Research Article

Guangyu An, Xueli Zhang, and Jun He*

Characterizations of *-antiderivable mappings on operator algebras

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Abstract: Let $\mathcal{A}$ be a $*$-algebra, $\mathcal{M}$ be a $*$-$\mathcal{A}$-bimodule, and δ be a linear mapping from $\mathcal{A}$ into $\mathcal{M}$. δ is called a $*$-*derivation* if $\delta(AB) = A\delta(B) + \delta(A)B$ and $\delta(A^*) = \delta(A)^*$ for each A, B in $\mathcal{A}$. Let G be an element in $\mathcal{A}$, δ is called a $*$-*antiderivable mapping* at G if $AB^* = G \Rightarrow \delta(G) = B^*\delta(A) + \delta(B)^*A$ for each A, B in $\mathcal{A}$. We prove that if $\mathcal{A}$ is a C^*-algebra, $\mathcal{M}$ is a Banach $*$-$\mathcal{A}$-bimodule and G in $\mathcal{A}$ is a separating point of $\mathcal{M}$ with $AG = GA$ for every A in $\mathcal{A}$, then every $*$-antiderivable mapping at G from $\mathcal{A}$ into $\mathcal{M}$ is a $*$-derivation. We also prove that if $\mathcal{A}$ is a zero product determined Banach $*$-algebra with a bounded approximate identity, $\mathcal{M}$ is an essential Banach $*$-$\mathcal{A}$-bimodule and δ is a continuous $*$-antiderivable mapping at the point zero from $\mathcal{A}$ into $\mathcal{M}$, then there exists a $*$-Jordan derivation Δ from $\mathcal{A}$ into $\mathcal{M}^{\sharp\sharp}$ and an element ξ in $\mathcal{M}^{\sharp\sharp}$ such that $\delta(A) = \Delta(A) + A\xi$ for every A in $\mathcal{A}$. Finally, we show that if $\mathcal{A}$ is a von Neumann algebra and δ is a $*$-antiderivable mapping (not necessary continuous) at the point zero from $\mathcal{A}$ into itself, then there exists a $*$-derivation Δ from $\mathcal{A}$ into itself such that $\delta(A) = \Delta(A) + A\delta(I)$ for every A in $\mathcal{A}$.

Keywords: *-derivation, *-antiderivable mapping, C*-algebra, von Neumann algebra

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1 Introduction

Throughout this paper, let $\mathcal{A}$ be an associative algebra over the complex field $\mathbb{C}$ and $\mathcal{M}$ be an $\mathcal{A}$-bimodule. A linear mapping δ from $\mathcal{A}$ into $\mathcal{M}$ is called a *derivation* if

$$\delta(AB) = A\delta(B) + \delta(A)B$$

for each A, B in $\mathcal{A}$; and δ is called a *Jordan derivation* if

$$\delta(A^2) = A\delta(A) + \delta(A)A$$

for every A in $\mathcal{A}$. It follows from [1, Corollary 17] that every Jordan derivation from a C^*-algebra $\mathcal{A}$ into a Banach $\mathcal{A}$-bimodule is a derivation.

Let G be an element in $\mathcal{A}$, δ is called a *derivable mapping at G* if

$$AB = G \Rightarrow \delta(G) = A\delta(B) + \delta(A)B$$

for each A, B in $\mathcal{A}$. In [2–9], the authors investigated derivable mappings at the point zero. In [10–16], the authors investigated derivable mappings at nonzero points.

A linear mapping δ from $\mathcal{A}$ into $\mathcal{M}$ is called an *antiderivable mapping at G* if

$$AB = G \Rightarrow \delta(G) = B\delta(A) + \delta(B)A$$

* **Corresponding author: Jun He,** Department of Mathematics, Anhui Polytechnic University, Wuhu 241000, China, e-mail: hejun_12@163.com
Guangyu An, Xueli Zhang: Department of Mathematics, Shaanxi University of Science and Technology, Xi'an 710021, China

for each A, B in $\mathcal{A}$. In [6,7] and [17,18], the authors characterized antiderivable mappings at the point zero on properly infinite von Neumann algebras, C^*-algebras and group algebras.

By an *involution* on an algebra $\mathcal{A}$, we mean a mapping $*$ from $\mathcal{A}$ into itself, such that

$$(\lambda A + \mu B)^* = \bar{\lambda}A^* + \bar{\mu}B^*, \quad (AB)^* = B^*A^* \quad \text{and} \quad (A^*)^* = A,$$

whenever A, B in $\mathcal{A}$, λ, μ in $\mathbb{C}$ and $\bar{\lambda}, \bar{\mu}$ denote the conjugate complex numbers. An algebra $\mathcal{A}$ equipped with an involution is called a $*$-algebra. Moreover, let $\mathcal{A}$ be a $*$-algebra, an $\mathcal{A}$-bimodule $\mathcal{M}$ is called a $*$-$\mathcal{A}$-bimodule if $\mathcal{M}$ equipped with a $*$-mapping from $\mathcal{M}$ into itself, such that

$$(\lambda M + \mu N)^* = \bar{\lambda}M^* + \bar{\mu}N^*, \quad (AM)^* = M^*A^*, \quad (MA)^* = A^*M^* \quad \text{and} \quad (M^*)^* = M,$$

whenever A in $\mathcal{A}$, M, N in $\mathcal{M}$ and λ, μ in $\mathbb{C}$.

An element A in a $*$-algebra $\mathcal{A}$ is called *Hermitian* if $A^* = A$; an element P in $\mathcal{A}$ is called an *idempotent* if $P^2 = P$; and P is called a *projection* if P is both a self-adjoint element and an idempotent.

In [19], Kishimoto studied the $*$-derivations on a C^*-algebra and proved that the closure of a normal $*$-derivation on a UHF algebra satisfying a special condition is a generator of a one-parameter group of $*$-automorphisms. Let $\mathcal{A}$ be a $*$-algebra and $\mathcal{M}$ be a $*$-$\mathcal{A}$-bimodule. A derivation δ from $\mathcal{A}$ into $\mathcal{M}$ is called a $*$-*derivation* if $\delta(A^*) = \delta(A)^*$ for every A in $\mathcal{A}$. Obviously, every derivation δ is a linear combination of two $*$-derivations. In fact, we can define a linear mapping $\delta^\sharp$ from $\mathcal{A}$ into $\mathcal{M}$ by $\delta^\sharp(A) = \delta(A^*)^*$ for every A in $\mathcal{A}$; therefore, $\delta = \delta_1 + \mathrm{i}\delta_2$, where $\delta_1 = \frac{1}{2}(\delta + \delta^\sharp)$ and $\delta_2 = \frac{1}{2\mathrm{i}}(\delta - \delta^\sharp)$. It is easy to show that δ_1 and δ_2 are both $*$-derivations.

Similar to derivable and antiderivable mappings, we can consider $*$-derivable and $*$-antiderivable mappings. Let $\mathcal{A}$ be a $*$-algebra, $\mathcal{M}$ be a $*$-$\mathcal{A}$-bimodule and G be an element in $\mathcal{A}$. A linear mapping δ from $\mathcal{A}$ into $\mathcal{M}$ is called a $*$-*derivable mapping at* G if

$$AB^* = G \Rightarrow \delta(G) = A\delta(B)^* + \delta(A)B^*$$

for each A, B in $\mathcal{A}$ and δ is called a $*$-*antiderivable mapping at* G if

$$AB^* = G \Rightarrow \delta(G) = B^*\delta(A) + \delta(B)^*A$$

for each A, B in $\mathcal{A}$.

In [6], Ghahramani supposed that $\mathcal{G}$ is a locally compact group, $L^1(\mathcal{G})$ and $M(\mathcal{G})$ denote the the group algebra and the measure convolution algebra of $\mathcal{G}$, respectively, and showed that if δ is a $*$-derivable mapping or a $*$-antiderivable mapping at the point zero from $L^1(\mathcal{G})$ into $M(\mathcal{G})$, then there exist two elements B, C in $M(\mathcal{G})$ such that $\delta(A) = AB - CA$ for every A in $L^1(\mathcal{G})$. In [7], Ghahramani and Pan supposed that $\mathcal{A}$ is a properly infinite W^*-algebra or a simple C^*-algebra with a nontrivial idempotent, and proved that if δ is a $*$-derivable mapping at the point zero from $\mathcal{A}$ into itself, then there exist two elements B, C in $\mathcal{A}$ such that $\delta(A) = AB - CA$ for every A in $\mathcal{A}$; if δ is a $*$-antiderivable mapping at the point zero from $\mathcal{A}$ into itself, then $\delta(A) = \delta(I)A$ for every A in $\mathcal{A}$. In [17], Abulhamil et al. supposed that $\mathcal{A}$ is a C^*-algebra and $\mathcal{M}$ is an essentially Banach $\mathcal{A}$-bimodule, and proved that if δ is a continuous $*$-antiderivable mapping at the point zero from $\mathcal{A}$ into $\mathcal{M}$, then there exists a $*$-derivation Δ from $\mathcal{A}$ into $\mathcal{M}^{\sharp\sharp}$ and ξ in $\mathcal{M}^{\sharp\sharp}$ such that $\delta(A) = \Delta(A) + A\xi$ for every A in $\mathcal{A}$, where $\mathcal{M}^{\sharp\sharp}$ is the second dual of $\mathcal{M}$. In [18], Fadaee and Ghahramani supposed that $\mathcal{A}$ is a von Neumann algebra or a simple unital C^*-algebra, and proved that if δ is a $*$-derivable mapping or a $*$-antiderivable mapping at the point zero from $\mathcal{A}$ into itself, then there exist two elements B, C in $\mathcal{A}$ such that $\delta(A) = AB - CA$ for every A in $\mathcal{A}$.

For an algebra $\mathcal{A}$ and an $\mathcal{A}$-bimodule $\mathcal{M}$, we call an element G in $\mathcal{A}$ a *left* (*right*) *separating point* of $\mathcal{M}$ if $GM = 0$ ($MG = 0$) implies $M = 0$ for every M in $\mathcal{M}$. It is easy to see that every left(right) invertible element in $\mathcal{A}$ is a left(right) separating point of $\mathcal{M}$. If $G \in \mathcal{A}$ is both the left and right separating point, then G is called a *separating point* of $\mathcal{M}$.

In Section 2, we prove that if $\mathcal{A}$ is a C^*-algebra, $\mathcal{M}$ is a Banach $*$-$\mathcal{A}$-bimodule and G in $\mathcal{A}$ is a separating point of $\mathcal{M}$ with $AG = GA$ for every $A \in \mathcal{A}$, then every $*$-antiderivable mapping at G from $\mathcal{A}$ into $\mathcal{M}$ is a $*$-derivation.

In Section 3, we investigate $*$-antiderivable mappings at the point zero and prove that if $\mathcal{A}$ is a zero product determined Banach $*$-algebra with a bounded approximate identity, $\mathcal{M}$ is an essential Banach

$*$-$\mathcal{A}$-bimodule and δ is a continuous $*$-antiderivable mapping at the point zero from $\mathcal{A}$ into $\mathcal{M}$, then there exists a $*$-Jordan derivation Δ from $\mathcal{A}$ into $\mathcal{M}^{\sharp\sharp}$ and ξ in $\mathcal{M}^{\sharp\sharp}$, such that $\delta(A) = \Delta(A) + A\xi$ for every A in $\mathcal{A}$, where $\mathcal{M}^{\sharp\sharp}$ stands for the second dual of M. Thus, we generalize [6, Theorem 3.2(2)] and [17, Theorem 9]. Finally, we prove that every $*$-antiderivable mapping at the point zero from a von Neumann algebra $\mathcal{A}$ into itself satisfies that $\delta(A) = \Delta(A) + A\delta(I)$ for every A in $\mathcal{A}$, where Δ is a $*$-derivation from $\mathcal{A}$ into itself.

2 $*$-Antiderivable mappings at a separating points

Before we give the main result in this section, we need to prove the following proposition.

Proposition 2.1. *Suppose that $\mathcal{A}$ is a unital Banach algebra, $\mathcal{M}$ is a unital Banach $\mathcal{A}$-bimodule and G in $\mathcal{A}$ is a separating point of $\mathcal{M}$ with $AG = GA$ for every A in $\mathcal{A}$. If δ and τ are two linear mappings from $\mathcal{A}$ into $\mathcal{M}$ such that*

$$AB = G \Rightarrow \delta(G) = B\delta(A) + \tau(B)A$$

for each A, B in $\mathcal{A}$, then τ is a Jordan derivation, δ is a generalized Jordan derivation, that is, for every A in $\mathcal{A}$, $\delta(A^2) = A\delta(A) + \delta(A)A - A\delta(I)A$. Moreover, the following identities hold:

$$\tau(AG) = \tau(G)A + G\delta(A) - AG\delta(I)$$

and

$$\delta(GA) = A\delta(G) + \tau(A)G$$

for every A in $\mathcal{A}$.

Proof. By $IG = GI = G$, we have that

$$\delta(G) = G\delta(I) + \tau(G) \tag{2.1}$$

and

$$\delta(G) = \delta(G) + \tau(I)G. \tag{2.2}$$

Since G is a separating point for $\mathcal{M}$, by (2.2), we have $\tau(I) = 0$. Let T be a invertible element in $\mathcal{A}$. By $GT^{-1}T = T^{-1}GT = G$, we obtain

$$\delta(G) = T\delta(GT^{-1}) + \tau(T)GT^{-1} \tag{2.3}$$

and

$$\delta(G) = T^{-1}G\delta(T) + \tau(T^{-1}G)T. \tag{2.4}$$

Multiplying by T^{-1} from the left-hand side of (2.3), we can obtain that

$$\delta(GT^{-1}) = T^{-1}\delta(G) - T^{-1}\tau(T)GT^{-1}. \tag{2.5}$$

Multiplying by T^{-1} from the right-hand side of (2.4), we have that

$$\tau(T^{-1}G) = \delta(G)T^{-1} - T^{-1}G\delta(T)T^{-1}. \tag{2.6}$$

Let A be in $\mathcal{A}$, n be a positive integer with $n > (\|A\| + 1)$ and $B = nI + A$. Then, both B and $I - B$ are invertible in $\mathcal{A}$. By replacing T with B in (2.3), by (2.5) and $\tau(I) = 0$, we obtain

$$\begin{aligned}\tau(B)GB^{-1} &= \delta(G) - B\delta(GB^{-1})\\ &= \delta(G) - B\delta(GB^{-1}(I - B) + G)\\ &= (I - B)\delta(G) - B\delta(GB^{-1}(I - B))\\ &= (I - B)\delta(G) - B[B^{-1}(I - B)\delta(G) - B^{-1}(I - B)\tau((I - B)^{-1}B)GB^{-1}(I - B)]\end{aligned}$$

$$
\begin{aligned}
&= (I - B)\tau((I - B)^{-1}B)GB^{-1}(I - B)\\
&= (I - B)\tau((I - B)^{-1} - I)GB^{-1}(I - B)\\
&= (I - B)\tau((I - B)^{-1})GB^{-1}(I - B).
\end{aligned}
$$

Since $AG = GA$ for every A in $\mathcal{A}$, it follows that

$$\tau(B)B^{-1}G = (I - B)\tau((I - B)^{-1})B^{-1}(I - B)G. \tag{2.7}$$

By replacing T with B in (2.4), we obtain

$$
\begin{aligned}
B^{-1}G\delta(B) &= \delta(G) - \tau(B^{-1}G)B\\
&= \delta(G) - \tau(B^{-1}(I - B)G + G)B\\
&= \delta(G) - \tau(G)B - \tau(B^{-1}(I - B)G)B.
\end{aligned}
$$

By (2.6), it implies that

$$
\begin{aligned}
B^{-1}G\delta(B) &= \delta(G) - \tau(G)B - \tau(B^{-1}(I - B)G)B\\
&= \delta(G) - \tau(G)B - [\delta(G)B^{-1}(I - B) - B^{-1}(I - B)G\delta((I - B)^{-1}B)B^{-1}(I - B)]B\\
&= \delta(G) - \tau(G)B - \delta(G)(I - B) + B^{-1}(I - B)G\delta((I - B)^{-1}B)(I - B)\\
&= (\delta(G) - \tau(G))B + B^{-1}(I - B)G\delta((I - B)^{-1} - I)(I - B),
\end{aligned}
$$

and by (2.1), it follows that

$$
\begin{aligned}
B^{-1}G\delta(B) &= (\delta(G) - \tau(G))B + B^{-1}(I - B)G\delta((I - B)^{-1} - I)(I - B)\\
&= G\delta(I)B - B^{-1}(I - B)G\delta(I)(I - B) + B^{-1}(I - B)G\delta((I - B)^{-1})(I - B).
\end{aligned}
$$

By $AG = GA$ for every A in $\mathcal{A}$, we can obtain that

$$GB^{-1}\delta(B) = G[\delta(I)B - B^{-1}(I - B)\delta(I)(I - B) + B^{-1}(I - B)\delta((I - B)^{-1})(I - B)]. \tag{2.8}$$

Since G is a separating point of $\mathcal{M}$, by (2.7) and (2.8), we obtain

$$\tau(B)B^{-1} = (I - B)\tau((I - B)^{-1})B^{-1}(I - B) \tag{2.9}$$

and

$$B^{-1}\delta(B) = \delta(I)B - B^{-1}(I - B)\delta(I)(I - B) + B^{-1}(I - B)\delta((I - B)^{-1})(I - B). \tag{2.10}$$

Multiplying by B from the right-hand side of (2.9) and from the left-hand side of (2.10), we can obtain that

$$\tau(B) = (I - B)\tau((I - B)^{-1})(I - B) \tag{2.11}$$

and

$$\delta(B) = B\delta(I)B - (I - B)\delta(I)(I - B) + (I - B)\delta((I - B)^{-1})(I - B). \tag{2.12}$$

Multiplying by G from the right-hand side of (2.11) and by $AG = GA$, it follows that

$$\tau(B)G = (I - B)\tau((I - B)^{-1})(I - B)G = (I - B)\tau((I - B)^{-1})G(I - B).$$

By (2.3),

$$
\begin{aligned}
\tau(B)G &= (I - B)[\delta(G) - (I - B)^{-1}\delta(G(I - B))]\\
&= (I - B)\delta(G) - \delta(G - GB)\\
&= \delta(GB) - B\delta(G).
\end{aligned} \tag{2.13}
$$

Multiplying by G from the left of (2.12) and by $AG = GA$, it follows that

$$
\begin{aligned}
G\delta(B) &= GB\delta(I)B - G(I - B)\delta(I)(I - B) + G(I - B)\delta((I - B)^{-1})(I - B)\\
&= GB\delta(I)B - G(I - B)\delta(I)(I - B) + (I - B)G\delta((I - B)^{-1})(I - B),
\end{aligned}
$$

and by (2.4),

$$\begin{aligned}G\delta(B) &= GB\delta(I)B - G(I-B)\delta(I)(I-B) + [\delta(G) - \tau((I-B)G)(I-B)^{-1}](I-B)\\ &= GB\delta(I)B - G(I-B)\delta(I) + G(I-B)\delta(I)B + \delta(G) - \delta(G)B - \tau(G) + \tau(BG)\\ &= GB\delta(I)B - G\delta(I) + GB\delta(I) + G\delta(I)B - GB\delta(I)B + \delta(G) - \delta(G)B - \tau(G) + \tau(BG)\\ &= GB\delta(I) + [\delta(G) - \tau(G) - G\delta(I)] + [G\delta(I) - \delta(G)]B + \tau(BG),\end{aligned}$$

and by (2.1), it implies that

$$G\delta(B) = GB\delta(I) - \tau(G)B + \tau(BG). \tag{2.14}$$

By (2.13), (2.14) and $AG = GA$, we have that

$$\delta(GB) = B\delta(G) + \tau(B)G$$

and

$$\tau(BG) = G\delta(B) + \tau(G)B - BG\delta(I).$$

Since $B = nI + A$, we have the following two equations:

$$\delta(GA) = A\delta(G) + \tau(A)G \tag{2.15}$$

and

$$\tau(AG) = \tau(G)A + G\delta(A) - AG\delta(I). \tag{2.16}$$

By (2.15), we know that for every invertible element T in $\mathcal{A}$, it follows that

$$\begin{aligned}\delta(G) &= \delta(GTT^{-1})\\ &= T^{-1}\delta(GT) + \tau(T^{-1})GT\\ &= T^{-1}[T\delta(G) + \tau(T)G] + \tau(T^{-1})TG\\ &= \delta(G) + T^{-1}\tau(T)G + \tau(T^{-1})TG.\end{aligned}$$

Since G is a separating point,

$$T^{-1}\tau(T) + \tau(T^{-1})T = 0. \tag{2.17}$$

By (2.16), we know that for every invertible element T in $\mathcal{A}$, it follows that

$$\begin{aligned}\delta(G) &= \delta(T^{-1}TG)\\ &= TG\delta(T^{-1}) + \tau(TG)T^{-1}\\ &= TG\delta(T^{-1}) + [\tau(G)T + G\delta(T) - TG\delta(I)]T^{-1}\\ &= TG\delta(T^{-1}) + \tau(G) + G\delta(T)T^{-1} - TG\delta(I)T^{-1}\\ &= GT\delta(T^{-1}) + \tau(G) + G\delta(T)T^{-1} - GT\delta(I)T^{-1}.\end{aligned}$$

Thus,

$$\delta(G) - \tau(G) = GT\delta(T^{-1}) + G\delta(T)T^{-1} - GT\delta(I)T^{-1}.$$

By (2.1), we have that

$$G\delta(I) = GT\delta(T^{-1}) + G\delta(T)T^{-1} - GT\delta(I)T^{-1}.$$

Since G is a separating point, we know that

$$\delta(I) = T\delta(T^{-1}) + \delta(T)T^{-1} - T\delta(I)T^{-1}. \tag{2.18}$$

It follows from (2.17), (2.18) and [13, Lemma 2.1] that τ and $\Delta(A) := \delta(A) - A\delta(I)$ both are Jordan derivations, and hence, δ is a generalized Jordan derivation. □

Let $G = I$ in Proposition 2.1, we have the following result.

Corollary 2.2. *Suppose $\mathcal{A}$ is a unital Banach algebra and $\mathcal{M}$ is a unital Banach $\mathcal{A}$-bimodule. If δ and τ are two linear mappings from $\mathcal{A}$ into $\mathcal{M}$, such that*

$$AB = I \Rightarrow \delta(I) = B\delta(A) + \tau(B)A$$

for each A, B in $\mathcal{A}$, then τ is a Jordan derivation and δ is a generalized Jordan derivation. Moreover, for every A in $\mathcal{A}$, we have that

$$\delta(A) = A\delta(I) + \tau(A).$$

For every $*$-antiderivable mapping at unit element from a unital Banach $*$-algebra into its unital Banach $*$-$\mathcal{A}$-bimodule, we have the following result.

Corollary 2.3. *Suppose that $\mathcal{A}$ is a unital Banach $*$-algebra and $\mathcal{M}$ is a unital Banach $*$-$\mathcal{A}$-bimodule. If δ is a linear mapping from $\mathcal{A}$ into $\mathcal{M}$ such that*

$$AB^* = I \Rightarrow \delta(I) = B^*\delta(A) + \delta(B)^*A$$

for each A, B in $\mathcal{A}$, then δ is a $*$-Jordan derivation.

Proof. Let τ be the linear mapping from $\mathcal{A}$ into $\mathcal{M}$ such that for every A in $\mathcal{A}$,

$$\delta^\sharp(A) = \delta(A^*)^*.$$

It follows that for each A, B in $\mathcal{A}$, we have that

$$AB = I = A(B^*)^* = I \Rightarrow \delta(I) = B\delta(A) + \delta(B^*)^*A \Rightarrow \delta(I) = B\delta(A) + \delta^\sharp(B)A.$$

It follows from Proposition 2.1 that $\delta^\sharp$ is a Jordan derivation, and hence, δ is also a Jordan derivation.

Finally, we prove that δ is a $*$-Jordan derivation, that is, $\delta(A^*) = \delta(A)^*$ for every A in $\mathcal{A}$. In fact, by $\delta(I) = 0$ and Corollary 2.2, we have that $\delta(A) = \delta^\sharp(A) = \delta(A^*)^*$. It implies that $\delta(A)^* = \delta(A^*)$ for every A in $\mathcal{A}$. □

For every $*$-antiderivable mapping from a unital C^*-algebra into its Banach $*$-$\mathcal{A}$-bimodule, we have the following theorem.

Theorem 2.4. *Suppose that $\mathcal{A}$ is a unital C^*-algebra, $\mathcal{M}$ is a unital Banach $*$-$\mathcal{A}$-bimodule and G in $\mathcal{A}$ is a separating point of $\mathcal{M}$ with $AG = GA$ for every A in $\mathcal{A}$. If δ is a linear mapping from $\mathcal{A}$ into $\mathcal{M}$ such that*

$$AB^* = G \Rightarrow \delta(G) = B^*\delta(A) + \delta(B)^*A$$

for each A, B in $\mathcal{A}$, then δ is a $*$-derivation.

Proof. Let τ be a linear mapping from $\mathcal{A}$ into $\mathcal{M}$ such that for every A in $\mathcal{A}$

$$\tau(A) = \delta(A^*)^*.$$

It follows that for each A, B in $\mathcal{A}$, we have that

$$AB = G = A(B^*)^* = G \Rightarrow \delta(G) = B\delta(A) + \delta(B^*)^*A \Rightarrow \delta(G) = B\delta(A) + \tau(B)A.$$

By Proposition 2.1, τ is a Jordan derivation, and hence, δ is also a Jordan derivation. Since $\mathcal{A}$ is a C^*-algebra, δ is a derivation.

Finally, we show that δ is a $*$-derivation, that is, $\delta(A^*) = \delta(A)^*$ for every A in $\mathcal{A}$. Let A be an invertible element in $\mathcal{A}$, by $GA((A^{-1})^*)^* = G$, we have that

$$\delta(G) = A^{-1}\delta(GA) + \delta((A^{-1})^*)^*GA.$$

Since δ is a derivation and $AG = GA$, it follows that

$$\delta(G) = \delta((A^{-1})^*)^*AG + A^{-1}(A\delta(G) + \delta(A)G),$$

that is,

$$\delta((A^{-1})^*)^*AG + A^{-1}\delta(A)G = 0.$$

Since G is a separating point, we have that

$$\delta((A^{-1})^*)^*A + A^{-1}\delta(A) = 0. \tag{2.19}$$

On the other hand, we can obtain that

$$\delta(A^{-1})A + A^{-1}\delta(A) = \delta(I) = 0. \tag{2.20}$$

By (2.19) and (2.20), we know that $\delta((A^{-1})^*)^*A = \delta(A^{-1})A$, that is, $\delta((A^{-1})^*) = \delta(A^{-1})^*$. Thus, for every invertible element $A \in \mathcal{A}$, we have showed that $\delta(A)^* = \delta(A^*)$.

Since every element in a unital C^*-algebra is a linear combination of four unitaries [20], it follows that $\delta(A)^* = \delta(A^*)$ for every $A \in \mathcal{A}$. □

In particular, let $G = I$ in Theorem 2.4, the following corollary holds.

Corollary 2.5. *Suppose $\mathcal{A}$ is a unital C^*-algebra and $\mathcal{M}$ is a unital Banach $*$-$\mathcal{A}$-bimodule. If δ is a linear mapping from $\mathcal{A}$ into $\mathcal{M}$ such that*

$$AB^* = I \Rightarrow \delta(I) = B^*\delta(A) + \delta(B)^*A$$

for each A, B in $\mathcal{A}$, then δ is a $*$-derivation.

Remark 2.6. Suppose that $\mathcal{A}$ is a unital $*$-algebra, $\mathcal{M}$ is a unital Banach $*$-$\mathcal{A}$-bimodule and δ is a linear mapping from $\mathcal{A}$ into $\mathcal{M}$. We should notice that the following two conditions are not equivalent:
(1) $A, B \in \mathcal{A}, AB^* = G \Rightarrow B^*\delta(A) + \delta(B)^*A = \delta(G)$;
(2) $A, B \in \mathcal{A}, A^*B = G \Rightarrow B\delta(A)^* + \delta(B)A^* = \delta(G)$.

Hence, we also can define a $*$-derivable mapping at G in $\mathcal{A}$ from $\mathcal{A}$ into $\mathcal{M}$ by

$$A, B \in \mathcal{A}, A^*B = G \Rightarrow B\delta(A)^* + \delta(B)A^* = \delta(G).$$

Through the minor modifications, we can obtain the corresponding results.

3 $*$-Antiderivable mappings at the point zero

A (Banach) algebra $\mathcal{A}$ is said to be *zero product determined* if every (continuous) bilinear mapping ϕ from $\mathcal{A} \times \mathcal{A}$ into any (Banach) linear space $\mathcal{X}$ satisfying

$$\phi(A, B) = 0, \text{ whenever } AB = 0$$

can be written as $\phi(A, B) = T(AB)$, for some (continuous) linear mapping T from $\mathcal{A}$ into $\mathcal{X}$. In [21], Brešar showed that if $\mathcal{A} = \mathfrak{J}(\mathcal{A})$, then $\mathcal{A}$ is a zero product determined, where $\mathfrak{J}(\mathcal{A})$ is the subalgebra of $\mathcal{A}$ generated by all idempotents in $\mathcal{A}$, and in [2], the authors proved that every C^*-algebra $\mathcal{A}$ is zero product determined.

Suppose that $\mathcal{A}$ is a Banach algebra and $\mathcal{M}$ is a Banach-$\mathcal{A}$-bimodule. $\mathcal{M}$ is called an *essential Banach $\mathcal{A}$-bimodule* if

$$\mathcal{M} = \overline{\text{span}}\{ANB : A, B \in \mathcal{A}, N \in \mathcal{M}\},$$

where $\overline{\text{span}}\{\cdot\}$ denotes the norm closure of the linear span of the set $\{\cdot\}$.

Let $\mathcal{A}$ be a Banach $*$-algebra, a *bounded approximate identity* for $\mathcal{A}$ is a net $(e_i)_{i\in\Gamma}$ of self-adjoint elements in $\mathcal{A}$ such that $\lim_i\|Ae_i - A\| = \lim_i\|e_iA - A\| = 0$ for every A in $\mathcal{A}$ and $\sup_{i\in\Gamma}\|e_i\| \leq K$ for some $K > 0$.

Theorem 3.1. *Suppose $\mathcal{A}$ is a zero product determined Banach $*$-algebra with a bounded approximate identity and $\mathcal{M}$ is an essential Banach $*$-$\mathcal{A}$-bimodule. If δ is a continuous linear mapping from $\mathcal{A}$ into $\mathcal{M}$ such that*

$$AB^* = 0 \Rightarrow B^*\delta(A) + \delta(B)^*A = 0$$

for each A, B in $\mathcal{A}$, then there are a $*$-Jordan derivation Δ from $\mathcal{A}$ into $\mathcal{M}^{\sharp\sharp}$ and an element ξ in $\mathcal{M}^{\sharp\sharp}$, such that

$$\delta(A) = \Delta(A) + A\xi$$

for every A in $\mathcal{A}$. Furthermore, ξ can be chosen in $\mathcal{M}$ in each of the following cases:
(1) *$\mathcal{A}$ has an identity.*
(2) *$\mathcal{M}$ is a dual $*$-$\mathcal{A}$-bimodule.*

In [17, Section 4] and in [22, p. 720], the authors showed that $\mathcal{M}^{\sharp\sharp}$ is also a Banach $*$-$\mathcal{A}$-bimodule, where $\mathcal{M}^{\sharp\sharp}$ is the second dual space of $\mathcal{M}$. But, for the sake of completeness, we recall the argument here.

In fact, since $\mathcal{M}$ is a Banach $*$-$\mathcal{A}$-bimodule, $\mathcal{M}^{\sharp\sharp}$ turns into a dual Banach $\mathcal{A}$-bimodule with the operation defined by

$$A \cdot M^{\sharp\sharp} = \lim_{\mu} AM_\mu \text{ and } M^{\sharp\sharp} \cdot A = \lim_{\mu} M_\mu A$$

for every A in $\mathcal{A}$ and every $M^{\sharp\sharp}$ in $\mathcal{M}^{\sharp\sharp}$, where (M_μ) is a net in $\mathcal{M}$ with $\|M_\mu\| \leqslant \|M^{\sharp\sharp}\|$ and $(M_\mu) \to M^{\sharp\sharp}$ in the weak*-topology $\sigma(\mathcal{M}^{\sharp\sharp}, \mathcal{M}^{\sharp})$.

We define an involution $*$ in $\mathcal{M}^{\sharp\sharp}$ by

$$(M^{\sharp\sharp})^*(\rho) = \overline{M^{\sharp\sharp}(\rho^*)}, \rho^*(M) = \overline{\rho(M^*)},$$

where $M^{\sharp\sharp}$ in $\mathcal{M}^{\sharp\sharp}$, ρ in $\mathcal{M}^{\sharp}$ and M in $\mathcal{M}$. Moreover, if (M_μ) is a net in $\mathcal{M}$ and $M^{\sharp\sharp}$ is an element in $\mathcal{M}^{\sharp\sharp}$ such that $M_\mu \to M^{\sharp\sharp}$ in $\sigma(\mathcal{M}^{\sharp\sharp}, \mathcal{M}^{\sharp})$, then for every ρ in $\mathcal{M}^{\sharp}$, we have that

$$\rho(M_\mu) = M_\mu(\rho) \to M^{\sharp\sharp}(\rho).$$

It follows that

$$(M_\mu^*)(\rho) = \rho(M_\mu^*) = \overline{\rho^*(M_\mu)} \to \overline{M^{\sharp\sharp}(\rho^*)} = (M^{\sharp\sharp})^*(\rho)$$

for every ρ in $\mathcal{M}^{\sharp}$. It means that the involution $*$ in $\mathcal{M}^{\sharp\sharp}$ is continuous in $\sigma(\mathcal{M}^{\sharp\sharp}, \mathcal{M}^{\sharp})$. Thus, we can obtain that

$$(A \cdot M^{\sharp\sharp})^* = (\lim_{\mu} AM_\mu)^* = \lim_{\mu} M_\mu^* A^* = (M^{\sharp\sharp})^* \cdot A^*.$$

Similarly, we can show that $(M^{\sharp\sharp} \cdot A)^* = A^* \cdot (M^{\sharp\sharp})^*$. It implies that $\mathcal{M}^{\sharp\sharp}$ is a Banach $*$-$\mathcal{A}$-bimodule.

In the following, we prove that Theorem 3.1.

Proof. Let $(e_i)_{i\in\Gamma}$ be a bounded approximate identity of $\mathcal{A}$. Since δ is a continuous mapping, $(\delta(e_i))_{i\in\Gamma}$ is bounded in $\mathcal{M}$. Moreover, $(\delta(e_i))_{i\in\Gamma}$ is also bounded in $\mathcal{M}^{\sharp\sharp}$. By the Alaoglu-Bourbaki theorem, we may assume that $(\delta(e_i))_{i\in\Gamma}$ converges to the element ξ in $\mathcal{M}^{\sharp\sharp}$ with the weak*-topology $\sigma(\mathcal{M}^{\sharp\sharp}, \mathcal{M}^{\sharp})$.

Since $\mathcal{M}$ is an essential Banach-$\mathcal{A}$-bimodule, Me_i converges to M with respect to the weak*-topology $\sigma(\mathcal{M}^{\sharp\sharp}, \mathcal{M}^{\sharp})$ for every M in $\mathcal{M}$. In fact, since $\mathcal{M} = \overline{\text{span}}\{ANB : A, B \in \mathcal{A}, N \in \mathcal{M}\}$, there exists a sequence $M_n = \Sigma_{k=1}^{m_n} A_k^n N_k^n B_k^n \in \mathcal{M}$ converging to M in the norm topology, where $A_k^n, B_k^n \in \mathcal{A}$ and $N_k^n \in \mathcal{M}$, $k = 1, 2, \ldots, m_n$, $n = 1, 2, \ldots$. Since $(ANBe_i)$ converges to ANB in the norm topology for each $A, B \in \mathcal{A}$ and $N \in \mathcal{M}$, it follows that Me_i converges to $\mathcal{M}$ in the norm topology for every M in $\mathcal{M}$.

Define a continuous bilinear mapping from $\mathcal{A} \times \mathcal{A}$ into $\mathcal{M}$ by

$$\phi(A, B) = \delta(B^*)^*A + B\delta(A)$$

for every A, B in $\mathcal{A}$. It follows that

$$AB = 0 \Rightarrow \phi(A, B) = 0.$$

Since $\mathcal{A}$ is a zero product determined Banach algebra, there exists a continuous linear mapping $T : \mathcal{A} \to \mathcal{M}$ such that $\phi(A, B) = T(AB)$ for every $A, B \in \mathcal{A}$. Moreover, for every $A, B, C \in \mathcal{A}$, we have that

$$\phi(AB, C) = \phi(A, BC).$$

That is,

$$\delta(C^*)^*AB + C\delta(AB) = \delta(C^*B^*)^*A + BC\delta(A). \tag{3.1}$$

Let $A = e_i$ in (3.1) and take the limit on both sides with the weak*-topology $\sigma(\mathcal{M}^{\sharp\sharp}, \mathcal{M}^{\sharp})$, we can obtain that

$$\delta(C^*)^*B + C\delta(B) = \delta(C^*B^*)^* + BC\xi. \tag{3.2}$$

Take the involution on both sides in (3.2), it implies that

$$B^*\delta(C^*) + \delta(B)^*C^* = \delta(C^*B^*) + \xi^*C^*B^*. \tag{3.3}$$

Let $C = e_i$ in (3.3) and take the limit on both sides of (3.3) with the weak*-topology $\sigma(\mathcal{M}^{\sharp\sharp}, \mathcal{M}^{\sharp})$, we can obtain that

$$B^*\xi + \delta(B)^* = \delta(B^*) + \xi^*B^*,$$

that is,

$$\delta(B^*) - B^*\xi = \delta(B)^* - \xi^*B^*. \tag{3.4}$$

Define a linear mapping Δ from $\mathcal{A}$ into $\mathcal{M}^{\sharp\sharp}$ by

$$\Delta(A) = \delta(A) - A\xi$$

for every A in $\mathcal{A}$. Next, we prove that Δ is a $*$-Jordan derivation. By (3.4), we have that $\Delta(A^*) = \Delta(A)^*$ for every $A \in \mathcal{A}$.

By replacing C^*, B^* with A, B in (3.3), respectively, we can obtain that

$$B\delta(A) + \delta(B^*)^*A = \delta(AB) + \xi^*AB,$$

that is,

$$\delta(AB) = B\delta(A) + \delta(B^*)^*A - \xi^*AB. \tag{3.5}$$

In the following, we prove that

$$\Delta(A^2) = A\Delta(A) + \Delta(A)A$$

for every A in $\mathcal{A}$. By the definition of Δ and (3.5), we have the following two equations:

$$\Delta(A^2) = \delta(A^2) - A^2\xi = A\delta(A) + \delta(A^*)^*A - \xi^*A^2 - A^2\xi \tag{3.6}$$

and

$$A\Delta(A) + \Delta(A)A = A(\delta(A) - A\xi) + (\delta(A) - A\xi)A = A\delta(A) - A^2\xi + \delta(A)A - A\xi A. \tag{3.7}$$

By $\Delta(A^*) = \Delta(A)^*$, it implies that $\delta(A^*) - A^*\xi = (\delta(A) - A\xi)^*$, and

$$\delta(A^*)^* - \xi^*A = \delta(A) - A\xi. \tag{3.8}$$

Multiplying by A from the right side of (3.8), we have that

$$(\delta(A^*)^* - \xi^*A)A = (\delta(A) - A\xi)A. \tag{3.9}$$

Finally, by (3.6), (3.7), and (3.9), it follows that $\Delta(A^2) = A\Delta(A) + \Delta(A)A$. Thus, Δ is a $*$-Jordan derivation.

Suppose that $\mathcal{A}$ is a unital Banach algebra, we can assume that $\xi = \delta(I)$.

Suppose that $\mathcal{M}$ is a dual essential Banach $*$-$\mathcal{A}$-bimodule and $\mathcal{M}_\sharp$ is the pre-dual space of $\mathcal{M}$, since δ is continuous, we can assume that the net $(\delta(e_i))_{i\in\Gamma}$ converges to element $\xi \in \mathcal{M}$ with the weak*-topology $\sigma(\mathcal{M}^{\sharp\sharp}, \mathcal{M}^{\sharp})$. □

Let G be a locally compact group. The group algebra and the measure convolution algebra of G, are denoted by $L^1(G)$ and $M(G)$, respectively. The convolution product is denoted by $\cdot$, and the involution is denoted by $*$. It is well known that $M(G)$ is a unital Banach $*$-algebra, and $L^1(G)$ is a closed ideal in $M(G)$ with a bounded approximate identity. By [23, Lemma 1.1], we know that $L^1(G)$ is zero product determined. By [24, Theorem 3.3.15(ii)], it follows that $M(G)$ with respect to convolution product is the dual of $C_0(G)$ as a Banach $M(G)$-bimodule.

Since $L^1(\mathcal{G})$ is a semisimple algebra, we know from [25] that every continuous Jordan derivation from $L^1(\mathcal{G})$ into itself is a derivation. By [26, Corollary 1.2], we know that every continuous derivation Δ from $L^1(G)$ into $M(G)$ is an inner derivation, that is, there exists μ in $M(G)$ such that $\Delta(f) = f \cdot \mu - \mu \cdot f$ for every f in $L^1(G)$. Thus, by Theorem 3.1, we can rediscover [6, Theorem 3.2(ii)] as follows:

Corollary 3.2. [6, Theorem 3.2(ii)] *Let $\mathcal{G}$ be a locally compact group. If δ is a continuous linear mapping from $L^1(\mathcal{G})$ into $M(\mathcal{G})$ such that*

$$f \cdot g^* = 0 \Rightarrow \delta(g)^* \cdot f + g^* \cdot \delta(f) = 0,$$

for each f, g in $L^1(\mathcal{G})$, then there exist two-element $\mu, \nu \in M(\mathcal{G})$ such that

$$\delta(f) = f \cdot \nu - \mu \cdot f$$

for every f in $L^1(\mathcal{G})$ and $\mathrm{Re}\mu \in \mathcal{Z}(M(\mathcal{G}))$.

Proof. By Theorem 3.1, we know that there exist a $*$-derivation Δ from $L^1(G)$ into $M(G)$ and an element ξ in $M(G)$ such that

$$\delta(f) = \Delta(f) + \xi \cdot f$$

for every f in $L^1(G)$. By [26, Corollary 1.2], it follows that there exists μ in $M(G)$ such that $\Delta(f) = f \cdot \mu - \mu \cdot f$. Since $\Delta(f^*) = \Delta(f)^*$, we have that

$$f^* \cdot \mu - \mu \cdot f^* = \mu^* \cdot f^* - f^* \cdot \mu^*$$

for every f in $L^1(G)$. By [23, Lemma 1.3(ii)], we know $\mathrm{Re}\mu = \frac{1}{2}(\mu + \mu^*) \in \mathcal{Z}(M(\mathcal{G}))$. Let $\nu = \mu - \xi$, from the definition of Δ, we have that $\delta(f) = f \cdot \mu - \nu \cdot f$ for every f in $L^1(G)$. ☐

In [2], the authors proved that every C^*-algebra $\mathcal{A}$ is zero product determined, and by [27, Corollary 7.5], we know that $\mathcal{A}$ has a bounded approximate identity. Thus, by Theorem 3.1, we can obtain a new proof of [17, Theorem 9] as follows:

Corollary 3.3. [17, Theorem 9] *Let $\mathcal{A}$ be a C^*-algebra and $\mathcal{M}$ an essential Banach $*$-$\mathcal{A}$-bimodule. If δ is a continuous linear mapping from $\mathcal{A}$ into $\mathcal{M}$ such that*

$$AB^* = 0 \Rightarrow B^*\delta(A) + \delta(B)^*A = 0$$

for every A, B in $\mathcal{A}$, then there exists a $*$-derivation Δ from $\mathcal{A}$ into $\mathcal{M}^{\sharp\sharp}$ and ξ in $\mathcal{M}^{\sharp\sharp}$ such that

$$\delta(A) = \Delta(A) + A\xi$$

for every A in $\mathcal{A}$. Furthermore, ξ can be chosen in $\mathcal{M}$ in each of the following cases:
(1) *$\mathcal{A}$ has an identity.*
(2) *$\mathcal{M}$ is a dual $*$-$\mathcal{A}$-bimodule.*

Suppose that $\mathcal{A}$ is a zero product determined unital $*$-algebra and δ is a $*$-antiderivable mapping from $\mathcal{A}$ into a $*$-$\mathcal{A}$-bimodule. Let $(e_i)_{i\in\Gamma} = I$ and $\xi = \delta(I)$ in Theorem 3.1, we can obtain the following conclusion.

Corollary 3.4. *Let $\mathcal{A}$ be a zero product determined unital $*$-algebra and $\mathcal{M}$ be a $*$-$\mathcal{A}$-bimodule. If δ is a linear mapping* (*continuity is not necessary*) *from $\mathcal{A}$ into $\mathcal{M}$ such that*

$$AB^* = 0 \Rightarrow B^*\delta(A) + \delta(B)^*A = 0$$

for each A, B *in* $\mathcal{A}$, *then there exists a* $*$*-Jordan derivation* Δ *from* $\mathcal{A}$ *into* $\mathcal{M}$ *such that*

$$\delta(A) = \Delta(A) + A\delta(I)$$

for every A *in* $\mathcal{A}$.

Finally, we investigate $*$-antiderivable mappings at the zero point on a von Neumann algebra. The following result is the second main theorem in this section.

Theorem 3.5. *Let* $\mathcal{A}$ *be a von Neumann algebra. If* δ *is a linear mapping from* $\mathcal{A}$ *into itself, such that*

$$AB^* = 0 \Rightarrow B^*\delta(A) + \delta(B)^*A = 0$$

for each A, B *in* $\mathcal{A}$, *then there exists a* $*$*-derivation* Δ *from* $\mathcal{A}$ *into* $\mathcal{M}$ *such that*

$$\delta(A) = \Delta(A) + A\delta(I)$$

for every A *in* $\mathcal{A}$. *In particular,* δ *is a* $*$*-derivation when* $\delta(I) = 0$.

Proof. Suppose that $\mathcal{B}$ is a commutative von Neumann subalgebra of $\mathcal{A}$. For each A, B in $\mathcal{B}$, we have that

$$AB = 0 \Leftrightarrow AB^* = 0 \Leftrightarrow A^*B = 0 \Leftrightarrow A^*B^* = 0.$$

Let A, B, C be in $\mathcal{B}$ satisfying $AB = BC = 0$. Since $A^*B^* = 0$, we obtain $B^*\delta(A^*) + \delta(B)^*A^* = 0$. By multiplying the previous identity by C^* form the left-hand side, we have $C^*\delta(B)^*A^* = 0$, equivalently, $A\delta(B)C = 0$. Therefore, [28, Theorem 2.12] implies that δ is automatically continuous, and by [17, Theorem 9], we can prove this theorem. □

Remark 3.6. Let $\mathcal{A}$ be a unital $*$-algebra and $\mathcal{M}$ be a unital Banach $*$-$\mathcal{A}$-bimodule. δ is a linear mapping from $\mathcal{A}$ into $\mathcal{M}$ such that

$$A^*B = 0 \Rightarrow B\delta(A^*) + \delta(B)A^* = 0.$$

Through the minor modifications of Theorems 3.1 and 3.5, we can obtain the corresponding results.

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