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S-shaped connected component of radial positive solutions for a prescribed mean curvature problem in an annular domain

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Abstract: In this paper, we show the existence of an S-shaped connected component in the set of radial positive solutions of boundary value problem

$$\begin{cases} -\operatorname{div}\big(\phi_N(\nabla y)\big) = \lambda a(|x|)f(y) \quad \text{in } \mathcal{A}, \\ \dfrac{\partial y}{\partial \nu} = 0 \quad \text{on } \Gamma_1, \qquad y = 0 \quad \text{on } \Gamma_2, \end{cases}$$

where $R_2 \in (0, \infty)$ and $R_1 \in (0, R_2)$ is a given constant, $\mathcal{A} = \{x \in \mathbb{R}^N : R_1 < |x| < R_2\}$, $\Gamma_1 = \{x \in \mathbb{R}^N : |x| = R_1\}$, $\Gamma_2 = \{x \in \mathbb{R}^N : |x| = R_2\}$, $\phi_N(s) = \frac{s}{\sqrt{1-|s|^2}}$, $s \in \mathbb{R}^N$, λ is a positive parameter, $a \in C[R_1, R_2]$, $f \in C[0, \infty)$, $\frac{\partial y}{\partial \nu}$ denotes the outward normal derivative of y and $|\cdot|$ denotes the Euclidean norm in $\mathbb{R}^N$. The proof of main result is based upon bifurcation techniques.

Keywords: S-shaped connected component, positive radial solutions, mean curvature operator, annular domain, Minkowski space, bifurcation

MSC: 34B10, 34B18, 34C23, 35B40, 35J65

1 Introduction

Hypersurfaces of prescribed mean curvature in flat Minkowski space $\mathbb{L}^{N+1} = \{(x, t) : x \in \mathbb{R}^N, t \in \mathbb{R}\}$, with the Lorentzian metric $\sum\limits_{i=1}^{N} dx_i^2 - dt^2$, where $(x, t) = (x_1, x_2, \cdots, x_N, t)$, are of interest in differential geometry and in general relativity. It is well-known that the study of spacelike submanifolds of codimension one in $\mathbb{L}^{N+1}$ with prescribed mean extrinsic curvature leads to Dirichlet problems of the type

$$\begin{cases} -\operatorname{div}\Big(\dfrac{\nabla u}{\sqrt{1-|\nabla u|^2}}\Big) = f(x, u) \quad \text{in } \Omega, \\ u = 0 \quad \text{on } \partial\Omega, \end{cases} \tag{1}$$

where Ω is a bounded domain in $\mathbb{R}^N$ and the nonlinearity $f : \Omega \times \mathbb{R} \to \mathbb{R}$ is continuous, see [1, 2].

The existence, multiplicity and qualitative properties of solutions of (1) have been extensively studied by many authors in recent years, see Coelho et al. [3], Treibergs [4], Cano-Casanova et al. [5], Pan et al. [6],

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López [7], Corsato et al. [8, 9], Korman [10] as well as Ma et al. [11, 12], and the references therein. It is worth pointing out that the starting point of this type of problems is the seminal paper [13], and from Bartnik and Simon [2] as well as Bereanu and Mawhin [14], we know (1) has a solution whatever f is. This can be seen as a universal existence result for the above problem. However, in our study problem (1) generally admits the null solution, it may be interesting to investigate the existence of non-trivial solutions, especially the positive solutions. However, there are few works on positive solutions of (1), see Coelho et al. [15], Bereanu et al. [16, 17], Ma et al. [18] and Dai [19].

Specifically, depending on the behaviour of $f = f(x, s)$ near $s = 0$, Coelho et al. [15] discussed the existence of either one, or two, or three, or infinitely many positive solutions of the quasilinear two-point boundary value problem

$$\begin{cases} -\left(\frac{u'}{\sqrt{1-u'^2}}\right)' = f(x,u), \quad x \in (0,1), \\ u(0) = u(1) = 0, \end{cases}$$

where f is L^p-Carathéodory function, and the proof of main results are based upon the variational and topological methods. Bereanu et al. [16, 17] obtained some important existence, nonexistence and multiplicity results for the positive radial solutions of problem (1) in a ball by using Leray-Schauder degree argument and critical point theory. Recently, Ma et al. [18] are concerned with the global structure of radial positive solutions for the problem (1) in a ball by using global bifurcation techniques, and extended the results of [16, 17] to more general cases, all results, depending on the behavior of nonlinear term f near 0. Dai [19] investigated the intervals of the parameter λ in which the problem (1) has zero, one or two positive radial solutions corresponding to sublinear, linear, and superlinear nonlinearities f at zero, respectively. However, [18, 19] only give a full description of the set of radial positive solutions of (1) for certain classes of nonlinearities f, and give no any information about the directions of a bifurcation.

In 2015, Sim and Tanaka [20] proved the existence of S-shaped connected component in the set of positive solutions for the one-dimensional p-Laplacian problem with sign-changing weight

$$\begin{cases} -(|u'|^{p-2}u')' = \mu m(x) f(u), \quad x \in (0,1), \\ u(0) = u(1) = 0, \end{cases} \tag{2}$$

where $p > 1$, $m \in C[0, 1]$, $f \in C[0, \infty)$ and μ is a positive parameter. They obtained the following result by bifurcation techniques.

Theorem A. ([20, Thoerem 1.1]) Assume
(H1) there exist $x_1, x_2 \in [0, 1]$ such that $x_1 < x_2$, $m(x) > 0$ on (x_1, x_2) and $m(x) \le 0$ on $[0, 1]\backslash[x_1, x_2]$,
(F1') there exist $\alpha > 0$, $f_0 > 0$ and $f_1 > 0$ such that $\lim_{s\to 0^+} \frac{f(s)-f_0 s^{p-1}}{s^{p-1+\alpha}} = -f_1$,
(F2') $f_\infty := \lim_{s\to\infty} \frac{f(s)}{s^{p-1}} = 0$,
(F3') there exists $s_0 > 0$ such that

$$\min_{s\in[s_0,2s_0]} \frac{f(s)}{s^{p-1}} \ge \frac{f_0(p-1)}{\mu_1 m_0} \left(\frac{\pi_p}{x_2 - x_1}\right)^p,$$

where $\mu_1 > 0$ is the first eigenvalue of the linear problem associated to (2), and

$$\pi_p := \frac{2\pi}{p\sin(\frac{\pi}{p})}, \quad m_0 = \min_{x\in[\frac{3x_1+x_2}{4}, \frac{x_1+3x_2}{4}]} m(x).$$

Then there exist $\mu_\star \in (0, \frac{\mu_1}{f_0})$ and $\mu^\star > \frac{\mu_1}{f_0}$ such that
(i) (2) has at least one positive solution if $\mu = \mu_\star$;
(ii) (2) has at least two positive solutions if $\mu_\star < \mu \le \frac{\mu_1}{f_0}$;
(iii) (2) has at least three positive solutions if $\frac{\mu_1}{f_0} < \mu < \mu^\star$;
(iv) (2) has at least two positive solutions if $\mu = \mu^\star$;
(v) (2) has at least one positive solution if $\mu > \mu^\star$.

Of course, the natural question is whether or not the similar result can be established for the prescribed mean curvature problem (1)?

The purpose of this paper is to show the existence of the S-shaped connected component in the set of radial positive solutions for a prescribed mean curvature problem in an annular domain

$$\begin{cases} -\operatorname{div}\big(\phi_N(\nabla y)\big) = \lambda a(|x|)f(y) & \text{in } \mathcal{A}, \\ \dfrac{\partial y}{\partial \nu} = 0 \quad \text{on } \Gamma_1, & y = 0 \quad \text{on } \Gamma_2, \end{cases} \tag{3}$$

where $R_2 \in (0,\infty)$ and $R_1 \in (0, R_2)$ is a given constant, $\mathcal{A} = \{x \in \mathbb{R}^N : R_1 < |x| < R_2\}$, $\Gamma_1 = \{x \in \mathbb{R}^N : |x| = R_1\}$, $\Gamma_2 = \{x \in \mathbb{R}^N : |x| = R_2\}$, $\phi_N(s) = \frac{s}{\sqrt{1-|s|^2}}$, $s \in \mathbb{R}^N$, λ is a positive parameter, $a \in C[R_1, R_2]$, $f \in C[0,\infty)$, $\frac{\partial y}{\partial \nu}$ denotes the outward normal derivative of y and $|\cdot|$ denotes the Euclidean norm in $\mathbb{R}^N$. To the best of our knowledge, for problem (3), such bifurcation curve is completely new and has not been practically described before.

Setting, as usual $|x| = r$ and $y(x) = u(r)$, the problem (3) reduces to the mixed boundary value problem

$$\begin{cases} -(r^{N-1}\phi_1(u'))' = \lambda r^{N-1}a(r)f(u), & r \in (R_1, R_2), \\ u'(R_1) = u(R_2) = 0, \end{cases} \tag{4}$$

where $\phi_1(s) = \frac{s}{\sqrt{1-s^2}}$, $s \in \mathbb{R}$, $\phi_1 : (-1,1) \to \mathbb{R}$ is an odd, increasing homeomorphism and $\phi_1(0) = 0$. To find a positive radial solution of (3), it is enough to find a positive solution of (4). We say that a function $u \in C^1[R_1, R_2]$ is a solution of (4) if $\max\limits_{r\in[R_1,R_2]} |u'(r)| < 1$, $r^{N-1}\phi_1(u') \in C^1[R_1, R_2]$, and satisfies (4). Let $X = C[R_1, R_2]$ with the norm $||u||_\infty := \max\limits_{s\in[R_1,R_2]} |u(s)|$. Let $E = \{u \in C^1[R_1, R_2] : u'(R_1) = u(R_2) = 0\}$ with the norm $||u|| := ||u'||_\infty$.

Let $\lambda_k(a, R_1)$ be the k-th eigenvalue of the eigenvalue problem

$$\begin{cases} -(r^{N-1}u')' = \lambda r^{N-1}a(r)u, & r \in (R_1, R_2), \\ u'(R_1) = u(R_2) = 0, \end{cases} \tag{5}$$

and φ_k be the eigenfunction corresponding to $\lambda_k(a, R_1)$. It is well-known that

$$0 < \lambda_1(a, R_1) < \lambda_2(a, R_1) < \cdots < \lambda_k(a, R_1) < \cdots \to +\infty \quad \text{as } k \to +\infty,$$

and no other eigenvalues. Moreover, the algebraic multiplicity of $\lambda_k(a, R_1)$ is 1, and the eigenfunction φ_k has exactly $k-1$ zeros in (R_1, R_2), see [21].

The first eigenvalue $\lambda_1(a, R_1)$ is the minimum of the Rayleigh quotient, namely,

$$\lambda_1(a, R_1) = \inf\left\{ \frac{\int_{R_1}^{R_2} s^{N-1}(u'(s))^2\,ds}{\int_{R_1}^{R_2} s^{N-1}a(s)(u(s))^2\,ds} : u \in H_0^1(R_1, R_2), \int_{R_1}^{R_2} s^{N-1}a(s)(u(s))^2\,ds > 0 \right\}.$$

Assume that:

(A1) $a : [R_1, R_2] \to [0,\infty)$ is a continuous function and $a(s) > 0$ for $s \in (R_1, R_2)$;

(F1) $f \in C([0,\infty), [0,\infty))$ with $f(s) > 0$ for $s > 0$;

(F2) there exist $f_0 \in (0,\infty)$, $\delta \in (0, \frac{R_2-R_1}{32})$ and $g \in C([0,\infty), [0,\infty))$ with $g(s) > 0$ for $s \in (0, \delta]$ such that

$$f(s) = f_0 s - g(s) \quad \text{for} \quad s \in [0, \delta],$$

where $\lim\limits_{s\to 0^+} \frac{g(s)}{s} = 0$;

(F3) there exists $s_0 : s_0 \in (\frac{R_2-R_1}{32}, \frac{3(R_2-R_1)}{32})$, such that

$$\min_{s\in[s_0,4s_0]} \frac{f(s)}{s} \geq \frac{64 f_0}{7\sqrt{7}\lambda_1(a, R_1)a_0} \cdot \eta_1,$$

where

$$a_0 = \min_{s \in [R_1, \frac{R_1+3R_2}{4}]} a(s),$$

η_1 is the first positive eigenvalue of the problem

$$\begin{cases} (r^{N-1}v'(r))' + \eta_1 r^{N-1}v(r) = 0, \quad r \in \left(R_1, \dfrac{R_1+3R_2}{4}\right), \\ v'(R_1) = v\left(\dfrac{R_1+3R_2}{4}\right) = 0. \end{cases}$$

The main result of this paper is the following.

Theorem 1.1. Assume that (A1) and (F1)-(F3) hold. Then there exist $\lambda_\star \in (0, \frac{\lambda_1(a,R_1)}{f_0})$ and $\lambda^\star > \frac{\lambda_1(a,R_1)}{f_0}$ such that
(i) (3) has at least one radial positive solution if $\lambda = \lambda_\star$;
(ii) (3) has at least two radial positive solutions if $\lambda_\star < \lambda \leq \frac{\lambda_1(a,R_1)}{f_0}$;
(iii) (3) has at least three radial positive solutions if $\frac{\lambda_1(a,R_1)}{f_0} < \lambda < \lambda^\star$;
(iv) (3) has at least two radial positive solutions if $\lambda = \lambda^\star$;
(v) (3) has at least one radial positive solution if $\lambda > \lambda^\star$;
(vi) $\lim_{\lambda\to\infty} \|u\|_\infty = R_2 - R_1$ and $\lim_{\lambda\to\infty} \|u\| = 1$.

Remark 1.1. Let (λ, u) be a solution of (4), then it follows from $|u'(r)| < 1$ that

$$||u||_\infty < R_2 - R_1.$$

This leads to the bifurcation diagrams mainly depend on the behavior of $f = f(s)$ near $s = 0$. This is a significant difference between the Minkowski-curvature problems and the p-Laplacian problems.

Remark 1.2. In the special case $p = 2$, (F1') reduces to

$$f(s) = f_0 s - f_1 s^{1+\alpha} \quad \text{for} \quad s \in [0, \chi], \tag{6}$$

where $\chi > 0$ is a sufficiently small constant. It is easy to see that condition (F2) is weaker than (6), in fact $f_1 s^{1+\alpha}$ is a special case of $g(s)$.

The main result is obtained by reducing the problem (4) to an equivalent problem and use the Rabinowitz global bifurcation techniques [22]. Indeed, under (F1) and (F2) we get an unbounded connected component which is bifurcating from $(\frac{\lambda_1(a,R_1)}{f_0}, 0)$, and condition (F2) pushes the bifurcation to the right near $u = 0$. Condition (F3) leads the bifurcation curve to the left at some point, and finally to the right near $\lambda = \infty$.

For other results concerning the existence of an S-shaped connected component in the set of solutions for diverse boundary value problems, see [23-26] for the semilinear boundary value problems, and [27] for the p-Laplacian boundary value problems.

The rest of the paper is organized as follows. In Section 2, we give an equivalent formulation of problem (4) and some preliminary results to show the change of direction of a bifurcation. Section 3 is devoted to proving the main result.

2 Some preliminary results

2.1 An equivalent formulation

Let us define a function $\widetilde{f} : \mathbb{R} \to \mathbb{R}$ by setting

$$\widetilde{f}(s)=\begin{cases} f(s), & 0\le s\le R_2-R_1,\\ \text{linear}, & R_2-R_1<s<(R_2-R_1)+1,\\ 0, & s\ge (R_2-R_1)+1,\\ -\widetilde{f}(-s), & s<0.\end{cases} \tag{7}$$

Notice that, within the context of positive solutions, problem (4) is equivalent to the same problem with f replaced by $\widetilde{f}$. In the sequel, we shall replace f with $\widetilde{f}$, however, for the sake of simplicity, the modified function $\widetilde{f}$ will still be denoted by f. Next, let us define h as follows

$$h(s)=\begin{cases} (1-s^2)^{3/2}, & |s|\le 1,\\ 0, & |s|>1.\end{cases} \tag{8}$$

Lemma 2.1. A function $u\in C^1[R_1,R_2]$ is a positive solution of (4) if and only if it is a positive solution of the problem

$$\begin{cases} -(r^{N-1}u')'=r^{N-1}\left[\lambda a(r)f(u)h(u')-\dfrac{N-1}{r}u'^3\right], & r\in(R_1,R_2),\\ u'(R_1)=u(R_2)=0.\end{cases} \tag{9}$$

Proof. It is clear that a positive solution $u\in C^1[R_1,R_2]$ of (4) is a positive solution of (9). Conversely, assume that $u\in C^1[R_1,R_2]$ is a positive solution of (9). We aim to show that $\|u'\|_\infty<1$. Assume on the contrary that this is not true. Then we can easily find an interval $[c,d]\subseteq[R_1,R_2]$ such that, either $u'(c)=0$, $0<|u'(r)|<1$ in (c,d) and $|u'(d)|=1$, or $|u'(c)|=1$, $0<|u'(r)|<1$ in (c,d) and $u'(d)=0$. Assume the former case occurs. The function u satisfies the equation

$$(r^{N-1}\phi_1(u'))'+\lambda r^{N-1}a(r)f(u)=0$$

in $[c,d]$. For each $r\in(c,d)$, integrating over the interval $[c,r]$, we obtain

$$|\phi_1(u'(r))|=\left|\frac{1}{r^{N-1}}\int_c^r \lambda t^{N-1}a(t)f(u(t))dt\right|\le C_1$$

for some constant $C_1>0$ and hence

$$|u'(r)|\le \phi_1^{-1}(C_1)$$

for every $r\in[c,d]$. Since $\phi_1^{-1}(C_1)<1$, taking the limit as $r\to d^-$ we obtain the contradiction $|u'(d)|<1$. Therefore $\|u'\|_\infty<1$ and accordingly, u is a positive solution of (4). □

Lemma 2.2. Assume that (A1) and (F1) hold. Let u be a nontrivial solution of (4). Then $u>0$ on $[R_1,R_2)$ and u is strictly decreasing.

Proof. From

$$\phi_1(u')=-\frac{\lambda}{r^{N-1}}\int_{R_1}^r s^{N-1}a(s)f(u(s))ds, \tag{10}$$

it follows $u'\le 0$ because (A1) and (F1), so u is decreasing. Since $u(R_2)=0$, we have $u\ge 0$ on $[R_1,R_2]$. As u is not identically zero, one has $u(R_1)>0$ and, from (10) we deduce that $u'<0$ on $(R_1,R_2]$, which ensures that actually u is strictly decreasing and $u>0$ on $[R_1,R_2)$. □

Lemma 2.3. Assume that (A1) and (F1) hold. Let u be a positive solution of (4). Then

$$\frac{1}{4}\|u\|_\infty\le u(r)\le\|u\|_\infty,\quad r\in\left[R_1,\frac{R_1+3R_2}{4}\right].$$

Proof. Since $(r^{N-1}\phi_1(u'))'=-\lambda r^{N-1}a(r)f(u)$, condition (A1) and (F1) imply that $u'(r)$ is decreasing on (R_1,R_2). Since $u'(R_1)=u(R_2)=0$ and $u(r)>0$ on (R_1,R_2), we have $u'(R_2)<0$. Therefore, u is concave on (R_1,R_2).

Hence,

$$u(r) \geq \frac{||u||_\infty}{R_2 - R_1}(R_2 - r), \quad r \in [R_1, R_2].$$

Note that $\frac{R_2-r}{R_2-R_1} \geq \frac{1}{4}$ is equivalent to $r \leq \frac{R_1+3R_2}{4}$. Therefore, we get the conclusion. □

Next, we give some property of concave functions.

Lemma 2.4. Let $\nu \in (0,1)$ and $\beta_0 \in \left(0, \frac{(R_2-R_1)(1-\nu)}{8}\right)$ be given. Let $I_{\nu,\beta_0} := \left[R_1,\, R_2 - \frac{4\beta_0}{1-\nu}\right]$. Then

$$|u'(s)| \leq 1-\nu, \quad \forall\, u \in \mathcal{A},\ \forall\, s \in I_{\nu,\beta_0},$$

where

$$\mathcal{A} := \{u \in E \,|\, u \text{ is concave and strictly decreasing on } (R_1, R_2),\ u'(R_2) > -1,\ ||u||_\infty \leq 4\beta_0\}.$$

Proof. Assume on the contrary that for any $\left[R_1, R_1 + \frac{R_2-R_1}{n}\right]$, there exist sequence $u_n \in E$ with u_n is concave and strictly decreasing on (R_1, R_2),

$$u_n'(R_2) > -1, \quad ||u_n||_\infty \leq 4\beta_0,$$

and $x_n \in \left[R_1, R_1 + \frac{R_2-R_1}{n}\right]$, such that

$$|u_n'(x_n)| > 1-\nu,$$

that is to say

$$-u_n'(x_n) > 1-\nu.$$

From the concavity and monotonicity of u_n on (R_1, R_2), we have

$$-u_n'(x) > 1-\nu, \quad x \in [x_n, R_2].$$

Hence

$$u_n(x_n) = \int_{x_n}^{R_2} -u_n'(s)ds > (1-\nu)(R_2 - x_n) > (R_2 - R_1 - \frac{R_2-R_1}{n})(1-\nu) > \frac{R_2-R_1}{2}(1-\nu).$$

This is a contradiction since $||u||_\infty \leq 4\beta_0 < \frac{R_2-R_1}{2}(1-\nu)$.

And it is easy to see that $(1-\nu)(R_2-\theta) \leq 4\beta_0$, it follows that $\theta \geq R_2 - \frac{4\beta_0}{1-\nu}$, hence we can take $I_{\nu,\beta_0} = \left[R_1, R_2 - \frac{4\beta_0}{1-\nu}\right]$. □

If we let $\nu = \frac{1}{4}$ and $\beta_0 = \frac{3(R_2-R_1)}{64} \in (0, \frac{3(R_2-R_1)}{32})$. Then we have the following

Corollary 2.1. For any concave function $u \in E$ with

$$u'(R_2) > -1, \quad ||u||_\infty \leq \frac{3(R_2-R_1)}{16},$$

we have

$$|u'(r)| \leq \frac{3}{4}, \quad r \in \left[R_1, \frac{R_1+3R_2}{4}\right].$$

2.2 The direction of bifurcation

Let $h \in X$ be given. It is well-known that the solution u of problem

$$\begin{cases} -(r^{N-1}u')' = r^{N-1}h(r), \quad r \in (R_1, R_2), \\ u'(R_1) = u(R_2) = 0 \end{cases} \tag{11}$$

can be expressed by

$$u(r) = \int_{R_1}^{R_2} G(r, s)s^{N-1}h(s)ds := \mathcal{K}(h),$$

where the Green's function of (11) for $N = 2$ is explicitly given by

$$G(t, s) = \begin{cases} \ln \frac{R_2}{t}, & R_1 \le s \le t \le R_2, \\ \ln \frac{R_2}{s}, & R_1 \le t \le s \le R_2, \end{cases}$$

and the Green's function of (11) for $N \ge 3$ is explicitly given by

$$G(t, s) = \begin{cases} \frac{1}{2-N}[R_2^{2-N} - t^{2-N}], & R_1 \le s \le t \le R_2, \\ \frac{1}{2-N}[R_2^{2-N} - s^{2-N}], & R_1 \le t \le s \le R_2. \end{cases}$$

Let $\mathcal{L} : X \to E$ be defined by $\mathcal{L}(u) = \mathcal{K}(au)$. Both $\mathcal{K}$ and $\mathcal{L}$ are completely continuous and (5) is equivalent to

$$u = \lambda \mathcal{L}(u),$$

so that the eigenvalues of (5) are the characteristic values of $\mathcal{L}$.

If (F2) holds, then $\lim_{s\to 0^+} \frac{f(s)}{s} = f_0$. Moreover,

$$f(s) = \left(f_0 - \frac{g(s)}{s}\right)s,$$

where $g(s)$ is a continuous function and

$$\lim_{s\to 0} \frac{g(s)}{s} = 0. \tag{12}$$

Let us set, for convenience, $k(y) = h(y) - 1$ for $y \in \mathbb{R}$. We have

$$\lim_{y\to 0} \frac{k(y)}{y} = 0. \tag{13}$$

Define the operator $\mathcal{H} : \mathbb{R} \times E \to E$ by

$$\mathcal{H}(\lambda, u) = \mathcal{L}\left(\lambda\big((f_0 - \frac{g(u)}{u})k(u') - \frac{g(u)}{u}\big)u - v(\cdot)u'^3\right),$$

where $v(r) = \frac{N-1}{r}$. Clearly, $\mathcal{H}$ is completely continuous and, by (12) and (13), we have

$$\lim_{\|u\|\to 0} \frac{\|\mathcal{H}(\lambda, u)\|}{\|u\|} = 0$$

uniformly with respect to λ varying in bounded intervals. Observe that, for any λ, $(\lambda, u) \in \mathbb{R} \times E$, with $u > 0$, is a solution of the equation

$$u = \lambda f_0 \mathcal{L}(u) + \mathcal{H}(\lambda, u), \tag{14}$$

if and only if u is a positive solution of (9). Denote by $\mathcal{S}$ the closure in $\mathbb{R}\times E$ of the set of all non-trivial solutions (λ, u) of (14) with $\lambda > 0$. Let $P = \{u \in E : u(r) \ge 0, r \in [R_1, R_2]\}$. Then P is a positive cone of E and $\mathrm{int}P \ne \emptyset$.

Notice that

$$\|u'\|_\infty < 1, \quad (\lambda, u) \in \mathcal{S},$$

which implies that

$$\|u\|_\infty < R_2 - R_1, \quad (\lambda, u) \in \mathcal{S}.$$

Hence, by Theorem 1.3 in [22] or Theorem 1.1 in [18], we have the following result.

Lemma 2.5. Assume that (A1), (F1) and (F2) hold. Then there exists an unbounded connected component $\mathcal{C}$ in $\mathcal{S}$ which is bifurcating from $(\frac{\lambda_1(a,R_1)}{f_0}, 0)$ such that $\mathcal{C} \subseteq (([0, +\infty) \times \text{int}P) \cup \{(\frac{\lambda_1(a,R_1)}{f_0}, 0)\})$. Moreover, $\mathcal{C}$ joins $(\frac{\lambda_1(a,R_1)}{f_0}, 0)$ with infinity in λ direction.

We next give a Sturm-type comparison theorem.

Lemma 2.6. ([28, Lemma 2.1], [29, Lemma 4.1]) Let $b_2(r) > b_1(r) > 0$ for $r \in (R_1, R_2)$ and $b_i \in L^\infty(R_1, R_2)$, $i = 1, 2$. Also let u_1, u_2 be solutions of

$$-(r^{N-1}u_i')' = r^{N-1}b_i(r)u_i,$$

respectively. If u_1 has k zeros in (R_1, R_2), then u_2 has at least $k + 1$ zeros in (R_1, R_2).

Lemma 2.7. Assume that (A1), (F1) and (F2) hold. Let $\{(\lambda_n, u_n)\}$ be a sequence of positive solutions of (14) which satisfies $\|u_n\| \to 0$ and $\lambda_n \to \frac{\lambda_1(a,R_1)}{f_0}$. Let $\varphi_1(r)$ be the first eigenfunction of (5) which satisfies $\|\varphi_1\| = 1$. Then there exists a subsequence of $\{u_n\}$, again denoted by $\{u_n\}$, such that $\frac{u_n}{\|u_n\|}$ converges uniformly to φ_1 on $[0, 1]$.

Proof. Let $v_n := \frac{u_n}{\|u_n\|}$. Then $\|v_n\| = \|v_n'\|_\infty = 1$, consequently, $\|v_n\|_\infty$ is bounded. By the Ascoli-Arzela theorem, there exists a subsequence of v_n which uniformly converges to $v \in X$. We again denote the subsequence by v_n. For any (λ_n, u_n), we have

$$u_n(r) = \int_{R_1}^{R_2} G(r, s)s^{N-1}\Big[\lambda_n a(s)f(u_n(s))h(u_n'(s)) - \frac{N-1}{s}(u_n'(s))^3\Big]ds. \tag{15}$$

Multiplying both sides of (15) by $\|u_n\|^{-1}$, we have

$$v_n(r) = \int_{R_1}^{R_2} G(r, s)s^{N-1}\Big[\lambda_n a(s)\frac{f(u_n(s))}{u_n(s)}h(u_n'(s))v_n(s) - \frac{N-1}{s}\frac{(u_n'(s))^3}{\|u_n\|}\Big]ds.$$

Since $\|u_n\| \to 0$ implies $\|u_n\|_\infty \to 0$. From (F2) and (8), we conclude that $\frac{f(u_n(s))}{u_n(s)} \to f_0$ and $h(u_n'(s)) \to 1$ as $n \to \infty$ uniformly for $s \in [0, 1]$. By Lebesgue's dominated convergence theorem we know that

$$v(r) = \frac{\lambda_1(a, R_1)}{f_0} \int_{R_1}^{R_2} G(r, s)s^{N-1}f_0 a(s)v(s)ds,$$

which means that v is a nontrivial solution of (5) with $\lambda = \lambda_1(a, R_1)$, and hence $v \equiv \varphi_1$. □

Lemma 2.8. Assume that (A1), (F1) and (F2) hold. Let $\mathcal{C}$ be as in Lemma 2.5. Then there exists $\sigma > 0$ such that $(\lambda, u) \in \mathcal{C}$ and $|\lambda - \frac{\lambda_1(a,R_1)}{f_0}| + ||u|| \le \sigma$ imply $\lambda > \frac{\lambda_1(a,R_1)}{f_0}$.

Proof. By (F1) and (F2), there exists a sufficiently small $\delta > 0$ such that

$$0 \le f(s) \le f_0 s \ \text{ for } \ s \in [0, \delta].$$

This fact together with the definition of h, we have

$$0 \le \frac{f(s)}{s}h < f_0 \ \text{ in } \ (0, \delta). \tag{16}$$

Fixed $\sigma = \frac{\delta}{R_2 - R_1}$, then for any $(\lambda, u) \in \mathcal{C}$ which satisfies $|\lambda - \frac{\lambda_1(a,R_1)}{f_0}| + ||u|| \le \sigma$, we known u is a positive solution of the problem

$$\begin{cases} -(r^{N-1}u')' = r^{N-1}\Big[\lambda a(r)f(u)h(u') - \frac{N-1}{r}u'^3\Big], & r \in (R_1, R_2), \\ u'(R_1) = u(R_2) = 0, \end{cases} \tag{17}$$

and

$$0 \le u \le \delta.$$

Assume that there exists a sequence $\{(\lambda_n, u_n)\}$ such that $(\lambda_n, u_n) \in \mathcal{C}$, $|\lambda_n - \frac{\lambda_1(a,R_1)}{f_0}| + ||u_n|| \le \sigma$, $\lambda_n \to \lambda$ and $||u_n|| \to 0$. Let $v_n = \frac{u_n}{||u_n||}$. By Lemma 2.7, there exists a subsequence of v_n, again denoted by v_n, such that v_n converges uniformly to v on $[R_1, R_2]$, where $v > 0$ and satisfies $||v|| = 1$. Dividing both sides of the equation of (17) with $(\lambda, u) = (\lambda_n, u_n)$ by $||u_n||$, we obtain

$$\begin{cases} -(r^{N-1}v_n')' = r^{N-1}\left[\lambda_n a(r)\dfrac{f(u_n)}{u_n}h(u_n')v_n - \dfrac{N-1}{r}v_n'(u_n')^2\right], & r \in (R_1, R_2), \\ v_n'(R_1) = v_n(R_2) = 0. \end{cases}$$

Since $\|u_n\| \to 0$ implies $\|u_n\|_\infty \to 0$. Combining this fact with (16), we have

$$-(r^{N-1}v')' < \lambda r^{N-1}a(r)f_0 v,$$

and

$$v'(R_1) = v(R_2) = 0.$$

On the other hand

$$\begin{cases} -(r^{N-1}\varphi_1')' = \lambda_1(a, R_1)r^{N-1}a(r)\varphi_1, & r \in (R_1, R_2), \\ \varphi_1'(R_1) = \varphi_1(R_2) = 0, \end{cases}$$

therefore

$$\begin{aligned} \lambda_1(a, R_1)\int_{R_1}^{R_2} s^{N-1}a(s)\varphi_1(s)v(s)ds &= \int_{R_1}^{R_2} s^{N-1}v'(s)\varphi_1'(s)ds \\ &< \lambda f_0 \int_{R_1}^{R_2} s^{N-1}a(s)\varphi_1(s)v(s)ds, \end{aligned}$$

that is $\lambda_1(a, R_1) - \lambda f_0 < 0$, and accordingly, $\lambda > \frac{\lambda_1(a,R_1)}{f_0}$. □

Lemma 2.9. Assume that (A1), (F1) and (F2) hold. Let $\mathcal{C}$ be as in Lemma 2.5. Then there exists $\lambda_\diamond > 0$ such that $\mathrm{Proj}_{\mathbb{R}}\mathcal{C} = [\lambda_\diamond, \infty) \subset (0, \infty)$.

Proof. Suppose on the contrary that $\lambda_\diamond = 0$. Then there exists a sequence $\{(\mu_n, u_n)\} \subset \mathcal{C}$ satisfying $u_n > 0$ such that

$$\lim_{n\to\infty}(\mu_n, u_n) = (0, u^*) \ \text{in} \ \mathbb{R}\times X$$

for some $u^* \ge 0$. Then by the fact

$$-(r^{N-1}\phi_1(u_n'))' = \mu_n r^{N-1}a(r)f(u_n), \ \ r \in (R_1, R_2), \ \ u_n'(R_1) = u_n(R_2) = 0,$$

after taking a subsequence and relabeling, if necessary, we have $u_n \to 0$.

From the fact $u_n'(R_1) = 0$, it follows that

$$\phi_1(u_n'(r)) = -\frac{\mu_n}{r^{N-1}}\int_{R_1}^{r} s^{N-1}a(s)f(u_n(s))ds \ \ \text{for} \ \ r \in (R_1, R_2).$$

This fact together with $f(0) = 0$ yield that

$$\lim_{n\to\infty}\|u_n'\|_\infty = 0. \tag{18}$$

On the other hand

$$\begin{cases} -(r^{N-1}u_n')' = r^{N-1}\left[\mu_n a(r)f(u_n)h(u_n') - \dfrac{N-1}{r}u_n'^3\right], & r \in (R_1, R_2), \\ u_n'(R_1) = u_n(R_2) = 0. \end{cases}$$

Let, for all n, $v_n = \frac{u_n}{||u_n||}$. Then we have

$$\begin{cases} -(r^{N-1}v_n'(r))' = r^{N-1}\Big[\mu_n a(r)\dfrac{f(u_n(r))}{u_n(r)}h(u_n'(r))v_n(r) - \dfrac{N-1}{r}\dfrac{(u_n'(r))^3}{||u_n||}\Big], \quad r \in (R_1, R_2), \\ v_n'(R_1) = v_n(R_2) = 0. \end{cases} \tag{19}$$

Similar to the proof of Lemma 2.7, by (18) and (19), it concludes that $\mu_n \to \frac{\lambda_1(a,R_1)}{f_0}$, which contradicts $\mu_n \to 0$. □

Lemma 2.10. Assume that (A1), (F1) and (F3) hold. Let $(\lambda, u) \in \mathcal{C}$ with $||u||_\infty = 4s_0$. Then $\lambda < \frac{\lambda_1(a,R_1)}{f_0}$.

Proof. Let $(\lambda, u) \in \mathcal{C}$. Then by Lemma 2.3, we obtain

$$s_0 \le u(r) \le 4s_0, \quad r \in \Big[R_1, \frac{R_1+3R_2}{4}\Big]. \tag{20}$$

Fixed $s_0 = \frac{3(R_2-R_1)}{64}$, then from Corollary 2.1, for any $(\lambda, u) \in \mathcal{C}$ with $||u||_\infty = 4s_0$, we have

$$0 \le |u'(r)| \le \frac{3}{4}, \quad r \in \Big[R_1, \frac{R_1+3R_2}{4}\Big].$$

Now we assume on the contrary that $\lambda \ge \frac{\lambda_1(a,R_1)}{f_0}$. Then for $r \in \big[R_1, \frac{R_1+3R_2}{4}\big]$, by (20) and (F3), we have

$$\lambda a(r)\frac{f(u(r))h(u'(r))}{u(r)} \ge \frac{\lambda_1(a,R_1)}{f_0}\cdot a_0 \cdot \frac{64f_0\eta_1}{7\sqrt{7}\lambda_1(a,R_1)a_0}\cdot\frac{7\sqrt{7}}{64} = \eta_1,$$

where η_1 is the first positive eigenvalue of the problem

$$\begin{cases} (r^{N-1}v'(r))' + \eta_1 r^{N-1}v(r) = 0, \quad r \in \Big(R_1, \dfrac{R_1+3R_2}{4}\Big), \\ v'(R_1) = v\big(\dfrac{R_1+3R_2}{4}\big) = 0. \end{cases}$$

Let v be the corresponding eigenfunction of η_1. Then

$$v(r) > 0, \quad r \in \Big[R_1, \frac{R_1+3R_2}{4}\Big].$$

We notice that u is a solution of

$$-(r^{N-1}u'(r))' = r^{N-1}\Big[\lambda a(r)\frac{f(u(r))h(u'(r))}{u(r)}u(r) - \frac{N-1}{r}u'^3(r)\Big]$$

on $\big[R_1, \frac{R_1+3R_2}{4}\big]$. Lemmas 2.2 and 2.6 imply that u has at least one zero on $\big[R_1, \frac{R_1+3R_2}{4}\big]$. This contradicts the fact that $u(r) > 0$ on $\big[R_1, \frac{R_1+3R_2}{4}\big]$. □

Lemma 2.11. Assume that (A1) and (F1) hold. Let $\mathcal{C}$ be as in Lemma 2.5. Then $\lim\limits_{(\lambda,u)\in\mathcal{C},\lambda\to\infty} ||u|| = 1$ and $\lim\limits_{(\lambda,u)\in\mathcal{C},\lambda\to\infty} ||u||_\infty = R_2 - R_1$.

Proof. We divide the proof into four steps.

Step 1. We claim that there exists a constant $B_0 > 0$ such that for every $(\lambda, u) \in \mathcal{C}$, if $\lambda \ge B_0$, then

$$||u||_\infty \ge \rho_\star$$

for some $\rho_\star > 0$.

Suppose on the contrary that there exists a sequence $\{(\mu_n, u_n)\} \subset \mathcal{C}$ satisfying

$$(\mu_n, u_n) \to (\infty, 0) \text{ in } (0, +\infty) \times X.$$

Then, similar to the proof of Lemma 2.9, we have u_n' converges to 0 as $n \to \infty$. From this fact and (19), after taking a subsequence and relabeling, if necessary, we have $v_n \to v_\star$ in X for some $v_\star \in X$, and,

$$\begin{cases} -(r^{N-1}v_\star'(r))' = r^{N-1}\mu_n a(r) f_0 v_\star(r), & r \in (R_1, R_2), \\ v_\star'(R_1) = v_\star(R_2) = 0, \end{cases}$$

that is $\mu_n \to \frac{\lambda_1(a,R_1)}{f_0}$. This contradicts with the fact $\mu_n \to \infty$. Therefore, the claim is valid.

Step 2. Fixed $\varepsilon \in (0, \frac{R_2-R_1}{4})$, we can show that there exists $\gamma > 0$ such that for every $(\lambda, u) \in \mathcal{C}$ with $\lambda > B_0$,

$$\min_{x\in[R_1,R_2-\varepsilon]} u(x) \geq \gamma\rho_\star. \tag{21}$$

In fact, it is an immediate consequence of the fact

$$u(r) = \int_{R_1}^{R_2} G(r,s)s^{N-1}\left[\lambda a(s)f(u(s))h(u'(s)) - \frac{N-1}{s}u'^3(s)\right]ds$$

and

$$G(r,s) \geq \gamma G(s,s), \quad (r,s) \in [R_1, R_2-\varepsilon] \times [R_1, R_2].$$

Step 3. It follows from (21) and (F1), there exists some constant $M_0 > 0$ such that

$$f(u(r)) \geq M_0 > 0 \quad \text{for} \quad r \in [R_1, R_2-\varepsilon],$$

and accordingly,

$$\lim_{\lambda\to\infty} \frac{\lambda}{r^{N-1}} \int_{R_1}^{r} s^{N-1}a(s)f(u(s))ds = +\infty \ \text{ uniformly in } \ r \in [R_1+\varepsilon_1, R_2-\varepsilon]$$

for arbitrary fixed $\varepsilon_1 \in (0, \frac{R_2-\varepsilon-R_1}{4})$. This together with (A1), (F1) and the relation

$$u'(r) = -\phi_1^{-1}\left(\frac{\lambda}{r^{N-1}} \int_{R_1}^{r} s^{N-1}a(s)f(u(s))ds\right)$$

imply that

$$u' \to -1 \ \text{ in } \ C[R_1+\varepsilon_1, R_2-\varepsilon] \ \text{ as } \ \lambda \to +\infty. \tag{22}$$

Therefore, by the arbitrariness of ε and ε_1, we get $\lim_{(\lambda,u)\in\mathcal{C},\lambda\to\infty} ||u|| = 1$.

Step 4. Since

$$-u'(r) \geq 0, \quad r \in (R_1, R_2].$$

This fact together with (22) imply that for $(\lambda, u) \in \mathcal{C}$,

$$\lim_{\lambda\to\infty} ||u||_\infty = \lim_{\lambda\to\infty} u(R_1) = \lim_{\lambda\to\infty} \int_{R_1}^{R_2} -u'(s)ds \geq \lim_{\lambda\to\infty} \int_{R_1+\varepsilon_1}^{R_2-\varepsilon} -u'(s)ds = R_2 - \varepsilon - R_1 - \varepsilon_1. \tag{23}$$

By the arbitrariness of ε and ε_1, we have

$$\lim_{\lambda\to\infty} ||u||_\infty \geq R_2 - R_1. \tag{24}$$

On the other hand

$$u(R_1) = \int_{R_1}^{R_2} -u'(s)ds \leq R_2 - R_1. \tag{25}$$

Therefore, from (24) and (25), we have

$$\lim_{\lambda\to\infty} ||u||_\infty = R_2 - R_1.$$

□

3 Proof of the main result

In this section, we shall prove Theorem 1.1. We divide the proof into three steps.

Step 1. Rightward bifurcation.

By Lemmas 2.5 and 2.8, there exists an unbounded connected component $\mathcal{C}$ in the set of radial positive solutions of (3), which is bifurcating from $(\frac{\lambda_1(a,R_1)}{f_0}, 0)$, and for any $(\lambda, u) \in \mathcal{C}$ which satisfies $|\lambda - \frac{\lambda_1(a,R_1)}{f_0}| + ||u|| \leq \sigma$, where $\sigma > 0$ is a sufficiently small constant, $\mathcal{C}$ goes rightward. Moreover, from Lemma 2.11, it follows that $\mathcal{C}$ joins $(\frac{\lambda_1(a,R_1)}{f_0}, 0)$ with infinity in λ direction.

Step 2. Direction turn of bifurcation.

By Lemma 2.11 again, we have $\lim_{(\lambda,u)\in\mathcal{C},\lambda\to\infty} ||u|| = 1$ and $\lim_{(\lambda,u)\in\mathcal{C},\lambda\to\infty} ||u||_\infty = R_2 - R_1$. Then there exists $(\lambda_0, u_0) \in \mathcal{C}$ such that $\|u_0\|_\infty = 4s_0$. Lemma 2.10 implies that $\mathcal{C}$ goes leftward.

Step 3. Existence of $\lambda_\star$ and $\lambda^\star$.

By Lemma 2.9, if $(\lambda, u) \in \mathcal{C}$, then there exists $\lambda_\diamond > 0$ such that $\lambda \geq \lambda_\diamond$. And by Lemmas 2.8, 2.10 and 2.11 again, $\mathcal{C}$ passes through some points $(\frac{\lambda_1(a,R_1)}{f_0}, v_1)$ and $(\frac{\lambda_1(a,R_1)}{f_0}, v_2)$ with $\|v_1\|_\infty < 4s_0 < \|v_2\|_\infty$, and there exists $\underline{\lambda}$ and $\bar{\lambda}$ which satisfy $0 < \underline{\lambda} < \frac{\lambda_1(a,R_1)}{f_0} < \bar{\lambda}$ and both (i) and (ii):

(i) if $\lambda \in (\frac{\lambda_1(a,R_1)}{f_0}, \bar{\lambda}]$, then there exist u and v such that $(\lambda, u), (\lambda, v) \in \mathcal{C}$ and $\|u\|_\infty < \|v\|_\infty < 4s_0$;
(ii) if $\lambda \in [\underline{\lambda}, \frac{\lambda_1(a,R_1)}{f_0}]$, then there exist u and v such that $(\lambda, u), (\lambda, v) \in \mathcal{C}$ and $\|u\|_\infty < 4s_0 < \|v\|_\infty$.

Define $\lambda^\star = \sup\{\bar{\lambda} : \bar{\lambda}$ satisfies (i)$\}$ and $\lambda_\star = \inf\{\underline{\lambda} : \underline{\lambda}$ satisfies (ii)$\}$. Then by the standard argument, (3) has a radial positive solution at $\lambda = \lambda_\star$ and $\lambda = \lambda^\star$, respectively. This completes the proof. □

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