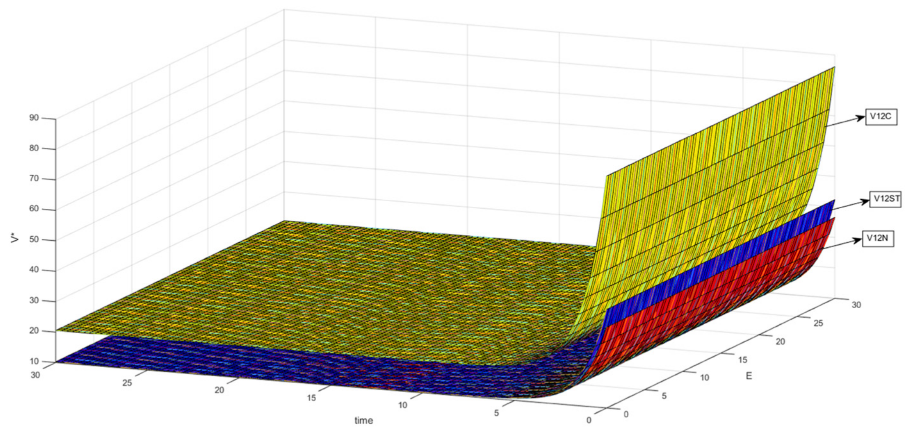

Figure 3: Comparison of total benefits under three games in digital ecosystem.

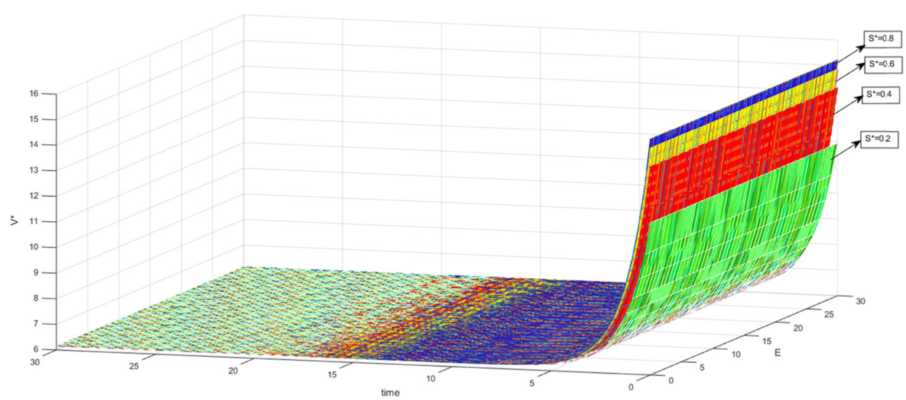

Figure 4: The change of government's optimal benefit under different optimal subsidy ratios.

not only improve its own efficiency but also help enterprises to improve their efficiency. This is also consistent with the conclusion of $V_2^{ST\star} > V_2^{N\star}$ in Theorem 5.

As shown in Figure 3, in the digital ecosystem composed of government and enterprises in three cases, the overall optimal benefit is the largest in cooperation and the minimum in noncooperation. When the government gives certain subsidies, it can achieve Pareto improvement of the overall benefits of the digital

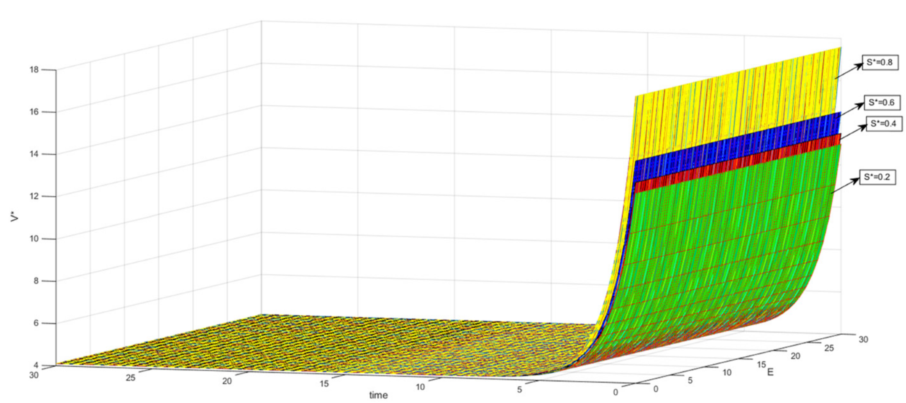

Figure 5: The change of enterprise's optimal benefit under different optimal subsidy ratios.

ecosystem. In the case of cooperative cooperation between the government and enterprises, the total benefit of the digital ecosystem is much greater than that of the two noncooperative systems, which verifies the result of $V_{12}^{C*} > V_1^{ST*} + V_2^{ST} > V_1^{N*} + V_2^{N*}$ in Theorem 6, which fully shows that cooperative decision-making is better than noncooperative decision-making. It can provide a reference for government–enterprise cooperation to promote the digital transformation of enterprises and develop the digital economy together.

Figures 4 and 5 show the sensitivity analysis of the change of government optimal subsidy coefficient S^* to the optimal benefits of government and enterprises. As shown in this figure with the continuous improvement of the digital level of enterprises E, the increase of enterprise subsidies will increase the optimal benefits of the government and enterprises at the same time. From this point of view, within the scope of its own capacity, increasing subsidies will help to promote the digital transformation of enterprises and better develop the digital economy. This does not contradict the conclusion of Theorem 4, which is from the point of view of the effort cost of the government and enterprises, while this conclusion only analyzes the government subsidy and does not consider other factors.

6 Conclusion and discussion

This paper takes the government and enterprises as the research objects and uses the differential game method to study the government subsidy behavior and the cooperation behavior with enterprises in three different game situations in the process of digital transformation using digital technology. The results show that (1) under the three kinds of games, the greater the influence of the marginal income and the digitization level of the government and the enterprise on the total income of the digital ecosystem, the higher the degree of effort, while the smaller the effort cost of the government and the enterprise, the higher the degree of effort. (2) To obtain the best benefit, the optimal effort degree of the government and the enterprise in the cooperative game is better than that in the noncooperative game and the Stackelberg game. (3) From the perspective of the overall benefit of the digital ecosystem, the cooperative game between the government and enterprises is better than the noncooperative game and the Stackelberg game in which the government subsidizes the enterprises and achieves the Pareto optimality of the system. As an effective adjustment mechanism, the subsidy factor can urge enterprises to make more efforts to promote their own digital transformation, and increase the total benefits of the government and enterprises. However, in the actual development, there are a large number of enterprises and unique, and so it is difficult for local governments to fully communicate with all enterprises to achieve ideal cooperation. It is more practical for the government to give certain subsidies, which is an effective way for our government to promote the digital transformation of enterprises and promote the development of digital economy at the present stage.

Currently, governments all over the world have put promoting the development of digital economy in a very important position. The digital transformation of enterprises requires not only the efforts of enterprises themselves but also the strong support of the government and the whole society. From the previous analysis, there is no doubt that the government plays an important role in promoting the development process of enterprise digital transformation. The government can effectively integrate resources, provide financial support, concentrate social forces, quickly build an industrial chain, and provide support for the digital transformation of enterprises is the practical embodiment of the government to fully play a guiding role. At the same time, it should be noted that the government should not act as the main attacker, but should act as a guide and play a major role in the field of digital economy development. For example, give a certain proportion of subsidies to the digital transformation of enterprises and guide the direction of the digital transformation of enterprises through the formulation of policies such as recommending the use of digital technology. By comparing the cost coefficient of effort, the influence coefficient of digitization level, and the optimal benefit, we can find the right focus, reasonably grasp the subsidy for the digital transformation of enterprises, improve the total benefit of the digital ecosystem with twice the result with half the effort, and promote enterprises to successfully complete digital transformation and promote the development of digital economy in China.

The disadvantage of this paper is that it only considers the impact of the government and enterprises on their respective benefits and the overall benefits of the digital ecosystem; however, many factors affect the total benefits, and the possible interaction and impact among these factors need to be further studied. To facilitate the solution, this paper assumes that all the parameters in the model are not related to time. Future research can further expand the model, add consumer factors, consider more reality, and further study the impact of consumer participation behavior on the digital transformation of enterprises.

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