Open Mathematics

Research Article

Fengde Chen*, Xinyu Guan, Xiaoyan Huang, and Hang Deng

Dynamic behaviors of a Lotka-Volterra type predator-prey system with Allee effect on the predator species and density dependent birth rate on the prey species

https://doi.org/10.1515/math-2019-0082
Received May 10, 2019; accepted June 8, 2019

Abstract: A Lotka-Volterra type predator-prey system with Allee effect on the predator species and density dependent birth rate on the prey species is proposed and studied. For non-delay case, such topics as the persistent of the system, the local stability property of the equilibria, the global stability of the positive equilibrium are investigated. For the system with infinite delay, by using the iterative method, a set of sufficient conditions which ensure the global attractivity of the positive equilibrium is obtained. By introducing the density dependent birth rate, the dynamic behaviors of the system becomes complicated. The system maybe collapse in the sense that both the species will be driven to extinction, or the two species could be coexist in a stable state. Numeric simulations are carried out to show the feasibility of the main results.

Keywords: predator, prey, Allee effect, global stability, density dependent birth rate

MSC: 34C25, 92D25, 34D20

1 Introduction

As was pointed out by Berryman [1], the dynamic relationship between predators and their prey has long been and will continue to be one of the dominant themes in both ecology and mathematical ecology due to its universal existence and importance. Already, the influence of the Allee effect [2-6], the influence of the mutual interferences [7-8], the influence of the stage structure [9-13], the stability of the positive equilibrium [12-17], the existence and stability of the almost periodic solution [18], the existence of the positive periodic solution [19, 20], the persistent of the system [21] have been extensively studied, and many excellent results were obtained.

Allee effect, which reflects the fact that the population growth rate is reduced at low population size, due to its importance, the ecosystem subject to Allee effect has recently been extensively studied by many scholars, see [2-6, 22-26] and the references cited therein.

***Corresponding Author: Fengde Chen:** College of Mathematics and Computer Science, Fuzhou University, Fuzhou, Fujian 350108, China, E-mail: fdchen@fzu.edu.cn
Xinyu Guan: College of Mathematics and Computer Science, Fuzhou University, Fuzhou, Fujian 350108, China, E-mail: n160320014@fzu.edu.cn
Xiaoyan Huang: College of Mathematics and Computer Science, Fuzhou University, Fuzhou, Fujian 350108, China, E-mail: 1224319298@qq.com
Hang Deng: College of Mathematics and Computer Science, Fuzhou University, Fuzhou, Fujian 350108, China, E-mail: 2451871575@qq.com

Hüseyin Merdan [2] investigated the influence of the Allee effect on the Lotka-Volterra type predator-prey system. To do so, the author proposed the following predator-prey with Allee effect system

$$\frac{dx}{dt} = \frac{rx}{\beta + x}x(1-x) - axy, \quad \frac{dy}{dt} = ay(x-y). \tag{1.1}$$

Hüseyin Merdan showed that if $r{-}a\beta > 0$ hold, the model (1.1) has three steady-state solutions: $A(0,0)$, $B(1,0)$ and $C(x^*, y^*)$. the first two are locally unstable, while the third one is locally asymptotically stable. By carrying out a series of numeric simulations, the author found the following two phenomenon. (1) The system subject to an Allee effect takes a longer time to reach its steady-state solution; (2) The Allee effect reduces the population densities of both predator and prey at the steady-state.

In [17], Guan, Liu and Xie argued that "It seems interesting to consider the influence of the Allee effect on the predator species, since generally speaking, the higher the hierarchy in the food chain, the more likely it is to become extinct" and they proposed the following model with the Allee effect on the predator species:

$$\frac{dx}{dt} = rx(1-x) - axy, \quad \frac{dy}{dt} = \frac{ay}{\beta + y}y(x-y), \tag{1.2}$$

where r, a are positive constants. They showed that if $r > a$ holds, then system (1.2) admits a unique positive equilibrium, and the Allee effect has no influence on the final density of the species.

It bring to our attention that in system (1.1) and (1.2), without consider the influence of the predator species and the Allee effect, the prey species satisfies the traditional Logistic equation

$$\frac{dx}{dt} = rx(1-x), \tag{1.3}$$

where r is the intrinsic growth rate, which is equal to the birth rate minus death rate. Hence system (1.3) could be revised as

$$\frac{dx}{dt} = x\Big(a_1 - d_1 - e_1 x\Big). \tag{1.4}$$

where a_1 is the birth rate of the species and d_1 is the death rate of the species. Already, Brauer and Castillo-Chavez [26], Tang and Chen [27] and Berezansky, Braverman, et al. [28] had showed that in some case, the density dependent birth rate of the species is more suitable. If we take the famous Beverton-Holt function [28] as the birth rate, then system (1.4) should be revised to

$$\frac{dx}{dt} = x\Big(\frac{a_1}{b_1 + c_1 x} - d_1 - e_1 x\Big). \tag{1.5}$$

System (1.5) combines with the idea of Merdan [2] and Guan et al. [17], will lead to the following Lotka-Volterra type predator-prey system with Allee effect on the predator species and density dependent birth rate on the prey species

$$\begin{aligned} \frac{dx}{dt} &= x\Big(\frac{a_1}{b_1 + c_1 x} - d_1 - e_1 x\Big) - axy, \\ \frac{dy}{dt} &= \frac{ay}{\beta + y}y\Big(x - y\Big). \end{aligned} \tag{1.6}$$

It is well known that in a more realistic model the delay effect should be an average over past populations. This results in an equation with a distributed delay or an infinite delay [29-41]. Here, if we incorporate the infinite delay to system (1.6), then we will have the following system

$$\begin{aligned} \frac{dx}{dt} &= x\Big(\frac{a_1}{b_1 + c_1 x} - d_1 - e_1 x\Big) - ax\int_{-\infty}^{t} K_1(t-s)y(s)ds, \\ \frac{dy}{dt} &= \frac{ay}{\beta + y}y\Big(\int_{-\infty}^{t} K_2(t-s)x(s)ds - y\Big). \end{aligned} \tag{1.7}$$

The delay kernels $K_i : [0, +\infty) \to (0, +\infty)$, $i = 1, 2$ are continuous functions such that

$$\int_{0}^{+\infty} K_i(s)ds = 1. \tag{1.8}$$

We shall consider (1.7) together with the initial conditions

$$x(s) = \phi(s),\ \ s \in (-\infty, 0],\ \ y(s) = \psi(s),\ \ s \in (-\infty, 0], \tag{1.9}$$

where $\phi, \psi \in BC^+$. It is well known that by the fundamental theory of functional differential equations [37], system (1.7) has a unique solution $(x(t), y(t))$ satisfying the initial condition (1.9). We easily prove $x(t) > 0, y(t) > 0$ in maximal interval of existence of the solution. In this paper, the solution of system (1.7) satisfying the initial conditions (1.9) is said to be positive.

We mention here that to this day, though there are many scholars investigated the dynamic behaviors of the ecosystem with Allee effect [1-6, 22-26], none of them considered the density dependent birth rate of the species. Also, to the best of the authors knowledge, to this day, still no scholars propose a ecosystem with infinite delay and Allee effect at the same time. It seems that this is the first time such kind of model are proposed and studied.

The paper is arranged as follows. In section 2 we investigate the persistent and extinct property of the system, based on this, we are able to investigate the locally stability property of the equilibrium solutions of system (1.6). In section 3, by applying the Dulac criterion, we are able to show that under some assumption, the positive equilibrium is globally asymptotically stable. Section 4 presents some numerical simulations concerning the stability of our model. We end this paper by a briefly discussion.

2 Persistence and local stability of the equilibria

We need several Lemmas to prove the persistent property of the system.

Lemma 2.1 [40] *Consider the following equation*

$$\frac{dy}{dt} = y\Big(\frac{a}{b+cy} - d - ey\Big). \tag{2.1}$$

Assume that $a > bd$, then the unique positive equilibrium y^* of system (2.1) is globally asymptotically stable, where

$$y^* = \frac{-(eb+dc) + \sqrt{(eb+dc)^2 - 4ec(db-a)}}{2ec}.$$

Lemma 2.2 [22] *Consider the following equation*

$$\frac{dy}{dt} = \frac{ay}{\beta + y}y(b-y). \tag{2.2}$$

The unique positive equilibrium $y^* = b$ is global stability.

Theorem 2.1. *Assume that*

$$\frac{a_1}{b_1} > d_1 + au^* \tag{2.3}$$

holds, where u^* is defined by (2.6), then system (1.6) is permanent.

Proof. It follows from (2.3) that there exists a $\varepsilon > 0$ enough small such that

$$\frac{a_1}{b_1} > d_1 + a(u^* + \varepsilon). \tag{2.4}$$

Let $(x(t), y(t))$ be any positive solution of system (1.6). From system (1.6) it follows that

$$\frac{dx}{dt} \le x\Big(\frac{a_1}{b_1 + c_1 x} - d_1 - e_1 x\Big).$$

Consider the equation

$$\frac{du_1}{dt} = u_1\Big(\frac{a_1}{b_1 + c_1 u_1} - d_1 - e_1 u_1\Big). \tag{2.5}$$

It follows from Lemma 2.1 that (2.5) admits a unique globally stable positive equilibrium u^*, where

$$u^* = \frac{-(e_1 b_1 + d_1 c_1) + \sqrt{(e_1 b_1 + d_1 c_1)^2 - 4 e_1 c_1 (d_1 b_1 - a_1)}}{2 e_1 c_1}. \tag{2.6}$$

By using the differential inequality theory, any solution of (2.5) satisfies

$$\limsup_{t \to +\infty} x(t) \le \lim_{t \to +\infty} u(t) = u^*. \tag{2.7}$$

Hence, there exists a $T_1 > 0$ such that

$$x(t) < u^* + \tfrac{\varepsilon}{2}. \tag{2.8}$$

For $t > T_1$, it follows from the second equation of system (1.6) that

$$\frac{dy}{dt} \le \frac{ay}{\beta + y} y\left(u^* + \frac{\varepsilon}{2} - y\right). \tag{2.9}$$

Consider the equation

$$\frac{du_2}{dt} = \frac{au_2}{\beta + u_2} u_2\left(u^* + \frac{\varepsilon}{2} - u_2\right). \tag{2.10}$$

It follows from Lemma 2.2 that (2.10) admits a unique globally stable positive equilibrium

$$u_2^* = u^* + \frac{\varepsilon}{2}. \tag{2.11}$$

By differential inequality theory, any solution of (2.9) satisfies

$$\limsup_{t \to +\infty} y(t) \le u^* + \frac{\varepsilon}{2}. \tag{2.12}$$

Hence, there exists a $T_2 > T_1$ such that

$$y(t) < u^* + \varepsilon. \tag{2.13}$$

For $t > T_2$, it follows from the first equation of system (1.6) that

$$\frac{dx}{dt} \ge x\left(\frac{a_1}{b_1 + c_1 x} - d_1 - e_1 x - a(u^* + \varepsilon)\right). \tag{2.14}$$

Now let's consider the equation

$$\frac{dv_1}{dt} = v_1\left(\frac{a_1}{b_1 + c_1 v_1} - d_1 - e_1 v_1 - a(u^* + \varepsilon)\right). \tag{2.15}$$

Since

$$a_1 > b_1\left(d_1 + a(u^* + \varepsilon)\right),$$

it follows from Lemma 2.1 that system (2.15) admits a unique positive equilibrium v_1^*, which is globally asymptotically stable. Applying the differential inequality theory to (2.14) leads to

$$\liminf_{t \to +\infty} x(t) \ge \lim_{t \to +\infty} v(t) = v_1^*.$$

It follows from above inequality that there exists an enough large $T_3 > T_2$ such that

$$x(t) > v_1^* - \frac{\varepsilon}{4} \quad \text{for all} \quad t \ge T_3,$$

and so, from the second equation of system (1.6), we have

$$\frac{dy}{dt} \ge \frac{ay}{\beta + y} y\left(v_1^* - \frac{\varepsilon}{4} - y\right). \tag{2.16}$$

Consider the equation

$$\frac{dv_2}{dt} = \frac{av_2}{\beta + v_2} v_2\left(v_1^* - \frac{\varepsilon}{4} - v_2\right). \tag{2.17}$$

It follows from Lemma 2.2 that (2.17) admits a unique globally stable positive equilibrium

$$v_2^\star = v_1^\star - \frac{\varepsilon}{4}. \tag{2.18}$$

By using the differential inequality theory, any solution of (2.16) satisfies

$$\liminf_{t\to+\infty} y(t) \geq \lim_{t\to+\infty} v_2(t) = v_1^\star - \frac{\varepsilon}{4}. \tag{2.19}$$

(2.7), (2.12), (2.15) and (2.19) show that system (1.6) is permanent. This ends the proof of Theorem 2.1.

Remark 2.1. By using the software Maple, for the fixed coefficients, one could always compute $u^\star$ easily, however, condition (2.3) could be replaced by some more restricted but easily verified condition, indeed, we could have the following results.

Corollary 2.1. *Assume that*

$$\frac{a_1}{b_1} > d_1 + a\frac{\frac{a_1}{b_1} - d_1}{e_1} \tag{2.20}$$

holds, then system (1.6) is permanent.

One interesting problem is to investigate the extinction property of system (1.6), for this, we have the following result.

Theorem 2.2. *Assume that*

$$\frac{a_1}{b_1} < d_1$$

holds, then

$$\lim_{t\to+\infty} x(t) = 0, \quad \lim_{t\to+\infty} y(t) = 0.$$

Proof. From the first equation of system (1.6) we have

$$\begin{aligned}\frac{dx}{dt} &= x\Big(\frac{a_1}{b_1 + c_1 x} - d_1 - e_1 x\Big) - axy \\ &\leq x\Big(\frac{a_1}{b_1 + c_1 x} - d_1 - e_1 x\Big) \\ &\leq x\Big(\frac{a_1}{b_1} - d_1\Big).\end{aligned}$$

Hence

$$x(t) \leq x(0)\exp\Big\{\Big(\frac{a_1}{b_1} - d_1\Big)t\Big\} \to 0 \ \text{ as } \ t \to +\infty.$$

For any positive constant $\varepsilon > 0$ enough small, there exists a $T > 0$ such that

$$x(t) < \varepsilon \ \text{ for all } \ t \geq T.$$

Hence, from the second equation of system (1.6), we have

$$\frac{dy}{dt} \leq \frac{ay}{\beta + y}y\big(\varepsilon - y\big).$$

Consider the equation

$$\frac{du}{dt} = \frac{au}{\beta + u}u\big(\varepsilon - u\big).$$

It follows from Lemma 2.2 that above equation admits a unique globally stable positive equilibrium $u^\star = \varepsilon$. By using the differential inequality theory, we have

$$\limsup_{t\to+\infty} y(t) \leq \varepsilon.$$

Hence

$$0 \le \liminf_{t\to+\infty} y(t) \le \limsup_{t\to+\infty} y(t) \le \varepsilon.$$

Since ε is any small positive constant, setting $\varepsilon \to 0$ in above inequality leads to

$$\lim_{t\to+\infty} y(t) = 0.$$

This ends the proof of Theorem 2.2.

Now we are in the position of investigate the stability property of steady-state solutions of the model (1.6). Defining

$$f(x,y) := x\left(\frac{a_1}{b_1 + c_1 x} - d_1 - e_1 x\right) - axy,$$
$$g(x,y) := \frac{ay}{\beta + y} y(x-y).$$

The steady-state solutions of (1.6) are obtained by solving the equations $f(x,y) = 0$ and $g(x,y) = 0$. The model has three steady-state solutions: $A(0,0)$, $B(u^*,0)$ and $C(x^*,y^*)$.

Theorem 2.3. *If $a_1 > b_1 d_1$ holds, then $C(x^*,y^*)$ is non-negative equilibrium and it is locally asymptotically stable. If inequality (2.3) holds, then $A(0,0)$ and $B(u^*,0)$ is unstable.*

Proof. The variation matrix of the continuous-time system (1.6) at an equilibrium solution (x,y) is

$$J(x,y) = \begin{pmatrix} f_x(x,y) & f_y(x,y) \\ g_x(x,y) & g_y(x,y) \end{pmatrix} = \begin{pmatrix} K_1 & -ax \\ \frac{ay^2}{\beta+y} & K_2 \end{pmatrix},$$

where

$$K_1 = \frac{a_1}{c_1 x + b_1} - d_1 - e_1 x + x\left(-\frac{a_1 c_1}{(c_1 x + b_1)^2} - e_1\right) - ay,$$
$$K_2 = 2\frac{ay(x-y)}{\beta+y} - \frac{ay^2}{\beta+y} - \frac{ay^2(x-y)}{(\beta+y)^2}.$$

Noting that (x^*,y^*) satisfies the equation

$$\left(\frac{a_1}{b_1 + c_1 x^*} - d_1 - e_1 x^*\right) - ay^* = 0,$$
$$\frac{ay^*}{\beta + y^*} y^*(x^* - y^*) = 0.$$

Hence, at $C(x^*,y^*)$

$$J(x^*,y^*) = \begin{pmatrix} -x^*\left(\frac{a_1 c_1}{(c_1 x^* + b_1)^2} + e_1\right) & -ax^* \\ \frac{a(y^*)^2}{\beta + y^*} & -\frac{a(y^*)^2}{\beta + y^*} \end{pmatrix}.$$

Noting that

$$tr(J(x^*,y^*)) = -x^*\left(\frac{a_1 c_1}{(c_1 x^* + b_1)^2} + e_1\right) - \frac{a(y^*)^2}{\beta + y^*} < 0,$$

and

$$det(J(x^*,y^*)) = \left(\frac{a_1 c_1}{(c_1 x^* + b_1)^2} + e_1\right)\frac{ax^*(y^*)^2}{\beta + y^*} + \frac{a^2 x^*(y^*)^2}{\beta + y^*} > 0.$$

So that both eigenvalues of $J(x^*,y^*)$ have negative real parts, and hence this steady-state solution is locally asymptotically stable.

From Theorem 2.1 we know that under the assumption (2.3) holds, system (1.6) is permanent, hence no solution could approach to $A(0,0)$ and $B(u^*,0)$, which means that $A(0,0)$ and $B(u^*,0)$ are locally unstable.

This ends the proof of Theorem 2.3.

3 Global stability

We had showed that the positive equilibrium is locally stable, in this section, we further give sufficient conditions to ensure the global stability of the positive equilibrium.

Theorem 3.1. *Assume that (2.3) holds, then the unique positive equilibrium is globally asymptotically stable.*

Proof. Set

$$\begin{aligned} P_1 &= x\Big(\frac{a_1}{b_1+c_1x} - d_1 - e_1x\Big) - axy, \\ Q_1 &= \frac{ay}{\beta+y}y(x-y). \end{aligned} \tag{3.1}$$

From Theorem 2.2 system (1.6) admits an unique local stable positive equilibrium $C(x^*,y^*)$. Also, from Theorem 2.3, $A(0,0)$ and $B(u^*,0)$ is unstable. To ensure $C(x^*,y^*)$ is globally asymptotically stable, we consider the Dulac function $u_1(x,y) = x^{-1}y^{-2}$, then

$$\begin{aligned} \frac{\partial(u_1P_1)}{\partial x} + \frac{\partial(u_1Q_1)}{\partial y} &= \frac{1}{xy^2}\left(\frac{a_1}{c_1x+b_1} - d_1 - e_1x + x\left(-\frac{a_1c_1}{(c_1x+b_1)^2} - e_1\right) - ay\right) - \frac{a}{(\beta+y)x} - \frac{a(x-y)}{(\beta+y)^2x} \\ &\quad -\frac{1}{x^2y^2}\left(x\left(\frac{a_1}{c_1x+b_1} - d_1 - e_1x\right) - axy\right) \\ &= -\frac{1}{x(c_1x+b_1)^2y^2(\beta+y)^2}K(x,y), \end{aligned}$$

where

$$\begin{aligned} K(x,y) = {} & a\beta {c_1}^2x^2y^2 + a{c_1}^2x^3y^2 + \beta^2{c_1}^2e_1x^3 + 2\beta {c_1}^2e_1x^3y + {c_1}^2e_1x^3y^2 + 2ab_1\beta c_1xy^2 \\ & +2ab_1c_1x^2y^2 + 2b_1\beta^2c_1e_1x^2 + 4b_1\beta c_1e_1x^2y + 2b_1c_1e_1x^2y^2 \\ & +a{b_1}^2\beta y^2 + a{b_1}^2xy^2 + {b_1}^2\beta^2e_1x + 2{b_1}^2\beta e_1xy + {b_1}^2e_1xy^2 + a_1\beta^2c_1x \\ & +2a_1\beta c_1xy + a_1c_1xy^2. \end{aligned}$$

Hence

$$\frac{\partial(u_1P_1)}{\partial x} + \frac{\partial(u_1Q_1)}{\partial y} < 0 \ \text{ for all } \ x>0, y>0.$$

By Dulac Theorem [41], there is no closed orbit in area R_2^+. So $C(x^*,y^*)$ is globally asymptotically stable. This completes the proof of Theorem 3.1.

4 Global attractivity of system (1.7)

As far as system (1.7) is concerned, one of the most important topics is to obtain a set of sufficient conditions to ensure the global attractivity of the positive equilibrium, since which means the stale coexistence of the two species. Before we state and prove the main result of this section, we need to introduce two lemmas.

Lemma 4.1. [35] *Let $x: R\to R$ be a bounded nonnegative continuous function, and let $k:[0,+\infty)\to(0,+\infty)$ be a continuous kernel such that $\int_0^\infty k(s)ds = 1$. Then*

$$\liminf_{t\to+\infty} x(t) \le \liminf_{t\to+\infty}\int_{-\infty}^{t} k(t-s)x(s)ds \le \limsup_{t\to+\infty}\int_{-\infty}^{t} k(t-s)x(s)ds \le \limsup_{t\to+\infty} x(t).$$

Lemma 4.2. [35] *If $a > 0$, $b > 0$ and $\dot{x} \geq x(b - ax)$, when $t \geq 0$ and $x(0) > 0$, we have*

$$\liminf_{t\to+\infty} x(t) \geq \frac{b}{a}.$$

If $a > 0$, $b > 0$ and $\dot{x} \leq x(b - ax)$, when $t \geq 0$ and $x(0) > 0$, we have

$$\limsup_{t\to+\infty} x(t) \leq \frac{b}{a}.$$

Lemma 4.3. *Assume that $\frac{a_1}{b_1} > d_1$, then equation $F(x) = \dfrac{a_1}{b_1 + c_1 x} - d_1 - e_1 x = 0$ admits unique positive solution x^*, also, x^* is the decreasing function of d_1.*

Proof. One could easily see that the equation $F(x) = 0$ admits a unique positive solution

$$x^* = \frac{1}{2}\frac{-b_1 e_1 - c_1 d_1 + \sqrt{\Delta}}{c_1 e_1},$$

where

$$\Delta = {b_1}^2{e_1}^2 - 2\, b_1\, c_1\, d_1\, e_1 + {c_1}^2{d_1}^2 + 4\, a_1\, c_1\, e_1.$$

It immediately follows from the fact

$$\frac{dx^*}{dd_1} = -\frac{1}{2}\frac{b_1\, e_1 - c_1\, d_1 + \sqrt{\Delta}}{\sqrt{\Delta} e_1} < 0$$

that x^* is the decreasing function of d_1. This ends the proof of Lemma 4.3.

Concerned with the global attractivity of the positive equilibrium of system (1.7), we have the following result.

Theorem 4.1. *Assume that*

$$\frac{a_1}{b_1} > d_1 + au^*$$

holds, where u^* is defined by (2.6), then system (1.7) admits a unique positive equilibrium which is globally attractive.

Proof. The positive solution of system (1.7) satisfies the equation

$$\begin{aligned} &\tfrac{a_1}{b_1+c_1 x} - d_1 - e_1 x = ay = 0,\\ &x = y. \end{aligned} \tag{4.1}$$

Obviously, under the assumption of Theorem 4.1, system (4.1) admits a unique positive solution $C(x^*, y^*)$.

To end the proof of Theorem 4.1, it is enough to show that $C(x^*, y^*)$ is globally attractive.
It follows from (4.1) that there exists a $\varepsilon > 0$ enough small such that

$$\frac{a_1}{b_1} > d_1 + a(u^* + \varepsilon). \tag{4.2}$$

Let $(x(t), y(t))$ be any positive solution of system (1.7). From system (1.7) it follows that

$$\frac{dx}{dt} \leq x\Big(\frac{a_1}{b_1 + c_1 x} - d_1 - e_1 x\Big).$$

Consider the equation

$$\frac{du_1}{dt} = u_1\Big(\frac{a_1}{b_1 + c_1 u_1} - d_1 - e_1 u_1\Big). \tag{4.3}$$

It follows from Lemma 2.1 that (4.3) admits a unique globally stable positive equilibrium u^*, where u^* is defined by (2.6). By using the differential inequality theory, any positive solution of (1.7) satisfies

$$\limsup_{t\to+\infty} x(t) \leq \lim_{t\to+\infty} u(t) = u^*, \tag{4.4}$$

and so, from Lemma 4.1 we have

$$\limsup_{t\to+\infty} \int_{-\infty}^{t} K_2(t-s)x(s)ds \le u^*. \tag{4.5}$$

Hence, there exists a $T_{11} > 0$ such that

$$x(t) < u^* + \tfrac{\varepsilon}{2} \stackrel{\text{def}}{=} M_1^{(1)}, \tag{4.6}$$

and

$$\int_{-\infty}^{t} K_2(t-s)x(s)ds < u^* + \frac{\varepsilon}{2} \stackrel{\text{def}}{=} M_1^{(1)}. \tag{4.7}$$

For $t > T_{11}$, it follows from the second equation of system (1.7) and (4.7) that

$$\frac{dy}{dt} \le \frac{ay}{\beta+y} y\left(M_1^{(1)} - y\right). \tag{4.8}$$

Consider the equation

$$\frac{du_2}{dt} = \frac{au_2}{\beta+u_2} u_2\left(M_1^{(1)} - u_2\right). \tag{4.9}$$

It follows from Lemma 2.2 that (4.9) admits a unique globally stable positive equilibrium

$$u_2^* = u^* + \frac{\varepsilon}{2}. \tag{4.10}$$

By differential inequality theory, any positive solution of (1.7) satisfies

$$\limsup_{t\to+\infty} y(t) \le M_1^{(1)}, \tag{4.11}$$

and so, from Lemma 4.1 we have

$$\limsup_{t\to+\infty} \int_{-\infty}^{t} K_1(t-s)y(s)ds \le M_1^{(1)}. \tag{4.12}$$

Hence, there exists a $T_{12} > T_{11}$ such that

$$y(t) < M_1^{(1)} + \tfrac{\varepsilon}{2} \stackrel{\text{def}}{=} M_2^{(1)}, \tag{4.13}$$

and

$$\int_{-\infty}^{t} K_1(t-s)y(s)ds < M_1^{(1)} + \frac{\varepsilon}{2} \stackrel{\text{def}}{=} M_2^{(1)}. \tag{4.14}$$

For $t > T_{12}$, it follows from the first equation of system (1.7) and (4.14) that

$$\frac{dx}{dt} \ge x\left(\frac{a_1}{b_1+c_1x} - d_1 - e_1x - aM_2^{(1)}\right). \tag{4.15}$$

Now let's consider the equation

$$\frac{dv_1}{dt} = v_1\left(\frac{a_1}{b_1+c_1v_1} - d_1 - e_1v_1 - aM_2^{(1)}\right). \tag{4.16}$$

Since

$$a_1 > b_1\left(d_1 + a(u^*+\varepsilon)\right) = b_1\left(d_1 + aM_2^{(1)}\right),$$

it follows from Lemma 2.1 that system (4.16) admits a unique positive equilibrium v_1^*, which is globally asymptotically stable. Applying the differential inequality theory to (4.15) leads to

$$\liminf_{t\to+\infty} x(t) \ge \lim_{t\to+\infty} v(t) = v_1^*,$$

and so, from Lemma 4.1 we have

$$\liminf_{t\to+\infty}\int_{-\infty}^{t} K_2(t-s)x(s)ds \geq v_1^*.$$

It follows from above inequality that there exists an enough large $T_{13} > T_{12}$ such that for all $t \geq T_{13}$, the following inequalities hold.

$$x(t) > v_1^* - \frac{\varepsilon}{4} \overset{\text{def}}{=} m_1^{(1)}, \tag{4.17}$$

$$\int_{-\infty}^{t} K_2(t-s)x(s)ds > v_1^* - \frac{\varepsilon}{4} \overset{\text{def}}{=} m_1^{(1)}. \tag{4.18}$$

From the second equation of system (1.7), for $t \geq T_{13}$, we have

$$\frac{dy}{dt} \geq \frac{ay}{\beta + y} y\left(m_1^{(1)} - y\right). \tag{4.19}$$

Consider the equation

$$\frac{dv_2}{dt} = \frac{av_2}{\beta + v_2} v_2\left(m_1^{(1)} - v_2\right). \tag{4.20}$$

It follows from Lemma 2.2 that (4.20) admits a unique globally stable positive equilibrium

$$v_2^* = m_1^{(1)}. \tag{4.21}$$

By using the differential inequality theory, any solution of (4.19) satisfies

$$\liminf_{t\to+\infty} y(t) \geq \lim_{t\to+\infty} v_2(t) = m_1^{(1)},$$

and so, from Lemma 4.1 we have

$$\liminf_{t\to+\infty}\int_{-\infty}^{t} K_1(t-s)y(s)ds \geq m_1^{(1)}.$$

It follows from above inequality that there exists an enough large $T_{14} > T_{13}$ such that for all $t \geq T_{14}$, the following inequalities hold

$$y(t) > m_1^{(1)} - \frac{\varepsilon}{2} \overset{\text{def}}{=} m_2^{(1)}, \tag{4.22}$$

$$\int_{-\infty}^{t} K_1(t-s)y(s)ds > m_1^{(1)} - \frac{\varepsilon}{2} \overset{\text{def}}{=} m_2^{(1)}. \tag{4.23}$$

For $t > T_{14}$, it follows from (4.23) and the first equation of system (1.7) that

$$\frac{dx}{dt} \leq x\left(\frac{a_1}{b_1 + c_1 x} - d_1 - e_1 x\right) - axm_2^{(1)}.$$

Consider the equation

$$\frac{du_1}{dt} = u_1\left(\frac{a_1}{b_1 + c_1 u_1} - d_1 - am_2^{(1)} - e_1 u_1\right). \tag{4.24}$$

It follows from Lemma 2.1 that (4.24) admits a unique globally stable positive equilibrium $u^*_{m_2^{(1)}}$, from Lemma 4.3, one could see that $u^*_{m_2^{(1)}} < u^*$. By using the differential inequality theory, any positive solution of (1.7) satisfies

$$\limsup_{t\to+\infty} x(t) \leq \lim_{t\to+\infty} u(t) = u^*_{m_2^{(1)}}, \tag{4.25}$$

and so, from Lemma 4.1 we have

$$\limsup_{t\to+\infty} \int_{-\infty}^{t} K_2(t-s)x(s)ds \leq u^*_{m_2^{(1)}}. \tag{4.26}$$

Hence, there exists a $T_{21} > 0$ such that

$$x(t) < u^{\star}_{m_2^{(1)}} + \tfrac{\varepsilon}{4} \stackrel{\text{def}}{=} M_1^{(2)}, \tag{4.27}$$

and

$$\int_{-\infty}^{t} K_2(t-s)x(s)ds < u^{\star}_{m_2^{(1)}} + \frac{\varepsilon}{4} \stackrel{\text{def}}{=} M_1^{(2)}. \tag{4.28}$$

For $t > T_{21}$, it follows from the second equation of system (1.7) and (4.28) that

$$\frac{dy}{dt} \le \frac{ay}{\beta + y} y\left(M_1^{(2)} - y\right). \tag{4.29}$$

Consider the equation

$$\frac{du_2}{dt} = \frac{au_2}{\beta + u_2} u_2\left(M_1^{(2)} - u_2\right). \tag{4.30}$$

It follows from Lemma 2.2 that (4.30) admits a unique globally stable positive equilibrium $M_1^{(2)}$. By using the differential inequality theory, any positive solution of (1.7) satisfies

$$\limsup_{t\to+\infty} y(t) < M_1^{(2)}, \tag{4.31}$$

and so, from Lemma 4.1 we have

$$\limsup_{t\to+\infty} \int_{-\infty}^{t} K_1(t-s)y(s)ds \le M_1^{(2)}. \tag{4.32}$$

Hence, there exists a $T_{22} > T_{21}$ such that

$$y(t) < M_1^{(2)} + \varepsilon \stackrel{\text{def}}{=} M_2^{(2)}, \tag{4.33}$$

and

$$\int_{-\infty}^{t} K_1(t-s)y(s)ds < M_1^{(2)} + \varepsilon \stackrel{\text{def}}{=} M_2^{(2)}. \tag{4.34}$$

For $t > T_{22}$, it follows from the first equation of system (1.7) and (4.34) that

$$\frac{dx}{dt} \ge x\left(\frac{a_1}{b_1 + c_1 x} - d_1 - e_1 x - aM_2^{(2)}\right). \tag{4.35}$$

Now let's consider the equation

$$\frac{dv_1}{dt} = v_1\left(\frac{a_1}{b_1 + c_1 v_1} - d_1 - e_1 v_1 - aM_2^{(2)}\right). \tag{4.36}$$

Since

$$M_2^{(2)} < M_2^{(1)},$$

it follows from (4.1) that

$$a_1 > b_1\left(d_1 + aM_2^{(2)}\right).$$

Hence, applying Lemma 2.1 to system (4.36), one could see that (4.36) admits a unique positive equilibrium $v^{\star}_{M_2^{(2)}}$, which is globally asymptotically stable. Also, from Lemma 4.3, we have

$$v^{\star}_{M_2^{(2)}} > v_1^{\star}.$$

Applying the differential inequality theory to (4.35) leads to

$$\liminf_{t\to+\infty} x(t) \ge \lim_{t\to+\infty} v(t) = v^{\star}_{M_2^{(2)}},$$

and so, from Lemma 4.1 we have

$$\liminf_{t\to+\infty}\int_{-\infty}^{t} K_2(t-s)x(s)ds \geq v^{*}_{M_2^{(2)}}.$$

It follows from above inequality that there exists an enough large $T_{13} > T_{12}$ such that for all $t \geq T_{13}$, the following inequalities hold.

$$x(t) > v^{*}_{M_2^{(2)}} - \frac{\varepsilon}{4} \overset{\text{def}}{=} m_1^{(2)}, \tag{4.37}$$

$$\int_{-\infty}^{t} K_2(t-s)x(s)ds > v^{*}_{M_2^{(2)}} - \frac{\varepsilon}{4} \overset{\text{def}}{=} m_1^{(2)}. \tag{4.38}$$

From the second equation of system (1.7), we have

$$\frac{dy}{dt} \geq \frac{ay}{\beta+y} y\left(m_1^{(2)} - y\right). \tag{4.39}$$

Consider the equation

$$\frac{dv_2}{dt} = \frac{av_2}{\beta+v_2} v_2\left(m_1^{(2)} - v_2\right). \tag{4.40}$$

It follows from Lemma 2.2 that (4.40) admits a unique globally stable positive equilibrium $m_1^{(2)}$. By using the differential inequality theory, any solution of (4.39) satisfies

$$\liminf_{t\to+\infty} y(t) \geq \lim_{t\to+\infty} v_2(t) = m_1^{(2)}, \tag{4.41}$$

and so, from Lemma 4.1 we have

$$\liminf_{t\to+\infty}\int_{-\infty}^{t} K_1(t-s)y(s)ds \geq m_1^{(2)}.$$

It follows from above inequality that there exists an enough large $T_{24} > T_{23}$ such that for all $t \geq T_{24}$, the following inequalities hold.

$$y(t) > m_1^{(2)} - \frac{\varepsilon}{2} \overset{\text{def}}{=} m_2^{(2)}, \tag{4.42}$$

$$\int_{-\infty}^{t} K_1(t-s)y(s)ds > m_1^{(2)} - \frac{\varepsilon}{2} \overset{\text{def}}{=} m_2^{(2)}. \tag{4.43}$$

One could easily see that

$$\begin{aligned}
M_1^{(2)} &= u^{*}_{m_2^{(1)}} + \tfrac{\varepsilon}{4} < u^{*} + \tfrac{\varepsilon}{2} = M_1^{(1)};\\
M_2^{(2)} &= M_1^{(2)} + \varepsilon < u^{*} + \varepsilon = M_2^{(1)};\\
m_1^{(2)} &= v^{*}_{M_2^{(2)}} - \tfrac{\varepsilon}{4} > v_1^{*} - \tfrac{\varepsilon}{4} = m_1^{(1)};\\
m_2^{(2)} &= m_1^{(2)} - \tfrac{\varepsilon}{2} > m_1^{(1)} - \tfrac{\varepsilon}{2} = m_2^{(1)}.
\end{aligned} \tag{4.44}$$

Repeating the above procedure, we get four sequences $M_i^{(n)}, m_i^{(n)}$, $i = 1, 2$, $n = 1, 2, \cdots$, such that for $n \geq 2$

$$\begin{aligned}
&\frac{a_1}{b_1 + c_1\left(M_1^{(n)} - \frac{\varepsilon}{2^n}\right)} - d_1 - e_1\left(M_1^{(n)} - \frac{\varepsilon}{2^n}\right) - am_2^{(n-1)} = 0;\\
&M_2^{(n)} = M_1^{(n)} + \varepsilon;\\
&\frac{a_1}{b_1 + c_1\left(m_1^{(n)} + \frac{\varepsilon}{2^n}\right)} - d_1 - e_1\left(m_1^{(n)} + \frac{\varepsilon}{2^n}\right) - aM_2^{(n)} = 0;\\
&m_2^{(n)} = m_1^{(n)} - \tfrac{\varepsilon}{2}.
\end{aligned} \tag{4.45}$$

Obviously

$$m_i^{(n)} < N_i(t) < M_i^{(n)}, \quad \text{for } t \geq T_{2n}, \quad i = 1, 2.$$

We claim that sequences $M_i^{(n)}$, $i = 1, 2$ are non-increasing, and sequences $m_i^{(n)}$, $i = 1, 2$ are non-decreasing. To prove this claim, we will carry out by induction. Firstly, from (4.44) we have

$$M_i^{(2)} < M_i^{(1)}, \quad m_i^{(2)} > m_i^{(1)}, \quad i = 1, 2.$$

Let us assume now that our claim is true for n, that is,

$$M_i^{(n)} < M_i^{(n-1)}, \quad m_i^{(n)} > m_i^{(n-1)}, \quad i = 1, 2.$$

Then, by Lemma 4.3, we immediately obtain

$$M_1^{(n+1)} < M_1^{(n)}; \quad M_2^{(n+1)} < M_2^{(n)};$$

$$m_1^{(n+1)} > m_1^{(n)}; \quad m_2^{(n+1)} > m_2^{(n)}.$$

Therefore

$$\lim_{t\to+\infty} M_1^{(n)} = \overline{x}, \quad \lim_{t\to+\infty} M_2^{(n)} = \overline{y},$$

$$\lim_{t\to+\infty} m_1^{(n)} = \underline{x}, \quad \lim_{t\to+\infty} m_2^{(n)} = \underline{y}.$$

Letting $n \to +\infty$ in (4.45), we obtain

$$\begin{aligned} &\frac{a_1}{b_1 + c_1\overline{x}} - d_1 - e_1\overline{x} - a\underline{y} = 0; \\ &\overline{y} = \overline{x}; \\ &\frac{a_1}{b_1 + c_1\underline{x}} - d_1 - e_1\underline{x} - a\overline{y} = 0; \\ &\underline{y} = \underline{x}. \end{aligned} \tag{4.46}$$

(4.46) shows that $(\overline{x}, \underline{y})$ and $(\underline{x}, \overline{y})$ are solutions of (4.1), which (4.1) has a unique positive solution $C(x^*, y^*)$. Hence, we conclude that

$$\overline{x} = \underline{x} = x^*, \quad \overline{y} = \underline{y} = y^*,$$

that is

$$\lim_{t\to+\infty} x(t) = x^* \quad \lim_{t\to+\infty} y(t) = y^*.$$

Thus, the unique interior equilibrium $C(x^*, y^*)$ is globally attractive. This completes the proof of Theorem 4.1.

5 Numeric simulations

Now let's consider the following four examples.

Example 5.1

$$\begin{aligned} \frac{dx}{dt} &= x\Big(\frac{1}{2+x} - 1 - x\Big) - xy, \\ \frac{dy}{dt} &= y\frac{y}{1+y}\Big(x - y\Big). \end{aligned} \tag{5.1}$$

In this system, corresponding to system (1.6), we take $a_1 = c_1 = d_1 = e_1 = a = \beta = 1$, $b_1 = 2$, since $a_1 < b_1d_1$, it follows from Theorem 2.2 that the boundary equilibrium $A(0, 0)$ is globally asymptotically stable. Figure 1 supports this assertion.

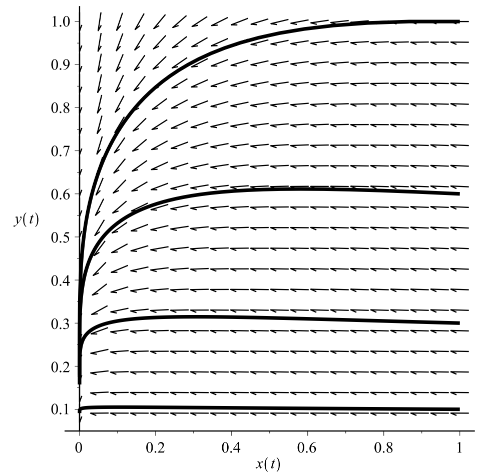

Figure 1: Dynamic behavior of system (5.1), here the initial condition $(x(0), y(0)) = (1, 1), (1, 0.3), (1, 0.1)$ and $(1, 0.6)$, respectively.

Example 5.2

$$\begin{aligned}\frac{dx}{dt} &= x\Big(\frac{5}{2+x} - 1 - 6x\Big) - xy,\\ \frac{dy}{dt} &= y\frac{y}{1+y}\Big(x-y\Big).\end{aligned} \tag{5.2}$$

In this system, corresponding to system (1.6), we take $a_1 = c_1 = d_1 = e_1 = a = \beta = 1, b_1 = 2, e_1 = 6$, since $\frac{a_1}{b_1} = \frac{5}{2} > 1 + \frac{5}{12} = d_1 + a\frac{\frac{a_1}{b_1} - d_1}{e_1}$, it follows from corollary 2.1 that system (4.2) is permanent. Figure 2 supports this assertion.

Example 5.3

$$\begin{aligned}\frac{dx}{dt} &= x\Big(\frac{2}{1+x} - 1 - x\Big) - 4xy,\\ \frac{dy}{dt} &= 4y\frac{y}{1+y}\Big(x-y\Big).\end{aligned} \tag{5.3}$$

In this system, corresponding to system (1.6), we take $b_1 = c_1 = d_1 = e_1 = \beta = 1, a_1 = 2, a = 4$, by computation, $u^* = \sqrt{2} - 1$, and so, $\frac{a_1}{b_1} = 2 < 1 + 4(\sqrt{2} - 1) = d_1 + au^*$, Hence, the conditions of Theorem 2.1 could not satisfied, however, numeric simulation (Figure 3) shows that the system also admits a unique positive equilibrium which is globally asymptotically stable.

6 Discussion

During the last decades, many scholars [2-6, 22-25] investigated the influence of Allee effect on the dynamic behaviors of ecosystem. Also, there are several scholars [32-38] investigated the almost periodic solution of the ecosystem. However, all of those studies are based on the traditional Logistic model.

In this paper, we argued that the nonlinear birth rate of the prey species is more suitable, and take Beverton-Holt function [28] as the birth rate, this leads to system (1.6).

We showed that depending on the range of the birth rate parameter, the system maybe collapse or the two species could be coexist in a stable state. That is, the birth rate plays essential role on the dynamic behaviors of system (1.6).

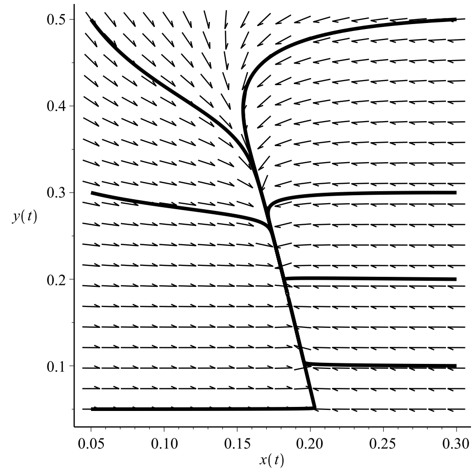

Figure 2: Dynamic behavior of system (5.2), here the initial condition $(x(0), y(0)) = (0.05, 0.5), (0.05, 0.3), (0.05, 0.5), (0.3, 0.1), (0.3, 0.3), (0.3, 0.5)$ and $(0.3, 0.2)$, respectively.

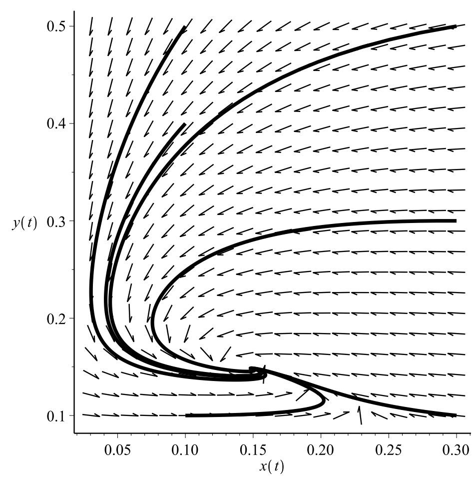

Figure 3: Dynamic behavior of system (5.3), here the initial condition $(x(0), y(0)) = (0.3, 0.5), (0.3, 0.1), (0.3, 0.3), (0.3, 0.4), (0.3, 0.2), (0.1, 0.3), (0.1, 0.1)$ and $(0.1, 0.5)$, respectively.

For the system with infinite delay, by using the iterative method, we could able to show that inequality (2.3) is enough to ensure the globally attractive of the positive equilibrium. We mentioned here that with the nonlinear birth rate, the method used in the paper [34] and [36] could not be applied to our system directly, to overcome this difficulty, we developing some new analysis technique.

At the end of the paper, we would like to point out that the results obtained in this paper are the sufficient ones, as was shown in Example 4.3, there are still have room to improve our results, we leave this for future study. Also, it seems interesting to investigate the dynamic behaviors of the non-autonomous case of system (1.6), specially focus on the permanence, extinction and almost periodic solution, we also leave this for future investigation.

Acknowledgements: The authors would like to thank Dr. Yu Liu for useful discussion about the mathematical modeling. The research was supported by the National Natural Science Foundation of China under Grant (11601085) and the Natural Science Foundation of Fujian Province (2017J01400).

References

[1] Berryman A. A., The origins and evolution of predator-prey theory, Ecology, 1992, 73(5), 1530–1535.
[2] Merdan H., Stability analysis of a Lotka-Volterra type predator-prey system involving Allee effect, The ANZIAM Journal, 2010, 52(2), 139–145.
[3] Çelik C., Duman O., Allee effect in a discrete-time predator-prey system, Chaos Solitons Fractals, 2009, 40(4), 1952–1956.
[4] Wang J., Shi J., Wei J., Predator-prey system with strong Allee effect in prey, J. Math. Biol., 2011, 62(3), 291–331.
[5] Wang W., Zhang Y., Liu C., Analysis of a discrete-time predator-prey system with Allee effect, Ecological Complexity, 2011, 8(1), 81–85.
[6] Gonzalez-Olivares E., Mena-Lorca J., Rojas-Palma A., Flores J.D., Dynamical complexities in the Leslie-Gower predator-prey model as consequences of the Allee effect on prey, Appl. Math. Model., 2011, 35(1), 366–381.
[7] Ma Z., Chen F., Wu C., Chen W., Dynamic behaviors of a Lotka-Volterra predator-prey model incorporating a prey refuge and predator mutual interference, Appl. Math. Comput., 2013, 219(15), 7945–7953.
[8] Chen L., Chen F., Wang Y., Influence of predator mutual interference and prey refuge on Lotka-Volterra predator-prey dynamics, Commun. Nonlinear Sci. Numer. Simul., 2013, 18(11), 3174–3180.
[9] Chen F., Chen W., Wu Y., Ma Z., Permanence of a stage-structured predator-prey system, Appl. Math. Comput., 2013, 219(17), 8856–8862.
[10] Chen F., Wang H., Lin Y., Chen W., Global stability of a stage-structured predator-prey system, Appl. Math. Comput., 2013, 223(15), 45–53.
[11] Chen F., Xie X., Li Z., Partial survival and extinction of a delayed predator-prey model with stage structure, Appl. Math. Comput., 2012, 219(8), 4157–4162.
[12] Li Z., Han M., Chen F., Global stability of a a stage-structured predator-prey model with modified Leslie-Gower and Holling-type II schemes, Int. J. Biomath., 2012, 5(06), 112–122.
[13] Lin Y., Xie X., Chen F., Li T., Convergences of a stage-structured predator-prey model with modified Leslie-Gower and Holling-type II schemes, Adv. Difference Equ., 2016, 2016:181.
[14] Yu S., Global asymptotic stability of a predator-prey model with modified Leslie-Gower and Holling-type II schemes, Discrete Dyn. Nat. Soc., 2012, Article ID 208167.
[15] Yue Q., Dynamics of a modified Leslie-Gower predator-prey model with Holling-type II schemes and a prey refuge, SpringerPlus, 2016, 5, 461–472.
[16] Chen L., Chen F., Global stability and bifurcation of a ratio-dependent predator-prey model with prey refuge, Acta Math. Sinica (Chin. Ser.), 2014, 57(2), 301–310.
[17] Guan X., Liu Y., Xie Y., Stability analysis of a Lotka-Volterra type predator-prey system with Allee effect on the predator species, Commun. Math. Biol. Neurosci., 2018, Article ID 9.
[18] Yu S., Chen F., Almost periodic solution of a modified Leslie-Gower predator-prey model with Holling-type II schemes and mutual interference, Int. J. Biomath., 2014, 7(3), 81–95.
[19] Chen L., Chen F., Dynamic behaviors of the periodic predator-prey system with distributed time delays and impulsive effect, Nonlinear Anal. Real World Appl., 2011, 12(4), 2467–2473.
[20] Yang L., Xie X., Chen F., Xue Y., Permanence of the periodic predator-prey-mutualist system, Adv. Difference Equ., 2015, 2015:331.
[21] Wu R., Li L., Permanence and global attractivity of the discrete predator-prey system with Hassell-Varley-Holling III type functional response, Discrete Dyn. Nat. Soc., 2013, 2013(2), 295–321.
[22] Wu R., Li L., Lin Q., A Holling type commensal symbiosis model involving Allee effect, Commun. Math. Biol. Neurosci., 2018, Article ID 6.
[23] Lin Q., Allee effect increasing the final density of the species subject to the Allee effect in a Lotka-Volterra commensal symbiosis model, Adv. Difference Equ., 2018, 2018:196.
[24] Lin Q., Stability analysis of a single species logistic model with Allee effect and feedback control, Adv. Difference Equ., 2018, 2018:190.
[25] Chen B., Dynamic behaviors of a commensal symbiosis model involving Allee effect and one party can not survive independently, Adv. Difference Equ., 2018, 2018:212.
[26] Brauer F., Castillo-Chavez C., Mathematical Models in Population Biology and Epidemiology, Springer, 2001.
[27] Tang S., Chen L., Density-dependent birth rate, birth pulses and their population dynamic consequences, J. Math. Biol., 2002, 44(2), 185–199.
[28] Berezansky L., Braverman E., Idels L., Nicholson's blowflies differential equations revisited: main results and open problems, Appl. Math. Model., 2010, 34(6), 1405–1417.
[29] Liao Y. Z., Dynamics of two-species harvesting model of almost periodic facultative mutualism with discrete and distributed delays, Engineering Letters, 2018, 26(1), 7–13.
[30] Chen F., Xie X., Miao Z., Pu L., Extinction in two species nonautonomous nonlinear competitive system, Appl. Math. Comput., 2016, 274, 119–124.

[31] Yang K., Miao Z., Chen F., Xie X., Influence of single feedback control variable on an autonomous Holling-II type cooperative system, J. Math. Anal. Appl., 2016, 435(1), 874–888.

[32] Li Z., Han M., Chen F., Influence of feedback controls on an autonomous Lotka-Volterra competitive system with infinite delays, Nonlinear Anal. Real World Appl., 2013, 14(1), 402–413.

[33] He M., Chen F., Extinction and stability of an impulsive system with pure delay, Appl. Math. Lett., 2019, 91(1), 123–136.

[34] Chen F., Xie X., Wang H., Global stability in a competition model of plankton allelopathy with infinite delay, J. Syst. Sci. Complex., 2015, 28(5), 1070–1079.

[35] Xue Y., Xie X., Lin Q., Chen F., Global attractivity and extinction of a discrete competitive system with infinite delays and single feedback control, Discrete Dyn. Nat. Soc., 2018, Article ID 1893181.

[36] Xie X., Xue Y., Wu R., Global attractivity of a discrete competition model of plankton allelopathy with infinite deviating argument, Adv. Difference Equ., 2018, 2018:303.

[37] Hale J. K., Theory of Functional Differential Equation, Springer-Verlag, Heidelberg, 1977.

[38] Chen L., Mathematical Models and Methods in Ecology, Beijing, China: Science Press, 1988.

[39] Chen F., Xue Y. L., Lin Q. F., Xie X. D., Dynamic behaviors of a Lotka-Volterra commensal symbiosis model with density dependent birth rate, Adv. Difference Equ., 2018, 2018:296.

[40] Zhou Y. C., Jin Z., Qin J. L., Ordinary differential equaiton and its application, Science Press, 2003.