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**Research Article**

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# Research on cooperation strategy between government and green supply chain based on differential game

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**Abstract:** Based on the "three bottom line" and stakeholder theory, the paper considers the relationship and cooperation strategy between the government and the supplier and manufacturer of the green supply chain. By constructing the dynamic differential game model, the paper discusses the differences in the optimal effort level, green degree of product, reputation and the optimal benefit under the three situations of non-cooperation, government promotion and collaborative cooperation. The results show that the optimal effort level, green degree of product, reputation and the optimal benefit in collaborative cooperation are obviously higher than the situations of non-cooperation and government promotion, and the cooperation of the three parties can promote the development of green supply chain. Government promotion is better than non-cooperation. The government plays an active role in improving the optimal benefit and reputation of green supply chain. Finally, the reliability of the proposed proposition is verified by an example analysis, which provides an important reference for improving the efficiency of green supply chain.

**Keywords:** green supply chain; government role; differential game; cooperation strategy

**MSC:** 03-XX, 03Cxx

## 1 Introduction

China's economic transformation is taking place. It has been recognized that the development of traditional rough type has caused serious environmental pollution and ecological damage [1]. The three bottom line (TBL) theory holds that enterprise development must take into account economic, environmental and social benefits [2]. It provides a basis for the transformation and development of Chinese enterprises. Therefore, in the "13th Five-Year plan" , China put forward the new development concept of innovation, coordination, green, openness and sharing. The formation of green supply chain fits the new development concept. "Actively promoting the green supply chain and strengthening the green supervision of the supply chain" is also clearly required in the guidance of the Chinese government to actively promote the innovation and application of supply chain. However, the green supply chain of China is still in the development stage, and the relevant governance mechanism is not perfect. Therefore, based on stakeholder theory, this paper discusses the cooperation strategy between the government and the green supply chain, and considers the

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differences between the government and the green supply chain in different game situations, which will help to improve the government's governance of the green supply chain.

Green Supply Chain Management (GSCM) has become a hot topic in academic circles, and many scholars have done related research [3, 4]. Among them, the green supply chain coordination, consumer willingness, government subsidies have attracted the attention of scholars. With regard to the coordination of green supply chain, Huang et al. [5] use game theory to study the relationship between green supply chain coordination and greenhouse gas emission reduction, and find that product line design, supplier selection, transport mode selection and pricing strategy will affect greenhouse gas emissions. Price and green level sensitivity will affect the coordination of green supply chain, and the cost sharing contract in green supply chain has a positive impact on improving product green level [6]. The development strategy selection of green supply chain is analyzed by differential game model, and it is found that centralized decision can be achieved through contract design to achieve the coordination of green supply chain [7]. On the study of the consumer behavior of the green supply chain, consumers' green sensitivity plays an important role in the green supply chain of smart phones, and consumers are highly sensitive to the green level of smart phones [8]. Wang et al. [9] find that the cost driven elements and consumer driven elements play an important role in the management practice of the green supply chain, and the role of consumer driven for small companies is more obvious. At the government level, Sheu [10] believes the government's fiscal intervention will have a significant impact on the strategy choices of members of the green supply chain, and government intervention may have a negative effect on corporate profits and social welfare. Yang and Xiao [11] discuss the relationship between product pricing and green level in green supply chain under government intervention through three game models, it is found that when government intervention increases, the level of green will rise, but government intervention is not always conducive to manufacturers of green supply chains. Sinayi and Rasti-Barzoki [12] construct a two-level model of government and supply chain, analyze the role of government in the green development of supply chain, and point out that different government policies have a significant effect on the profit and environment of supply chain members. Guo et al. [13] compare the impact of different government subsidy policies (subsidies for manufacturers' green efforts and price subsidies to green products) on the green manufacturing of supply chain, and find that the choice of government subsidy policy depends on the consumer sensitivity to the price of green products. Government subsidies can increase the profits of green supply chain entities and promote the development of green supply chain [14].

To sum up, the studies of green supply chain by scholars have laid a solid foundation for subsequent research, but there are still have some knowledge gaps. First of all, the previous researches usually show the role of government as an impact parameter in consideration of the role of the government, and do not bring the government as a stakeholder into the research framework, which lead to a certain degree of deviation in the analysis of the role of the government. Secondly, the green supply chain should reflect the combination of economic, social and environmental benefits. Most of the previous studies consider the problem of the coordination of green supply chain from the perspective of maximizing the economic profit. Finally, the cooperation of the green supply chain is a dynamic process. The green degree of product and reputation of the supplier and manufacturer will change according to the change of the state variables, and it is a long-term process. The previous researches are mostly based on the static perspective, and the dynamic changes of the characteristics of the green supply chain are not fully considered. Therefore, based on the differential game theory, this paper constructs the dynamic game model between the government and the supplier and manufacturer of the green supply chain, and analyzes the three situations of the non-cooperation, the government promotion and the collaborative cooperation. We hope to reveal the specific role of the government in the green supply chain, and then provide suggestions for the development of green supply chain.

This paper is arranged as follows. Section 2 presents the problem description and research assumptions. In Section 3, three differential game models are formulated according to the three situations. Section 4 presents a comparison of the different situations. Simulations and example analysis are conducted in Section 5, and conclusions are presented in Section 6.

## 2 Problem description and research assumptions

A simple system is used to describe the relationship between the government and the supplier and manufacturer of the green supply chain. In the green supply chain, the supplier and the manufacturer constitute a two-level supply chain. The main body of the supply chain needs to improve and update the raw materials and equipment, so as to better satisfy the relevant conditions of green innovation. Supplier provide raw materials to manufacturer, and manufacturer provide green products to consumers through production, assembly and sales [7]. Due to the growing awareness of consumers' environmental protection, a preference for green products is formed [15]. The consumers play an important role in the game because their environmental awareness has a significant influence on market demand. The role of government in green supply chain is to provide a series of policies and subsidies to supplier and manufacturer. The government's green supervision will also promote the green production of the supply chain, further reduce energy consumption and environmental pollution. We assume that government behavior will encourage green supply chain to carry out green innovation and enhance the green degree of product. It is conducive to the unification of economic, environmental and social benefits [15].

In green supply chain, supplier and manufacturer will make efforts to enhance the green degree of product and reputation. Let $N_S(t)$ denotes the effort level of supplier at time $t$, and let $N_M(t)$ denotes the effort level of manufacturer at time $t$. According to the assumption of cost in previous studies [17, 18], we assume that the cost of upgrading the green degree of product and reputation is a convex function of effort. Therefore, the cost of supplier and manufacturer of green supply chain can be written as $C_S(t) = \frac{1}{2}\lambda_S N_S^2(t)$ and $C_M(t) = \frac{1}{2}\lambda_M N_M^2(t)$, where $\lambda_S$, $\lambda_M$ are the cost coefficients of the supplier and manufacturer at time $t$, respectively. As a stakeholder, the government will make corresponding efforts to improve the green degree of product and reputation of the supplier and manufacturer, because the government wants the green supply chain to develop rapidly. Let $N_{G1}(t)$ and $N_{G2}(t)$ denote the effort level of government for supplier and manufacturer. The cost of government can be written as $C_G(t) = \frac{\lambda_G}{2}\left[N_{G1}^2(t) + N_{G2}^2(t)\right]$, where $\lambda_G$ is the cost coefficient of the government at time $t$.

It is proposed that the promotion of product green degree is a time-varied dynamic process. It depends on the efforts of the supplier, manufacturer and government toward the promotion of product green degree. The efforts of green supply chain and government during the production process contribute to the promotion of product green degree, which evolves according to the Nerlove and Arrow model (1962). When there is no effort, the green degree of product will decay because the related equipment will depreciate with time. In addition, the green degree of the supplier's product can affect the green degree of the manufacturer's product. Let $D_S(t)$ and $D_M(t)$ respectively denote the product green degree of supplier and manufacturer at time $t$. The differential equation of the product green degree of supplier can be expressed as

$$\dot{D_S}(t) = \alpha_1 N_S(t) + \beta_1 N_{G1}(t) - \eta_1 D_S(t) \tag{1}$$

For the manufacturer, the green degree of the product supplied by the supplier will affect the green degree of the manufacturer's product to a certain extent. According to formula (1), the differential equation of green degree of manufacturer's product can be expressed as

$$\begin{aligned} \dot{D_M}(t) &= \alpha_2 N_M(t) + \beta_2 N_{G2}(t) - \eta_2 D_M(t) + \mu\, \dot{D_S}(t) \\ &= \mu\left[\alpha_1 N_S(t) + \beta_1 N_{G1}(t) - \eta_1 D_S(t)\right] + \alpha_2 N_M(t) + \beta_2 N_{G2}(t) - \eta_2 D_M(t) \end{aligned} \tag{2}$$

where $\alpha_1$ and $\alpha_2$ are respectively the influence coefficient of supplier and manufacturer's effort level on product green degree, $\beta_1$ and $\beta_2$ are respectively the influence coefficient of government effort level on product green degree of green supply chain. $\eta_1$ and $\eta_2$ are respectively the attenuation coefficient of product green degree for supplier and manufacturer. $\mu$ is the promotion coefficient of supplier product to the green degree of manufacturer product. In the initial state, $D_S(0) = d_1 \geq 0$, $D_M(0) = d_2 \geq 0$.

Considering that green products are environmentally friendly and can be recognized by consumers, the promotion of reputation will also benefit the green supply chain and the whole system. Based on this, the consideration of reputation effect will further improve the research of green supply chain. Reputation also depends on the efforts of green supply chains and government [19]. Similar to the product green degree, reputation also decays over time without effort. Let $Q_S(t)$ and $Q_M(t)$ respectively denote the reputation of supplier and manufacturer at time $t$. The differential equation for the reputation of supplier and manufacturer can be written as

$$\dot{Q}_S(t) = \delta_1 N_S(t) + \varphi_1 N_{G1}(t) - \gamma_1 Q_S(t) \tag{3}$$

$$\dot{Q}_M(t) = \delta_2 N_M(t) + \varphi_2 N_{G2}(t) - \gamma_2 Q_M(t) \tag{4}$$

where $\delta_1$ and $\delta_2$ are respectively the influence coefficient of supplier and manufacturer effort level on reputation, $\varphi_1$ and $\varphi_2$ are respectively the influence coefficient of government effort level on reputation of green supply chain. $\gamma_1$ and $\gamma_2$ are respectively the attenuation coefficient of reputation for supplier and manufacturer. In the initial state, $Q_S(0) = q_1 \geq 0$ , $Q_M(0) = q_2 \geq 0$ .

Let $\pi_S(t)$ and $\pi_M(t)$ denote the benefits (including economic benefit, environmental benefit and social benefit) of supplier and manufacturer at time $t$. As we all know, the benefits come from the efforts of manufacturer, supplier and government, the green dgree of products and reputation. According to literature [15], the benefit function of supplier and manufacturer can be expressed as

$$\pi_S(t) = p_1 N_S(t) + k_1 N_{G1}(t) + l_1 D_S(t) + b_1 Q_S(t) \tag{5}$$

$$\pi_M(t) = p_2 N_M(t) + k_2 N_{G2}(t) + l_2 D_M(t) + b_2 Q_M(t) \tag{6}$$

where $p_1$ and $p_2$ are the influence coefficient of supplier and manufacturer effort level on benefit, $k_1$ and $k_2$ are the influence coefficient of government effort level on benefit of green supply chain. $l_1$ and $l_2$ are the influence coefficient of product green degree on benefit, $b_1$ and $b_2$ are the influence coefficient of reputation on benefit.

# 3 Differential game equilibrium analysis

Cooperation between government and green supply chain is a long-term dynamic process, and the effects of the supplier, manufacturer and government can be inter-temporal. Supplier, manufacturer and government are far sighted, they seek benefit maximization in the long run. It is useful to introduce the dynamic framework into the research on cooperation between government and green supply chain. Therefore, we use the differential game to analyze this problem and discuss the differences under the three situations of non-cooperation, government promotion and collaborative cooperation.

## 3.1 Situation of non-cooperation

In the situation of non-cooperation, both supplier and manufacturer will independently decide their level of effort to maximize their own benefit. The only regulatory mechanism in this situation is based on the market. Namely, the government will supervise green supply chain and provide corresponding efforts. However, the government does not subsidize the cost of product development of supplier and manufacturer of green supply chain. The benefit of green supply chain will be allocated between the government and the green supply chain. Let $\theta_i\ (0 \leq \theta_i \leq 1, i = 1, 2)$ respectively denote the benefit coefficient of the government from supplier and manufacturer, let $1 - \theta_1$ denotes the benefit coefficient of the supplier, let $1 - \theta_2$ denotes the benefit coefficient of the manufacturer. $\lambda_S$ , $\lambda_M$ and $\lambda_G$ are the cost coefficients of the supplier, manufacturer and government at time $t$, respectively. We assume that the discount rate $r$ is the same. In this situation, the objective functions (net benefit) of the supplier, manufacturer and government be expressed as

$$\prod_S = \int_0^{\infty} e^{-rt} \left\{ (1-\theta_1)\left[p_1 N_S(t) + k_1 N_{G1}(t) + l_1 D_S(t) + b_1 Q_S(t)\right] - \frac{1}{2}\lambda_S N_S^2(t) \right\} dt \tag{7}$$

$$\prod_M = \int_0^{\infty} e^{-rt} \left\{ (1-\theta_2)\left[p_2 N_M(t) + k_2 N_{G2}(t) + l_2 D_M(t) + b_2 Q_M(t)\right] - \frac{1}{2}\lambda_M N_M^2(t) \right\} dt \tag{8}$$

$$\begin{aligned} \prod_G = \int_0^{\infty} e^{-rt} \Big\{ & \theta_1\left[p_1 N_S(t) + k_1 N_{G1}(t) + l_1 D_S(t) + b_1 Q_S(t)\right] - \frac{1}{2}\lambda_G N_{G1}^2(t) \\ & + \theta_2\left[p_2 N_M(t) + k_2 N_{G2}(t) + l_2 D_M(t) + b_2 Q_M(t)\right] - \frac{1}{2}\lambda_G N_{G2}^2(t) \Big\} dt \end{aligned} \tag{9}$$

For convenience in writing and understanding, time $t$ is omitted in the next analysis [20].

**Proposition 1** In the situation of non-cooperation, the feedback non-cooperative game Nash equilibria are

$$N_S^{\star} = \frac{(1-\theta_1)\left[p_1 + \dfrac{\alpha_1 l_1}{\eta_1 + r} + \dfrac{\delta_1 b_1}{\gamma_1 + r}\right]}{\lambda_S} \tag{10}$$

$$N_M^{\star} = \frac{(1-\theta_2)\left[p_2 + \dfrac{\alpha_2 l_2}{\eta_2 + r} + \dfrac{\delta_2 b_2}{\gamma_2 + r}\right]}{\lambda_M} \tag{11}$$

$$N_{G1}^{\star} = \frac{\theta_1\left(k_1 + \dfrac{\beta_1 l_1}{\eta_1 + r} + \dfrac{\varphi_1 b_1}{\gamma_1 + r}\right) + \dfrac{\beta_1 \mu \theta_2 l_2 r}{(\eta_1 + r)(\eta_2 + r)}}{\lambda_G} \tag{12}$$

$$N_{G2}^{\star} = \frac{\theta_2\left(k_2 + \dfrac{\beta_2 l_2}{\eta_2 + r} + \dfrac{\varphi_2 b_2}{\gamma_2 + r}\right)}{\lambda_G} \tag{13}$$

**Proof** According to the optimal control theory and the sufficient conditions of the Nash equilibrium strategy, the three parties of the supplier, manufacturer of the green supply chain and government have benefit optimal value functions. $V_S(D_S, D_M, Q_S, Q_M)$, $V_M(D_S, D_M, Q_S, Q_M)$ and $V_G(D_S, D_M, Q_S, Q_M)$ respectively denote the benefit optimal value functions of the supplier, manufacturer and government. For all $D_i \geq 0$ and $Q_S \geq 0$, $i \in \{S, M\}$, the benefit optimal value functions satisfy the following Hamilton-Jacobi-Bellman equation

$$\begin{aligned} & rV_S(D_S, D_M, Q_S, Q_M) \\ = & \max_{N_S \geq 0} \Big\{ (1-\theta_1)\left[p_1 N_S + k_1 N_{G1} + l_1 D_S + b_1 Q_S\right] - \frac{1}{2}\lambda_S N_S^2 + \frac{\partial V_S}{\partial D_S}(\alpha_1 N_S + \beta_1 N_{G1} - \eta_1 D_S) \\ & + \frac{\partial V_S}{\partial D_M}\left[\mu(\alpha_1 N_S + \beta_1 N_{G1} - \eta_1 D_S) + \alpha_2 N_M + \beta_2 N_{G2} - \eta_2 D_M\right] \\ & + \frac{\partial V_S}{\partial Q_S}(\delta_1 N_S + \varphi_1 N_{G1} - \gamma_1 Q_S) + \frac{\partial V_S}{\partial Q_M}(\delta_2 N_M + \varphi_2 N_{G2} - \gamma_2 Q_M) \Big\} \end{aligned} \tag{14}$$

$$\begin{aligned} & rV_M(D_S, D_M, Q_S, Q_M) \\ = & \max_{N_M \geq 0} \Big\{ (1-\theta_2)\left[p_2 N_M + k_2 N_{G2} + l_2 D_M + b_2 Q_M\right] + \frac{\partial V_M}{\partial D_S}(\alpha_1 N_S + \beta_1 N_{G1} - \eta_1 D_S) \\ & + \frac{\partial V_M}{\partial D_M}\left[\mu(\alpha_1 N_S + \beta_1 N_{G1} - \eta_1 D_S) + \alpha_2 N_M + \beta_2 N_{G2} - \eta_2 D_M\right] - \frac{1}{2}\lambda_M N_M^2 \\ & + \frac{\partial V_M}{\partial Q_S}(\delta_1 N_S + \varphi_1 N_{G1} - \gamma_1 Q_S) + \frac{\partial V_M}{\partial Q_M}(\delta_2 N_M + \varphi_2 N_{G2} - \gamma_2 Q_M) \Big\} \end{aligned} \tag{15}$$

$$
\begin{aligned}
& rV_G\left(D_S, D_M, Q_S, Q_M\right) \\
&= \max_{\substack{N_S \geq 0 \\ N_M \geq 0}} \left\{\theta_1\left(p_1 N_S + k_1 N_{G1} + l_1 D_S + b_1 Q_S\right) + \theta_2\left(p_2 N_M + k_2 N_{G2} + l_2 D_M + b_2 Q_M\right)\right. \\
& -\frac{\lambda_G}{2}\left(N_{G1}^2 + N_{G2}^2\right) + \frac{\partial V_G}{\partial D_S}\left(\alpha_1 N_S + \beta_1 N_{G1} - \eta_1 D_S\right) \\
& + \frac{\partial V_G}{\partial D_M}\left[\mu\left(\alpha_1 N_S + \beta_1 N_{G1} - \eta_1 D_S\right) + \alpha_2 N_M + \beta_2 N_{G2} - \eta_2 D_M\right] \\
& \left. + \frac{\partial V_G}{\partial Q_S}\left(\delta_1 N_S + \varphi_1 N_{G1} - \gamma_1 Q_S\right) + \frac{\partial V_G}{\partial Q_M}\left(\delta_2 N_M + \varphi_2 N_{G2} - \gamma_2 Q_M\right)\right\}
\end{aligned} \tag{16}
$$

For solving formula (14), (15) and (17), using extreme conditions and searching for the optimal value of $N_S$, $N_M$, $N_{G1}$ and $N_{G2}$ by setting the all partial derivative equal to zero, we can get

$$
N_S = \frac{(1-\theta_1) p_1 + \alpha_1\left(\frac{\partial V_S}{\partial D_S} + \mu \frac{\partial V_S}{\partial D_M}\right) + \delta_1 \frac{\partial V_S}{\partial Q_S}}{\lambda_S} \tag{17}
$$

$$
N_M = \frac{(1-\theta_2) p_2 + \alpha_2 \frac{\partial V_M}{\partial D_M} + \delta_2 \frac{\partial V_M}{\partial Q_M}}{\lambda_M} \tag{18}
$$

$$
N_{G1} = \frac{\theta_1 k_1 + \beta_1\left(\frac{\partial V_G}{\partial D_S} + \mu \frac{\partial V_G}{\partial D_M}\right) + \varphi_1 \frac{\partial V_G}{\partial Q_S}}{\lambda_G} \tag{19}
$$

$$
N_{G2} = \frac{\theta_2 k_2 + \beta_2 \frac{\partial V_G}{\partial D_M} + \varphi_2 \frac{\partial V_G}{\partial Q_M}}{\lambda_G} \tag{20}
$$

Substituting the results of (17), (18), (19) and (20) into (14), (15) and (17), we can obtain

$$
\begin{aligned}
rV_S = & \left[(1-\theta_1) l_1 - \eta_1\left(\frac{\partial V_S}{\partial D_S} + \mu \frac{\partial V_S}{\partial D_M}\right)\right] D_S - \eta_2 \frac{\partial V_S}{\partial D_M} D_M + \left[(1-\theta_1) b_1 - \gamma_1 \frac{\partial V_S}{\partial Q_S}\right] Q_S \\
& - \gamma_2 \frac{\partial V_S}{\partial Q_M} Q_M + \frac{\left[(1-\theta_1) p_1 + \alpha_1\left(\frac{\partial V_S}{\partial D_S} + \mu \frac{\partial V_S}{\partial D_M}\right) + \delta_1 \frac{\partial V_S}{\partial Q_S}\right]^2}{2\lambda_S} \\
& + \frac{\left[\theta_1 k_1 + \beta_1\left(\frac{\partial V_G}{\partial D_S} + \mu \frac{\partial V_G}{\partial D_M}\right) + \varphi_1 \frac{\partial V_G}{\partial Q_S}\right]\left[(1-\theta_1) k_1 + \beta_1\left(\frac{\partial V_S}{\partial D_S} + \mu \frac{\partial V_S}{\partial D_M}\right) + \varphi_1 \frac{\partial V_S}{\partial Q_S}\right]}{\lambda_G} \\
& + \frac{\left(\frac{\partial V_S}{\partial D_M} \beta_2 + \frac{\partial V_S}{\partial Q_M} \varphi_2\right)\left(\theta_2 k_2 + \beta_2 \frac{\partial V_G}{\partial D_M} + \varphi_2 \frac{\partial V_G}{\partial Q_M}\right)}{\lambda_G} \\
& + \frac{\left(\frac{\partial V_S}{\partial D_M} \alpha_2 + \frac{\partial V_S}{\partial Q_M} \delta_2\right)\left[(1-\theta_2) p_2 + \alpha_2 \frac{\partial V_M}{\partial D_M} + \delta_2 \frac{\partial V_M}{\partial Q_M}\right]}{\lambda_M}
\end{aligned} \tag{21}
$$

$$\begin{aligned}
rV_M &= \left[(1-\theta_2)\,l_2 - \eta_2\frac{\partial V_M}{\partial D_M}\right] D_M - \eta_1\left(\frac{\partial V_M}{\partial D_S} + \mu\frac{\partial V_M}{\partial D_M}\right) D_S + \left[(1-\theta_2)\,b_2 - \gamma_2\frac{\partial V_M}{\partial Q_M}\right] Q_M \\
&\quad - \gamma_1\frac{\partial V_M}{\partial Q_S} Q_S + \frac{\left[(1-\theta_2)\,p_2 + \alpha_2\frac{\partial V_M}{\partial D_M} + \delta_2\frac{\partial V_M}{\partial Q_M}\right]^2}{2\lambda_M} \\
&\quad + \frac{\left(\theta_2 k_2 + \beta_2\frac{\partial V_G}{\partial D_M} + \varphi_2\frac{\partial V_G}{\partial Q_M}\right)\left[(1-\theta_2)\,k_2 + \beta_2\frac{\partial V_M}{\partial D_M} + \varphi_2\frac{\partial V_M}{\partial Q_M}\right]}{\lambda_G} \\
&\quad + \frac{\left[(1-\theta_1)\,p_1 + \alpha_1\left(\frac{\partial V_S}{\partial D_S} + \mu\frac{\partial V_S}{\partial D_M}\right) + \delta_1\frac{\partial V_S}{\partial Q_S}\right]\left[\alpha_1\frac{\partial V_M}{\partial D_S} + \alpha_1\mu\frac{\partial V_M}{\partial D_M} + \delta_1\frac{\partial V_M}{\partial Q_S}\right]}{\lambda_S} \\
&\quad + \frac{\left[\theta_1 k_1 + \beta_1\left(\frac{\partial V_G}{\partial D_S} + \mu\frac{\partial V_G}{\partial D_M}\right) + \varphi_1\frac{\partial V_G}{\partial Q_S}\right]\left[\beta_1\frac{\partial V_M}{\partial D_S} + \beta_1\mu\frac{\partial V_M}{\partial D_M} + \varphi_1\frac{\partial V_M}{\partial Q_S}\right]}{\lambda_G}
\end{aligned} \tag{22}$$

$$\begin{aligned}
rV_G &= \left[\theta_1 l_1 - \eta_1\left(\frac{\partial V_G}{\partial D_S} + \mu\frac{\partial V_G}{\partial D_M}\right)\right] D_S + \left[\theta_2 l_2 - \eta_2\frac{\partial V_G}{\partial D_M}\right] D_M + \left(\theta_1 b_1 - \gamma_1\frac{\partial V_G}{\partial Q_S}\right) Q_S \\
&\quad + \left(\theta_2 b_2 - \gamma_2\frac{\partial V_G}{\partial Q_M}\right) Q_M + \frac{\left[\theta_1 k_1 + \beta_1\left(\frac{\partial V_G}{\partial D_S} + \mu\frac{\partial V_G}{\partial D_M}\right) + \varphi_1\frac{\partial V_G}{\partial Q_S}\right]^2}{2\lambda_G} \\
&\quad + \frac{\left[\theta_2 k_2 + \beta_2\frac{\partial V_G}{\partial D_M} + \varphi_2\frac{\partial V_G}{\partial Q_M}\right]^2}{2\lambda_G} \\
&\quad + \frac{\left[(1-\theta_1)\,p_1 + \alpha_1\left(\frac{\partial V_S}{\partial D_S} + \mu\frac{\partial V_S}{\partial D_M}\right) + \delta_1\frac{\partial V_S}{\partial Q_S}\right]\left[\theta_1 p_1 + \alpha_1\left(\frac{\partial V_G}{\partial D_S} + \mu\frac{\partial V_G}{\partial D_M}\right) + \delta_1\frac{\partial V_G}{\partial Q_S}\right]}{\lambda_S} \\
&\quad + \frac{\left[(1-\theta_2)\,p_2 + \alpha_2\frac{\partial V_M}{\partial D_M} + \delta_2\frac{\partial V_M}{\partial Q_M}\right]\left[\theta_2 p_2 + \alpha_2\frac{\partial V_G}{\partial D_M} + \delta_2\frac{\partial V_G}{\partial Q_M}\right]}{\lambda_M}
\end{aligned} \tag{23}$$

Through the above formulas and the optimal control theory, we can see that the linear optimal function of the $D_S, D_M, Q_S, Q_M$ is the solution of the HJB equation. Based on the predecessor's literature [16, 21], we can get

$$\begin{aligned}
V_S\left(D_S, D_M, Q_S, Q_M\right) &= \sigma_1 D_S + \sigma_2 D_M + \sigma_3 Q_S + \sigma_4 Q_M + \omega_1 \\
V_M\left(D_S, D_M, Q_S, Q_M\right) &= h_1 D_S + h_2 D_M + h_3 Q_S + h_4 Q_M + \omega_2 \\
V_G\left(D_S, D_M, Q_S, Q_M\right) &= g_1 D_S + g_2 D_M + g_3 Q_S + g_4 Q_M + \omega_3
\end{aligned} \tag{24}$$

where $\sigma_i, q_i, g_i\,(i = 1, 2, 3, 4)$ and $\omega_i\,(i = 1, 2, 3)$ are the constants to be solved. We can have

$$\begin{aligned}
\frac{\partial V_S}{\partial D_S} = \sigma_1,\ \frac{\partial V_S}{\partial D_M} = \sigma_2,\ \frac{\partial V_S}{\partial Q_S} = \sigma_3,\ \frac{\partial V_S}{\partial Q_M} = \sigma_4 \\
\frac{\partial V_M}{\partial D_S} = h_1,\ \frac{\partial V_M}{\partial D_M} = h_2,\ \frac{\partial V_M}{\partial Q_S} = h_3,\ \frac{\partial V_M}{\partial Q_M} = h_4 \\
\frac{\partial V_G}{\partial D_S} = g_1,\ \frac{\partial V_G}{\partial D_M} = g_2,\ \frac{\partial V_G}{\partial Q_S} = g_3,\ \frac{\partial V_G}{\partial Q_M} = g_4
\end{aligned} \tag{25}$$

Substituting the result of (24) and (25) into (21), (22) and (23), we can get

$$\begin{aligned}
& r\left(\sigma_1 D_S+\sigma_2 D_M+\sigma_3 Q_S+\sigma_4 Q_M+\omega_1\right) \\
= & \left[\left(1-\theta_1\right) l_1-\eta_1\left(\sigma_1+\mu \sigma_2\right)\right] D_S-\eta_2 \sigma_2 D_M+\left[\left(1-\theta_1\right) b_1-\gamma_1 \sigma_3\right] Q_S \\
& -\gamma_2 \sigma_4 Q_M+\frac{\left[\left(1-\theta_1\right) p_1+\alpha_1\left(\sigma_1+\mu \sigma_2\right)+\delta_1 \sigma_3\right]^2}{2 \lambda_S} \\
& +\frac{\left[\theta_1 k_1+\beta_1\left(g_1+\mu g_2\right)+\varphi_1 g_3\right]\left[\left(1-\theta_1\right) k_1+\beta_1\left(\sigma_1+\mu \sigma_2\right)+\varphi_1 \sigma_3\right]}{\lambda_G} \\
& +\frac{\left(\sigma_2 \beta_2+\sigma_4 \varphi_2\right)\left(\theta_2 k_2+\beta_2 g_2+\varphi_2 g_4\right)}{\lambda_G}+\frac{\left(\sigma_2 \alpha_2+\sigma_4 \delta_2\right)\left[\left(1-\theta_2\right) p_2+\alpha_2 h_2+\delta_2 h_4\right]}{\lambda_M}
\end{aligned} \tag{26}$$

$$\begin{aligned}
& r\left(h_1 D_S+h_2 D_M+h_3 Q_S+h_4 Q_M+\omega_2\right) \\
= & \left[\left(1-\theta_2\right) l_2-\eta_2 h_2\right] D_M-\eta_1\left(h_1+\mu h_2\right) D_S+\left[\left(1-\theta_2\right) b_2-\gamma_2 h_4\right] Q_M-\gamma_1 h_3 Q_S \\
& +\frac{\left[\left(1-\theta_2\right) p_2+\alpha_2 h_2+\delta_2 h_4\right]^2}{2 \lambda_M}+\frac{\left(\theta_2 k_2+\beta_2 g_2+\varphi_2 g_4\right)\left[\left(1-\theta_2\right) k_2+\beta_2 h_2+\varphi_2 h_4\right]}{\lambda_G} \\
& +\frac{\left[\left(1-\theta_1\right) p_1+\alpha_1\left(\sigma_1+\mu \sigma_2\right)+\delta_1 \sigma_3\right]\left[\alpha_1 h_1+\alpha_1 \mu h_2+\delta_1 h_3\right]}{\lambda_S} \\
& +\frac{\left[\theta_1 k_1+\beta_1\left(g_1+\mu g_2\right)+\varphi_1 g_3\right]\left[\beta_1 h_1+\beta_1 \mu h_2+\varphi_1 h_3\right]}{\lambda_G}
\end{aligned} \tag{27}$$

$$\begin{aligned}
& r\left(g_1 D_S+g_2 D_M+g_3 Q_S+g_4 Q_M+\omega_3\right) \\
= & \left[\theta_1 l_1-\eta_1\left(g_1+\mu g_2\right)\right] D_S+\left[\theta_2 l_2-\eta_2 g_2\right] D_M+\left(\theta_1 b_1-\gamma_1 g_3\right) Q_S+\left(\theta_2 b_2-\gamma_2 g_4\right) Q_M \\
& +\frac{\left[\theta_1 k_1+\beta_1\left(g_1+\mu g_2\right)+\varphi_1 g_3\right]^2}{2 \lambda_G}+\frac{\left[\theta_2 k_2+\beta_2 g_2+\varphi_2 g_4\right]^2}{2 \lambda_G} \\
& +\frac{\left[\left(1-\theta_1\right) p_1+\alpha_1\left(\sigma_1+\mu \sigma_2\right)+\delta_1 \sigma_3\right]\left[\theta_1 p_1+\alpha_1\left(g_1+\mu g_2\right)+\delta_1 g_3\right]}{\lambda_S} \\
& +\frac{\left[\left(1-\theta_2\right) p_2+\alpha_2 h_2+\delta_2 h_4\right]\left[\theta_2 p_2+\alpha_2 g_2+\delta_2 g_4\right]}{\lambda_M}
\end{aligned} \tag{28}$$

Using the $D_S, D_M, Q_S, Q_M \geq 0$ to (26), (27) and (28), parameter values of the optimal value function can be expressed as follows

$$\begin{array}{llll}
\sigma_1=\dfrac{\left(1-\theta_1\right) l_1}{\eta_1+r}, & \sigma_2=0, & \sigma_3=\dfrac{\left(1-\theta_1\right) b_1}{\gamma_1+r}, & \sigma_4=0 \\
h_1=-\dfrac{\eta_1 \mu\left(1-\theta_2\right) l_2}{\left(\eta_1+r\right)\left(\eta_2+r\right)}, & h_2=\dfrac{\left(1-\theta_2\right) l_2}{\eta_2+r}, & h_3=0, & h_4=\dfrac{\left(1-\theta_2\right) b_2}{\gamma_2+r} \\
g_1=\dfrac{\theta_1 l_1}{\eta_1+r}-\dfrac{\eta_1 \mu \theta_2 l_2}{\left(\eta_1+r\right)\left(\eta_2+r\right)}, & g_2=\dfrac{\theta_2 l_2}{\eta_2+r}, & g_3=\dfrac{\theta_1 b_1}{\gamma_1+r}, & g_4=\dfrac{\theta_2 b_2}{\gamma_2+r}
\end{array} \tag{29}$$

$$\begin{aligned}
\omega_1= & \frac{\left(1-\theta_1\right)^2\left[p_1+\dfrac{\alpha_1 l_1}{\eta_1+r}+\dfrac{\delta_1 b_1}{\gamma_1+r}\right]^2}{2 r \lambda_S} \\
& +\frac{\left(1-\theta_1\right)\left[\theta_1\left(k_1+\dfrac{\beta_1 l_1}{\eta_1+r}+\dfrac{\varphi_1 b_1}{\gamma_1+r}\right)+\dfrac{\beta_1 \mu \theta_2 l_2 r}{\left(\eta_1+r\right)\left(\eta_2+r\right)}\right]\left[k_1+\dfrac{\beta_1 l_1}{\eta_1+r}+\dfrac{\varphi_1 b_1}{\gamma_1+r}\right]}{r \lambda_G}
\end{aligned} \tag{30}$$

$$\begin{aligned}
\omega_2= & \frac{\left(1-\theta_2\right)^2\left[p_2+\dfrac{\alpha_2 l_2}{\eta_2+r}+\dfrac{\delta_2 b_2}{\eta_2+r}\right]^2}{2 r \lambda_M}+\frac{\theta_2\left(1-\theta_2\right)\left(k_2+\dfrac{\beta_2 l_2}{\eta_2+r}+\dfrac{\varphi_2 b_2}{\gamma_2+r}\right)^2}{r \lambda_G} \\
& +\frac{\left(1-\theta_1\right)\left(1-\theta_2\right)\left[p_1+\dfrac{\alpha_1 l_1}{\eta_1+r}+\dfrac{\delta_1 b_1}{\gamma_1+r}\right]\left[\dfrac{\alpha_1 \mu l_2}{\left(\eta_1+r\right)\left(\eta_2+r\right)}\right]}{\lambda_S} \\
& +\frac{\left[\theta_1\left(k_1+\dfrac{\beta_1 l_1}{\eta_1+r}+\dfrac{\varphi_1 b_1}{\gamma_1+r}\right)+\dfrac{\beta_1 \mu \theta_2 l_2 r}{\left(\eta_1+r\right)\left(\eta_2+r\right)}\right]\left[\dfrac{\beta_1 \mu l_2\left(1-\theta_2\right)}{\left(\eta_1+r\right)\left(\eta_2+r\right)}\right]}{\lambda_G}
\end{aligned} \tag{31}$$

$$\omega_3 = \frac{\left[\theta_1\left(k_1 + \frac{\beta_1 l_1}{\eta_1 + r} + \frac{\varphi_1 b_1}{\gamma_1 + r}\right) + \frac{\beta_1 \mu \theta_2 l_2 r}{(\eta_1 + r)(\eta_2 + r)}\right]^2}{2r\lambda_G} + \frac{\left[\theta_2 k_2 + \frac{\beta_2 \theta_2 l_2}{\eta_2 + r} + \frac{\varphi_2 \theta_2 b_2}{\gamma_2 + r}\right]^2}{2r\lambda_G}$$
$$+ \frac{(1-\theta_1)\left[p_1 + \frac{\alpha_1 l_1}{\eta_1 + r} + \frac{\delta_1 b_1}{\gamma_1 + r}\right]\left[\theta_1\left(p_1 + \frac{\alpha_1 l_1}{\eta_1 + r} + \frac{\delta_1 b_1}{\gamma_1 + r}\right) + \frac{\alpha_1 \mu \theta_2 l_2 r}{(\eta_1 + r)(\eta_2 + r)}\right]}{r\lambda_S}$$
$$+ \frac{\theta_2(1-\theta_2)\left[p_2 + \frac{\alpha_2 l_2}{\eta_2 + r} + \frac{\delta_2 b_2}{\gamma_2 + r}\right]^2}{r\lambda_M} \tag{32}$$

Substituting the results of (29), (30), (31) and (32) into (24), we can obtain

$$V_S^\star = \frac{(1-\theta_1) l_1}{\eta_1 + r} D_S + \frac{(1-\theta_1) b_1}{\gamma_1 + r} Q_S + \frac{(1-\theta_1)^2\left[p_1 + \frac{\alpha_1 l_1}{\eta_1 + r} + \frac{\delta_1 b_1}{\gamma_1 + r}\right]^2}{2r\lambda_S}$$
$$+ \frac{(1-\theta_1)\left[\theta_1\left(k_1 + \frac{\beta_1 l_1}{\eta_1 + r} + \frac{\varphi_1 b_1}{\gamma_1 + r}\right) + \frac{\beta_1 \mu \theta_2 l_2 r}{(\eta_1 + r)(\eta_2 + r)}\right]\left[k_1 + \frac{\beta_1 l_1}{\eta_1 + r} + \frac{\varphi_1 b_1}{\gamma_1 + r}\right]}{r\lambda_G} \tag{33}$$

$$V_M^\star = -\frac{\eta_1 \mu (1-\theta_2) l_2}{(\eta_1 + r)(\eta_2 + r)} D_S + \frac{(1-\theta_2) l_2}{\eta_2 + r} D_M + \frac{(1-\theta_2) b_2}{\gamma_2 + r} Q_M$$
$$+ \frac{(1-\theta_2)^2\left[p_2 + \frac{\alpha_2 l_2}{\eta_2 + r} + \frac{\delta_2 b_2}{\gamma_2 + r}\right]^2}{2r\lambda_M} + \frac{\theta_2(1-\theta_2)\left(k_2 + \frac{\beta_2 l_2}{\eta_2 + r} + \frac{\varphi_2 b_2}{\gamma_2 + r}\right)^2}{r\lambda_G}$$
$$+ \frac{(1-\theta_1)(1-\theta_2)\left[p_1 + \frac{\alpha_1 l_1}{\eta_1 + r} + \frac{\delta_1 b_1}{\gamma_1 + r}\right]\left[\frac{\alpha_1 \mu l_2}{(\eta_1 + r)(\eta_2 + r)}\right]}{\lambda_S}$$
$$+ \frac{\left[\theta_1\left(k_1 + \frac{\beta_1 l_1}{\eta_1 + r} + \frac{\varphi_1 b_1}{\gamma_1 + r}\right) + \frac{\beta_1 \mu \theta_2 l_2 r}{(\eta_1 + r)(\eta_2 + r)}\right]\left[\frac{\beta_1 \mu l_2 (1-\theta_2)}{(\eta_1 + r)(\eta_2 + r)}\right]}{\lambda_G} \tag{34}$$

$$V_G^\star = \left[\frac{\theta_1 l_1}{\eta_1 + r} - \frac{\eta_1 \mu \theta_2 l_2}{(\eta_1 + r)(\eta_2 + r)}\right] D_S + \frac{\theta_2 l_2}{\eta_2 + r} D_M + \frac{\theta_1 b_1}{\gamma_1 + r} Q_S + \frac{\theta_2 b_2}{\gamma_2 + r} Q_M$$
$$+ \frac{\left[\theta_1\left(k_1 + \frac{\beta_1 l_1}{\eta_1 + r} + \frac{\varphi_1 b_1}{\gamma_1 + r}\right) + \frac{\beta_1 \mu \theta_2 l_2 r}{(\eta_1 + r)(\eta_2 + r)}\right]^2}{2r\lambda_G} + \frac{\left[\theta_2 k_2 + \frac{\beta_2 \theta_2 l_2}{\eta_2 + r} + \frac{\varphi_2 \theta_2 b_2}{\gamma_2 + r}\right]^2}{2r\lambda_G}$$
$$+ \frac{(1-\theta_1)\left[p_1 + \frac{\alpha_1 l_1}{\eta_1 + r} + \frac{\delta_1 b_1}{\gamma_1 + r}\right]\left[\theta_1\left(p_1 + \frac{\alpha_1 l_1}{\eta_1 + r} + \frac{\delta_1 b_1}{\gamma_1 + r}\right) + \frac{\alpha_1 \mu \theta_2 l_2 r}{(\eta_1 + r)(\eta_2 + r)}\right]}{r\lambda_S}$$
$$+ \frac{\theta_2(1-\theta_2)\left[p_2 + \frac{\alpha_2 l_2}{\eta_2 + r} + \frac{\delta_2 b_2}{\gamma_2 + r}\right]^2}{r\lambda_M} \tag{35}$$

In this case, the benefit of the whole system is

$$V_T^\star = V_S^\star + V_M^\star + V_G^\star \tag{36}$$

Solving the partial derivative of (33), (34) and (35), and substituting the results into (17), (18), (19) and (20), we can get the optimal effort level of supplier and manufacturer and government as the proposition 1. In the situation of non-cooperation, according to formula (1) and (3), we use the general solution method of differential equations, and can get the supplier's product green degree and reputation as

$$D_S^\star = \frac{\alpha_1 N_S^\star + \beta_1 N_{G1}^\star}{\eta_1} + \left(d_1 - \frac{\alpha_1 N_S^\star + \beta_1 N_{G1}^\star}{\eta_1}\right) e^{-\eta_1 t} \tag{37}$$

$$Q_S^\star = \frac{\delta_1 N_S^\star + \varphi_1 N_{G1}^\star}{\gamma_1} + \left( q_1 - \frac{\delta_1 N_S^\star + \varphi_1 N_{G1}^\star}{\gamma_1} \right) e^{-\gamma_1 t} \tag{38}$$

Similar to supplier, according to formulas (2) and (4), the manufacturer's product green degree and reputation are

$$\begin{aligned} D_M^\star = {} & \frac{\alpha_2 N_M^\star + \beta_2 N_{G2}^\star}{\eta_2} + \frac{\mu \left( \alpha_1 N_S^\star + \beta_1 N_{G1}^\star - d_1 \eta_1 \right)}{\eta_2 - \eta_1} e^{-\eta_1 t} \\ & + \left( d_2 - \frac{\alpha_2 N_M^\star + \beta_2 N_{G2}^\star}{\eta_2} - \frac{\mu \left( \alpha_1 N_S^\star + \beta_1 N_{G1}^\star - d_1 \eta_1 \right)}{\eta_2 - \eta_1} \right) e^{-\eta_2 t} \end{aligned} \tag{39}$$

$$Q_M^\star = \frac{\delta_2 N_M^\star + \varphi_2 N_{G2}^\star}{\gamma_2} + \left( q_2 - \frac{\delta_2 N_M^\star + \varphi_2 N_{G2}^\star}{\gamma_2} \right) e^{-\gamma_2 t} \tag{40}$$

From proposition 1, we can find that when supplier, manufacturer and government only consider maximizing their own benefits, the optimal effort level is related to some parameters. For the supplier and manufacturer of the green supply chain, $p_i, \alpha_i, l_i, \delta_i, b_i (i = 1, 2)$ are positively related to the optimal level of effort, $\eta_i, \gamma_i (i = 1, 2)$ , $\lambda_S$ , $\lambda_M$, $r$ are negatively related to the optimal level of effort, $k_i (i = 1, 2)$ are not related to the optimal level of effort, it shows that government efforts have no impact on the optimal level of effort of supplier and manufacturer.

For the government, $\theta_i, k_i, \beta_i, l_i, \varphi_i, b_i (i = 1, 2)$ and $\mu$ are positively related to the optimal level of effort, $\eta_i, \gamma_i (i = 1, 2)$ , $\lambda_G$ and $r$ are negatively related to the optimal level of effort. Government needs to integrate various factors to formulate corresponding incentive policies and regulatory mechanisms.

## 3.2 Situation of government promotion

The government plays an important role in promoting green supply chain cooperation. Cost subsidy is a common measure for the government to promote the development of green supply chain. Therefore, in the situation of government promotion, we focus on the cost sharing of the government to the green supply chain. We assume that the cost sharing coefficient for the supplier and manufacturer are $\varepsilon_1$ and $\varepsilon_2$. The government is a leader, the first to make the cost sharing coefficient, the supplier and manufacturer in the green supply chain as the followers, and then decide how much effort should be made. So, the situation of the Stackelberg master-slave game is formed. In this situation, the objective functions (net benefit) of the supplier, manufacturer and government be expressed as

$$\prod_S = \int_0^\infty e^{-rt} \left\{ (1 - \theta_1) \left[ p_1 N_S + k_1 N_{G1} + l_1 D_S + b_1 Q_S \right] - \frac{\lambda_S}{2} (1 - \varepsilon_1) N_S^2 \right\} dt \tag{41}$$

$$\prod_M = \int_0^\infty e^{-rt} \left\{ (1 - \theta_2) \left[ p_2 N_M + k_2 N_{G2} + l_2 D_M + b_2 Q_M \right] - \frac{\lambda_M}{2} (1 - \varepsilon_2) N_M^2 \right\} dt \tag{42}$$

$$\begin{aligned} \prod_G = {} & \int_0^\infty e^{-rt} \{ \theta_1 \left[ p_1 N_S + k_1 N_{G1} + l_1 D_S + b_1 Q_S \right] \\ & + \theta_2 \left[ p_2 N_M + k_2 N_{G2} + l_2 D_M + b_2 Q_M \right] - \frac{\lambda_G}{2} \left( N_{G1}^2 + N_{G2}^2 \right) - \frac{1}{2} \left( \lambda_S \varepsilon_1 N_S^2 + \lambda_M \varepsilon_2 N_M^2 \right) \Big\} dt \end{aligned} \tag{43}$$

**Proposition 2** In the situation of government promotion, the feedback Stackelberg master-slave equilibria are (for the proof, see the Appendix)

$$N_S^{\star\star} = \frac{(\theta_1 + 1) \left( p_1 + \dfrac{\alpha_1 l_1}{\eta_1 + r} + \dfrac{\delta_1 b_1}{\gamma_1 + r} \right) + \dfrac{2 \alpha_1 \mu \theta_2 l_2 r}{(\eta_1 + r)(\eta_2 + r)}}{2 \lambda_S} \tag{44}$$

$$N_M^{**} = \frac{(\theta_2 + 1)\left(p_2 + \frac{\alpha_2 l_2}{\eta_2 + r} + \frac{\delta_2 b_2}{\gamma_2 + r}\right)}{2\lambda_M} \tag{45}$$

$$N_{G1}^{**} = \frac{\theta_1\left(k_1 + \frac{\beta_1 l_1}{\eta_1 + r} + \frac{\varphi_1 b_1}{\gamma_1 + r}\right) + \frac{\beta_1 \mu \theta_2 l_2 r}{(\eta_1 + r)(\eta_2 + r)}}{\lambda_G} \tag{46}$$

$$N_{G2}^{**} = \frac{\theta_2\left(k_2 + \frac{\beta_2 l_2}{\eta_2 + r} + \frac{\varphi_2 b_2}{\gamma_2 + r}\right)}{\lambda_G} \tag{47}$$

$$\varepsilon_1^{**} = \begin{cases} 0 & 0 \le \theta_1 \le \frac{1}{3} \\ \dfrac{(3\theta_1 - 1)\left(p_1 + \frac{\alpha_1 l_1}{\eta_1 + r} + \frac{b_1 \delta_1}{\gamma_1 + r}\right) + \frac{2\alpha_1 \mu \theta_2 l_2 r}{(\eta_1 + r)(\eta_2 + r)}}{(\theta_1 + 1)\left(p_1 + \frac{\alpha_1 l_1}{\eta_1 + r} + \frac{\delta_1 b_1}{\gamma_1 + r}\right) + \frac{2\alpha_1 \mu \theta_2 l_2 r}{(\eta_1 + r)(\eta_2 + r)}} & \frac{1}{3} < \theta_1 \le 1 \end{cases} \tag{48}$$

$$\varepsilon_2^{**} = \begin{cases} 0 & 0 \le \theta_2 \le \frac{1}{3} \\ \dfrac{3\theta_2 - 1}{\theta_2 + 1} & \frac{1}{3} < \theta_2 \le 1 \end{cases} \tag{49}$$

Similar to proposition 1, in the situation of government promotion, we can get the supplier's product green degree and reputation as

$$D_S^{**} = \frac{\alpha_1 N_S^{**} + \beta_1 N_{G1}^{**}}{\eta_1} + \left(d_1 - \frac{\alpha_1 N_S^{**} + \beta_1 N_{G1}^{**}}{\eta_1}\right) e^{-\eta_1 t} \tag{50}$$

$$Q_S^{**} = \frac{\delta_1 N_S^{**} + \varphi_1 N_{G1}^{**}}{\gamma_1} + \left(q_1 - \frac{\delta_1 N_S^{**} + \varphi_1 N_{G1}^{**}}{\gamma_1}\right) e^{-\gamma_1 t} \tag{51}$$

The manufacturer's product green degree and reputation are

$$\begin{aligned} D_M^{**} = {} & \frac{\alpha_2 N_M^{**} + \beta_2 N_{G2}^{**}}{\eta_2} + \frac{\mu\left(\alpha_1 N_S^{**} + \beta_1 N_{G1}^{**} - d_1 \eta_1\right)}{\eta_2 - \eta_1} e^{-\eta_1 t} \\ & + \left(d_2 - \frac{\alpha_2 N_M^{**} + \beta_2 N_{G2}^{**}}{\eta_2} - \frac{\mu\left(\alpha_1 N_S^{**} + \beta_1 N_{G1}^{**} - d_1 \eta_1\right)}{\eta_2 - \eta_1}\right) e^{-\eta_2 t} \end{aligned} \tag{52}$$

$$Q_M^{**} = \frac{\delta_2 N_M^{**} + \varphi_2 N_{G2}^{**}}{\gamma_2} + \left(q_2 - \frac{\delta_2 N_M^{**} + \varphi_2 N_{G2}^{**}}{\gamma_2}\right) e^{-\gamma_2 t} \tag{53}$$

From proposition 2, in the case of government promotion, we find that the efforts of supplier and manufacturer in green supply chain are positively related to the government's benefit distribution coefficient. The more benefit the government gains, the higher the efforts of the supplier and manufacturer, which reflects the characteristics of the three parties driven by the government. The government is a leader, the first to make the cost sharing coefficient, the supplier and manufacturer in the green supply chain as the followers, if the government expects to get more benefit, it will enhance the subsidy level and promote the efficiency of green supply chain. When the benefit distribution coefficient is too small, the government will not provide a cost subsidy strategy to the green supply chain. The other influence coefficients are the same as the situation of non-cooperation.

### 3.3 Situation of collaborative cooperation

In the situation of collaborative cooperation, the government and the supplier and manufacturer of the green supply chain will choose their optimal effort levels based on maximization of their total benefit. Therefore, product green degree and reputation can be further improved through cooperation between the government and the supplier and manufacturer of the green supply chain.

**Proposition 3** In the situation of collaborative cooperation, the feedback cooperative game equilibria are (for the proof, see the Appendix)

$$N_S^{***}=\frac{p_1+\frac{\delta_1 b_1}{\gamma_1+r}+\frac{\alpha_1 l_1}{\eta_1+r}+\frac{\alpha_1\mu l_2 r}{(\eta_1+r)(\eta_2+r)}}{\lambda_S} \tag{54}$$

$$N_M^{***}=\frac{p_2+\frac{\alpha_2 l_2}{\eta_2+r}+\frac{\delta_2 b_2}{\gamma_2+r}}{\lambda_M} \tag{55}$$

$$N_{G1}^{***}=\frac{k_1+\frac{\varphi_1 b_1}{\gamma_1+r}+\frac{\beta_1 l_1}{\eta_1+r}+\frac{\mu\beta_1 l_2 r}{(\eta_1+r)(\eta_2+r)}}{\lambda_G} \tag{56}$$

$$N_{G2}^{***}=\frac{k_2+\frac{\beta_2 l_2}{\eta_2+r}+\frac{\varphi_2 b_2}{\gamma_2+r}}{\lambda_G} \tag{57}$$

Similar to proposition 1, in the situation of collaborative cooperation, we can get the supplier's product green degree and reputation as

$$D_S^{***}=\frac{\alpha_1 N_S^{***}+\beta_1 N_{G1}^{***}}{\eta_1}+\left(d_1-\frac{\alpha_1 N_S^{***}+\beta_1 N_{G1}^{***}}{\eta_1}\right)e^{-\eta_1 t} \tag{58}$$

$$Q_S^{***}=\frac{\delta_1 N_S^{***}+\varphi_1 N_{G1}^{***}}{\gamma_1}+\left(q_1-\frac{\delta_1 N_S^{***}+\varphi_1 N_{G1}^{***}}{\gamma_1}\right)e^{-\gamma_1 t} \tag{59}$$

The manufacturer's product green degree and reputation are

$$\begin{aligned}D_M^{***}=&\frac{\alpha_2 N_M^{***}+\beta_2 N_{G2}^{***}}{\eta_2}+\frac{\mu\left(\alpha_1 N_S^{***}+\beta_1 N_{G1}^{***}-d_1\eta_1\right)}{\eta_2-\eta_1}e^{-\eta_1 t}\\&+\left(d_2-\frac{\alpha_2 N_M^{***}+\beta_2 N_{G2}^{***}}{\eta_2}-\frac{\mu\left(\alpha_1 N_S^{***}+\beta_1 N_{G1}^{***}-d_1\eta_1\right)}{\eta_2-\eta_1}\right)e^{-\eta_2 t}\end{aligned} \tag{60}$$

$$Q_M^{***}=\frac{\delta_2 N_M^{***}+\varphi_2 N_{G2}^{***}}{\gamma_2}+\left(q_2-\frac{\delta_2 N_M^{***}+\varphi_2 N_{G2}^{***}}{\gamma_2}\right)e^{-\gamma_2 t} \tag{61}$$

From proposition 3, in the case of collaborative cooperation, we find that the benefit allocation coefficient no longer affects the decisions of supplier, manufacturer and government on the optimal effort level. The reason is that supplier, manufacturer and government work together to maximize the benefit of the whole system. It is no longer binding on the benefit obtained by each other. Therefore, benefit allocation coefficient does not affect the optimal effort level. The other influence coefficients are the same as the situation of non-cooperation and government promotion.

## 4 Comparison between situations

According to the analysis of the three situations, we get different optimal effort level, optimal benefit, product green degree and reputation. So, we develop four propositions for the different game equilibriums in the aspects of different optimal effort level, optimal benefit, product green degree and reputation.

**Proposition 4** If the benefit distribution coefficient from the government satisfies the condition $\frac{1}{3} < \theta_1 \le 1$, the steady-state efforts by the supplier hold for $N_S^* < N_S^{**} \le N_S^{***}$, the steady-state efforts by the manufacturer hold for $N_M^* < N_M^{**} \le N_M^{***}$, the steady-state efforts by the government hold for $N_{G1}^* = N_{G1}^{**} \le N_{G1}^{***}$, $N_{G2}^* = N_{G2}^{**} \le N_{G2}^{***}$, the cost sharing coefficient of the government are $\varepsilon_1^{**} = \frac{N_S^{**} - N_S^*}{N_S^{**}}$, $\varepsilon_2^{**} = \frac{N_M^{**} - N_M^*}{N_M^{**}}$.

**Proof** From formula (10), (44) and (54), we can get

$$\begin{aligned} & N_S^{**} - N_S^* \\ & = \frac{(\theta_1 + 1)\left(p_1 + \frac{\alpha_1 l_1}{\eta_1 + r} + \frac{\delta_1 b_1}{\gamma_1 + r}\right) + \frac{2\alpha_1 \mu \theta_2 l_2 r}{(\eta_1 + r)(\eta_2 + r)}}{2\lambda_S} - \frac{(1 - \theta_1)\left[p_1 + \frac{\alpha_1 l_1}{\eta_1 + r} + \frac{\delta_1 b_1}{\gamma_1 + r}\right]}{\lambda_S} \\ & = \frac{(3\theta_1 - 1)\left(p_1 + \frac{\delta_1 b_1}{\gamma_1 + r} + \frac{\alpha_1 l_1}{\eta_1 + r}\right) + \frac{2\alpha_1 \mu \theta_2 l_2 r}{(\eta_1 + r)(\eta_2 + r)}}{2\lambda_S} \end{aligned} \tag{62}$$

$$\begin{aligned} & N_S^{***} - N_S^{**} \\ & = \frac{p_1 + \frac{\delta_1 b_1}{\gamma_1 + r} + \frac{\alpha_1 l_1}{\eta_1 + r} + \frac{\alpha_1 \mu l_2 r}{(\eta_1 + r)(\eta_2 + r)}}{\lambda_S} - \frac{(\theta_1 + 1)\left(p_1 + \frac{\alpha_1 l_1}{\eta_1 + r} + \frac{\delta_1 b_1}{\gamma_1 + r}\right) + \frac{2\alpha_1 \mu \theta_2 l_2 r}{(\eta_1 + r)(\eta_2 + r)}}{2\lambda_S} \\ & = \frac{(1 - \theta_1)\left[p_1 + \frac{\alpha_1 l_1}{\eta_1 + r} + \frac{\delta_1 b_1}{\gamma_1 + r}\right] + \frac{2(1 - \theta_2)\alpha_1 \mu l_2 r}{(\eta_1 + r)(\eta_2 + r)}}{2\lambda_S} \end{aligned} \tag{63}$$

According to the $\frac{1}{3} < \theta_1 \le 1$, we can get$N_S^{**} - N_S^* > 0$ ,$N_S^{***} - N_S^{**} \ge 0$ .

From formula (11), (45) and (55), we can get

$$\begin{aligned} & N_M^{**} - N_M^* \\ & = \frac{(\theta_2 + 1)\left(p_2 + \frac{\alpha_2 l_2}{\eta_2 + r} + \frac{\delta_2 b_2}{\gamma_2 + r}\right)}{2\lambda_M} - \frac{(1 - \theta_2)\left[p_2 + \frac{\alpha_2 l_2}{\eta_2 + r} + \frac{\delta_2 b_2}{\gamma_2 + r}\right]}{\lambda_M} \\ & = \frac{(3\theta_2 - 1)\left(p_2 + \frac{\alpha_2 l_2}{\eta_2 + r} + \frac{\delta_2 b_2}{\gamma_2 + r}\right)}{2\lambda_M} \end{aligned} \tag{64}$$

$$\begin{aligned} & N_M^{***} - N_M^{**} \\ & = \frac{p_2 + \frac{\alpha_2 l_2}{\eta_2 + r} + \frac{\delta_2 b_2}{\gamma_2 + r}}{\lambda_M} - \frac{(\theta_2 + 1)\left(p_2 + \frac{\alpha_2 l_2}{\eta_2 + r} + \frac{\delta_2 b_2}{\gamma_2 + r}\right)}{2\lambda_M} \\ & = \frac{(1 - \theta_2)\left(p_2 + \frac{\alpha_2 l_2}{\eta_2 + r} + \frac{\delta_2 b_2}{\gamma_2 + r}\right)}{2\lambda_M} \end{aligned} \tag{65}$$

According to the $\frac{1}{3} < \theta_1 \le 1$, we can get $N_M^{**} - N_M^* > 0$ , $N_M^{***} - N_M^{**} \ge 0$. From formula (12), (46) and (56), we can get $N_{G1}^* = N_{G1}^{**}$ ,

$$\begin{aligned} & N_{G1}^{***} - N_{G1}^{**} \\ & = \frac{k_1 + \frac{\varphi_1 b_1}{\gamma_1 + r} + \frac{\beta_1 l_1}{\eta_1 + r} + \frac{\mu \beta_1 l_2 r}{(\eta_1 + r)(\eta_2 + r)}}{\lambda_G} - \frac{\theta_1\left(k_1 + \frac{\beta_1 l_1}{\eta_1 + r} + \frac{\varphi_1 b_1}{\gamma_1 + r}\right) + \frac{\beta_1 \mu \theta_2 l_2 r}{(\eta_1 + r)(\eta_2 + r)}}{\lambda_G} \\ & = \frac{(1 - \theta_1)\left[k_1 + \frac{\beta_1 l_1}{\eta_1 + r} + \frac{\varphi_1 b_1}{\gamma_1 + r}\right] + (1 - \theta_2)\frac{\mu \beta_1 l_2 r}{(\eta_1 + r)(\eta_2 + r)}}{\lambda_G} \end{aligned} \tag{66}$$

From formula (13), (47) and (57), we can get $N_{G2}^{*} = N_{G2}^{**}$,

$$\begin{aligned} & N_{G2}^{***} - N_{G2}^{**} \\ & = \frac{k_2 + \dfrac{\beta_2 l_2}{\eta_2 + r} + \dfrac{\varphi_2 b_2}{\gamma_2 + r}}{\lambda_G} - \frac{\theta_2 \left( k_2 + \dfrac{\beta_2 l_2}{\eta_2 + r} + \dfrac{\varphi_2 b_2}{\gamma_2 + r} \right)}{\lambda_G} \\ & = \frac{(1 - \theta_2) \left( k_2 + \dfrac{\beta_2 l_2}{\eta_2 + r} + \dfrac{\varphi_2 b_2}{\gamma_2 + r} \right)}{\lambda_G} \end{aligned} \tag{67}$$

and $N_{G1}^{*} = N_{G1}^{**} \le N_{G1}^{***}$, $N_{G2}^{*} = N_{G2}^{**} \le N_{G2}^{***}$ .

$$\begin{aligned} N_S^{**} - N_S^{*} = N_S^{**} \varepsilon_1^{**} \\ N_M^{**} - N_M^{*} = N_M^{**} \varepsilon_1^{**} \end{aligned} \tag{68}$$

So, the cost sharing coefficient of the government are $\varepsilon_1^{**} = \dfrac{N_S^{**} - N_S^{*}}{N_S^{**}}$ , $\varepsilon_2^{**} = \dfrac{N_M^{**} - N_M^{*}}{N_M^{**}}$ .

In proposition 4, we can find that the optimal effort of supplier and manufacturer has been improved in the situation of government promotion, and this promotion has a significant correlation with the government's cost subsidy coefficient, but the optimal effort level of government has not changed. The reason is that the government will work out the level of effort based on the related factors of its own benefit, which is the same as that in the non-cooperative situation, so the optimal effort level of the government has not changed. In the situation of collaborative cooperation, government, supplier and manufacturer will both make higher levels of effort to improve the product green degree and reputation.

**Proposition 5** If the benefit distribution coefficient from the government satisfies the condition $\frac{1}{3} < \theta_1 \le 1$ , then the steady-state product green degree by the supplier hold for $D_S^{*} \le D_S^{**} \le D_S^{***}$ , the steady-state product green degree by the manufacturer hold for $D_M^{*} \le D_M^{**} \le D_M^{***}$ (for the proof, see the Appendix).

In proposition 5, we can see that government cost subsidies play an important role in promoting the product green degree of green supply chain. Both supplier and manufacturer's product green degree has been increased. Collaborative cooperation is optimal in three situations.

**Proposition 6** If the benefit distribution coefficient from the government satisfies the condition $\frac{1}{3} < \theta_1 \le 1$ , then the steady-state reputation by the supplier hold for $Q_S^{*} \le Q_S^{**} \le Q_S^{***}$ , the steady-state reputation by the manufacturer hold for $Q_M^{*} \le Q_M^{**} \le Q_M^{***}$ (for the proof, see the Appendix).

In proposition 6, we can see that government cost subsidies play an important role in promoting the reputation of green supply chain. Both supplier and manufacturer's reputation has been increased. Collaborative cooperation is optimal in three situations.

**Proposition 7** If the benefit distribution coefficient from the government satisfies the condition $\frac{1}{3} < \theta_1 \le 1$ , then the steady-state benefit by the supplier hold for $V_S^{*} \le V_S^{**}$ , the steady-state benefit by the manufacturer hold for $V_M^{*} \le V_M^{**}$ , the steady-state benefit by the government hold for $V_G^{*} < V_G^{**}$ , the steady-state benefit by the whole system hold for $V_T^{*} \le V_T^{**} \le V_T^{***}$ (for the proof, see the Appendix).

In proposition 7, we can see that the benefits of government, supplier, manufacturer and the whole system have been increased in the situations of government promotion and collaborative cooperation. Government cost subsidies play an important role in promoting the reputation of green supply chain.

# 5 Simulation and example analysis

To better understand the long-term relationship between the equilibriums of three situations, simulation and example analysis of the models are conducted in this section. According to the assignment method of relevant literature [22, 23], through the assignment of relevant parameters, three situations can be compared. The specific parameter assignment is shown in Table 1.

**Table 1:** parameter assignment.

| $\lambda_S$ | $\lambda_M$ | $\lambda_G$ | $\alpha_1$ | $\alpha_2$ | $\beta_1$ | $\beta_2$ | $\eta_1$ | $\eta_2$ | $\mu$ | $\delta_1$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 1 | 3 | 4 | 1 | 1 | 0.3 | 0.4 | 0.2 | 2 |
| $\delta_2$ | $\varphi_1$ | $\varphi_2$ | $\gamma_1$ | $\gamma_2$ | $d_1$ | $d_2$ | $q_1$ | $q_2$ | $p_1$ | $p_2$ |
| 2 | 3 | 3 | 0.2 | 0.25 | 0 | 0 | 0 | 0 | 2 | 3 |
| $k_1$ | $k_2$ | $l_1$ | $l_2$ | $b_1$ | $b_2$ | $r$ | $\theta_1$ | $\theta_2$ | | |
| 1 | 2 | 2 | 3 | 2 | 3 | 0.1 | 0.4 | 0.5 | | |

We calculate the optimal effort level of the government and the supplier and manufacturer of the green supply chain in three situations, as shown in Table 2.

**Table 2:** Green supply chain and government effort level under different situations.

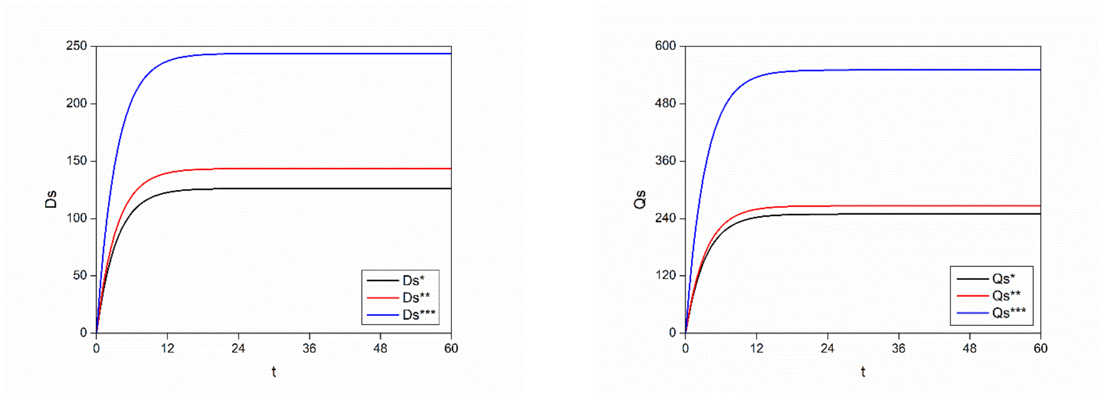
| | $N_S$ | $N_M$ | $N_G1$ | $N_G2$ |
|---|---|---|---|---|
| non cooperation | 9.100 | 7.357 | 10.550 | 16.857 |
| government promotion | 10.842 | 11.036 | 10.550 | 16.857 |
| collaborative cooperation | 15.617 | 14.714 | 26.300 | 33.714 |
| size relation | $N_S^* < N_S^{**} < N_S^{***}$ | $N_M^* < N_M^{**} < N_M^{***}$ | $N_{G1}^* = N_{G1}^{**} < N_{G1}^{***}$ | $N_{G2}^* = N_{G2}^{**} < N_{G2}^{***}$ |

In the three situations, we can find that the level of effort of supplier, manufacturer and government is the lowest in the non-cooperative differential game. The government's cost subsidy strategy enhances the level of effort of supplier and manufacturer in green supply chain. The level of effort supplier, manufacturer and government have greatly improved in the situation of collaborative cooperation. This verifies the truth of proposition 4. Based on the results of Table 2, we can further calculate the product green degree, reputation and benefit, as shown in Table 3.

In order to make more intuitive comparison, we use Origin to draw out the trend of product green degree and reputation changing with time, as shown in Figures 1 and 2. As time goes on, the green degree of product and reputation of the supplier and manufacturer are growing rapidly in the initial period, then the speed decreases, and finally tends to be smooth. In the case of collaborative cooperation, supplier and manufacturer in green supply chain have the highest level of effort, and government promotion is higher than non-cooperation. This verifies propositions 5 and 6.

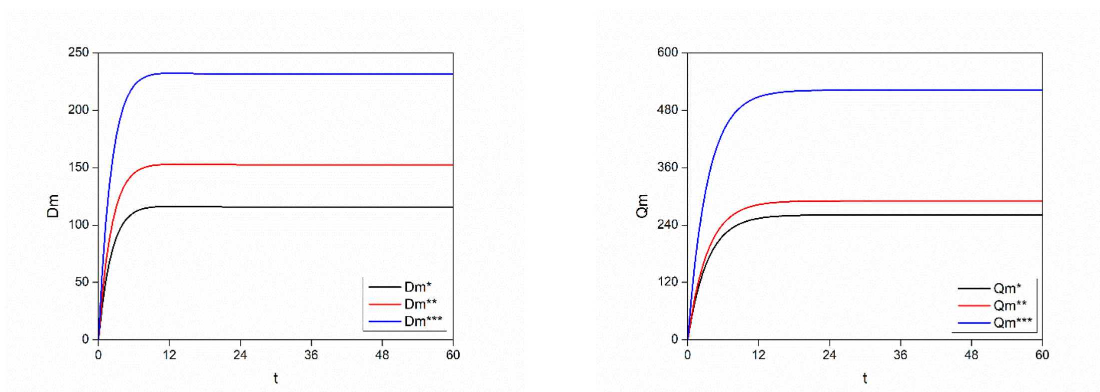
**Table 3:** Product green degree, reputation and benefit in different situations.

| Situation | Product green degree, reputation and benefit |
|---|---|
| non cooperation | $D_S^{\star}=126.167-126.167e^{-0.3t}$ |
| | $Q_S^{\star}=249.25-249.25e^{-0.2t}$ |
| | $V_S^{\star}=3D_S^{\star}+4Q_S^{\star}+919.9$ |
| | $D_M^{\star}=115.714+75.7e^{-0.3t}-191.414e^{-0.4t}$ |
| | $Q_M^{\star}=261.143-261.143e^{-0.25t}$ |
| | $V_M^{\star}=-0.45D_S^{\star}+3D_M^{\star}+4.286Q_M^{\star}+3710.321$ |
| | $V_G^{\star}=1.55D_S^{\star}+3D_M^{\star}+2.667Q_S^{\star}+4.286Q_M^{\star}+4746.239$ |
| | $V_T^{\star}=4.1D_S^{\star}+6D_M^{\star}+6.667Q_S^{\star}+8.571Q_M^{\star}+9376.46$ |
| government promotion | $D_S^{\star\star}=143.583-143.583e^{-0.3t}$ |
| | $Q_S^{\star\star}=266.667-266.667e^{-0.2t}$ |
| | $V_S^{\star\star}=3D_S^{\star\star}+4Q_S^{\star\star}+1068.154$ |
| | $D_M^{\star\star}=152.5+86.15e^{-0.3t}-238.65e^{-0.4t}$ |
| | $Q_M^{\star\star}=290.571-290.571e^{-0.25t}$ |
| | $V_M^{\star\star}=-0.45D_S^{\star\star}+3D_M^{\star\star}+4.286Q_M^{\star\star}+4124.115$ |
| | $V_G^{\star\star}=1.55D_S^{\star\star}+3D_M^{\star\star}+2.667Q_S^{\star\star}+4.286Q_M^{\star\star}+4979.551$ |
| | $V_T^{\star\star}=4.1D_S^{\star\star}+6D_M^{\star\star}+6.667Q_S^{\star\star}+8.571Q_M^{\star\star}+10171.82$ |
| collaborative cooperation | $D_S^{\star\star\star}=243.833-243.833e^{-0.3t}$ |
| | $Q_S^{\star\star\star}=550.667-550.667e^{-0.2t}$ |
| | $D_M^{\star\star\star}=231.429+146.3e^{-0.3t}-377.729e^{-0.4t}$ |
| | $Q_M^{\star\star\star}=522.286-522.286e^{-0.25t}$ |
| | $V_T^{\star\star\star}=4.1D_S^{\star\star\star}+6D_M^{\star\star\star}+6.667Q_S^{\star\star\star}+8.571Q_M^{\star\star\star}+14828.171$ |




(a) (b)

**Figure 1:** Change trend of supplier's product green degree and reputation.




(a) (b)

**Figure 2:** Change trend of manufacturer's product green degree and reputation.

We compare the benefits of supplier, manufacturer, government and the whole system in different situations, as shown in Figure 3. When the government adopts the cost subsidy strategy, the benefit of the supplier and manufacturer of the green supply chain will be greatly improved than the non-cooperation situation. It shows that the government plays an important role in promoting the development of green supply chain. A relatively suitable strategy can effectively improve the benefits of the green supply chain and the government. Figure 3 (d) shows the whole system benefits of government, supplier and manufacturer in different situations. It can be found that the whole system benefit reaches the maximum in the situation of collaborative cooperation, followed by the situation of government promotion, and the lowest one is the non-cooperation situation. Figure 3 verifies the related conclusions of proposition 7.

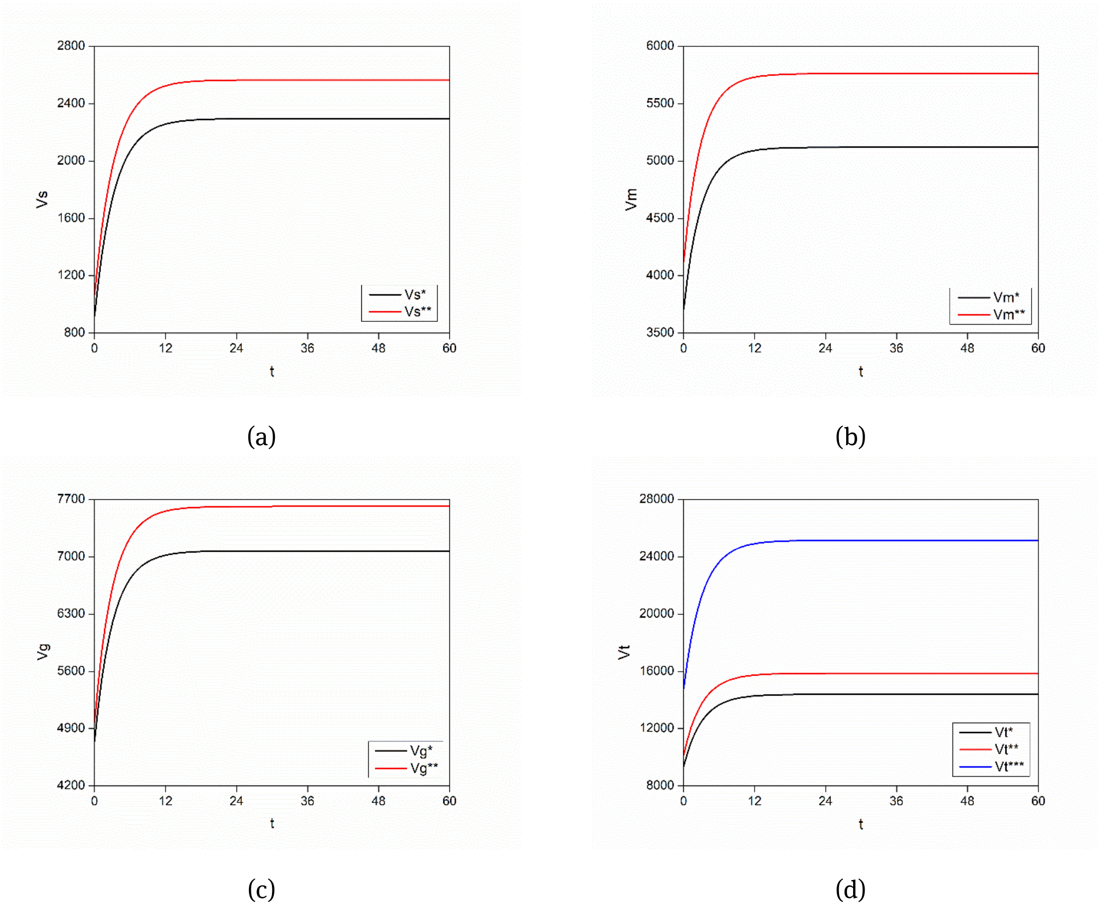



**Figure 3:** Change trend in the benefits of supplier, manufacturer and government.

## 6 Conclusion

In this paper, we have shown a differential game model under the three situations of non-cooperation, government promotion and cooperative cooperation. We consider the relationship and cooperation strategy between the government and the supplier and manufacturer of the green supply chain, and discuss the differences in the optimal effort level, green degree of product, reputation and the optimal benefit of each subject. By comparing and analyzing of equilibrium results, we have known that the optimal effort level, product green degree, reputation and optimal benefit are obviously higher in the situation of collaborative cooperation than that under non-cooperation and government promotion. The government's cost subsidy strategy enhances the level of effort of supplier and manufacturer in green supply chain. The cost coefficient, the natural attenuation coefficient of product green degree, the discount rate and the negative influence factors of reputation are negatively correlated with the optimal level of effort. The influence coefficient of the level of effort, the influence coefficient of product green degree on the benefit, the influence coefficient of reputation on the benefit, the influence coefficient of the degree of effort on the green degree of the product, and the influence coefficient of the effort on the reputation are positively related to the optimal level of effort.

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# Appendix

## Proof of Proposition 2

In order to obtain the Stackelberg equilibrium, there exist the benefit optimal value function of the green supply chain, $V_S(D_S, D_M, Q_S, Q_M)$ and $V_M(D_S, D_M, Q_S, Q_M)$, which are continuous differentiable functions. First, we use backward induction to solve optimal control problem. For all $D_i \geq 0$ and $Q_S \geq 0$, $i \in \{S, M\}$, the benefit optimal value functions satisfy the following Hamilton-Jacobi-Bellman equation

$$\begin{aligned} &rV_S(D_S, D_M, Q_S, Q_M) \\ &= \max_{N_S \geq 0} \Big\{ (1-\theta_1)[p_1 N_S + k_1 N_{G1} + l_1 D_S + b_1 Q_S] + \frac{\partial V_S}{\partial D_S}(\alpha_1 N_S + \beta_1 N_{G1} - \eta_1 D_S) \\ &+ \frac{\partial V_S}{\partial D_M}[\mu(\alpha_1 N_S + \beta_1 N_{G1} - \eta_1 D_S) + \alpha_2 N_M + \beta_2 N_{G2} - \eta_2 D_M] - \frac{\lambda_S}{2}(1-\varepsilon_1) N_S^2 \\ &+ \frac{\partial V_S}{\partial Q_S}(\delta_1 N_S + \varphi_1 N_{G1} - \gamma_1 Q_S) + \frac{\partial V_S}{\partial Q_M}(\delta_2 N_M + \varphi_2 N_{G2} - \gamma_2 Q_M) \Big\} \end{aligned} \tag{A1}$$

$$\begin{aligned} &rV_M(D_S, D_M, Q_S, Q_M) \\ &= \max_{N_M \geq 0} \Big\{ (1-\theta_2)[p_2 N_M + k_2 N_{G2} + l_2 D_M + b_2 Q_M] + \frac{\partial V_M}{\partial D_S}(\alpha_1 N_S + \beta_1 N_{G1} - \eta_1 D_S) \\ &+ \frac{\partial V_M}{\partial D_M}[\mu(\alpha_1 N_S + \beta_1 N_{G1} - \eta_1 D_S) + \alpha_2 N_M + \beta_2 N_{G2} - \eta_2 D_M] - \frac{\lambda_M}{2}(1-\varepsilon_2) N_M^2 \\ &+ \frac{\partial V_M}{\partial Q_S}(\delta_1 N_S + \varphi_1 N_{G1} - \gamma_1 Q_S) + \frac{\partial V_M}{\partial Q_M}(\delta_2 N_M + \varphi_2 N_{G2} - \gamma_2 Q_M) \Big\} \end{aligned} \tag{A2}$$

For solving formula (A1) and (A2), using extreme conditions and searching for the optimal value of $N_S$ and $N_M$ by setting the all partial derivative equal to zero, we can get

$$N_S = \frac{(1-\theta_1)p_1 + \alpha_1\left(\frac{\partial V_S}{\partial D_S} + \mu\frac{\partial V_S}{\partial D_M}\right) + \delta_1\frac{\partial V_S}{\partial Q_S}}{\lambda_S(1-\varepsilon_1)} \tag{A3}$$

$$N_M = \frac{(1-\theta_2)p_2 + \alpha_2\frac{\partial V_M}{\partial D_M} + \delta_2\frac{\partial V_M}{\partial Q_M}}{\lambda_M(1-\varepsilon_2)} \tag{A4}$$

Second, the benefit optimal value function, $V_G(D_S, D_M, Q_S, Q_M)$ satisfies the following Hamilton-Jacobi-Bellman equation

$$\begin{aligned} &rV_G(D_S, D_M, Q_S, Q_M) \\ &= \max_{\substack{N_S \geq 0 \\ N_M \geq 0}} \Big\{ \theta_1(p_1 N_S + k_1 N_{G1} + l_1 D_S + b_1 Q_S) + \theta_2(p_2 N_M + k_2 N_{G2} + l_2 D_M + b_2 Q_M) \\ &- \frac{\lambda_G}{2}\left(N_{G1}^2 + N_{G2}^2\right) - \frac{1}{2}\left(\lambda_S\varepsilon_1 N_S^2 + \lambda_M\varepsilon_2 N_M^2\right) + \frac{\partial V_G}{\partial D_S}(\alpha_1 N_S + \beta_1 N_{G1} - \eta_1 D_S) \\ &+ \frac{\partial V_G}{\partial D_M}[\mu(\alpha_1 N_S + \beta_1 N_{G1} - \eta_1 D_S) + \alpha_2 N_M + \beta_2 N_{G2} - \eta_2 D_M] \\ &+ \frac{\partial V_G}{\partial Q_S}(\delta_1 N_S + \varphi_1 N_{G1} - \gamma_1 Q_S) + \frac{\partial V_G}{\partial Q_M}(\delta_2 N_M + \varphi_2 N_{G2} - \gamma_2 Q_M) \Big\} \end{aligned} \tag{A5}$$

Substituting the results of (A3) and (A4) into (54), we can obtain

$$N_{G1} = \frac{\theta_1 k_1 + \beta_1\left(\frac{\partial V_G}{\partial D_S} + \mu\frac{\partial V_G}{\partial D_M}\right) + \varphi_1\frac{\partial V_G}{\partial Q_S}}{\lambda_G} \tag{A6}$$

$$N_{G2}=\frac{\theta_2 k_2+\beta_2\frac{\partial V_G}{\partial D_M}+\varphi_2\frac{\partial V_G}{\partial Q_M}}{\lambda_G} \tag{A7}$$

$$\varepsilon_1=\frac{(3\theta_1-1)\,p_1+\alpha_1\left[2\left(\frac{\partial V_G}{\partial D_S}+\mu\frac{\partial V_G}{\partial D_M}\right)-\left(\frac{\partial V_S}{\partial D_S}+\mu\frac{\partial V_S}{\partial D_M}\right)\right]+\delta_1\left(2\frac{\partial V_G}{\partial Q_S}-\frac{\partial V_S}{\partial Q_S}\right)}{(\theta_1+1)\,p_1+\alpha_1\left[2\left(\frac{\partial V_G}{\partial D_S}+\mu\frac{\partial V_G}{\partial D_M}\right)+\left(\frac{\partial V_S}{\partial D_S}+\mu\frac{\partial V_S}{\partial D_M}\right)\right]+\delta_1\left(2\frac{\partial V_G}{\partial Q_S}+\frac{\partial V_S}{\partial Q_S}\right)} \tag{A8}$$

$$\varepsilon_2=\frac{(3\theta_2-1)\,p_2+\alpha_2\left(2\frac{\partial V_G}{\partial D_M}-\frac{\partial V_M}{\partial D_M}\right)+\delta_2\left(2\frac{\partial V_G}{\partial Q_M}-\frac{\partial V_M}{\partial Q_M}\right)}{(\theta_2+1)\,p_2+\alpha_2\left(2\frac{\partial V_G}{\partial D_M}+\frac{\partial V_M}{\partial D_M}\right)+\delta_2\left(2\frac{\partial V_G}{\partial Q_M}+\frac{\partial V_M}{\partial Q_M}\right)} \tag{A9}$$

Substituting the results of (A3), (A4), (A6), (A7), (A8) and (A9) into (A1), (A2) and (A5), we can obtain

$$\begin{aligned}
rV_S=&\left[(1-\theta_1)\,l_1-\eta_1\left(\frac{\partial V_S}{\partial D_S}+\mu\frac{\partial V_S}{\partial D_M}\right)\right]D_S-\eta_2\frac{\partial V_S}{\partial D_M}D_M+\left[(1-\theta_1)\,b_1-\gamma_1\frac{\partial V_S}{\partial Q_S}\right]Q_S\\
&-\gamma_2\frac{\partial V_S}{\partial Q_M}Q_M+\frac{\left(\frac{\partial V_S}{\partial D_M}\beta_2+\frac{\partial V_S}{\partial Q_M}\varphi_2\right)\left(\theta_2 k_2+\beta_2\frac{\partial V_G}{\partial D_M}+\varphi_2\frac{\partial V_G}{\partial Q_M}\right)}{\lambda_G}\\
&+\frac{\left[\theta_1 k_1+\beta_1\left(\frac{\partial V_G}{\partial D_S}+\mu\frac{\partial V_G}{\partial D_M}\right)+\varphi_1\frac{\partial V_G}{\partial Q_S}\right]\left[(1-\theta_1)\,k_1+\beta_1\left(\frac{\partial V_S}{\partial D_S}+\mu\frac{\partial V_S}{\partial D_M}\right)+\varphi_1\frac{\partial V_S}{\partial Q_S}\right]}{\lambda_G}\\
&+\frac{\left(\frac{\partial V_S}{\partial D_M}\alpha_2+\frac{\partial V_S}{\partial Q_M}\delta_2\right)\left[(\theta_2+1)\,p_2+\alpha_2\left(2\frac{\partial V_G}{\partial D_M}+\frac{\partial V_M}{\partial D_M}\right)+\delta_2\left(2\frac{\partial V_G}{\partial Q_M}+\frac{\partial V_M}{\partial Q_M}\right)\right]}{2\lambda_M}\\
&+\frac{\left[(1-\theta_1)\,p_1+\alpha_1\left(\frac{\partial V_S}{\partial D_S}+\mu\frac{\partial V_S}{\partial D_M}\right)+\delta_1\frac{\partial V_S}{\partial Q_S}\right]\left[(\theta_1+1)\,p_1+\delta_1\left(2\frac{\partial V_G}{\partial Q_S}+\frac{\partial V_S}{\partial Q_S}\right)\right]}{4\lambda_S}\\
&+\frac{\alpha_1\left[2\left(\frac{\partial V_G}{\partial D_S}+\mu\frac{\partial V_G}{\partial D_M}\right)+\left(\frac{\partial V_S}{\partial D_S}+\mu\frac{\partial V_S}{\partial D_M}\right)\right]\left[(1-\theta_1)\,p_1+\alpha_1\left(\frac{\partial V_S}{\partial D_S}+\mu\frac{\partial V_S}{\partial D_M}\right)+\delta_1\frac{\partial V_S}{\partial Q_S}\right]}{4\lambda_S}
\end{aligned} \tag{A10}$$

$$\begin{aligned}
rV_M=&\left[(1-\theta_2)\,l_2-\eta_2\frac{\partial V_M}{\partial D_M}\right]D_M-\eta_1\left(\frac{\partial V_M}{\partial D_S}+\mu\frac{\partial V_M}{\partial D_M}\right)D_S+\left[(1-\theta_2)\,b_2-\gamma_2\frac{\partial V_M}{\partial Q_M}\right]Q_M\\
&-\gamma_1\frac{\partial V_M}{\partial Q_S}Q_S+\frac{\left(\theta_2 k_2+\beta_2\frac{\partial V_G}{\partial D_M}+\varphi_2\frac{\partial V_G}{\partial Q_M}\right)\left[(1-\theta_2)\,k_2+\beta_2\frac{\partial V_M}{\partial D_M}+\varphi_2\frac{\partial V_M}{\partial Q_M}\right]}{\lambda_G}\\
&+\frac{\left[(1-\theta_2)\,p_2+\alpha_2\frac{\partial V_M}{\partial D_M}+\delta_2\frac{\partial V_M}{\partial Q_M}\right]\left[(\theta_2+1)\,p_2+\alpha_2\left(2\frac{\partial V_G}{\partial D_M}+\frac{\partial V_M}{\partial D_M}\right)+\delta_2\left(2\frac{\partial V_G}{\partial Q_M}+\frac{\partial V_M}{\partial Q_M}\right)\right]}{4\lambda_M}\\
&+\frac{\left[(\theta_1+1)\,p_1+\delta_1\left(2\frac{\partial V_G}{\partial Q_S}+\frac{\partial V_S}{\partial Q_S}\right)\right]\left[\alpha_1\frac{\partial V_M}{\partial D_S}+\alpha_1\mu\frac{\partial V_M}{\partial D_M}+\delta_1\frac{\partial V_M}{\partial Q_S}\right]}{2\lambda_S}\\
&+\frac{\alpha_1\left[2\left(\frac{\partial V_G}{\partial D_S}+\mu\frac{\partial V_G}{\partial D_M}\right)+\left(\frac{\partial V_S}{\partial D_S}+\mu\frac{\partial V_S}{\partial D_M}\right)\right]\left[\alpha_1\frac{\partial V_M}{\partial D_S}+\alpha_1\mu\frac{\partial V_M}{\partial D_M}+\delta_1\frac{\partial V_M}{\partial Q_S}\right]}{2\lambda_S}\\
&+\frac{\left[\theta_1 k_1+\beta_1\left(\frac{\partial V_G}{\partial D_S}+\mu\frac{\partial V_G}{\partial D_M}\right)+\varphi_1\frac{\partial V_G}{\partial Q_S}\right]\left[\beta_1\frac{\partial V_M}{\partial D_S}+\beta_1\mu\frac{\partial V_M}{\partial D_M}+\varphi_1\frac{\partial V_M}{\partial Q_S}\right]}{\lambda_G}
\end{aligned} \tag{A11}$$

$$\begin{aligned} rV_G = &\left[\theta_1 l_1 - \eta_1\left(\frac{\partial V_G}{\partial D_S} + \mu\frac{\partial V_G}{\partial D_M}\right)\right] D_S + \left[\theta_2 l_2 - \eta_2\frac{\partial V_G}{\partial D_M}\right] D_M + \left(\theta_1 b_1 - \gamma_1\frac{\partial V_G}{\partial Q_S}\right) Q_S \\ &+ \left(\theta_2 b_2 - \gamma_2\frac{\partial V_G}{\partial Q_M}\right) Q_M + \frac{\left[\theta_1 k_1 + \beta_1\left(\frac{\partial V_G}{\partial D_S} + \mu\frac{\partial V_G}{\partial D_M}\right) + \varphi_1\frac{\partial V_G}{\partial Q_S}\right]^2}{2\lambda_G} + \frac{\left[\theta_2 k_2 + \beta_2\frac{\partial V_G}{\partial D_M} + \varphi_2\frac{\partial V_G}{\partial Q_M}\right]^2}{2\lambda_G} \\ &+ \frac{\left\{(\theta_1+1)p_1 + \alpha_1\left[2\left(\frac{\partial V_G}{\partial D_S} + \mu\frac{\partial V_G}{\partial D_M}\right) + \left(\frac{\partial V_S}{\partial D_S} + \mu\frac{\partial V_S}{\partial D_M}\right)\right] + \delta_1\left(2\frac{\partial V_G}{\partial Q_S} + \frac{\partial V_S}{\partial Q_S}\right)\right\}^2}{8\lambda_S} \\ &+ \frac{\left[(\theta_2+1)p_2 + \alpha_2\left(2\frac{\partial V_G}{\partial D_M} + \frac{\partial V_M}{\partial D_M}\right) + \delta_2\left(2\frac{\partial V_G}{\partial Q_M} + \frac{\partial V_M}{\partial Q_M}\right)\right]^2}{8\lambda_M} \end{aligned} \tag{A12}$$

Through the above formulas and the optimal control theory, we can see that the linear optimal function of the $D_S, D_M, Q_S, Q_M$ is the solution of the HJB equation. We can assume

$$\begin{aligned} V_S(D_S, D_M, Q_S, Q_M) &= \sigma_1 D_S + \sigma_2 D_M + \sigma_3 Q_S + \sigma_4 Q_M + \omega_1 \\ V_M(D_S, D_M, Q_S, Q_M) &= h_1 D_S + h_2 D_M + h_3 Q_S + h_4 Q_M + \omega_2 \\ V_G(D_S, D_M, Q_S, Q_M) &= g_1 D_S + g_2 D_M + g_3 Q_S + g_4 Q_M + \omega_3 \end{aligned} \tag{A13}$$

where $\sigma_i, q_i, g_i\,(i = 1, 2, 3, 4)$ and $\omega_i\,(i = 1, 2, 3)$ are the constants to be solved. We can have

$$\begin{aligned} &\frac{\partial V_S}{\partial D_S} = \sigma_1, \frac{\partial V_S}{\partial D_M} = \sigma_2, \frac{\partial V_S}{\partial Q_S} = \sigma_3, \frac{\partial V_S}{\partial Q_M} = \sigma_4 \\ &\frac{\partial V_M}{\partial D_S} = h_1, \frac{\partial V_M}{\partial D_M} = h_2, \frac{\partial V_M}{\partial Q_S} = h_3, \frac{\partial V_M}{\partial Q_M} = h_4 \\ &\frac{\partial V_G}{\partial D_S} = g_1, \frac{\partial V_G}{\partial D_M} = g_2, \frac{\partial V_G}{\partial Q_S} = g_3, \frac{\partial V_G}{\partial Q_M} = g_4 \end{aligned} \tag{A14}$$

Substituting the result of (A13) and (A14) into (A10), (A11) and (A12), we can get

$$\begin{aligned} &r(\sigma_1 D_S + \sigma_2 D_M + \sigma_3 Q_S + \sigma_4 Q_M + \omega_1) \\ = &\left[(1-\theta_1) l_1 - \eta_1(\sigma_1 + \mu\sigma_2)\right] D_S - \eta_2\sigma_2 D_M + \left[(1-\theta_1) b_1 - \gamma_1\sigma_3\right] Q_S \\ &- \gamma_2\sigma_4 Q_M + \frac{(\sigma_2\beta_2 + \sigma_4\varphi_2)(\theta_2 k_2 + \beta_2 g_2 + \varphi_2 g_4)}{\lambda_G} \\ &+ \frac{[\theta_1 k_1 + \beta_1(g_1 + \mu g_2) + \varphi_1 g_3][(1-\theta_1)k_1 + \beta_1(\sigma_1 + \mu\sigma_2) + \varphi_1\sigma_3]}{\lambda_G} \\ &+ \frac{(\sigma_2\alpha_2 + \sigma_4\delta_2)[(\theta_2+1)p_2 + \alpha_2(2g_2 + h_2) + \delta_2(2g_4 + h_4)]}{2\lambda_M} \\ &+ \frac{[(1-\theta_1)p_1 + \alpha_1(\sigma_1 + \mu\sigma_2) + \delta_1\sigma_3]\{(\theta_1+1)p_1 + \alpha_1[2(g_1 + \mu g_2) + (\sigma_1 + \mu\sigma_2)] + \delta_1(2g_3 + \sigma_3)\}}{4\lambda_S} \end{aligned} \tag{A15}$$

$$\begin{aligned} &r(h_1 D_S + h_2 D_M + h_3 Q_S + h_4 Q_M + \omega_2) \\ = &\left[(1-\theta_2) l_2 - \eta_2 h_2\right] D_M - \eta_1(h_1 + \mu h_2) D_S + \left[(1-\theta_2) b_2 - \gamma_2 h_4\right] Q_M \\ &- \gamma_1 h_3 Q_S + \frac{(\theta_2 k_2 + \beta_2 g_2 + \varphi_2 g_4)[(1-\theta_2)k_2 + \beta_2 h_2 + \varphi_2 h_4]}{\lambda_G} \\ &+ \frac{[(1-\theta_2)p_2 + \alpha_2 h_2 + \delta_2 h_4][(\theta_2+1)p_2 + \alpha_2(2g_2 + h_2) + \delta_2(2g_4 + h_4)]}{4\lambda_M} \\ &+ \frac{\{(\theta_1+1)p_1 + \alpha_1[2(g_1 + \mu g_2) + (\sigma_1 + \mu\sigma_2)] + \delta_1(2g_3 + \sigma_3)\}[\alpha_1 h_1 + \alpha_1\mu h_2 + \delta_1 h_3]}{2\lambda_S} \\ &+ \frac{[\theta_1 k_1 + \beta_1(g_1 + \mu g_2) + \varphi_1 g_3][\beta_1 h_1 + \beta_1\mu h_2 + \varphi_1 h_3]}{\lambda_G} \end{aligned} \tag{A16}$$

$$
\begin{aligned}
& r\left(g_1 D_S+g_2 D_M+g_3 Q_S+g_4 Q_M+\omega_3\right) \\
= & \left[\theta_1 l_1-\eta_1\left(g_1+\mu g_2\right)\right] D_S+\left[\theta_2 l_2-\eta_2 g_2\right] D_M+\left(\theta_1 b_1-\gamma_1 g_3\right) Q_S \\
& +\left(\theta_2 b_2-\gamma_2 g_4\right) Q_M+\frac{\left[\theta_1 k_1+\beta_1\left(g_1+\mu g_2\right)+\varphi_1 g_3\right]^2}{2 \lambda_G}+\frac{\left[\theta_2 k_2+\beta_2 g_2+\varphi_2 g_4\right]^2}{2 \lambda_G} \\
& +\frac{\left\{\left(\theta_1+1\right) p_1+\alpha_1\left[2\left(g_1+\mu g_2\right)+\left(\sigma_1+\mu \sigma_2\right)\right]+\delta_1\left(2 g_3+\sigma_3\right)\right\}^2}{8 \lambda_S} \\
& +\frac{\left[\left(\theta_2+1\right) p_2+\alpha_2\left(2 g_2+h_2\right)+\delta_2\left(2 g_4+h_4\right)\right]^2}{8 \lambda_M}
\end{aligned} \tag{A17}
$$

Using the $D_S, D_M, Q_S, Q_M \geq 0$ to (A15), (A16) and (A17), parameter values of the optimal value function can be expressed as follows

$$
\begin{array}{llll}
\sigma_1=\dfrac{\left(1-\theta_1\right) l_1}{\eta_1+r}, & \sigma_2=0, & \sigma_3=\dfrac{\left(1-\theta_1\right) b_1}{\gamma_1+r}, \ \sigma_4=0 & \\
h_1=-\dfrac{\eta_1 \mu\left(1-\theta_2\right) l_2}{\left(\eta_1+r\right)\left(\eta_2+r\right)}, & h_2=\dfrac{\left(1-\theta_2\right) l_2}{\eta_2+r}, \ h_3=0, & & h_4=\dfrac{\left(1-\theta_2\right) b_2}{\gamma_2+r} \\
g_1=\dfrac{\theta_1 l_1}{\eta_1+r}-\dfrac{\eta_1 \mu \theta_2 l_2}{\left(\eta_1+r\right)\left(\eta_2+r\right)}, & g_2=\dfrac{\theta_2 l_2}{\eta_2+r}, & g_3=\dfrac{\theta_1 b_1}{\gamma_1+r}, & g_4=\dfrac{\theta_2 b_2}{\gamma_2+r}
\end{array} \tag{A18}
$$

$$
\begin{aligned}
\omega_1= & \frac{\left[\theta_1\left(k_1+\dfrac{\beta_1 l_1}{\eta_1+r}+\dfrac{\varphi_1 b_1}{\gamma_1+r}\right)+\dfrac{\beta_1 \mu \theta_2 l_2 r}{\left(\eta_1+r\right)\left(\eta_2+r\right)}\right]\left(1-\theta_1\right)\left(k_1+\dfrac{\beta_1 l_1}{\eta_1+r}+\dfrac{\varphi_1 b_1}{\gamma_1+r}\right)}{r \lambda_G} \\
& +\frac{\left(1-\theta_1\right)\left(p_1+\dfrac{\alpha_1 l_1}{\eta_1+r}+\dfrac{\delta_1 b_1}{\gamma_1+r}\right)\left\{\left(\theta_1+1\right)\left(p_1+\dfrac{\alpha_1 l_1}{\eta_1+r}+\dfrac{\delta_1 b_1}{\gamma_1+r}\right)+\dfrac{\alpha_1 \mu \theta_2 l_2 r}{\left(\eta_1+r\right)\left(\eta_2+r\right)}\right\}}{4 r \lambda_S}
\end{aligned} \tag{A19}
$$

$$
\begin{aligned}
\omega_2= & \frac{\theta_2\left(1-\theta_2\right)\left(k_2+\dfrac{\beta_2 l_2}{\eta_2+r}+\dfrac{\varphi_2 b_2}{\gamma_2+r}\right)^2}{r \lambda_G}+\frac{\left(1-\theta_2\right)\left(\theta_2+1\right)\left[p_2+\dfrac{\alpha_2 l_2}{\eta_2+r}+\dfrac{\delta_2 b_2}{\gamma_2+r}\right]^2}{4 r \lambda_M} \\
& +\frac{\left[\left(\theta_1+1\right)\left(p_1+\dfrac{\alpha_1 l_1}{\eta_1+r}+\dfrac{\delta_1 b_1}{\gamma_1+r}\right)+\dfrac{2 \alpha_1 \mu \theta_2 l_2 r}{\left(\eta_1+r\right)\left(\eta_2+r\right)}\right]\left[\dfrac{\alpha_1 \mu\left(1-\theta_2\right) l_2}{\left(\eta_1+r\right)\left(\eta_2+r\right)}\right]}{2 \lambda_S} \\
& +\frac{\left[\theta_1\left(k_1+\dfrac{\beta_1 l_1}{\eta_1+r}+\dfrac{\varphi_1 b_1}{\gamma_1+r}\right)+\dfrac{\beta_1 \mu \theta_2 l_2 r}{\left(\eta_1+r\right)\left(\eta_2+r\right)}\right]\left[\dfrac{\beta_1 \mu\left(1-\theta_2\right) l_2}{\left(\eta_1+r\right)\left(\eta_2+r\right)}\right]}{\lambda_G}
\end{aligned} \tag{A20}
$$

$$
\begin{aligned}
\omega_3= & \frac{\left[\theta_1\left(k_1+\dfrac{\beta_1 l_1}{\eta_1+r}+\dfrac{\varphi_1 b_1}{\gamma_1+r}\right)+\dfrac{\beta_1 \mu \theta_2 l_2 r}{\left(\eta_1+r\right)\left(\eta_2+r\right)}\right]^2}{2 r \lambda_G}+\frac{\left[\theta_2 k_2+\dfrac{\beta_2 \theta_2 l_2}{\eta_2+r}+\dfrac{\varphi_2 \theta_2 b_2}{\gamma_2+r}\right]^2}{2 r \lambda_G} \\
& +\frac{\left\{\left(\theta_1+1\right)\left(p_1+\dfrac{\alpha_1 l_1}{\eta_1+r}+\dfrac{\delta_1 b_1}{\gamma_1+r}\right)+\dfrac{2 \alpha_1 \mu \theta_2 l_2 r}{\left(\eta_1+r\right)\left(\eta_2+r\right)}\right\}^2}{8 r \lambda_S} \\
& +\frac{\left[\left(\theta_2+1\right) p_2+\dfrac{\alpha_2\left(1+\theta_2\right) l_2}{\eta_2+r}+\dfrac{\delta_2\left(1+\theta_2\right) b_2}{\gamma_2+r}\right]^2}{8 r \lambda_M}
\end{aligned} \tag{A21}
$$

Substituting the results of (A18), (A19), (A20) and (A21) into (A13), we can obtain

$$
\begin{aligned}
V_S^{\star\star}= & \frac{\left(1-\theta_1\right) l_1}{\eta_1+r} D_S+\frac{\left(1-\theta_1\right) b_1}{\gamma_1+r} Q_S \\
& +\frac{\left[\theta_1\left(k_1+\dfrac{\beta_1 l_1}{\eta_1+r}+\dfrac{\varphi_1 b_1}{\gamma_1+r}\right)+\dfrac{\beta_1 \mu \theta_2 l_2 r}{\left(\eta_1+r\right)\left(\eta_2+r\right)}\right]\left(1-\theta_1\right)\left(k_1+\dfrac{\beta_1 l_1}{\eta_1+r}+\dfrac{\varphi_1 b_1}{\gamma_1+r}\right)}{r \lambda_G} \\
& +\frac{\left(1-\theta_1\right)\left(p_1+\dfrac{\alpha_1 l_1}{\eta_1+r}+\dfrac{\delta_1 b_1}{\gamma_1+r}\right)\left\{\left(\theta_1+1\right)\left(p_1+\dfrac{\alpha_1 l_1}{\eta_1+r}+\dfrac{\delta_1 b_1}{\gamma_1+r}\right)+\dfrac{\alpha_1 \mu \theta_2 l_2 r}{\left(\eta_1+r\right)\left(\eta_2+r\right)}\right\}}{4 r \lambda_S}
\end{aligned} \tag{A22}
$$

$$\begin{aligned} V_M^{**} =& -\frac{\eta_1\mu(1-\theta_2)l_2}{(\eta_1+r)(\eta_2+r)}D_S+\frac{(1-\theta_2)l_2}{\eta_2+r}D_M+\frac{(1-\theta_2)b_2}{\gamma_2+r}Q_M \\ &+\frac{\theta_2(1-\theta_2)\left(k_2+\frac{\beta_2 l_2}{\eta_2+r}+\frac{\varphi_2 b_2}{\gamma_2+r}\right)^2}{r\lambda_G}+\frac{(1-\theta_2)(\theta_2+1)\left[p_2+\frac{\alpha_2 l_2}{\eta_2+r}+\frac{\delta_2 b_2}{\gamma_2+r}\right]^2}{4r\lambda_M} \\ &+\frac{\left[(\theta_1+1)\left(p_1+\frac{\alpha_1 l_1}{\eta_1+r}+\frac{\delta_1 b_1}{\gamma_1+r}\right)+\frac{2\alpha_1\mu\theta_2 l_2 r}{(\eta_1+r)(\eta_2+r)}\right]\left[\frac{\alpha_1\mu(1-\theta_2)l_2}{(\eta_1+r)(\eta_2+r)}\right]}{2\lambda_S} \\ &+\frac{\left[\theta_1\left(k_1+\frac{\beta_1 l_1}{\eta_1+r}+\frac{\varphi_1 b_1}{\gamma_1+r}\right)+\frac{\beta_1\mu\theta_2 l_2 r}{(\eta_1+r)(\eta_2+r)}\right]\left[\frac{\beta_1\mu(1-\theta_2)l_2}{(\eta_1+r)(\eta_2+r)}\right]}{\lambda_G} \end{aligned} \tag{A23}$$

$$\begin{aligned} V_G^{**} =& \left[\frac{\theta_1 l_1}{\eta_1+r}-\frac{\eta_1\mu\theta_2 l_2}{(\eta_1+r)(\eta_2+r)}\right]D_S+\frac{\theta_2 l_2}{\eta_2+r}D_M+\frac{\theta_1 b_1}{\gamma_1+r}Q_S+\frac{\theta_2 b_2}{\gamma_2+r}Q_M \\ &+\frac{\left[\theta_1\left(k_1+\frac{\beta_1 l_1}{\eta_1+r}+\frac{\varphi_1 b_1}{\gamma_1+r}\right)+\frac{\beta_1\mu\theta_2 l_2 r}{(\eta_1+r)(\eta_2+r)}\right]^2}{2r\lambda_G}+\frac{\left[\theta_2 k_2+\frac{\beta_2\theta_2 l_2}{\eta_2+r}+\frac{\varphi_2\theta_2 b_2}{\gamma_2+r}\right]^2}{2r\lambda_G} \\ &+\frac{\left\{(\theta_1+1)\left(p_1+\frac{\alpha_1 l_1}{\eta_1+r}+\frac{\delta_1 b_1}{\gamma_1+r}\right)+\frac{2\alpha_1\mu\theta_2 l_2 r}{(\eta_1+r)(\eta_2+r)}\right\}^2}{8r\lambda_S}+\frac{(\theta_2+1)^2\left[p_2+\frac{\alpha_2 l_2}{\eta_2+r}+\frac{\delta_2 b_2}{\gamma_2+r}\right]^2}{8r\lambda_M} \end{aligned} \tag{A24}$$

In this case, the benefit of the whole system is

$$V_T^{**} = V_S^{**} + V_M^{**} + V_G^{**} \tag{A25}$$

Solving the partial derivative of (A22), (A23) and (A24), and substituting the results into (A3), (A4), (A6), (A7), (A8) and (A9), we can get the Stackelberg master-slave equilibria of supplier and manufacturer and government as the proposition 2.

## Proof of Proposition 3

$$\begin{aligned} V_T &= V_S + V_M + V_G \\ &= \int_0^{\infty} e^{-rt}\left\{p_1 N_S + k_1 N_{G1} + l_1 D_S + b_1 Q_S - \frac{1}{2}\left(\lambda_G N_{G1}^2 + \lambda_S N_S^2\right)\right. \\ &\qquad \left. p_2 N_M + k_2 N_{G2} + l_2 D_M + b_2 Q_M - \frac{1}{2}\left(\lambda_G N_{G2}^2 + \lambda_M N_M^2\right)\right\} dt \end{aligned} \tag{A26}$$

In order to obtain the cooperative equilibrium, there exists a benefit optimal value function of $V_T(D_S, D_M, Q_S, Q_M)$, which is a continuous differentiable function. For all $D_S, D_M, Q_S, Q_M \geq 0$, the benefit optimal value function satisfies the following Hamilton-Jacobi-Bellman equation

$$\begin{aligned} & rV_T(D_S, D_M, Q_S, Q_M) \\ =& \max_{\substack{N_S\geq 0, N_M\geq 0 \\ N_{G1}\geq 0, N_{G2}\geq 0}} \left\{p_1 N_S + k_1 N_{G1} + l_1 D_S + b_1 Q_S + p_2 N_M + k_2 N_{G2} + l_2 D_M + b_2 Q_M\right. \\ &- \frac{\lambda_G}{2}\left(N_{G1}^2 + N_{G2}^2\right) - \frac{1}{2}\left(\lambda_S N_S^2 + \lambda_M N_M^2\right) + \frac{\partial V_T}{\partial D_S}(\alpha_1 N_S + \beta_1 N_{G1} - \eta_1 D_S) \\ &+ \frac{\partial V_T}{\partial D_M}\left[\mu(\alpha_1 N_S + \beta_1 N_{G1} - \eta_1 D_S) + \alpha_2 N_M + \beta_2 N_{G2} - \eta_2 D_M\right] \\ &\left.+ \frac{\partial V_T}{\partial Q_S}(\delta_1 N_S + \varphi_1 N_{G1} - \gamma_1 Q_S) + \frac{\partial V_T}{\partial Q_M}(\delta_2 N_M + \varphi_2 N_{G2} - \gamma_2 Q_M)\right\} \end{aligned} \tag{A27}$$

For solving formula (A27), using extreme conditions and searching for the optimal value of $N_S$ , $N_M$ , $N_G1$ and $N_G2$ by setting the partial derivative equal to zero, we can get

$$N_S = \frac{p_1 + \alpha_1\left(\frac{\partial V_T}{\partial D_S} + \mu\frac{\partial V_T}{\partial D_M}\right) + \delta_1\frac{\partial V_T}{\partial Q_S}}{\lambda_S} \tag{A28}$$

$$N_M = \frac{p_2 + \alpha_2\frac{\partial V_T}{\partial D_M} + \delta_2\frac{\partial V_T}{\partial Q_M}}{\lambda_M} \tag{A29}$$

$$N_{G1} = \frac{k_1 + \beta_1\left(\frac{\partial V_T}{\partial D_S} + \mu\frac{\partial V_T}{\partial D_M}\right) + \varphi_1\frac{\partial V_T}{\partial Q_S}}{\lambda_G} \tag{A30}$$

$$N_{G2} = \frac{k_2 + \beta_2\frac{\partial V_T}{\partial D_M} + \varphi_2\frac{\partial V_T}{\partial Q_M}}{\lambda_G} \tag{A31}$$

Substituting the results of (A28), (A29), (A30) and (A31) into (A27), we can obtain

$$\begin{aligned}
&rV_T(D_S, D_M, Q_S, Q_M)\\
&= \left[l_1 - \eta_1\left(\frac{\partial V_T}{\partial D_S} + \mu\frac{\partial V_T}{\partial D_M}\right)\right]D_S + \left(l_2 - \eta_2\frac{\partial V_T}{\partial D_M}\right)D_M + \left(b_1 - \gamma_1\frac{\partial V_T}{\partial Q_S}\right)Q_S\\
&+ \left(b_2 - \gamma_2\frac{\partial V_T}{\partial Q_M}\right)Q_M + \frac{\left[p_1 + \alpha_1\left(\frac{\partial V_T}{\partial D_S} + \mu\frac{\partial V_T}{\partial D_M}\right) + \delta_1\frac{\partial V_T}{\partial Q_S}\right]^2}{2\lambda_S}\\
&+ \frac{\left(p_2 + \alpha_2\frac{\partial V_T}{\partial D_M} + \delta_2\frac{\partial V_T}{\partial Q_M}\right)^2}{2\lambda_M} + \frac{\left[k_1 + \beta_1\left(\frac{\partial V_T}{\partial D_S} + \mu\frac{\partial V_T}{\partial D_M}\right) + \varphi_1\frac{\partial V_T}{\partial Q_S}\right]^2}{2\lambda_G}\\
&+ \frac{\left(k_2 + \beta_2\frac{\partial V_T}{\partial D_M} + \varphi_2\frac{\partial V_T}{\partial Q_M}\right)^2}{2\lambda_G}
\end{aligned} \tag{A32}$$

Through the above formulas and the optimal control theory, we can see that the linear optimal function of the $D_S, D_M, Q_S, Q_M$ is the solution of the HJB equation. We can get

$$V_T(D_S, D_M, Q_S, Q_M) = t_1D_S + t_2D_M + t_3Q_S + t_4Q_M + \omega_1 \tag{A33}$$

$$\frac{\partial V_T}{\partial D_S} = t_1, \frac{\partial V_T}{\partial D_M} = t_2, \frac{\partial V_T}{\partial Q_S} = t_3, \frac{\partial V_T}{\partial Q_M} = t_4 \tag{A34}$$

Substituting the results of (A33) and (A34) into (A32), we can get

$$\begin{aligned}
&r(t_1D_S + t_2D_M + t_3Q_S + t_4Q_M + \omega_1)\\
&= [l_1 - \eta_1(t_1 + \mu t_2)]D_S + (l_2 - \eta_2t_2)D_M + (b_1 - \gamma_1t_3)Q_S + (b_2 - \gamma_2t_4)Q_M\\
&+ \frac{[p_1 + \alpha_1(t_1 + \mu t_2) + \delta_1t_3]^2}{2\lambda_S} + \frac{(p_2 + \alpha_2t_2 + \delta_2t_4)^2}{2\lambda_M}\\
&+ \frac{[k_1 + \beta_1(t_1 + \mu t_2) + \varphi_1t_3]^2}{2\lambda_G} + \frac{(k_2 + \beta_2t_2 + \varphi_2t_4)^2}{2\lambda_G}
\end{aligned} \tag{A35}$$

Using the $D_S, D_M, Q_S, Q_M \geq 0$ to (A35), parameter values of the optimal value function can be expressed as follows

$$t_1 = \frac{l_1}{\eta_1 + r} - \frac{\mu\eta_1l_2}{(\eta_1 + r)(\eta_2 + r)}, t_2 = \frac{l_2}{\eta_2 + r}, t_3 = \frac{b_1}{\gamma_1 + r}, t_4 = \frac{b_2}{\gamma_2 + r} \tag{A36}$$

$$\omega_1 = \frac{\left[p_1 + \frac{\delta_1 b_1}{\gamma_1 + r} + \frac{\alpha_1 l_1}{\eta_1 + r} + \frac{\mu\alpha_1 l_2 r}{(\eta_1 + r)(\eta_2 + r)}\right]^2}{2r\lambda_S} + \frac{\left(p_2 + \frac{\alpha_2 l_2}{\eta_2 + r} + \frac{\delta_2 b_2}{\gamma_2 + r}\right)^2}{2r\lambda_M} + \frac{\left[k_1 + \frac{\varphi_1 b_1}{\gamma_1 + r} + \frac{\beta_1 l_1}{\eta_1 + r} + \frac{\mu\beta_1 l_2 r}{(\eta_1 + r)(\eta_2 + r)}\right]^2}{2r\lambda_G} + \frac{\left(k_2 + \frac{\beta_2 l_2}{\eta_2 + r} + \frac{\varphi_2 b_2}{\gamma_2 + r}\right)^2}{2r\lambda_G} \tag{A37}$$

Substituting the results of (A36) and (A37) into (A35), we can get

$$\begin{aligned} &V_T^{***}(D_S, D_M, Q_S, Q_M) \\ &= \left(\frac{l_1}{\eta_1 + r} - \frac{\mu\eta_1 l_2}{(\eta_1 + r)(\eta_2 + r)}\right) D_S + \left(\frac{l_2}{\eta_2 + r}\right) D_M + \left(\frac{b_1}{\gamma_1 + r}\right) Q_S + \left(\frac{b_2}{\gamma_2 + r}\right) Q_M \\ &+ \frac{\left[p_1 + \frac{\delta_1 b_1}{\gamma_1 + r} + \frac{\alpha_1 l_1}{\eta_1 + r} + \frac{\mu\alpha_1 l_2 r}{(\eta_1 + r)(\eta_2 + r)}\right]^2}{2r\lambda_S} + \frac{\left(p_2 + \frac{\alpha_2 l_2}{\eta_2 + r} + \frac{\delta_2 b_2}{\gamma_2 + r}\right)^2}{2r\lambda_M} \\ &+ \frac{\left[k_1 + \frac{\varphi_1 b_1}{\gamma_1 + r} + \frac{\beta_1 l_1}{\eta_1 + r} + \frac{\mu\beta_1 l_2 r}{(\eta_1 + r)(\eta_2 + r)}\right]^2}{2r\lambda_G} + \frac{\left(k_2 + \frac{\beta_2 l_2}{\eta_2 + r} + \frac{\varphi_2 b_2}{\gamma_2 + r}\right)^2}{2r\lambda_G} \end{aligned} \tag{A38}$$

Solving the partial derivative of (A38), and substituting the results into (A28), (A29), (A30) and (A31), we can get the cooperative game equilibria of supplier and manufacturer and government as the proposition 3.

## Proof of Proposition 5

From formula (37), (50) and (58), we can get

$$\begin{aligned} &D_S^{**} - D_S^{*} \\ &= \frac{\alpha_1 N_S^{**} + \beta_1 N_{G1}^{**}}{\eta_1} + \left(d_1 - \frac{\alpha_1 N_S^{**} + \beta_1 N_{G1}^{**}}{\eta_1}\right) e^{-\eta_1 t} - \frac{\alpha_1 N_S^{*} + \beta_1 N_{G1}^{*}}{\eta_1} + \left(d_1 - \frac{\alpha_1 N_S^{*} + \beta_1 N_{G1}^{*}}{\eta_1}\right) e^{-\eta_1 t} \\ &= \frac{\alpha_1\left(N_S^{**} - N_S^{*}\right) + \beta_1\left(N_{G1}^{**} - N_{G1}^{*}\right)}{\eta_1} + \left[\frac{\alpha_1\left(N_S^{*} - N_S^{**}\right) + \beta_1\left(N_{G1}^{*} - N_{G1}^{**}\right)}{\eta_1}\right] e^{-\eta_1 t} \\ &= \frac{\alpha_1\left(N_S^{**} - N_S^{*}\right) + \beta_1\left(N_{G1}^{**} - N_{G1}^{*}\right)}{\eta_1}\left(1 - e^{-\eta_1 t}\right) \end{aligned} \tag{A39}$$

$$D_S^{***} - D_S^{**} = \frac{\alpha_1\left(N_S^{***} - N_S^{**}\right) + \beta_1\left(N_{G1}^{***} - N_{G1}^{**}\right)}{\eta_1}\left(1 - e^{-\eta_1 t}\right) \tag{A40}$$

According to the $\frac{1}{3} < \theta_1 \le 1$, we can get $D_S^{**} - D_S^{*} \ge 0$, $D_S^{***} - D_S^{**} \ge 0$.

From formula (39), (52) and (60), we can get

$$\begin{aligned}
&D_M^{**} - D_M^{*} \\
&= \frac{\alpha_2 N_M^{**} + \beta_2 N_{G2}^{**}}{\eta_2} + \frac{\mu\left(\alpha_1 N_S^{**} + \beta_1 N_{G1}^{**} - d_1\eta_1\right)}{\eta_2 - \eta_1} e^{-\eta_1 t} \\
&+ \left(d_2 - \frac{\alpha_2 N_M^{**} + \beta_2 N_{G2}^{**}}{\eta_2} - \frac{\mu\left(\alpha_1 N_S^{**} + \beta_1 N_{G1}^{**} - d_1\eta_1\right)}{\eta_2 - \eta_1}\right) e^{-\eta_2 t} \\
&- \frac{\alpha_2 N_M^{*} + \beta_2 N_{G2}^{*}}{\eta_2} + \frac{\mu\left(\alpha_1 N_S^{*} + \beta_1 N_{G1}^{*} - d_1\eta_1\right)}{\eta_2 - \eta_1} e^{-\eta_1 t} \\
&- \left(d_2 - \frac{\alpha_2 N_M^{*} + \beta_2 N_{G2}^{*}}{\eta_2} - \frac{\mu\left(\alpha_1 N_S^{*} + \beta_1 N_{G1}^{*} - d_1\eta_1\right)}{\eta_2 - \eta_1}\right) e^{-\eta_2 t} \\
&= \frac{\alpha_2\left(N_M^{**} - N_M^{*}\right) + \beta_2\left(N_{G2}^{**} - N_{G2}^{*}\right)}{\eta_2}\left(1 - e^{-\eta_2 t}\right) \\
&+ \frac{\mu\left[\alpha_1\left(N_M^{**} - N_M^{*}\right) + \beta_1\left(N_{G2}^{**} - N_{G2}^{*}\right)\right]}{\eta_2 - \eta_1}\left(e^{-\eta_1 t} - e^{-\eta_2 t}\right)
\end{aligned} \tag{A41}$$

$$\begin{aligned}
&D_M^{***} - D_M^{**} \\
&= \frac{\alpha_2\left(N_M^{***} - N_M^{**}\right) + \beta_2\left(N_{G2}^{***} - N_{G2}^{**}\right)}{\eta_2}\left(1 - e^{-\eta_2 t}\right) \\
&+ \frac{\mu\left[\alpha_1\left(N_M^{***} - N_M^{**}\right) + \beta_1\left(N_{G2}^{***} - N_{G2}^{**}\right)\right]}{\eta_2 - \eta_1}\left(e^{-\eta_1 t} - e^{-\eta_2 t}\right)
\end{aligned} \tag{A42}$$

According to the $\frac{1}{3} < \theta_1 \le 1$ , we can get $D_M^{**} - D_M^{*} \ge 0$ , $D_M^{***} - D_M^{**} \ge 0$ .

## Proof of Proposition 6

From formula (38), (51) and (59), we can get

$$\begin{aligned}
&Q_S^{**} - Q_S^{*} \\
&= \frac{\delta_1 N_S^{**} + \varphi_1 N_{G1}^{**}}{\gamma_1} + \left(q_1 - \frac{\delta_1 N_S^{**} + \varphi_1 N_{G1}^{**}}{\gamma_1}\right) e^{-\gamma_1 t} - \frac{\delta_1 N_S^{*} + \varphi_1 N_{G1}^{*}}{\gamma_1} - \left(q_1 - \frac{\delta_1 N_S^{*} + \varphi_1 N_{G1}^{*}}{\gamma_1}\right) e^{-\gamma_1 t} \\
&= \frac{\delta_1\left(N_S^{**} - N_S^{*}\right) + \varphi_1\left(N_{G1}^{**} - N_{G1}^{*}\right)}{\gamma_1}\left(1 - e^{-\gamma_1 t}\right)
\end{aligned} \tag{A43}$$

$$Q_S^{***} - Q_S^{**} = \frac{\delta_1\left(N_S^{***} - N_S^{**}\right) + \varphi_1\left(N_{G1}^{***} - N_{G1}^{**}\right)}{\gamma_1}\left(1 - e^{-\gamma_1 t}\right) \tag{A44}$$

According to the $\frac{1}{3} < \theta_1 \le 1$ , we can get $Q_S^{**} - Q_S^{*} \ge 0$ , $Q_S^{***} - Q_S^{*} \ge 0$ .
From formula (40), (53) and (61), we can get

$$\begin{aligned}
&Q_M^{**} - Q_M^{*} \\
&= \frac{\delta_2 N_M^{**} + \varphi_2 N_{G2}^{**}}{\gamma_2} + \left(q_2 - \frac{\delta_2 N_M^{**} + \varphi_2 N_{G2}^{**}}{\gamma_2}\right) e^{-\gamma_2 t} \\
&- \frac{\delta_2 N_M^{*} + \varphi_2 N_{G2}^{*}}{\gamma_2} - \left(q_2 - \frac{\delta_2 N_M^{*} + \varphi_2 N_{G2}^{*}}{\gamma_2}\right) e^{-\gamma_2 t} \\
&= \frac{\delta_2\left(N_M^{**} - N_M^{*}\right) + \varphi_2\left(N_{G2}^{**} - N_{G2}^{*}\right)}{\gamma_2}\left(1 - e^{-\gamma_2 t}\right)
\end{aligned} \tag{A45}$$

$$Q_M^{***} - Q_M^{**} = \frac{\delta_2\left(N_M^{***} - N_M^{**}\right) + \varphi_2\left(N_{G2}^{***} - N_{G2}^{**}\right)}{\gamma_2}\left(1 - e^{-\gamma_2 t}\right) \tag{A46}$$

According to the $\frac{1}{3} < \theta_1 \le 1$ , we can get $Q_M^{**} - Q_M^{*} \ge 0$ , $Q_M^{***} - Q_M^{*} \ge 0$ .

## Proof of Proposition 7

From formula (33) and (A22), we can get

$$V_S^{**} - V_S^{*} = \frac{(1-\theta_1)\,l_1}{\eta_1+r}\left(D_S^{**}-D_S^{*}\right) + \frac{(1-\theta_1)\,b_1}{\gamma_1+r}\left(Q_S^{**}-Q_S^{*}\right) + \frac{(1-\theta_1)\left[p_1+\frac{\alpha_1 l_1}{\eta_1+r}+\frac{\delta_1 b_1}{\gamma_1+r}\right]\left[(3\theta_1-1)\left(p_1+\frac{\alpha_1 l_1}{\eta_1+r}+\frac{\delta_1 b_1}{\gamma_1+r}\right)+\frac{2\alpha_1\mu\theta_2 l_2 r}{(\eta_1+r)(\eta_2+r)}\right]}{4r\lambda_S} \tag{A47}$$

From formula (34) and (A23), we can get

$$\begin{aligned} V_M^{**} - V_M^{*} = {} & \frac{(1-\theta_2)\,l_2}{\eta_2+r}\left(D_M^{**}-D_M^{*}\right) + \frac{(1-\theta_2)\,b_2}{\gamma_2+r}\left(Q_M^{**}-Q_M^{*}\right) \\ & + \frac{(1-\theta_2)(3\theta_2-1)\left[p_2+\frac{\alpha_2 l_2}{\eta_2+r}+\frac{\delta_2 b_2}{\gamma_2+r}\right]^2}{4r\lambda_M} - \frac{\eta_1\mu(1-\theta_2)\,l_2}{(\eta_1+r)(\eta_2+r)}\left(D_S^{**}-D_S^{*}\right) \\ & + \frac{\frac{(1-\theta_2)\,\alpha_1\mu l_2}{(\eta_1+r)(\eta_2+r)}\left[(3\theta_1-1)\left(p_1+\frac{\alpha_1 l_1}{\eta_1+r}+\frac{\delta_1 b_1}{\gamma_1+r}\right)+\frac{2\alpha_1\mu\theta_2 l_2 r}{(\eta_1+r)(\eta_2+r)}\right]}{2\lambda_S} \end{aligned} \tag{A48}$$

From formula (35) and (A24), we can get

$$\begin{aligned} & V_G^{**} - V_G^{*} \\ & = \left[\frac{\theta_1 l_1}{\eta_1+r} - \frac{\eta_1\mu\theta_2 l_2}{(\eta_1+r)(\eta_2+r)}\right]\left(D_S^{**}-D_S^{*}\right) + \frac{\theta_2 l_2}{\eta_2+r}\left(D_M^{**}-D_M^{*}\right) \\ & + \frac{\theta_1 b_1}{\gamma_1+r}\left(Q_S^{**}-Q_S^{*}\right) + \frac{\theta_2 b_2}{\gamma_2+r}\left(Q_M^{**}-Q_M^{*}\right) \\ & + \frac{\left\{(3\theta_1-1)\left(p_1+\frac{\alpha_1 l_1}{\eta_1+r}+\frac{\delta_1 b_1}{\gamma_1+r}\right)+\frac{2\alpha_1\mu\theta_2 l_2 r}{(\eta_1+r)(\eta_2+r)}\right\}^2}{8r\lambda_S} \\ & + \frac{(3\theta_2-1)^2\left[p_2+\frac{\alpha_2 l_2}{\eta_2+r}+\frac{\delta_2 b_2}{\gamma_2+r}\right]^2}{8r\lambda_M} \end{aligned} \tag{A49}$$

According to the $\frac{1}{3} < \theta_1 \le 1$, we can get $V_S^{**} - V_S^{*} \ge 0$, $V_M^{**} - V_M^{*} \ge 0$, $V_G^{**} - V_G^{*} \ge 0$.
Based on the above analysis, we can find $V_T^{*} \le V_T^{**}$.
From the formula (A25) and (A38), we can get

$$\begin{aligned} & V_T^{***} - V_T^{**} \\ & = \left(\frac{l_1}{\eta_1+r} - \frac{\mu\eta_1 l_2}{(\eta_1+r)(\eta_2+r)}\right)\left(D_S^{***}-D_S^{**}\right) + \left(\frac{l_2}{\eta_2+r}\right)\left(D_M^{***}-D_M^{**}\right) + \left(\frac{b_1}{\gamma_1+r}\right)\left(Q_S^{***}-Q_S^{**}\right) \\ & + \left(\frac{b_2}{\gamma_2+r}\right)\left(Q_M^{***}-Q_M^{**}\right) + \frac{\left[(1-\theta_1)\left(k_1+\frac{\beta_1 l_1}{\eta_1+r}+\frac{\varphi_1 b_1}{\gamma_1+r}\right)+\frac{\beta_1\mu\theta_2 l_2 r}{(\eta_1+r)(\eta_2+r)}\right]^2}{2r\lambda_G} \\ & + \frac{(1-\theta_2)^2\left[k_2+\frac{\beta_2 l_2}{\eta_2+r}+\frac{\varphi_2 b_2}{\gamma_2+r}\right]^2}{2r\lambda_G} + \frac{\left\{(1-\theta_1)\left(p_1+\frac{\alpha_1 l_1}{\eta_1+r}+\frac{\delta_1 b_1}{\gamma_1+r}\right)+\frac{2\alpha_1\mu\theta_2 l_2 r}{(\eta_1+r)(\eta_2+r)}\right\}^2}{8r\lambda_S} \\ & + \frac{(1-\theta_2)^2\left[p_2+\frac{\alpha_2 l_2}{\eta_2+r}+\frac{\delta_2 b_2}{\gamma_2+r}\right]^2}{8r\lambda_M} \end{aligned} \tag{A50}$$

According to the $\frac{1}{3} < \theta_1 \le 1$, we can get $V_T^{**} - V_T^{*} \ge 0$, $V_T^{***} - V_T^{**} \ge 0$.

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