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Research Article

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Dynamic of a nonautonomous two-species impulsive competitive system with infinite delays

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Abstract: In this paper, we consider a nonautonomous two-species impulsive competitive system with infinite delays. By the impulsive comparison theorem and some mathematical analysis, we investigate the permanence, extinction and global attractivity of the system, as well as the influence of impulse perturbation on the dynamic behaviors of this system. For the logistic type impulsive equation with infinite delay, our results improve those of Xuxin Yang, Weibing Wang and Jianhua Shen [Permanence of a logistic type impulsive equation with infinite delay, Applied Mathematics Letters, 24(2011), 420-427]. For the corresponding nonautonomous two-species impulsive competitive system without delays, we discuss its permanence, extinction and global attractivity, which weaken and complement the results of Zhijun Liu and Qinglong Wang [An almost periodic competitive system subject to impulsive perturbations, Applied Mathematics and Computation, 231(2014), 377-385].

Keywords: Competitive system; Impulses; Permanence; Extinction; Infinite delays

MSC: 34D23; 34A37

1 Introduction

The logistic system is considered to be one of the most important systems in mathematical ecology, and a great deal of research works have been done based on this system. Because of the seasonal fluctuations in the environment and hereditary factors, many scholars have investigated the logistic system with time delays (see [1-8]). Noticing that the disturbance of environmental factors at certain time moments can give rise to instantaneous and changes of population density, many scholars have investigated the dynamic behaviors of impulsive differential equations (see [9-21]). Especially, Yang [21] investigated the following logistic system with infinite delay

$$\begin{aligned} &\dot{x}(t) = x(t)\left(a(t) - b(t)\int_0^{+\infty} K(s)x(t-s)\mathrm{d}s\right), \quad t \ge 0,\ t \ne t_k, \\ &x(t_k^+) = h_k x(t_k), \quad k = 1,\ 2, \cdots, \end{aligned} \tag{1.1}$$

with the initial condition $x(t) = \phi(t),\ t \le 0$, which is continuous and bounded on $(-\infty, 0]$ to $[0, +\infty)$ with $\phi(0) > 0$. Here $a(t)$ and $b(t)$ are continuous functions, bounded above and below by positive constants;

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$K : [0, +\infty) \to (0, +\infty)$ is a continuous kernel such that $\int_0^{+\infty} K(s)ds = 1$; t_k $(k = 1, 2, \cdots)$ are impulse points with $\lim_{k\to+\infty} t_k = +\infty$; the impulse perturbations $\{h_k : k = 1, 2, \cdots\}$ is positive sequences bounded above and below by positive constants. The authors discussed the permanence and global asymptotical stability of system (1.1) under the following condition

$$0 < \inf_{k\geq 1} h_k \leq h_k \leq 1 \ \ (k = 1, 2, \cdots) \text{ and } \inf_{k\geq 1}(h_k - h_{k-1}) > 0,$$

which implies $h_k \leq 1$ is an increase sequence.

On the other hand, competition for limited resources among ecologically similar species has been intensively investigated by many scholars due to its extensive prevalence and its importance on determining the structure of animal and plant communities, the diversity and the evolution of species. The famous Lotka-Volterra competition system has been studied extensively (see [22-24]). Naturally, impulse perturbations have been introduced into competitive systems and many excellent results have been obtained (see [13, 18, 25-32]). Recently, Liu and Wang [32] considered an almost periodic impulsive competitive system of the form

$$\begin{aligned}
\dot{x_1}(t) &= x_1(t)\left(r_1(t) - a_1(t)x_1(t) - \frac{b_1(t)x_2(t)}{1 + x_2(t)}\right),\\
\dot{x_2}(t) &= x_2(t)\left(r_2(t) - a_2(t)x_2(t) - \frac{b_2(t)x_1(t)}{1 + x_1(t)}\right), \quad t \geq 0,\ t \neq t_k,\\
x_1(t_k^+) &= h_{1k}x_1(t_k),\\
x_2(t_k^+) &= h_{2k}x_2(t_k), \quad k = 1,\ 2, \cdots .
\end{aligned} \tag{1.2}$$

For any given continuous function $f(t)$, let f_L and f_M denote $\inf_{0\leq t<+\infty} f(t)$ and $\sup_{0\leq t<+\infty} f(t)$, respectively. The authors discussed the permanence of system (1.2) under the following conditions:

(H1) $\Pi_{0<t_k<t} h_{ik}$, $i = 1,\ 2$, are bounded above and below by positive constants for all $t > 0$;

(H2) $r_{iL} - b_{iM} > 0$, $i = 1,\ 2$.

But the authors did not consider its competition exclusion, global attractivity and extinction. For the permanence of system (1.2), we also want to know whether conditions $(H1)$ and $(H2)$ can be weakened? To answer this question, we first introduce the following example.

Example 1.1 For system (1.2), let $r_1(t) = \frac{0.2t+0.4}{t+1}$, $r_2(t) = \frac{t+2}{2t+1}$, $a_1(t) = 5 + 4\sin\sqrt{2}t$, $a_2(t) = 11.5 + 8.5\sin 2t$, $b_1(t) = 2 + \sin t$, $b_2(t) = 1 + 0.5\sin t$, $h_{1k} = 5.5 - 0.5\cos k$, $h_{2k} = 3.5 + 0.5\sin 2k$ and $t_k = k + \frac{1}{k}$. Obviously, condition (H2) does not hold, but Figure 1 shows that system (1.2) is permanent.

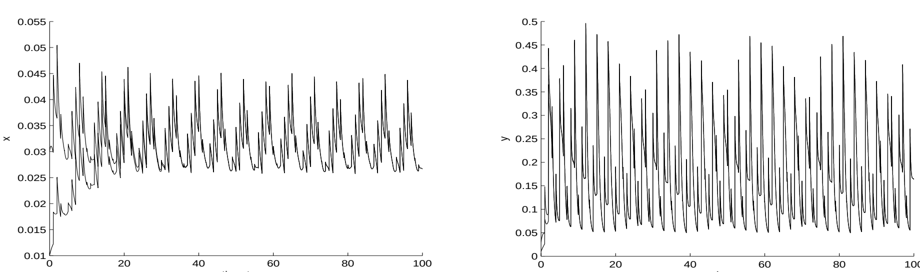

Figure 1: System (1.2) with the initial conditions $(0.01, 0.01)^T$ and $(0.03, 0.03)^T$ respectively.

This example gives a certain answer to the above question. So it requires us to give its strict mathematical verification and to discuss the competition exclusion, global attractivity and extinction of (1.2). Our results improve and complement the corresponding results of Liu and Wang [32].

Motivated by the above papers, in this paper we consider the following system

$$\begin{aligned} \dot{x_1}(t) &= x_1(t)\left(r_1(t) - a_1(t)\int_0^{+\infty} K_1(s)x_1(t-s)\mathrm{d}s - \frac{b_1(t)x_2(t)}{1+x_2(t)}\right),\\ \dot{x_2}(t) &= x_2(t)\left(r_2(t) - a_2(t)\int_0^{+\infty} K_2(s)x_2(t-s)\mathrm{d}s - \frac{b_2(t)x_1(t)}{1+x_1(t)}\right), \quad t \geq 0,\ t \neq t_k,\\ x_1(t_k^+) &= h_{1k}x_1(t_k),\\ x_2(t_k^+) &= h_{2k}x_2(t_k), \quad k = 1,\ 2, \cdots, \end{aligned} \tag{1.3}$$

under an initial condition

$$x_i(u) = \phi_i(u) \geq 0, \text{ for } u \in (-\infty, 0] \text{ with } \phi_i \in C((-\infty, 0], [0, +\infty)) \text{ and } \phi_i(0) > 0. \tag{1.4}$$

Here $x_1(t)$ and $x_2(t)$ are population densities of species x_1 and x_2 at time t respectively; $r_1(t) > 0$ and $r_2(t) > 0$ are the growth rates; $a_1(t) > 0$ and $a_2(t) > 0$ are the effects of intra-specific competition; $r_i(t)$ and $a_i(t)$ are continuous functions, bounded above and below by positive constants for all $t > 0$; the continuous functions $b_1(t) \geq 0$ and $b_2(t) \geq 0$ are the rates of inter-specific competition, which are bounded for all $t > 0$; $K_i : [0, +\infty) \to (0, +\infty)$ $(i = 1,\ 2)$ are continuous kernels such that $\int_0^{+\infty} K_i(s)\mathrm{d}s = 1$; $0 < t_1 < t_2 < \cdots < t_k < t_{k+1} < \cdots$ are impulse points with $\lim_{k\to+\infty} t_k = +\infty$; the impulse perturbations $\{h_{ik} : k = 1, 2, \cdots\}$ $(i = 1,\ 2)$ are positive sequences bounded above and below by positive constants.

2 Preliminaries

In this section, we present the following definitions and lemmas which are useful in proving our main results.

Let $PC([0, +\infty), R^2)$=$\{\phi : [0, +\infty) \to R^+ \times R^+$, ϕ is continuous for $t \neq t_k$. Also $\phi(t_k^-)$ and $\phi(t_k^+)$ exist, and $\phi(t_k^-) = \phi(t_k)$, $k = 1,\ 2, \cdots\}$. By the basic theories of impulsive differential equations in [9][10], system (1.3) has a unique solution $X(t) = X(t, X_0) \in PC([0, +\infty), R^+ \times R^+)$. For any sequence $\{h_k\}$, let h_L and h_M denote $\inf_{k\in Z} h_k$ and $\sup_{k\in Z} h_k$, respectively. For the sequence $\{t_k\}$, denote $\sup_{k\in Z} t_k^1 = \sup_{k\in Z}(t_{k+1} - t_k) = \eta$ and $\inf_{k\in Z} t_k^1 = \theta$. Obviously $\eta \geq \theta > 0$.

Define $G_k = (t_{k-1}, t_k) \times R^+ \times R^+$, $k = 1, 2, \cdots$; $G = \bigcup_{k=1}^{+\infty} G_k$; $V_0 = \{V \in C[G, R^+]$, there exists the limits $V(t_k^-, X_0)$, $V(t_k^+, X_0)$, $V(t_k^-, X_0) = V(t_k, X_0)$, and V is locally Lipschitz continuous$\}$.

Definition 2.1 Let $V \in V_0$. For any $(t, X(t)) \in [t_{k-1}, t_k) \times R^+ \times R^+$, the right-hand derivative $D^+V(t, X(t))$ along the solution $X(t, X_0)$ of system (1.3) is defined by

$$D^+V(t, X(t)) = \liminf_{h\to0^+} \tfrac{1}{h}[V(t + h, X(t + h)) - V(t, X(t))].$$

Lemma 2.1 (see [10]) *Assume that $m \in PC[R^+, R]$ with points of discontinuity at $t = t_k$ is left continuous at $t = t_k$, $k = 1, 2, \cdots$, and that*

$$\begin{aligned} D^+m(t) &\leq g(t, m(t)), \quad t \neq t_k, \quad k = 1, 2, \cdots,\\ m(t_k^+) &\leq \phi_k(m(t_k)), \quad t = t_k, \quad k = 1, 2, \cdots, \end{aligned} \tag{2.1}$$

where $g \in C[R^+ \times R^+, R]$, $\phi_k \in C[R, R]$ *and* $\phi_k(u)$ *is nondecreasing in* u *for each* $k = 1, 2, \cdots$. *Let* $r(t)$ *be the maximal solution of the scalar impulsive differential equation*

$$\begin{aligned} &\dot{u} = g(t, u), && t \neq t_k, \quad k = 1, 2, \cdots, \\ &u(t_k^+) = \phi_k(u(t_k)) \geq 0, && t = t_k, \quad t_k > t_0, \quad k = 1, 2, \cdots, \\ &u(t_0^+) = u_0, && \end{aligned} \tag{2.2}$$

existing on $[t_0, +\infty)$, *then* $m(t_0^+) \leq u_0$ *implies* $m(t) \leq r(t)$, $t \geq t_0$.

Remark 2.1 (see [10]) In Lemma 2.1, assume inequalities (2.1) reverse. Let $p(t)$ be the minimal solution of (2.1) existing on $[t_0, +\infty)$, then $p(t_0^+) \geq u_0$ implies $p(t) \geq r(t)$, $t \geq t_0$.

Consider the following impulsive system

$$\begin{aligned} &\dot{y}(t) = y(t)(a - by(t)), \quad t \neq t_k, \\ &y(t_k^+) = h_k y(t_k), \quad k = 1, 2, \cdots, \end{aligned} \tag{2.3}$$

where a and b are positive constants.

Lemma 2.2 (see [17]), *Let* $y(t)$ *be any positive solution of system* (2.3). *It follows that:*

(i) *If* $h_L \geq 1$, *then*

$$\frac{a\eta + \ln h_L}{b\eta h_L} \leq \liminf_{t \to +\infty} y(t) \leq \limsup_{t \to +\infty} y(t) \leq \frac{(a\theta + \ln h_M)h_M}{b\theta}.$$

(ii) *If* $h_L < 1$, $h_M < 1$ *and* $a\theta + \ln h_L > 0$, *then*

$$\frac{(a\theta + \ln h_L)h_L}{b\theta} \leq \liminf_{t \to +\infty} y(t) \leq \limsup_{t \to +\infty} y(t) \leq \frac{a\eta + \ln h_M}{b\eta h_M}.$$

(iii) *If* $h_L < 1$, $h_M \geq 1$ *and* $a\theta + \ln h_L > 0$, *then*

$$\frac{(a\theta + \ln h_L)h_L}{b\theta} \leq \liminf_{t \to +\infty} y(t) \leq \limsup_{t \to +\infty} y(t) \leq \frac{(a\theta + \ln h_M)h_M}{b\theta}.$$

Lemma 2.3 *Let* $y(t)$ *be any positive solution of system* (2.3). *Assume that* $a\eta + \ln h_M \leq 0$. *Then* $\lim_{t \to +\infty} y(t) = 0$.

Proof. Let $z(t) = 1/y(t)$, then system (2.3) is transformed into

$$\begin{aligned} &\dot{z}(t) = -az(t) + b, \quad t \neq t_k, \\ &z(t_k^+) = \frac{1}{h_k} z(t_k), \quad k = 1, 2, \cdots, \end{aligned}$$

According to [9], for any $T > 0$, we can obtain

$$z(t) = \prod_{T \leq t_k < t} \frac{1}{h_k} e^{-a(t-T)} z(T^+) + b \int_T^t \prod_{s \leq t_k < t} \frac{1}{h_k} e^{-a(t-s)} \mathrm{d}s.$$

First we consider $a\eta + \ln h_M = 0$, that is $e^a h_M^{1/\eta} = 1$ and $h_M < 1$. According to [17], we obtain

$$\begin{aligned} z(t) &\geq \left(\frac{1}{h_M}\right)^{\frac{t-T}{\eta} - 1} e^{-a(t-T)} z(T^+) + b \int_T^t \left(\frac{1}{h_M}\right)^{\frac{t-s}{\eta} - 1} e^{-a(t-s)} \mathrm{d}s \\ &= h_M(z(T^+) + bt - bT) \to +\infty, \quad t \to +\infty. \end{aligned}$$

Next consider $a\eta + \ln h_M < 0$, that is $e^a h_M^{1/\eta} < 1$ and $h_M < 1$, then

$$\begin{aligned} z(t) &\geq \left(\frac{1}{h_M}\right)^{\frac{t-T}{\eta}-1} e^{-a(t-T)} z(T^+) + \frac{b\eta h_M}{a\eta + \ln h_M}\left[1 - \left(\frac{1}{h_M}\right)^{\frac{t-T}{\eta}} e^{-a(t-T)}\right] \\ &\geq \left(h_M z(T^+) - \frac{b\eta h_M}{a\eta + \ln h_M}\right)\left(\frac{1}{e^a h_M^{1/\eta}}\right)^{t-T} + \frac{b\eta h_M}{a\eta + \ln h_M} \to +\infty, \quad t \to +\infty, \end{aligned}$$

because of $\dfrac{b\eta h_M}{a\eta + \ln h_M} < 0$. Therefore, it follows from the positivity of $y(t)$ and the relationship between $z(t)$ and $y(t)$ that $\lim\limits_{t\to+\infty} y(t) = 0$. This completes the proof of Lemma 2.3.

Lemma 2.4 *Let $(x_1(t), x_2(t))^T$ be any solution of system* (1.3) *with* (1.4), *then $x_i(t) > 0$, $i = 1, 2$, for all $t \geq 0$.*
Proof. From the ith equation of (1.3) with (1.4) ($i = 1, 2$), we can obtain

$$x_i(t) = \phi_i(0)\Big(\prod_{0<t_k<t} h_{ik}\Big) \exp\int_0^t \Big(r_i(u) - a_i(u)\int_0^{+\infty} K_i(s)x_i(u-s)\mathrm{d}s - \frac{b_i(u)x_j(u)}{1 + x_j(u)}\Big)\mathrm{d}u > 0,$$

where $1 \leq j \leq 2$, $i \neq j$, which completes the proof of Lemma 2.4.

Lemma 2.5 *For any $y \in PC([0, +\infty), R^+)$, let $k : [0, +\infty) \to (0, +\infty)$ be a continuous kernel such that $\int_0^{+\infty} k(s)\mathrm{d}s = 1$. Then*

$$\liminf_{t\to+\infty} y(t) \leq \liminf_{t\to+\infty}\int_0^{+\infty} k(s)y(t-s)\mathrm{d}s \leq \limsup_{t\to+\infty}\int_0^{+\infty} k(s)y(t-s)\mathrm{d}s \leq \limsup_{t\to+\infty} y(t).$$

The proof is similar to that of Lemma 3 in [24], so we omit it.

3 Main results

In this section, we present the main results of this paper. First we study the coexistence of system (1.3).

Theorem 3.1 *Let $(x_1(t), x_2(t))^T$ be any solution of system* (1.3) *with* (1.4), *$i = 1, 2$. Assume that*

$$\begin{aligned} &\Big(r_{iL} - b_{iM}\frac{M_j}{1 + M_j}\Big)\theta + \ln h_{iL} > 0, \quad 1 \leq i, j \leq 2, \ i \neq j, \\ &r_{iL} - b_{iM}\frac{M_j}{1 + M_j} > 0, \end{aligned} \tag{3.1}$$

then $m_i \leq \liminf\limits_{t\to+\infty} x_i(t) \leq \limsup\limits_{t\to+\infty} x_i(t) \leq M_i$, $i = 1, 2$, where

$$\begin{aligned} M_i &= \max\left\{\frac{(r_{iM}\theta + \ln h_{iM})h_{iM}^2}{a_{iL}\theta R_{1i}}, \frac{r_{iM}\eta + \ln h_{iM}}{a_{iL}h_{iM}^2\eta R_{2i}}\right\}, \\ m_i &= \min\left\{\frac{(r_{iL} - \frac{b_{iM}M_j}{1+M_j})\eta + \ln h_{iL}}{a_{iM}H_{1i}\eta h_{iL}^2}, \frac{[(r_{iL} - \frac{b_{iM}M_j}{1+M_j})\theta + \ln h_{iL}]h_{iL}^2}{a_{iM}H_{2i}\theta}\right\} \end{aligned}$$

with

$$\begin{aligned} R_{1i} &= \int_0^{+\infty} (h_{iM}^{\frac{1}{\theta}} e^{r_{iM}})^{-s} K_i(s)\mathrm{d}s < +\infty, \quad R_{2i} = \int_0^{+\infty} (h_{iM}^{\frac{1}{\eta}} e^{r_{iM}})^{-s} K_i(s)\mathrm{d}s < +\infty, \\ H_{1i} &= \int_0^{+\infty} \left(\frac{1}{h_{iL}}\right)^{\frac{s}{\eta}} \exp\left\{-\Big(r_{iL} - a_{iM}M_i - \frac{b_{iM}M_j}{1 + M_j}\Big)s\right\}K_i(s)\mathrm{d}s < +\infty, \\ H_{2i} &= \int_0^{+\infty} \left(\frac{1}{h_{iL}}\right)^{\frac{s}{\theta}} \exp\left\{-\Big(r_{iL} - a_{iM}M_i - \frac{b_{iM}M_j}{1 + M_j}\Big)s\right\}K_i(s)\mathrm{d}s < +\infty. \end{aligned}$$

Proof. From (1.3), we can obtain for $i = 1, 2$ that

$$\dot{x}_i(t) \le r_{iM} x_i(t),$$

$$x_i(t_k^+) = h_{ik} x_i(t_k).$$

Then according to Lemma 2.1, we have

$$x_i(t-s) \ge \Big(\prod_{t-s\le t_k<t} \frac{1}{h_{ik}}\Big) e^{-r_{iM}s} x_i(t).$$

For $i = 1, 2$, substituting this into the ith equation of (1.3), we obtain

$$\dot{x}_i(t) \le x_i(t)\Big[r_{iM} - a_{iL}\int_0^{+\infty}\Big(\prod_{t-s\le t_k<t}\frac{1}{h_{ik}}\Big)e^{-r_{iM}s}K_i(s)\mathrm{d}s x_i(t)\Big]. \tag{3.2}$$

(1) If $h_{iM} \ge 1$, it follows that

$$\begin{aligned}\dot{x}_i(t) &\le x_i(t)\Big[r_{iM} - a_{iL}\int_0^{+\infty}\Big(\frac{1}{h_{iM}}\Big)^{\frac{s}{\theta}+1} e^{-r_{iM}s}K_i(s)\mathrm{d}s x_i(t)\Big]\\ &\le x_i(t)\Big[r_{iM} - \frac{a_{iL}R_{1i}}{h_{iM}} x_i(t)\Big],\end{aligned}$$

where $R_{1i} = \int_0^{+\infty}(h_{iM}^{\frac{1}{\theta}} e^{r_{iM}})^{-s}K_i(s)\mathrm{d}s$. According to Lemma 2.2, we obtain that

$$\limsup_{t\to+\infty} x_i(t) \le \frac{(r_{iM}\theta + \ln h_{iM})h_{iM}^2}{a_{iL}R_{1i}\theta}, \quad i = 1, 2.$$

(2) If $h_{iM} < 1$, we have

$$\begin{aligned}\dot{x}_i(t) &\le x_i(t)\Big[r_{iM} - a_{iL}\int_0^{+\infty}\Big(\frac{1}{h_{iM}}\Big)^{\frac{s}{\eta}-1} \exp\{-r_{iM}s\}K_i(s)\mathrm{d}s x_i(t)\Big]\\ &\le x_i(t)(r_{iM} - a_{iL}h_{iM}R_{2i}x_i(t)),\end{aligned}$$

where $R_{2i} = \int_0^{+\infty}(h_{iM}^{\frac{1}{\eta}} e^{r_{iM}})^{-s}K_i(s)\mathrm{d}s$. Again from Lemma 2.2 we have

$$\limsup_{t\to+\infty} x_i(t) \le \frac{r_{iM}\eta + \ln h_{iM}}{a_{iL}h_{iM}^2 R_{2i}\eta}.$$

All the above analysis show that

$$\limsup_{t\to+\infty} x_i(t) \le \max\Big\{\frac{(r_{iM}\theta + \ln h_{iM})h_{iM}^2}{a_{iL}R_{1i}\theta}, \frac{r_{iM}\eta + \ln h_{iM}}{a_{iL}h_{iM}^2 R_{2i}\eta}\Big\} \triangleq M_i, \quad i = 1, 2. \tag{3.3}$$

Therefore for any given $\varepsilon > 0$ satisfying

$$\begin{aligned}&\Big(r_{iL} - b_{iM}\frac{M_j+\varepsilon}{1+M_j+\varepsilon}\Big)\theta + \ln h_{iL} > 0, \quad 1 \le i, j \le 2, i \ne j,\\ &r_{iL} - b_{iM}\frac{M_j+\varepsilon}{1+M_j+\varepsilon} > 0,\end{aligned} \tag{3.4}$$

there exists a $T > 0$ such that for $t > T$, $x_i(t) \le M_i + \varepsilon$, $i = 1, 2$.

Substituting this into system (1.3), it follows from Lemma 2.5 that, for $1 \le i, j \le 2$ and $i \neq j$

$$\dot{x}_i(t) \ge x_i(t)\Big[r_{iL} - a_{iM}(M_i+\varepsilon) - \frac{b_{iM}(M_j+\varepsilon)}{1+M_j+\varepsilon}\Big],$$

$$x_i(t_k^+) = h_{ik}x_i(t_k).$$

We can easily obtain that

$$x_i(t-s) \le \Big(\prod_{t-s\le t_k<t}\frac{1}{h_{ik}}\Big)\exp\Big\{-\Big(r_{iL} - a_{iM}(M_i+\varepsilon) - \frac{b_{iM}(M_j+\varepsilon)}{1+M_j+\varepsilon}\Big)s\Big\}x_i(t).$$

Substituting this into the ith equation of system (1.3) gives rise to

$$\dot{x}_i(t) \ge x_i(t)\Big[r_{iL} - \frac{b_{iM}(M_j+\varepsilon)}{1+M_j+\varepsilon} - a_{iM}\int_0^{+\infty}\Big(\prod_{t-s\le t_k<t}\frac{1}{h_{ik}}\Big)\exp\Big\{-\Big(r_{iL}-a_{iM}(M_i+\varepsilon)-\frac{b_{iM}(M_j+\varepsilon)}{1+M_j+\varepsilon}\Big)s\Big\}K_i(s)\mathrm{d}s\,x_i(t)\Big].$$

Next we prove $\liminf_{t\to+\infty} x_i(t) \ge m_i$.

(3) If $h_{iL} \ge 1$, we deduce that

$$\dot{x}_i(t) \ge x_i(t)\Big[r_{iL} - \frac{b_{iM}(M_j+\varepsilon)}{1+M_j+\varepsilon} - a_{iM}\int_0^{+\infty}\Big(\frac{1}{h_{iL}}\Big)^{\frac{s}{\eta}-1}\exp\Big\{-\Big(r_{iL}-a_{iM}(M_i+\varepsilon)-\frac{b_{iM}(M_j+\varepsilon)}{1+M_j+\varepsilon}\Big)s\Big\}K_i(s)\mathrm{d}s\,x_i(t)\Big].$$

By setting $\varepsilon \to 0$, it follows from Lemma 2.2 that

$$\liminf_{t\to+\infty} x_i(t) \ge \frac{\Big(r_{iL} - \frac{b_{iM}M_j}{1+M_j}\Big)\eta + \ln h_{iL}}{a_{iM}H_{1i}h_{iL}^2\eta},$$

where

$$H_{1i} = \int_0^{+\infty}\Big(\frac{1}{h_{iL}}\Big)^{\frac{s}{\eta}}\exp\Big\{-\Big(r_{iL} - a_{iM}M_i - \frac{b_{iM}M_j}{1+M_j}\Big)s\Big\}K_i(s)\mathrm{d}s.$$

(4) If $h_{iL} < 1$, we obtain

$$\dot{x}_i(t) \ge x_i(t)\Big[r_{iL} - \frac{b_{iM}(M_j+\varepsilon)}{1+M_j+\varepsilon} - a_{iM}\int_0^{+\infty}\Big(\frac{1}{h_{iL}}\Big)^{\frac{s}{\theta}+1}\exp\Big\{-\Big(r_{iL}-a_{iM}(M_i+\varepsilon)-\frac{b_{iM}(M_j+\varepsilon)}{1+M_j+\varepsilon}\Big)s\Big\}K_i(s)\mathrm{d}s\,x_i(t)\Big].$$

By setting $\varepsilon \to 0$, it follows from Lemma 2.2 that

$$\liminf_{t\to+\infty} x_i(t) \ge \frac{\Big[\Big(r_{iL} - \frac{b_{iM}M_j}{1+M_j}\Big)\theta + \ln h_{iL}\Big]h_{iL}^2}{a_{iM}H_{2i}\theta},$$

where

$$H_{2i} = \int_0^{+\infty}\Big(\frac{1}{h_{iL}}\Big)^{\frac{s}{\theta}}\exp\Big\{-\Big(r_{iL} - a_{iM}M_i - \frac{b_{iM}M_j}{1+M_j}\Big)s\Big\}K_i(s)\mathrm{d}s.$$

Thus,

$$\liminf_{t\to+\infty} x_i(t) \ge \min\left\{\frac{[(r_{iL} - \frac{b_{iM}M_j}{1+M_j})\theta + \ln h_{iL}]h_{iL}^2}{a_{iM}\theta H_{2i}},\ \frac{(r_{iL} - \frac{b_{iM}M_j}{1+M_j})\eta + \ln h_{iL}}{a_{iM}\eta h_{iL}^2 H_{1i}}\right\} \triangleq m_i,\quad i = 1,\ 2.$$

This proves the permanence of (1.3).

Theorem 3.2 *Suppose that the conditions of* Theorem 3.1 *holds, and there exist* $\sigma_i > 0$ *and* $\rho_i > 0$ *such that*

$$\int_0^{+\infty} sK_i(s)\mathrm{d}s = \sigma_i,\quad i = 1,\ 2 \tag{3.5}$$

and

$$2\frac{a_{1L}}{M_1}\rho_1-\frac{b_{1M}\rho_1}{m_1^2}-b_{2M}\rho_2-2a_{1M}^2\rho_1\sigma_1-b_{1M}a_{1M}\rho_1\sigma_1-b_{2M}a_{2M}\rho_2\sigma_2>0,$$
$$2\frac{a_{2L}\rho_2}{M_2}-b_{1M}\rho_1-\frac{b_{2M}\rho_2}{m_2^2}-2a_{2M}^2\rho_2\sigma_2-b_{1M}a_{1M}\rho_1\sigma_1-b_{2M}a_{2M}\rho_2\sigma_2>0 \tag{3.6}$$

where M_i *and* m_i $(i = 1, 2)$ *are defined in* Theorem 3.1. *Then for any two solutions* $(x_1(t), x_2(t))^T$ *and* $(y_1(t), y_2(t))^T$ *of system* (1.3) *with* (1.4), *there are*

$$\lim_{t\to+\infty}|x_i(t)-y_i(t)|=0,\quad for\ i=1,\ 2.$$

Proof. Let $(x_1(t), x_2(t))^T$ and $(y_1(t), y_2(t))^T$ be any two solutions of system (1.3) with (1.4). From Theorem 3.1, for any $\varepsilon_1 > 0$ satisfying $0 < \varepsilon_1 < \min\{m_1, m_2\}$, there exist $\delta > 0$ such that

$$2\frac{a_{1L}\rho_1}{M_1+\varepsilon_1}-\frac{b_{1M}\rho_1}{(m_1-\varepsilon_1)^2}-b_{2M}\rho_2-2a_{1M}^2\rho_1\sigma_1-b_{1M}a_{1M}\rho_1\sigma_1-b_{2M}a_{2M}\rho_2\sigma_2\geq\delta,$$
$$2\frac{a_{2L}\rho_2}{M_2+\varepsilon_1}-b_{1M}\rho_1-\frac{b_{2M}\rho_2}{(m_2-\varepsilon_1)^2}-2a_{2M}^2\rho_2\sigma_2-b_{1M}a_{1M}\rho_1\sigma_1-b_{2M}a_{2M}\rho_2\sigma_2\geq\delta, \tag{3.7}$$

and $T_1 > 0$ such that for $t > T_1$,

$$m_i-\varepsilon_1\leq x_i,\ y_i\leq M_i+\varepsilon_1,\quad i=1,\ 2. \tag{3.8}$$

Define a Lyapunov function as follows

$$V_{1i}(t)=\Big(\ln x_i(t)-\ln y_i(t)-\int_0^{+\infty}\int_{t-s}^{t}K_i(s)a_i(v+s)(x_i(v)-y_i(v))\mathrm{d}v\mathrm{d}s\Big)^2,\quad i=1,\ 2.$$

For $t > T_1$ and $t \neq t_k$, $k = 1, 2, \cdots$, calculating the upper right derivatives of $V_{1i}(t)$ with $1 \leq i, j \leq 2$ and $i \neq j$, we have

$$\begin{aligned}
&D^+V_{1i}(t)\\
&=2\Big(\ln x_i(t)-\ln y_i(t)-\int_0^{+\infty}\int_{t-s}^{t}K_i(s)a_i(v+s)(x_i(v)-y_i(v))\mathrm{d}v\mathrm{d}s\Big)\\
&\quad\times\Big[-(x_i(t)-y_i(t))\int_0^{+\infty}a_i(t+s)K_i(s)\mathrm{d}s-b_i(t)\Big(\frac{x_j(t)}{1+x_j(t)}-\frac{y_j(t)}{1+y_j(t)}\Big)\Big]\\
&\leq-2a_{iL}(x_i(t)-y_i(t))(\ln x_i(t)-\ln y_i(t))+\frac{2b_{iM}}{(1+\xi_j(t))^2}|\ln x_i(t)-\ln y_i(t)||x_j(t)-y_j(t)|\\
&\quad+2\left[a_{iM}^2|x_i(t)-y_i(t)|+\frac{b_{iM}a_{iM}}{(1+\xi_j(t))^2}|x_j(t)-y_j(t)|\right]\int_0^{+\infty}\int_{t-s}^{t}K_i(s)|x_i(v)-y_i(v)|\mathrm{d}v\mathrm{d}s\\
&\leq-2a_{iL}(x_i(t)-y_i(t))(\ln x_i(t)-\ln y_i(t))+2b_{iM}|\ln x_i(t)-\ln y_i(t)||x_j(t)-y_j(t)|\\
&\quad+a_{iM}^2\sigma_i|x_i(t)-y_i(t)|^2+b_{iM}a_{iM}\sigma_i|x_j(t)-y_j(t)|^2\\
&\quad+(a_{iM}^2+b_{iM}a_{iM})\int_0^{+\infty}\int_{t-s}^{t}K_i(s)|x_i(v)-y_i(v)|^2\mathrm{d}v\mathrm{d}s,
\end{aligned}$$

where $\xi_j(t)$ lies between $x_j(t)$ and $y_j(t)$, $j = 1, 2$.

For $i = 1, 2$, define

$$V_{2i}(t) = (a_{iM}^2 + b_{iM}a_{iM}) \int_0^{+\infty} \int_{t-s}^{t} \int_{v}^{t} K_i(s)|x_i(u) - y_i(u)|^2 \mathrm{d}u\mathrm{d}v\mathrm{d}s.$$

For $t > T_1$ and $t \neq t_k$, $k = 1, 2, \cdots$, calculating the upper right derivatives of $V_{2i}(t)$, it follows that

$$D^+V_{2i}(t) = (a_{iM}^2 + b_{iM}a_{iM}) \left(\sigma_i|x_i(t) - y_i(t)|^2 - \int_0^{+\infty} \int_{t-s}^{t} K_i(s)|x_i(v) - y_i(v)|^2 \mathrm{d}v\mathrm{d}s \right).$$

Denote $V_i(t) = V_{1i}(t) + V_{2i}(t)$ for $i = 1, 2$. Therefore, for $t > T_1$ and $t \neq t_k$, $k = 1, 2, \cdots$,

$$\begin{aligned}
D^+V(t) &= D^+(\rho_1 V_1(t) + \rho_2 V_2(t)) \\
&\leq \sum_{i=1}^{2} \rho_i[-2a_{iL}(x_i(t)-y_i(t))(\ln x_i(t)-\ln y_i(t))+2b_{iM}|\ln x_i(t)-\ln y_i(t)||x_j(t)-y_j(t)| \\
&\quad +(2a_{iM}^2 + b_{iM}a_{iM})\sigma_i|x_i(t) - y_i(t)|^2 + b_{iM}a_{iM}\sigma_i|x_j(t) - y_j(t)|^2] \\
&= -2a_{1L}\rho_1(x_1(t)-y_1(t))(\ln x_1(t)-\ln y_1(t)) + 2b_{1M}\rho_1|\ln x_1(t)-\ln y_1(t)||x_2(t)-y_2(t)| \\
&\quad -2a_{2L}\rho_2(x_2(t)-y_2(t))(\ln x_2(t)-\ln y_2(t)) + 2b_{2M}\rho_2|\ln x_2(t)-\ln y_2(t)||x_1(t)-y_1(t)| \\
&\quad +(2a_{1M}^2\rho_1\sigma_1 + b_{1M}a_{1M}\rho_1\sigma_1 + b_{2M}a_{2M}\rho_2\sigma_2)|x_1(t) - y_1(t)|^2 \\
&\quad +(2a_{2M}^2\rho_2\sigma_2 + b_{1M}a_{1M}\rho_1\sigma_1 + b_{2M}a_{2M}\rho_2\sigma_2)|x_2(t) - y_2(t)|^2 \\
&\leq -2\frac{a_{1L}\rho_1}{\xi_{11}(t)}|x_1(t) - y_1(t)|^2 + \frac{b_{1M}\rho_1}{\xi_{12}^2(t)}|x_1(t) - y_1(t)|^2 + b_{1M}\rho_1|x_2(t) - y_2(t)|^2 \\
&\quad -2\frac{a_{2L}\rho_2}{\xi_{21}(t)}|x_2(t) - y_2(t)|^2 + \frac{b_{2M}\rho_2}{\xi_{22}^2(t)}|x_2(t) - y_2(t)|^2 + b_{2M}\rho_2|x_1(t) - y_1(t)|^2 \\
&\quad +(2a_{1M}^2\rho_1\sigma_1 + b_{1M}a_{1M}\rho_1\sigma_1 + b_{2M}a_{2M}\rho_2\sigma_2)|x_1(t) - y_1(t)|^2 \\
&\quad +(2a_{2M}^2\rho_2\sigma_2 + b_{1M}a_{1M}\rho_1\sigma_1 + b_{2M}a_{2M}\rho_2\sigma_2)|x_2(t) - y_2(t)|^2 \\
&\leq \left(-2\frac{a_{1L}\rho_1}{M_1+\varepsilon_1} + \frac{b_{1M}\rho_1}{(m_1-\varepsilon_1)^2} + b_{2M}\rho_2 + 2a_{1M}^2\rho_1\sigma_1 + b_{1M}a_{1M}\rho_1\sigma_1 + b_{2M}a_{2M}\rho_2\sigma_2\right)|x_1(t)-y_1(t)|^2 \\
&\quad \left(-2\frac{a_{2L}\rho_2}{M_2+\varepsilon_1} + \frac{b_{2M}\rho_2}{(m_2-\varepsilon_1)^2} + b_{1M}\rho_1 + 2a_{2M}^2\rho_2\sigma_2 + b_{1M}a_{1M}\rho_1\sigma_1 + b_{2M}a_{2M}\rho_2\sigma_2\right)|x_2(t)-y_2(t)|^2, \\
&\leq -\delta(|x_1(t) - y_1(t)|^2 + |x_2(t) - y_2(t)|^2),
\end{aligned}$$

where $\xi_{ij}(t)$ $(1 \leq i, j \leq 2; i \neq j)$ lies between $x_i(t)$ and $y_i(t)$, $i = 1, 2$.

For $t = t_k$, we can easily verify that $V(t_k^+) = V(t_k)$. Integrating both sides of the above inequality from T_1 to t, we obtain

$$V(t) + \delta \int_{T_1}^{t} (|x_1(s) - y_1(s)|^2 + |x_2(s) - y_2(s)|^2)\mathrm{d}s \leq V(T_1^+) < +\infty.$$

Therefore, $V(t)$ is bounded on $[T_1, +\infty)$ and there is

$$\int_{T_1}^{+\infty} (|x_1(s) - y_1(s)|^2 + |x_2(s) - y_2(s)|^2)\mathrm{d}s < +\infty.$$

Similarly to the analysis of [17], it is obvious that

$$\lim_{t\to+\infty}|x_1(t)-y_1(t)| = \lim_{t\to+\infty}|x_2(t)-y_2(t)| = 0.$$

This completes the proof of Theorem 3.2.

Next, we consider the competition exclusion of system (1.3).

Theorem 3.3 *Let* $(x_1(t), x_2(t))^T$ *be any solution of system* (1.3) *with* (1.4)*. Assume that*

$$r_{1L}\theta + \ln h_{1L} > 0, \tag{3.9}$$

$$r_{2M}\eta + \ln h_{2M} \le 0, \tag{3.10}$$

then the species x_1 *is permanent but the species* x_2 *is extinct, that is*

$$\bar{m}_1 \le \liminf_{t\to+\infty} x_1(t) \le \limsup_{t\to+\infty} x_1(t) \le M_1 \text{ and } \lim_{t\to+\infty} x_2(t) = 0,$$

where M_1 *is defined in Theorem* 3.1 *and*

$$\bar{m}_1 = \min\left\{\frac{r_{1L}\eta + \ln h_{1L}}{a_{1M}\bar{H}_{11}\eta h_{1L}^2}, \frac{(r_{1L}\theta + \ln h_{1L})h_{1L}^2}{a_{1M}\bar{H}_{21}\theta}\right\},$$

with

$$\bar{H}_{11} = \int_0^{+\infty}\left(\frac{1}{h_{1L}}\right)^{\frac{s}{\eta}} \exp\left\{-\left(r_{1L} - a_{1M}M_1\right)s\right\}K_1(s)\mathrm{d}s < +\infty,$$

$$\bar{H}_{21} = \int_0^{+\infty}\left(\frac{1}{h_{1L}}\right)^{\frac{s}{\theta}} \exp\left\{-\left(r_{1L} - a_{1M}M_1\right)s\right\}K_1(s)\mathrm{d}s < +\infty.$$

Proof. Since (3.9) implies that $r_{1M}\theta + \ln h_{1L} > 0$, according to the proof of Theorem 3.1 there is $\limsup_{t\to+\infty} x_1(t) \le M_1$. Condition (3.10) implies $h_{2M} \le 1$. Again from the proof of Theorem 3.1, we obtain

$$\dot{x_2}(t) \le x_2(t)\left[r_{2M} - a_{2L}h_{2M}\int_0^{+\infty}\left(h_{2M}^{\frac{1}{\eta}}e^{r_{2M}}\right)^{-s}K_2(s)\mathrm{d}s x_2(t)\right].$$

According to Lemmas 2.1 and 2.3, we have

$$\lim_{t\to+\infty} x_2(t) = 0.$$

Then for any $\varepsilon_2 > 0$ satisfying $\left(r_{1L} - \frac{b_{1M}\varepsilon_2}{1+\varepsilon_2}\right)\theta + \ln h_{1L} > 0$, there exists a $T_2 > 0$ such that for $t > T_2$,

$$x_1(t) < M_1 + \varepsilon_2, \quad x_2(t) < \varepsilon_2. \tag{3.11}$$

Substituting this into system (1.3), it follows from Lemma 2.5 that

$$\dot{x_1}(t) \ge x_1(t)\left[r_{1L} - a_{1M}(M_1+\varepsilon_2) - \frac{b_{1M}\varepsilon_2}{1+\varepsilon_2}\right],$$

$$x_i(t_k^+) = h_{ik}x_i(t_k).$$

Similarly we have

$$\dot{x_1}(t) \geq x_1(t)\left[r_{1L} - \frac{b_{1M}\varepsilon_2}{1+\varepsilon_2} - a_{1M}\int_0^{+\infty}\left(\prod_{t-s\le t_k<t}\frac{1}{h_{1k}}\right)\exp\left\{-\left(r_{1L} - a_{1M}(M_1+\varepsilon_2) - \frac{b_{1M}\varepsilon_2}{1+\varepsilon}\right)s\right\}K_1(s)\mathrm{d}s x_1(t)\right].$$

Then similarly to the analysis of Lemma 2.2, by setting $\varepsilon_2 \to 0$ we can easily obtain

$$\liminf_{t\to+\infty} x_1(t) \geq \min\left\{\frac{r_{1L}\eta + \ln h_{1L}}{a_{1M}\bar{H}_{11}\eta h_{1L}^2}, \frac{(r_{1L}\theta + \ln h_{1L})h_{1L}^2}{a_{1M}\bar{H}_{21}\theta}\right\} \triangleq \bar{m}_1$$

with

$$\bar{H}_{11} = \int_0^{+\infty}\left(\frac{1}{h_{1L}}\right)^{\frac{s}{\eta}} \exp\left\{-\left(r_{1L} - a_{1M}M_1\right)s\right\}K_1(s)\mathrm{d}s < +\infty,$$

$$\bar{H}_{21} = \int_0^{+\infty}\left(\frac{1}{h_{1L}}\right)^{\frac{s}{\theta}} \exp\left\{-\left(r_{1L} - a_{1M}M_1\right)s\right\}K_1(s)\mathrm{d}s < +\infty.$$

This completes the proof of the theorem.

Consider the following impulsive system

$$\begin{aligned}\dot{x}(t) &= x(t)\left(r_1(t) - a_1(t)\int_0^{+\infty} K_1(s)x(t-s)\mathrm{d}s\right),\\ x(t_k^+) &= h_{1k}x(t_k), \quad k = 1, 2, \cdots,\end{aligned} \tag{3.12}$$

Theorem 3.4 *Under the assumptions of* Theorem 3.3, *we further suppose that there exists a* $\sigma_1 > 0$ *such that*

$$\int_0^{+\infty} sK_1(s)\mathrm{d}s = \sigma_1 \ \text{and}\ \frac{a_{1L}}{M_1} - a_{1M}^2\sigma_1 > 0.$$

Then for any positive solution $(x_1(t), x_2(t))^T$ *of system* (1.3), *and any positive solution* $x(t)$ *of system* (3.12), *there is* $\lim_{t\to+\infty} |x_1(t) - x(t)| = 0$.

Proof. Let $(x_1(t), x_2(t))^T$ be any positive solution of system (1.3), and $x(t)$ be any positive solution of system (3.12). From the condition of Theorem 3.6, there exists a $\delta_1 > 0$ such that

$$\frac{a_{1L}}{M_1} - a_{1M}^2\sigma_1 \geq \delta_1.$$

According to Theorem 3.5, for any $0 < \varepsilon_3 < \bar{m}_1$ small enough, there exists a $T_3 > 0$ such that for $t > T_3$,

$$\bar{m}_1 - \varepsilon_3 \leq x_1 \leq M_1 + \varepsilon_3.$$

Define a Lyapunov function as follows

$$\bar{V}_1(t) = \left(\ln x_1(t) - \ln x(t) - \int_0^{+\infty}\int_{t-s}^{t} K_1(s)a_1(v+s)(x_1(v) - x(v))\mathrm{d}v\mathrm{d}s\right)^2.$$

Similarly to the analysis of Theorem 3.2, for $t > T_3$ and $t \neq t_k$, $k = 1, 2, \cdots$, calculating the upper right derivatives of $\bar{V}_1(t)$, we can obtain

$$\begin{aligned}D^+\bar{V}_1(t) \leq& -2a_{1L}(x_1(t) - x(t))(\ln x_1(t) - \ln x(t)) + a_{1M}^2\sigma_1|x_1(t) - x(t)|^2\\ &+ a_{1M}^2\int_0^{+\infty}\int_{t-s}^{t} K_1(s)|x_1(v) - x(v)|^2\mathrm{d}v\mathrm{d}s + \frac{2b_{1M}\varepsilon_3}{1+\varepsilon_3}|\ln x_1(t) - \ln x(t)|\\ &+ \frac{2b_{1M}a_{1M}\varepsilon_3}{1+\varepsilon_2}\int_0^{+\infty}\int_{t-s}^{t} K_1(s)|x_1(v) - x(v)|\mathrm{d}v\mathrm{d}s.\end{aligned}$$

Define

$$\bar V_2(t) = a_{1M}^2 \int_0^{+\infty}\int_{t-s}^{t}\int_v^t K_1(s)|x_1(u)-x(u)|^2 \mathrm{d}u\mathrm{d}v\mathrm{d}s + \frac{2b_{1M}a_{1M}\varepsilon_3}{1+\varepsilon_3}\int_0^{+\infty}\int_{t-s}^{t}\int_v^t K_1(s)|x_1(u)-x(u)|\mathrm{d}u\mathrm{d}v\mathrm{d}s.$$

For $t > T_3$ and $t \neq t_k$, $k = 1, 2, \cdots$, calculating the upper right derivatives of $\bar V_2(t)$ and denoting $\bar V(t) = \bar V_1(t) + \bar V_2(t)$, it follows that

$$\begin{aligned} D^+\bar V(t) &\le -2a_{1L}(x_1(t)-x(t))(\ln x_1(t) - \ln x(t)) + 2a_{1M}^2\sigma_1|x_1(t)-x(t)|^2 \\ &\quad + \frac{2b_{1M}a_{1M}\varepsilon_3\sigma_1}{1+\varepsilon_3}|x_1(t)-x(t)| + \frac{2b_{1M}\varepsilon_3}{1+\varepsilon_3}|\ln x_1(t) - \ln x(t)| \\ &\le \Big(-\frac{2a_{1L}}{M_1+\varepsilon_3} + 2a_{1M}^2\sigma_1\Big)|x_1(t)-x(t)|^2 + \frac{2b_{1M}\varepsilon_3}{1+\varepsilon_3}\Big(a_{1M}\sigma_1 + \frac{1}{\bar m_1 - \varepsilon_3}\Big)|x_1(t)-x(t)|. \end{aligned}$$

By the boundedness of $x_1(t)$ and $x(t)$ and setting $\varepsilon_3 \to 0$, we educe that

$$D^+\bar V(t) \le \Big(-\frac{2a_{1L}}{M_1} + 2a_{1M}^2\sigma_1\Big)|x_1(t)-x(t)|^2 < -\delta_1|x_1(t)-x(t)|^2.$$

For $t = t_k$, we can easily verify that $\bar V(t_k^+) = \bar V(t_k)$. Integrating both sides of the above inequality from T_3 to t, we obtain

$$\bar V(t) + \delta_1\int_{T_3}^{t}|x_1(s)-x(s)|^2\mathrm{d}s \le \bar V(T_3^+) < +\infty.$$

Therefore, $\bar V(t)$ is bounded on $[T_3, +\infty)$ and there is

$$\int_{T_3}^{+\infty}|x_1(s)-x(s)|^2\mathrm{d}s < +\infty.$$

Similarly to the analysis of [17], it is obvious that

$$\lim_{t\to+\infty}|x_1(t)-x(t)| = 0.$$

This completes the proof of Theorem 3.4.

Now we discuss the extinction of system (1.3).

Theorem 3.5 *Let* $(x_1(t), x_2(t))^T$ *be any positive solution of system* (1.3). *Assume that*

$$r_{iM}\eta + \ln h_{iM} \le 0, \quad 1 \le i \le 2,$$

then system (1.3) *is extinct, that is* $\lim_{t\to+\infty} x_1(t) = \lim_{t\to+\infty} x_2(t) = 0$.

Proof. The proof of the theorem is similar to the corresponding part of Theorem 3.3, so we omit the detail.

In the following part of this section, based on the above theorems, we gives some corresponding results for systems (1.1) and (1.2) respectively. First for system (1.1), similarly to the analysis of Theorems 3.1 and 3.2, we can easy obtain the following theorem.

Theorem 3.6 *Let* $x(t)$ *and* $y(t)$ *be any two positive solutions of system* (1.1). *Assume that*

$$a_L\theta + \ln h_L > 0, \quad \int_0^{+\infty} sK(s)\mathrm{d}s = \sigma \quad and \quad b_L > b_M^2 M\sigma.$$

Then system (1.1) *is permanent and globally attractive, that is*

$$m \le \liminf_{t\to+\infty} x(t) \le \limsup_{t\to+\infty} x(t) \le M \quad and \quad \lim_{t\to+\infty} |x(t)-y(t)| = 0,$$

where

$$M = \max\left\{\frac{(a_M\theta + \ln h_M)h_M^2}{b_L R_1 \theta}, \frac{a_M\eta + \ln h_M}{b_L h_M^2 R_2 \eta}\right\} \quad and \quad m = \min\left\{\frac{a_L\eta + \ln h_L}{b_M H_1 \eta h_L^2}, \frac{(a_L\theta + \ln h_L)h_L^2}{b_M H_2 \theta}\right\}$$

with

$$R_1 = \int_0^{+\infty} (h_M^{\frac{1}{\theta}} e^{a_M})^{-s} K(s)\mathrm{d}s, \qquad R_2 = \int_0^{+\infty} (h_M^{\frac{1}{\eta}} e^{a_M})^{-s} K(s)\mathrm{d}s,$$

$$H_1 = \int_0^{+\infty} \left(\frac{1}{h_L}\right)^{\frac{s}{\eta}} \exp\left\{-\left(a_L - b_M M\right)s\right\} K(s)\mathrm{d}s < +\infty,$$

$$H_2 = \int_0^{+\infty} \left(\frac{1}{h_L}\right)^{\frac{s}{\theta}} \exp\left\{-\left(a_L - b_M M\right)s\right\} K(s)\mathrm{d}s < +\infty.$$

Remark 3.1 In Corollary 3.1, we prove the global attractivity of (1.1), but under some weaker conditions than those in Yang [21]; especially, our result does not require the following unreasonable condition:

$$0 < \inf_{k\ge1} h_k \le h_k \le 1 \ \ (k = 1, 2, \cdots) \text{ and } \inf_{k\ge1}(h_k - h_{k-1}) > 0.$$

Next for system (1.2), similarly to the proof of Theorem 3.1, we can easily prove the following theorem.

Theorem 3.7 *Let* $(x_1(t), x_2(t))^T$ *be any solution of system* (1.2) *with* $x_i(0) > 0$, $i = 1,\ 2$. *Assume that*

$$\begin{aligned} &\Big(r_{iL} - \frac{b_{iM}M_j'}{1+M_j'}\Big)\theta + \ln h_{iL} > 0, \quad 1 \le i,\ j \le 2,\ i \ne j, \\ &r_{iL} - \frac{b_{iM}M_j'}{1+M_j'} > 0, \end{aligned} \tag{3.13}$$

then $m_i' \le \liminf_{t\to+\infty} x_i(t) \le \limsup_{t\to+\infty} x_i(t) \le M_i',\ i = 1,\ 2,$ *where*

$$M_i' = \max\left\{\frac{(r_{iM}\theta + \ln h_{iM})h_{iM}}{a_{iL}\theta}, \frac{r_{iM}\eta + \ln h_{iM}}{a_{iL}h_{iM}\eta}\right\},$$

$$m_i' = \min\left\{\frac{\Big(r_{iL} - \frac{b_{iM}M_j'}{1+M_j'}\Big)\eta + \ln h_{iL}}{a_{iM}h_{iL}\eta}, \frac{\Big[\Big(r_{iL} - \frac{b_{iM}M_j'}{1+M_j'}\Big)\theta + \ln h_{iL}\Big]h_{iL}}{a_{iM}\theta}\right\}.$$

Theorem 3.8 *Under the conditions of* Theorem 3.7, *we further assume that there exist* $\rho_1 > 0$ *and* $\rho_2 > 0$ *such that*

$$a_{1L}\rho_1 - b_{2M}\rho_2 > 0 \ \ and \ \ a_{2L}\rho_2 - b_{1M}\rho_1 > 0,$$

Then for any two positive solutions $(x_1(t), x_2(t))^T$ *and* $(y_1(t), y_2(t))^T$ *of system* (1.2), *there are* $\lim_{t\to+\infty} |x_i(t) - y_i(t)| = 0,\ for\ \ i = 1,\ 2.$

Proof. Let $(x_1(t), x_2(t))^T$ and $(y_1(t), y_2(t))^T$ be any two positive solutions of system (1.2). From Theorem 3.7, for any $\varepsilon_4 > 0$ small enough, there exist $\delta_2 > 0$ satisfying $m_i' - \varepsilon_4 > 0$ and $T_4 > 0$ such that for $t > T_4$

$$a_{1L}\rho_1 - b_{2M}\rho_2 \ge \delta_2 \ \ \text{and} \ \ a_{2L}\rho_2 - b_{1M}\rho_1 \ge \delta_2,$$

$$m_i' - \varepsilon_4 \le x_i,\ y_i \le M_i' + \varepsilon_4, \quad i = 1,\ 2.$$

Define a Lyapunov function as follows

$$\tilde{V}(t) = \sum_{i=1}^{2} \rho_i |\ln x_i(t) - \ln y_i(t)|.$$

For $t > T_4$ and $t \neq t_k$, $k = 1, 2, \cdots$, calculating the upper right derivatives of $\tilde{V}(t)$, for $j = 1, 2$ and $j \neq i$, we have

$$D^+\tilde{V}(t) = \sum_{i=1}^{2}\rho_i \operatorname{sgn}(x_i(t) - y_i(t))\left[a_i(t)(y_i(t) - x_i(t)) + b_i(t)\Big(\frac{y_j(t)}{1+y_j(t)} - \frac{x_j(t)}{1+x_j(t)}\Big)\right]$$
$$\leq \sum_{i=1}^{2}\rho_i\Big(-a_{iL}|x_i(t) - y_i(t)| + \frac{b_{iM}}{(1+\zeta_j(t))^2}|x_j(t) - y_j(t)|\Big)$$
$$\leq (-a_{1L}\rho_1 + b_{2M}\rho_2)|x_1(t) - y_1(t)| + (-a_{2L}\rho_2 + b_{1M}\rho_1)|x_2(t) - y_2(t)|$$
$$\leq -\delta_2(|x_1(t) - y_1(t)| + |x_2(t) - y_2(t)|),$$

where $\zeta_j(t)$ lies between $x_j(t)$ and $y_j(t)$.

For $t = t_k$, we can easily verify that $\tilde{V}(t_k^+) = \tilde{V}(t_k)$. Integrating both sides of the above inequality from T_4 to t, we obtain

$$\tilde{V}(t) + \delta_2\int_{T_4}^{t}(|x_1(s) - y_1(s)| + |x_2(s) - y_2(s)|)ds \leq \tilde{V}(T_4^+) < +\infty.$$

Therefore, $\tilde{V}(t)$ is bounded on $[T_4^+, +\infty)$ and there is

$$\int_{T_4}^{+\infty}(|x_1(s) - y_1(s)| + |x_2(s) - y_2(s)|)ds < +\infty.$$

Similarly to the analysis of [17], it is obvious that

$$\lim_{t\to+\infty}|x_1(t) - y_1(t)| = \lim_{t\to+\infty}|x_2(t) - y_2(t)| = 0.$$

This completes the proof of Theorem 3.8.

Consider the following impulsive system

$$\begin{aligned}\dot{x}(t) &= x(t)(r_1(t) - a_1(t)x(t)),\\ x(t_k^+) &= h_{1k}x(t_k), \quad k = 1, 2, \cdots,\end{aligned} \tag{3.14}$$

Similarly to the analysis of Theorems 3.7 and 3.8, we can easily prove the following theorem.

Theorem 3.9 *Let $(x_1(t), x_2(t))^T$ be any positive solution of system* (1.2), *$x(t)$ be any positive solution of system* (3.14). *Assume that*

$$r_{1L}\theta + \ln h_{1L} > 0 \ \ and\ \ r_{2M}\eta + \ln h_{2M} \leq 0$$

Then the species x_1 is permanent and globally attractive but the species x_2 is extinct, that is

$$m_1'' \leq \liminf_{t\to+\infty} x_1(t) \leq \limsup_{t\to+\infty} x_1(t) \leq M_1' \ \ and\ \ \lim_{t\to+\infty}|x_1(t) - x(t)| = 0; \ \lim_{t\to+\infty} x_2(t) = 0,$$

where M_1' is defined in Theorem 3.7 and

$$m_1'' = \min\left\{\frac{r_{1L}\eta + \ln h_{1L}}{a_{1M}h_{1L}\eta}, \frac{(r_{1L}\theta + \ln h_{1L})h_{1L}}{a_{1M}\theta}\right\}.$$

Theorem 3.10 *Let $(x_1(t), x_2(t))^T$ be any solution of system* (1.2) *with $x_i(0) > 0$, $i = 1, 2$. Assume that*

$$r_{iM}\eta + \ln h_{iM} \leq 0, \quad 1 \leq i, \leq 2,$$

then system (1.2) *is extinct, that is* $\lim_{t\to+\infty} x_1(t) = \lim_{t\to+\infty} x_2(t) = 0.$

Proof By impulsive comparison theorem and Lemma 2.3, these results can be easily obtained, so we omit the detail.

Remark 3.2 Obviously, condition (H_1) implies (3.13), but not vice versa. Thus Theorem 3.7 weakens Lemma 2.4 in [32]. Also Theorems 3.8-3.10 complement the results of [32].

4 Numerical simulation

In this section, we present some numerical simulations to show the influence of impulse perturbations on the dynamic behaviors of systems.

Table 1: Parameter values of system (1.3)

Parameter	Interpretation	Value
$r_1(t)$	Growth rate of species x_1	$\frac{0.8t+0.88}{t+1}$
$r_2(t)$	Growth rate of species x_2	$\frac{0.3t+0.36}{t+1}$
$a_1(t)$	Intra-specific competition of species x_1	$0.28+0.01\sin\sqrt{2}t$
$a_2(t)$	Intra-specific competition of species x_2	$0.11+0.01\sin 2t$
$b_1(t)$	Interspecific competition o of species x_2 on x_1	$0.011+0.001\sin t$
$b_2(t)$	Interspecific competition o of species x_1 on x_2	$0.021+0.001\sin t$
$K_1(t)$	Kernel function of species x_1	$10e^{-10t}$
$K_2(t)$	Kernel function of species x_2	$8e^{-8t}$
h_{1k}	Impulse perturbations on species x_1	$1.1-0.2\cos k$
h_{2k}	Impulse perturbations on species x_2	$1.05+0.04\sin 2k$
t_k	Impulse points	$k+\frac{1}{k}$

In Table 1, by calculation we have $r_{1L}=0.8$, $r_{1M}=0.88$, $a_{1L}=0.27$, $a_{1M}=0.29$, $b_{1L}=0.01$, $b_{1M}=0.012$, $h_{1L}=0.9$, $h_{1M}=1.3$, $r_{2L}=0.3$, $r_{2M}=0.36$, $a_{2L}=0.1$, $a_{2M}=0.12$, $b_{2L}=0.02$, $b_{2M}=0.022$, $h_{2L}=1.01$, $h_{2M}=1.09$, $\theta=0.5$, $\eta=1$. Choose $\rho_1=9$ and $\rho_2=5$. Therefore,

$$M_1=\frac{(r_{1M}\theta+\ln h_{1M})h_{1M}^2}{a_{1L}\theta\int_0^{+\infty}(h_{1M}^{\frac{1}{\theta}}e^{r_{1M}})^{-s}K_1(s)\mathrm{d}s}\approx 10.0277,$$

$$M_2=\frac{(r_{2M}\theta+\ln h_{2M})h_{2M}^2}{a_{2L}\theta\int_0^{+\infty}(h_{2M}^{\frac{1}{\theta}}e^{r_{2M}})^{-s}K_2(s)\mathrm{d}s}\approx 6.7458,$$

$$H_{21} = \int_0^{+\infty} \left(\frac{1}{h_{1L}}\right)^{\frac{s}{\theta}} \exp\left\{-\left(r_{1L} - a_{1M}M_1 - \frac{b_{1M}M_2}{1+M_2}\right)s\right\}K_1(s)\mathrm{d}s \approx 1.3036,$$

$$H_{12} = \int_0^{+\infty} \left(\frac{1}{h_{2L}}\right)^{\frac{s}{\eta}} \exp\left\{-\left(r_{2L} - a_{2M}M_2 - \frac{b_{2M}M_1}{1+M_1}\right)s\right\}K_2(s)\mathrm{d}s \approx 1.0695,$$

$$\sigma_1 = \int_0^{+\infty} sK_1(s)\mathrm{d}s = \frac{1}{10} \quad \text{and} \quad \sigma_2 = \int_0^{+\infty} sK_2(s)\mathrm{d}s = \frac{1}{8}.$$

We can easily verify that

$$r_{1L} - b_{1M}\frac{M_2}{1+M_2} \approx 0.7895 > 0, \quad \left(r_{1L} - b_{1M}\frac{M_2}{1+M_2}\right)\theta + \ln h_{1L} \approx 0.2894 > 0,$$

$$r_{2L} - b_{2M}\frac{M_1}{1+M_1} \approx 0.2800 > 0, \quad \left(r_{2L} - b_{2M}\frac{M_1}{1+M_1}\right)\theta + \ln h_{2L} \approx 0.1499 > 0,$$

$$m_1 = \frac{[(r_{1L} - \frac{b_{1M}M_2}{1+M_2})\theta + \ln h_{1L}]h_{1L}^2}{a_{1M}H_{21}\theta} \approx 1.2402,$$

$$m_2 = \frac{(r_{2L} - \frac{b_{2M}M_1}{1+M_1})\eta + \ln h_{2L}}{a_{2M}H_{12}\eta h_{2L}^2} \approx 2.2148,$$

$$2\frac{a_{1L}}{M_1}\rho_1 - \frac{b_{1M}\rho_1}{m_1^2} - b_{2M}\rho_2 - 2a_{1M}^2\rho_1\sigma_1 - b_{1M}a_{1M}\rho_1\sigma_1 - b_{2M}a_{2M}\rho_2\sigma_2 \approx 0.0603 > 0,$$

$$2\frac{a_{2L}\rho_2}{M_2} - b_{1M}\rho_1 - \frac{b_{2M}\rho_2}{m_2^2} - 2a_{2M}^2\rho_2\sigma_2 - b_{1M}a_{1M}\rho_1\sigma_1 - b_{2M}a_{2M}\rho_2\sigma_2 \approx 0.0430 > 0.$$

Thus all the conditions of Theorem 3.2 are satisfied. Therefore both species x_1 and x_2 are permanent and globally attractive, which is shown in Figure 2.

Table 2: Simulations of system (1.3)

Case	h_{1k}	h_{2k}	Species x_1	Species x_2	Figure
1	$1.1 - 0.2\cos k$	$0.2 + 0.1\sin 2k$	Permanence	Extinction	Figure 3
2	$0.2 - 0.2\cos k$	$1.5 + 0.1\sin 2k$	Extinction	Permanence	Figure 4
3	$0.3 - 0.1\cos k$	$0.5 + 0.1\sin 2k$	Extinction	Extinction	Figure 5

Furthermore, we keep the growth rates, the intra-specific competition and the kernel functions of all species unchanged in Table 1, but adjust the values of the impulse perturbations given in Table 2, then simulations (see Figures 3-5) show that the permanence and extinction of the species are significantly changed, which are in accordance with the results of Theorems 3.4 and 3.5, here we can verify the corresponding conditions similarly to those in Table 1.

5 Conclusion

In this paper, we are devoted to obtaining the major factors that affect the coexistence, competition exclusion and extinction of system (1.3). Table 1 shows that we can choose some suitable values of parameters

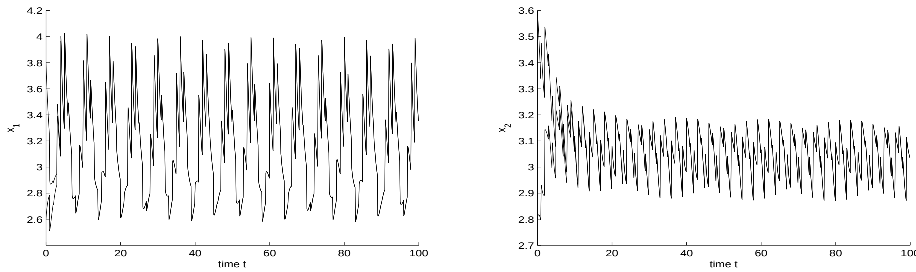

Figure 2: System (1.3) with $(\phi_1(t), \phi_2(t)) = (2.6, 2.8)^T$ and $(3.8, 3.6)^T$ for $t \le 0$ respectively.

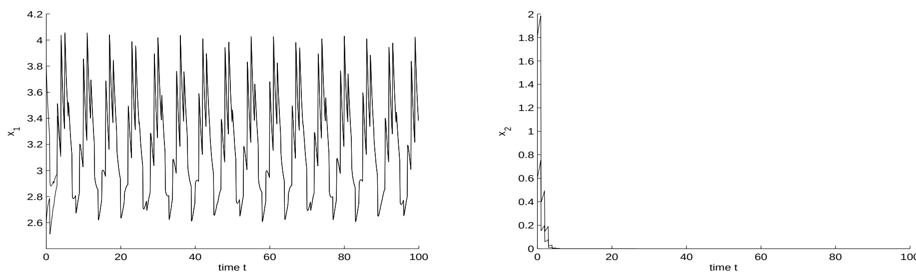

Figure 3: System (1.3) with $(\phi_1(t), \phi_2(t)) = (3.8, 0.6)^T$ and $(2.6, 1.8)^T$ for $t \le 0$ respectively.

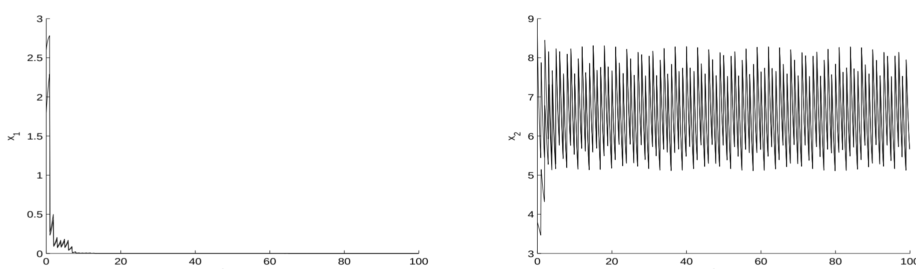

Figure 4: System (1.3) with $(\phi_1(t), \phi_2(t)) = (2.6, 3.8)^T$ and $(1.8, 8.6)^T$ for $t \le 0$ respectively.

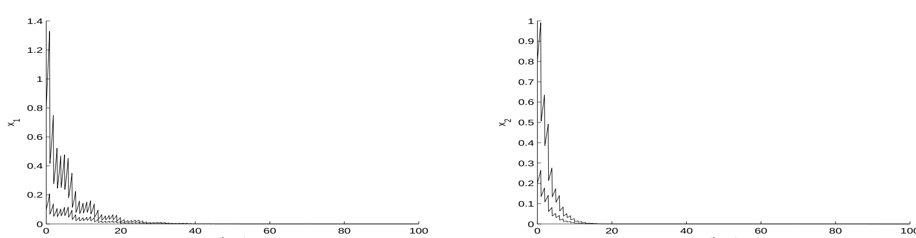

Figure 5: System (1.3) with $(\phi_1(t), \phi_2(t)) = (0.1, 0.8)^T$ and $(0.8, 0.2)^T$ for $t \le 0$ respectively.

of system (1.3) to guarantee the coexistence of both species. However, when we change the values of the impulse perturbations shown in Table 2, there is a significant variation of the survival of each species. When choosing the impulse perturbations $h_{ik} < 1$ small enough and keeping the value of the growth rate unchanged, it is hard to maintain the permanence of the species x_i. Moreover, this can result in the extinction of both species, which is different from the continuous system. The impulse perturbation plays an important role in the survival of the species and can deduce more situations of real ecosystems. Furthermore, for the logistic type impulsive equation with infinite delay, our results improve those of [21] and remove its unreasonable condition. For the corresponding nonautonomous two-species impulsive competitive system without delays, our results weaken and complement the results of [32].

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