Open Math. 2019; 17:716–727 DE GRUYTER

Open Mathematics

Research Article

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Nonfragile observer-based guaranteed cost finite-time control of discrete-time positive impulsive switched systems

https://doi.org/10.1515/math-2019-0058
Received September 4, 2018; accepted May 5, 2019

Abstract: This paper considers the nonfragile observer-based guaranteed cost finite-time control of discrete-time positive impulsive switched systems(DPISS). Firstly, the positive observer and nonfragile positive observer are designed to estimate the actual state of the underlying systems, respectively. Secondly, by using the average dwell time(ADT) approach and multiple linear co-positive Lyapunov function (MLCLF), two guaranteed cost finite-time controller are designed and sufficient conditions are obtained to guarantee the corresponding closed-loop systems are guaranteed cost finite-time stability(GCFTS). Such conditions can be solved by linear programming. Finally, a numerical example is provided to show the effectiveness of the proposed method.

Keywords: Guaranteed cost finite-time control, Nonfragile positive observer design, Discrete-time positive impulsive switched systems, Average dwell time

MSC: 15-xx, 65-xx, 93-xx

1 Introduction

The switched system is a type of hybrid systems. It comprises a set of a differential or difference equations and a switched controller, which designate the switching between subsystems at a specific interval of time. It has been studied very well, see [1-4]. As a special kind of switched systems, the positive switched systems whose output and state are non-negative have been paid much attention and adopted to many practical applications, such as communication networks [5], viral mutation [6], formation flying [7], and so on. There have been many available results about discrete-time positive switched systems [8-10]. Because of sudden changes in the state of the system at certain instants of switching, many practical switched systems exhibit impulsive dynamical behavior, these systems are usually called impulsive switched systems [11]. For discrete-time positive impulsive switched system, a few results have been obtained, see [12,13]. In [12], the exponential stability for a class of discrete-time positive impulsive switched linear systems was studied. In [13], the finite-time stability for a class of discrete-time positive impulsive switched time-delay systems under asynchronous switching was discussed. However, [12] and [13] are based on the assumption of the known state.

Moreover, it is necessary to design a state observer, because the states of the systems are not all measurable in practice [14]. Recently, the exponential stability property of the proposed switching observer

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was discussed, and an LMI-based algorithm was given for discrete-time impulsive switched nonlinear systems with time-varying delays in [15]. The problem of state observation for continuous-time and discrete-time impulsive switched systems was investigated in [16]. Furthermore, when the state observer gain variations could not be avoided, a kind of nonfragile state observers for discrete-time switched nonlinear systems was proposed in [17]. However, [15-17] were involved in non-positive systems and the design of finite time controller was not considered.

On the other hand, in most practical applications, the researchers are more interested in designing the control system, which is not only finite-time stable but also guarantees an adequate level of performance. One method to this problem is so-called guaranteed cost finite time control. Some remarkable results have been presented, see [18-23]. These results mainly focus on non-positive systems. Very recently, In [24], guaranteed cost finite-time control was extended to fractional-order positive switched systems and a cost function for fractional-order positive systems (or fractional-order positive switched systems) was proposed. In [25], the problem of guaranteed cost finite-time control for positive switched linear systems with time-varying delays was considered and a cost function of positive systems (or positive switched systems) was also presented. It is worth noting that [24] and [25] are involved in continuous-time positive switched systems with known states. To the best of our knowledge, the problem of observer-based guaranteed cost finite-time control of discrete-time impulsive switched systems has not been fully investigated, especially for DPISS, which motivates us for this study.

In this paper, the problem of nonfragile observer-based guaranteed cost finite-time control of DPISS is considered. The co-positive type Lyapunov function with average dwell time (ADT) technique is constructed. The main contributions lie in two aspects: 1) For DPISS, the design methods of positive observer and nonfragile positive observer are firstly proposed, respectively. 2) Two types of guaranteed cost finite-time controller are designed to guarantee the corresponding closed-loop systems are GCFTS, the obtained conditions can be easily solved by linear programming. The rest of the paper is organised as follows: Section 2 gives some necessary preliminaries and problem statements. In Section 3, the main results are given. In Section 4, a numerical example is provided. Section 5 concludes the paper.

Notations. The representation $A \succ 0$ ($\succeq 0, \prec 0, \preceq 0$) means that $a_{ij} > 0$ ($\geq 0, < 0, \leq 0$), which is also applying to a vector. $A \succ B$ ($A \succeq B$) means that $A - B \succ 0$ ($A - B \succeq 0$). R_+^n is the n-dimensional non-negative (positive) vector space. $R^{n\times n}$ denotes the space of $n \times n$ matrices with real entries. A^T denotes the transpose of matrix A. N and N^+ are the sets of non-negative and positive integers. Z^+ denotes the set of positive integers. Matrices are assumed to have compatible dimensions for calculating if their dimensions are not explicitly stated.

2 Preliminaries and problem statements

Consider the following DPISS:

$$\begin{cases} x(k+1) = A_{\sigma(k)}x(k) + B_{\sigma(k)}u(k), \ k \neq k_m - 1, m \in Z^+ \\ x(k+1) = E_{\sigma(k)}x(k), \ k = k_m - 1, m \in Z^+ \\ y(k) = C_{\sigma(k)}x(k) \\ x(k_0) = x_0 \end{cases} \tag{1}$$

where $k \in N$, $x(k) \in R^n$ is the system state, $u(k) \in R^m$ represents the control input. $\sigma(k)$ represents switching signal of system and takes values in a finite set $I = 1, 2, \dots, S$, $S \in N^+$. In general, A_i, B_i, C_i, E_i are the ith subsystem if $\sigma(k) = i \in I$. $k_0 = 0$ is the initial time. $k_m (m \in Z^+)$ denotes the bth impulsive switching instant. Moreover, $\sigma(k) = i \in I$ means that the ith subsystem is active. $\sigma(k-1) = j$ and $\sigma(k) = i (i \neq j)$ indicate that k is a switching instant at which the system is switched from the jth subsystem to the ith subsystem. At switching instants, there exist impulsive jumps described by (1). A_p, B_p, C_p, E_p are constant matrices with suitable dimensions.

Next, we will give some definitions and lemmas for the system (1).

Definition 1 [5]. System (1) with $u(k) \succeq 0$ is positive if $x(k_0) \succeq 0$ and any switching signals $\sigma(k)$, the corresponding trajectories $x(k) \succeq 0$, $y(k) \succeq 0$ hold for all $k \geq k_0$.
Lemma 1. System (1) is positive if and only if $A_i \succeq 0$, $B_i \succeq 0$, $C_i \succeq 0$, $E_i \succeq 0$, where $i \in I$.

2.1 Positive observer design

We construct the following DPISS (1):

$$\begin{cases} \hat{x}(k+1) = (A_{\sigma(k)} - L_{\sigma(k)}C_{\sigma(k)})\hat{x}(k) + B_{\sigma(k)}u(k) + L_{\sigma(k)}C_{\sigma(k)}x(k), \ k \neq k_m - 1, m \in Z^+ \\ \hat{x}(k+1) = H_{\sigma(k)}\hat{x}(k), \ k = k_m - 1, m \in Z^+ \\ u(k) = K_{\sigma(k)}\hat{x}(k) \\ x(k_0) = x_0 \end{cases} \tag{2}$$

where $\hat{x}(k) \in R^n$ is the estimated state vector of $x(k)$, $\hat{y}(k) \in R^p$ is the observer output vector. $L_i \in R^{n\times p}$ is the observer gain, $H_i \in R^{n\times n}$, $\forall i \in I$ is the matrix to be determined.
Remark 1. For DPISS (1), it not only requires that the state of the designed observer converges to that of the considered system, but also guarantees the positivity of the estimated state $\hat{x}(k)$ of system (2) with $u(k) \succeq 0$. To this end, it is naturally required, according to Lemma 1, $A_i - L_iC_i \succeq 0$, $B_i \succeq 0$, $L_iC_i \succeq 0$, $H_i \succeq 0$, $K_i \succeq 0$, where $i \in I$.

Let $e(k) = x(k) - \hat{x}(k)$ be the estimated error, then we can obtain the following error system:
When $k \neq k_m - 1$, we have

$$\begin{bmatrix} e(k+1) \\ \hat{x}(k+1) \end{bmatrix} = \begin{bmatrix} A_i - L_iC_i & 0 \\ L_iC_i & A_i + B_iK_i \end{bmatrix} \begin{bmatrix} e(k) \\ \hat{x}(k) \end{bmatrix} \tag{3}$$

When $k = k_m - 1$, we get

$$\begin{bmatrix} e(k+1) \\ \hat{x}(k+1) \end{bmatrix} = \begin{bmatrix} E_i & E_i - H_i \\ 0 & H_i \end{bmatrix} \begin{bmatrix} e(k) \\ \hat{x}(k) \end{bmatrix} \tag{4}$$

For the sake of convenience, we define $\tilde{x}(k+1) = \begin{bmatrix} e^T(k+1) & \hat{x}^T(k+1) \end{bmatrix}^T$. Thus, equation (3) and (4) can be rewritten as

$$\begin{cases} \tilde{x}(k+1) = \tilde{A}_i\tilde{x}(k), \ k \neq k_m - 1, m \in Z^+ \\ \tilde{x}(k+1) = \tilde{B}_i\tilde{x}(k), \ k = k_m - 1, m \in Z^+ \\ \tilde{x}(k_0) = \tilde{x}_0 \end{cases} \tag{5}$$

where

$$\tilde{A}_i = \begin{bmatrix} A_i - L_iC_i & 0 \\ L_iC_i & A_i + B_iK_i \end{bmatrix}, \tilde{B}_i = \begin{bmatrix} E_i & E_i - H_i \\ 0 & H_i \end{bmatrix}$$

Remark 2. According to Lemma 1, if error dynamic system (5) is positive, then it should guarantee that $\tilde{A}_i \geq 0$, $\tilde{B}_i \geq 0$, $\forall i \in I$(It means $A_i - L_iC_i \succeq 0$, $L_iC_i \succeq 0$, $A_i + B_iK_i \succeq 0$, $E_i \succeq 0$, $E_i - H_i \succeq 0$, $H_i \succeq 0$).
Definition 2. For any switching signal $\sigma(k)$ and any $k_2 \geq k_1 \geq 0$, let $N_\sigma(k_1, k_2)$ denote the switching numbers over the interval $[k_1, k_2)$. For given $\tau_\alpha > 0$ and $N_0 > 0$, if the inequality

$$N_\sigma(k_1, k_2) \leq N_0 + \frac{k_2 - k_1}{\tau_\alpha} \tag{6}$$

holds, then τ_α is called an average dwell time, and N_0 is called a chattering bound. Generally, we choose $N_0 = 0$.
Definition 3. (Finite-Time Stability(FTS)). For a given time T_f and two vectors $\alpha \succ \beta \succ 0$, discrete-time positive impulsive switched systems (5) with is said to be FTS with respect to $(\alpha, \beta, T_f, \sigma(k))$, if

$$\tilde{x}^T(0)\beta \leq 1 \Rightarrow \tilde{x}^T(k)\alpha < 1, \ \forall k \in [0, T_f] \tag{7}$$

Remark 3. In (5), the state $\tilde{x}^T(k)$ includes the $e(k)$ and $\hat{x}(k)$. From Definition 3, our goal is that the weighted system $\tilde{x}^T(k)\alpha$ does not exceed threshold 1 in a given time interval T_f, then the estimation error $e(k)$ might not converge to zero in a given time interval T_f. If a smaller threshold is chosen, then the estimation error will become very small.

Now we give some new definitions for our further study.

Definition 4 [24]. Define the cost function of DPISS (5) as follows:

$$J = \sum_{s=0}^{T_f - 1} (x^T(s)R_1 + u^T(s)R_2) \tag{8}$$

where $R_1 \succ 0$ and $R_2 \succ 0$ are two given vectors.

Definition 5 [24]. (GCFTS) For a given time constant T_f and two vectors $\alpha \succ \beta \succ 0$, consider DPISS (5) and cost function (8), if there exist a control law $u(k)$ and a positive scalar $J^\star$ such that the closed-loop system is FTS with respect to $(\alpha, \beta, T_f, \sigma(k))$ and the cost function satisfies $J \le J^\star$, then the closed-loop system is called GCFTS, where $J^\star$ is a guaranteed cost value and $u(k)$ is a guaranteed cost finite-time controller.

Remark 4. [14] noted that one could not stabilize any unstable positive system by using extended Luenberger type positive observers. But finite time stability means that the system state does not exceed the specified boundary within a given time interval and it is different from the asymptotic stability. So the problem of positive observer-based guaranteed cost finite-time control of DPISS is feasible.

2.2 Nonfragile positive observer design

If the state observer gain variations could not be avoided, a kind of nonfragile state observer will be designed as follows

$$\begin{cases} \hat{x}(k+1) = (A_{\sigma(k)} - (L_{\sigma(k)} + \triangle L_{\sigma(k)}))\hat{x}(k) + B_{\sigma(k)}u(k) + (L_{\sigma(k)} + \triangle L_{\sigma(k)})C_{\sigma(k)}x(k) \\ k \neq k_m - 1, m \in Z^+ \\ \hat{x}(k+1) = H_{\sigma(k)}\hat{x}(k), \ \ k = k_m - 1, m \in Z^+ \\ u(k) = K_{\sigma(k)}\hat{x}(k) \\ x(k_0) = x_0 \end{cases} \tag{9}$$

where $\triangle L_i \in R^{n\times p}$ are uncertain real-valued matrices which satisfy $\triangle L_i \in [L_1^{'}, L_2^{'}]$. $L_1^{'} \in R^{n\times p}$, $L_2^{'} \in R^{n\times p}$.

We can obtain the following error system

$$\begin{cases} \tilde{x}(k+1) = \tilde{A}_i\tilde{x}(k), \ \ k \neq k_m - 1, m \in Z^+ \\ \tilde{x}(k+1) = \tilde{B}_i\tilde{x}(k), \ \ k = k_m - 1, m \in Z^+ \\ \tilde{x}(k_0) = \tilde{x}_0 \end{cases} \tag{10}$$

where

$$\tilde{A}_i = \begin{bmatrix} A_i - (L_i + \triangle L_i)C_i & 0 \\ (L_i + \triangle L_i)C_i & A_i + B_iK_i \end{bmatrix}, \tilde{B}_i = \begin{bmatrix} E_i & E_i - H_i \\ 0 & H_i \end{bmatrix}$$

Remark 5. According to Lemma 1, if observer system (9) and error dynamic system (10) are positive, then it should guarantee that $\tilde{A}_i \geq 0, \tilde{B}_i \geq 0, \forall i \in I$(It means $A_i - (L_i + \triangle L_i)C_i \succeq 0, (L_i + \triangle L_i)C_i \succeq 0, A_i + B_iK_i \succeq 0, B_i \succeq 0, E_i \succeq 0, E_i - H_i \succeq 0, K_i \succeq 0, H_i \succeq 0$).

The aim of this paper is to design the positive observer and nonfragile positive observer based on state feedback controller, and find a class of switching signals $\sigma(k)$ for systems (2) and (6) such that the corresponding closed-loop systems are GCFTS, respectively.

3 Main results

3.1 Observer-based guaranteed cost finite-time stability analysis

In this subsection, we will focus on the problem of GCFTS for DPISS (5). The following theorem gives sufficient conditions of GCFTS for system (5).

Theorem 1. Consider the system (5), for a given time constant T_f, vectors $\alpha \succ \beta \succ 0$ and $R_1 \succ 0, R_2 \succ 0$, if there exist a set of positive vectors $v_i, v_j, \upsilon_i, \upsilon_j, i \neq j, i \in I$, positive matrices K_i, L_i and positive constants $\phi_1, \phi_2, \xi > 1, \mu > 1$, and such that the following inequalities hold:

$$(A_i^T - C_i^T L_i^T)v_i + R_1 + C_i^T L_i^T \upsilon_i - \xi v_i \prec 0 \tag{11}$$

$$(A_i^T + K_i^T B_i^T)\upsilon_i + R_1 + K_i^T R_2 - \xi \upsilon_i \prec 0 \tag{12}$$

$$E_i^T v_j - \mu v_i \prec 0 \tag{13}$$

$$(E_i^T - H_i^T)v_j + H_i^T \upsilon_j - \mu \upsilon_i \prec 0 \tag{14}$$

$$\phi_1 \alpha \prec \psi_i \prec \phi_2 \beta \tag{15}$$

$$A_i - L_i C_i \succ 0 \tag{16}$$

$$A_i + B_i K_i \succ 0 \tag{17}$$

$$E_i - H_i \succ 0 \tag{18}$$

$$\phi_1 > \phi_2 \xi^{T_f} \tag{19}$$

$\psi_i = [\psi_{i1}, \psi_{i2}, \ldots, \psi_{in}]^T$, ψ_{ir} represents the ith elements of the vectors ψ_i, respectively, then under the following ADT scheme

$$T_\alpha > T_\alpha^\star = \frac{T_f ln\mu}{ln\phi_1 - ln\phi_2 - T_f ln\xi} \tag{20}$$

the system (5) is GCFTS with respect to $(\alpha, \beta, T_f, \sigma(k))$ and the guaranteed cost value of system (5) is given by

$$J = \sum_{s=0}^{T_f-1}(x^T(s)R_1 + u^T(s)R_2) \le J^\star = \xi^{T_f} \mu^{\frac{T_f}{T_\alpha^\star}} \phi_2 \tag{21}$$

Proof. Construct the following multiple linear co-positive Lyapunov function for the systems (5) as follows:

$$V_i \tilde{x}(k) = \tilde{x}^T(k)\psi_i \tag{22}$$

where $\psi_i = \begin{bmatrix} v_i^T & \upsilon_i^T \end{bmatrix}^T, i \in I$.

Supposing a switching sequence $0 = k_0 \le k_1 \le k_m \le k_{m+1} \le \ldots \le T_f$. Without loss of generality, we assume that subsystem i is activated at the switching instant k_{m-1} and the subsystem j is activated at the switching instant k_m.

When $k \in [k_{m-1}, k_m - 1)$, $m \in N$, $\sigma(k) = \sigma(k+1) = i$, along the trajectory of system (5), the difference of the MLCLF is

$$\begin{aligned} &\triangle V_i(\tilde{x}(k) + x^T(k)R_1 + x^T(k)K_i^T R_2 \\ &= \tilde{x}^T(k+1)\psi_i - \tilde{x}^T(k)\psi_i + x^T(k)R_1 + x^T(k)K_i^T R_2 \\ &= \tilde{x}^T(k)\begin{bmatrix} (A_i^T - C_i^T L_i^T)v_i + R_1 + C_i^T L_i^T \upsilon_i \\ (A_i^T + K_i^T B_i^T)\upsilon_i + R_1 + K_i^T R_2 \end{bmatrix} - \tilde{x}^T(k)\psi_i \end{aligned} \tag{23}$$

From (11) and (12), we have

$$\triangle V_i(\tilde{x}(k)) + x^T(k)R_1 + x^T(k)K_i^T R_2 \le (\xi - 1)\tilde{x}^T(k)\psi_i \tag{24}$$

it implies

$$V_i(\tilde{x}(k+1)) \le \xi \tilde{x}^T(k)\psi_i \tag{25}$$

When $k = k_m - 1$, $\sigma(k+1) = \sigma(k_m) = j$, $\sigma(k) = \sigma(k_m - 1) = i$, $i \neq j$. Along the trajectory of system (5), the difference of MLCLF is

$$\begin{aligned} V_j(\tilde{x}(k+1)) - \mu V_i(\tilde{x}(k)) &= \tilde{x}^T(k+1)\psi_j - \mu\tilde{x}^T(k)\psi_i \\ &= \tilde{x}^T(k+1)\psi_j - \tilde{x}(k)\psi_i \\ &\le \tilde{x}^T(k)\begin{bmatrix} E_i^T \nu_j - \mu\nu_i \\ (E_i^T - H_i^T)\nu_j + H_i^T \upsilon_j - \mu\upsilon_i \end{bmatrix} \end{aligned} \tag{26}$$

From (13) and (14), we have

$$V_j(\tilde{x}(k+1)) \le \mu V_i(\tilde{x}(k)),\ i \neq j. \tag{27}$$

So, when $k \in [k_m, k_{m+1})$, from (27), we get

$$\begin{aligned} V_{\sigma(k)}(\tilde{x}(k)) &< \xi^{k-k_m} V_{\sigma(k_m)}(\tilde{x}(k_m)) \\ &< \mu\xi^{k-k_m} V_{\sigma(k_m-1)}(\tilde{x}(k_m - 1)) \end{aligned} \tag{28}$$

Repeating the procedure of (28) and noting $\xi > 1$, we obtain

$$\begin{aligned} V_{\sigma(k)}(\tilde{x}(k)) &< \xi^{k-k_m} V_{\sigma(k_m)}(\tilde{x}(k_m)) \\ &< \mu\xi^{k-k_m} V_{\sigma(k_m-1)}(\tilde{x}(k_m - 1)) \\ &< \mu\xi^{k-k_m}\xi^{k_m-1-k_{m-1}} V_{\sigma(k_{m-1})}(\tilde{x}(k_{m-1})) \\ &< \mu\xi^{k-k_{m-1}} V_{\sigma(k_{m-1})}(\tilde{x}(k_{m-1})) \end{aligned} \tag{29}$$

By iterative operation, we get

$$\begin{aligned} V_{\sigma(k)}(\tilde{x}(k)) &< \mu^2\xi^{k-k_{m-2}} V_{\sigma(k_{m-2})}(\tilde{x}(k_{m-2})) \\ &\le \ldots\ldots \le \\ &\le \mu^{N_\sigma(k,k_0)}\xi^{k-k_0} V_{\sigma(k_0)}(\tilde{x}(k_0)) \\ &= \mu^{N_\sigma(k,k_0)}\xi^{k-k_0} V_{\sigma(k_0)}(\tilde{x}(k_0)) \end{aligned} \tag{30}$$

From (15) and (22), we have

$$V_{\sigma(k)}(\tilde{x}(k)) = \tilde{x}^T(k)\psi_{\sigma(k)} \ge \phi_1 \tilde{x}^T(k)\alpha \tag{31}$$

$$V_{\sigma(k_0)}(\tilde{x}(k_0)) = \tilde{x}^T(0)\psi_{\sigma(k_0)} \le \phi_2 \tilde{x}^T(0)\beta \tag{32}$$

From (30)-(32), we obtain

$$\begin{aligned} \tilde{x}^T(k)\alpha &\le \tfrac{1}{\phi_1}\mu^{\frac{T_f}{T_\alpha}}\xi^{T_f}\phi_2 x^T(0)\beta \\ &\le \tfrac{1}{\phi_1}\mu^{\frac{T_f}{T_\alpha}}\xi^{T_f}\phi_2 \end{aligned} \tag{33}$$

Substituting (20) into (33), one has

$$\tilde{x}^T(k)\alpha < 1 \tag{34}$$

According to Definition 3, we conclude that the system (5) with $u(k) = 0$ is FTS with respect to $(\alpha, \beta, T_f, \sigma(k))$.

Next, we will give the guaranteed cost value of system (5).

When $k \in [k_{m-1}, k_m - 1)$, $m \in N$, according to (24), we know

$$V_i(\tilde{x}(k)) \le \xi\tilde{x}^T(k)\nu_i - x^T(k)R_1 - x^T(k)K_i^T R_2 \tag{35}$$

Similar to the proof process of (25)-(30), for any $k \in [0, T_f]$ and $\mu > 1$, we can obtain

$$V_{\sigma(k)}(\tilde{x}(k)) < \mu^{N_\sigma(k,k_0)}\xi^{k-k_0} V_{\sigma(k_0)}(\tilde{x}(k_0)) - \textstyle\sum_{s=k_0}^{k-1} x^T(s)(R_1 + K_i^T R_2) \tag{36}$$

Noting that $V_{\sigma(k)}(\tilde{x}(k)) > 0$, (36) can be rewritten as

$$0 < \mu^{N_\sigma(k,k_0)}\xi^{k-k_0}V_{\sigma(k_0)}(\tilde{x}(k_0)) - \sum_{s=0}^{k-1} x(s)^T(R_1 + K_i^T R_2) \tag{37}$$

Letting $k = T_f$, we get

$$\begin{aligned}\sum_{s=0}^{T_f-1} x(s)^T(R_1 + K_i^T R_2) &< \mu^{N_\sigma(k,k_0)}\xi^{k-k_0}(V_{\sigma(k_0)}\tilde{x}(k_0)) \\ &< \mu^{N_\sigma(T_f,k_0)}\xi^{T_f-k_0}(V_{\sigma(k_0)}\tilde{x}(k_0))\end{aligned} \tag{38}$$

Then we can obtain

$$J = \sum_{s=0}^{T_f-1} x^T(s)(R_1 + K_i^T R_2) \le J^\star = \xi^{T_f}\mu^{\frac{T_f}{T_\alpha^\star}}\phi_2 \tag{39}$$

Therefore, according to Definition 5, we can conclude that the system (5) is GCFTS. Thus, the proof is completed.

In Theorem 1, (11) and (12) are nonlinear matrix inequalities, there are no effective methods to solve K_i and L_i. So, an algorithm is presented to obtain the feedback gain matrices $K_i, L_i, i \in I$.

Algorithm 1

Step 1. By adjusting the parameters ξ, μ and Letting $f_p = K_p^T B_p^T \upsilon_p$, then solving (13)-(19) via linear programming, positive vectors ν_p, υ_p, K_p and L_p can be obtained.

Step 2. Substituting υ_p and K_p into $f_p = K_p^T B_p^T \upsilon_p$, $\tilde{f}_p$ can be obtained. The gain L_p, ν_p and υ_p are substituted into (12).

Step 3. If $\tilde{f}_p - f_p \preceq 0$ and (12) are all satisfied at the same time, then K_p and L_p are admissible. Otherwise, return to Step 1.

3.2 Nonfragile observer-based guaranteed cost finite-time stability analysis

Now we consider system (10); the following theorem gives sufficient conditions of GCFTS for system (10).

Theorem 2. Consider the system (10), for a given time constant T_f, vectors $\alpha \succ \beta \succ 0$, $L_1^{'}$, $L_2^{'}$ and $R_1 \succ 0$, $R_2 \succ 0$, if there exist a set of positive vectors $\nu_i, \nu_j, \upsilon_i, \upsilon_j, i \neq j, i \in I$, positive matrices K_i, L_i and positive constants $\phi_1, \phi_2, \xi > 1, \mu > 1$, such that (12)-(15), (17)-(19) and the following inequalities hold:

$$(A_i^T - C_i^T(L_i + \triangle L_i)^T\nu_i + R_1 + C_i^T(L_i + \triangle L_i)^T\upsilon_i - \xi\nu_i \prec 0 \tag{40}$$

$$A_i - (L_i + \triangle L_i)C_i \succ 0 \tag{41}$$

then under the ADT scheme (20), the system (10) is GCFTS with respect to $(\alpha, \beta, T_f, \sigma(k))$ and the guaranteed cost value of the system (10) is given by

$$J = \sum_{s=0}^{T_f-1}(x^T(s)R_1 + u^T(s)R_2) \le J^\star = \xi^{T_f}\mu^{\frac{T_f}{T_\alpha^\star}}\phi_2 \tag{42}$$

Proof. Replacing $(A_i^T - C_i^T L_i^T)\nu_i + R_1 + C_i^T L_i^T \upsilon_i$ in (23) with $(A_i^T - C_i^T(L_i + \triangle L_i)^T\nu_i + R_1 + C_i^T(L_i + \triangle L_i)^T\upsilon_i$, similar to the proof of Theorem 1, we easily obtain that the resulting closed-loop system (10) is GCFTS.

Thus, the proof is completed.

Theorem 3. Consider the system (10), for a given time constant T_f, vectors $\alpha \succ \beta \succ 0$, $L_1^{'}$, $L_2^{'}$ and $R_1 \succ 0$, $R_2 \succ 0$, if there exist a set of positive vectors $\nu_i, \nu_j, \upsilon_i, \upsilon_j, i \neq j, i \in I$, positive matrices K_i, L_i and positive constants $\phi_1, \phi_2, \xi > 1, \mu > 1$, such that (12)-(15), (17)-(19) and the following inequalities hold:

$$(A_i^T - C_i^T(L_i + L_1^{'})^T\nu_i + R_1 + C_i^T(L_i + L_2^{'})^T\upsilon_i - \xi\nu_i \prec 0 \tag{43}$$

$$A_i - (L_i + L_1^{'})C_i \succ 0 \tag{44}$$

then under the ADT scheme (20), the system (10) is GCFTS with respect to $(\alpha, \beta, T_f, \sigma(k))$ and the guaranteed cost value of the system (10) is given by

$$J = \sum_{s=0}^{T_f-1}(x^T(s)R_1 + u^T(s)R_2) \leq J^{\star} = \xi^{T_f}\mu^{\frac{T_f}{T_\alpha^{\star}}}\phi_2 \tag{45}$$

Proof. If $(A_i^T - C_i^T(L_i + L_1^{'})^T v_i + R_1 + C_i^T(L_i + L_2^{'})^T \upsilon_i - \xi v_i \prec 0$ and $A_i - (L_i + L_1^{'})C_i \succ 0$, we can easily get $(A_i^T - C_i^T(L_i + \triangle L_i)^T v_i + R_1 + C_i^T(L_i + \triangle L_i)^T \upsilon_i - \xi v_i \prec 0$ and $A_i - (L_i + \triangle L_i)C_i \succ 0$. Then, the conditions of Theorem 2 are satisfied. So the resulting closed-loop system (10) is GCFTS.

To obtain the feedback gain matrices K_i and L_i in Theorem 3, an algorithm is presented.

Algorithm 2

Step 1. By adjusting the parameters ξ, μ and Letting $f_p = K_p^T B_p^T \upsilon_p$, then solving (13)-(15), (17)-(19) and (44) via linear programming, positive vectors v_p, υ_p, K_p and L_p can be obtained.

Step 2. Substituting υ_p and K_p into $f_p = K_p^T B_p^T \upsilon_p$, $\tilde{f}_p$ can be obtained. The gain L_p, v_p and υ_p are substituted into (43).

Step 3. If $\tilde{f}_p - f_p \preceq 0$ and (43) are all satisfied at the same time, then K_p and L_p are admissible. Otherwise, return to Step 1.

4 Numerical example

We present a numerical example to show the effectiveness of the proposed approach. Without loss of generality, we consider the case of nonfragile positive observer of DPISS (10) with the parameters as follows:

$$A_1 = \begin{bmatrix} 0.1 & 0.3 \\ 0.2 & 0.3 \end{bmatrix}, B_1 = \begin{bmatrix} 0.3 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}, E_1 = \begin{bmatrix} 0.2 & 0.3 \\ 0.3 & 0.2 \end{bmatrix}, H_1 = \begin{bmatrix} 0.1 & 0.2 \\ 0.1 & 0.1 \end{bmatrix}, C_1 = \begin{bmatrix} 0.15 & 0.2 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.2 & 0.15 \\ 0.3 & 0.2 \end{bmatrix}, B_2 = \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & 0.1 \end{bmatrix}, E_2 = \begin{bmatrix} 0.3 & 0.4 \\ 0.2 & 0.5 \end{bmatrix}, H_2 = \begin{bmatrix} 0.2 & 0.2 \\ 0.1 & 0.4 \end{bmatrix}, C_2 = \begin{bmatrix} 0.1 & 0.3 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 0.5 \\ 0.6 \end{bmatrix}, R_2 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, L_1^{'} = \begin{bmatrix} 0.03 \\ 0.02 \end{bmatrix}, L_2^{'} = \begin{bmatrix} 0.04 \\ 0.03 \end{bmatrix}$$

$$\alpha = \begin{bmatrix} 0.2, 0.35, 0.1, 0.4 \end{bmatrix}^T, \beta = \begin{bmatrix} 0.4, 0.1, 0.2, 0.1 \end{bmatrix}^T,$$

Choosing $T_f = 5$, $\xi = 1.1$ and $\mu = 1.05$. Solving the inequalities in Theorem 3 by linear programming, we have

$$v_1 = \begin{bmatrix} 0.1522 \\ 0.1594 \end{bmatrix}, v_2 = \begin{bmatrix} 0.1522 \\ 0.1593 \end{bmatrix}, \upsilon_1 = \begin{bmatrix} 0.0817 \\ 0.0869 \end{bmatrix}, \upsilon_2 = \begin{bmatrix} 0.0815 \\ 0.0888 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} 0.0381 & 0.1498 \\ 0.0052 & 0.1234 \end{bmatrix}, K_2 = \begin{bmatrix} 0.0583 & 0.1505 \\ 0.0235 & 0.1244 \end{bmatrix}, L_1 = \begin{bmatrix} 0.3829 \\ 0.4170 \end{bmatrix}, L_2 = \begin{bmatrix} 0.3953 \\ 0.4258 \end{bmatrix},$$

$$f_1 = \begin{bmatrix} 0.0194 \\ 0.0825 \end{bmatrix}, f_2 = \begin{bmatrix} 0.0257 \\ 0.0929 \end{bmatrix},$$

It is easy to firm that $\tilde{f}_p - f_p \preceq 0$ and (10) are satisfied. then K_p and L_p are admissible. According to (20), we get $T_\alpha^{\star} = 1.7$.

The simulation results are shown in Figs. 1-5, where the initial conditions of the system (10) are $\tilde{x}(0) = \begin{bmatrix} 2, 2, 1, 1 \end{bmatrix}^T$, which meet the condition $\tilde{x}^T(0)\beta < 1$. The state trajectory of x_1 and state observation trajectory $\hat{x}_1$ are shown in Fig. 1. The state trajectory of x_2 and state observation trajectory $\hat{x}_2$ are depicted in Fig. 2. Fig. 3 plots the state of the error dynamic system. The switching signal $\sigma(k)$ is depicted in Fig. 4. Fig. 5 plots the evolution of $\tilde{x}(t)\alpha$, which implies that the corresponding closed-loop system is GCFTS with respect to $(\alpha, \beta, T_f, \sigma(k))$ to $(\alpha, \beta, T_f, \sigma(k))$, and the cost value $J^{\star} = 2.33$, which can be obtained by (45).

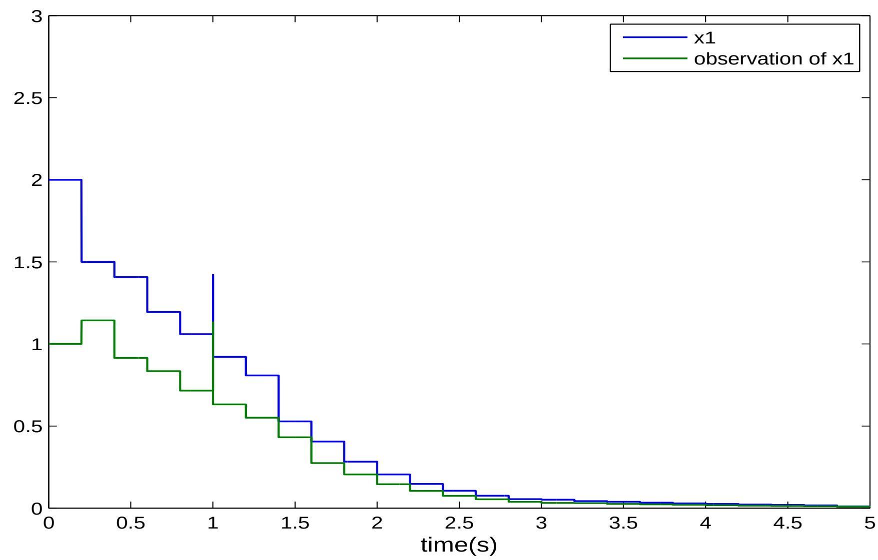

Fig. 1. The state trajectory of x_1 and state observation trajectory $\hat{x}_1$.

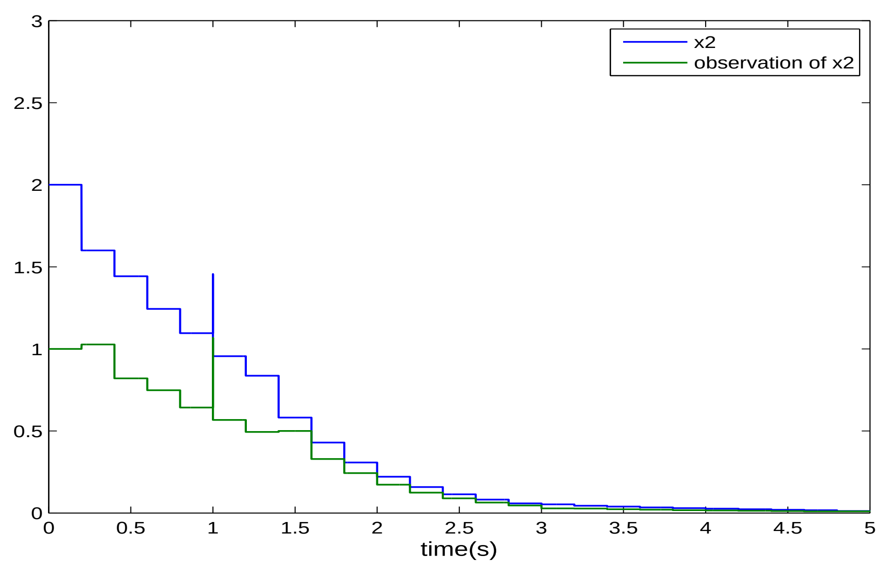

Fig. 2. The state trajectory of x_2 and state observation trajectory $\hat{x}_2$.

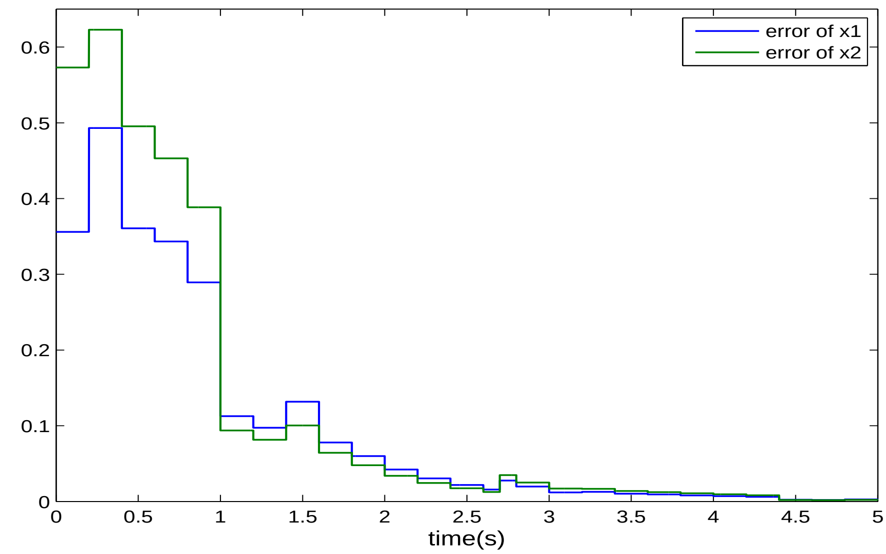

Fig. 3. State of the error dynamic system.

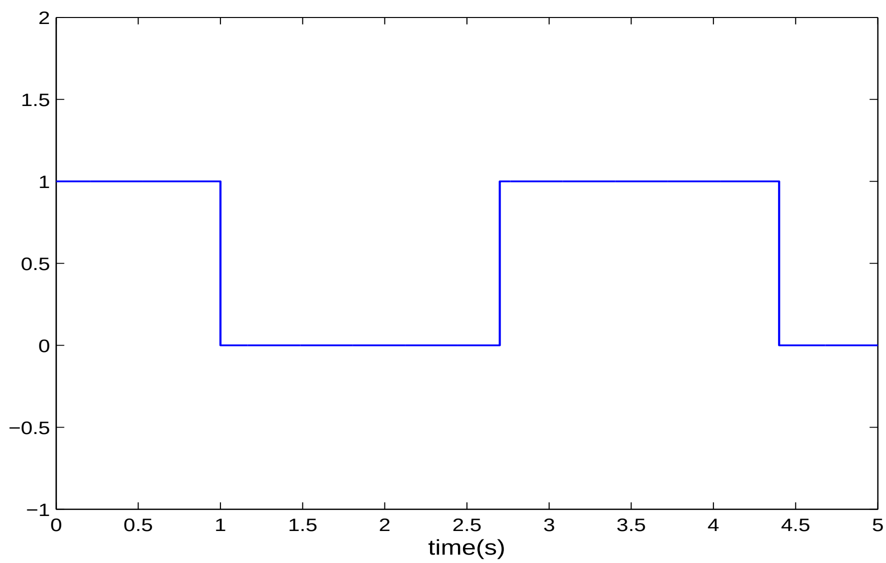

Fig. 4. Switching signal of system (5) with ADT.

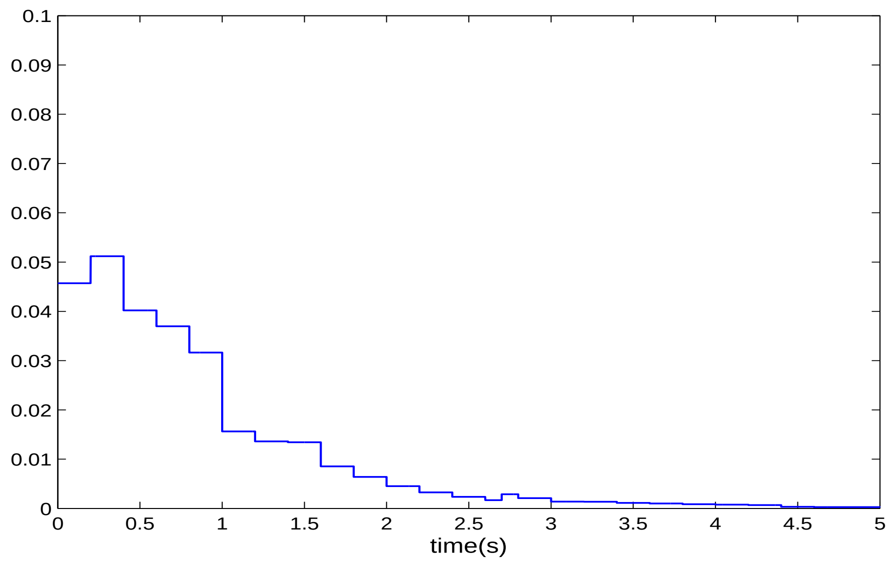

Fig. 5. The evolution of $\tilde{x}^T(k)\alpha$ of system (5).

5 Conclusions

In this paper, we have considered the issue of nonfragile observer-based guaranteed cost finite-time control for DPISS. Based on the ADT approach and co-positive type Lyapunov function technique, two types of guaranteed cost finite-time controller based on positive observer and nonfragile positive observer are designed, and sufficient conditions are obtained to guarantee the corresponding closed-loop systems are guaranteed cost finite-time stability(GCFTS), respectively. Such conditions can be solved by linear programming. Finally, a numerical example is given to illustrate the effectiveness of the proposed method.

Acknowledgement: The authors are thankful for the supports of the National Natural Science Foundation of China under grants U1404610, 61773350 and young key teachers plan of Henan province (2016GGJS-056).

Conflict of Interests: The authors declare that there is no conflict of interests regarding the publication of this paper.

References

[1] Niu B., Wang D., Alotaibi N.D., Alsaadi F.E., Adaptive neural state-feedback tracking control of stochastic nonlinear switched systems, IEEE Transactions on Neural Networks and Learing Systems, doi: 10.1109/TNNLS.2018.2860944

[2] Niu B., Li H., Zhang Z.H., Li J., Hayat T., Alsaadi F.E., An average dwell-time method and adaptive neural-network-based dynamic surface control for stochastic interconnected nonlinear non-strict-feedback systems with dead zone, IEEE Transactions on Systems, Man, and Cybernetics: Systems, doi: 10.1109/TSMC.2018.2866519

[3] Liu X., Li S.H., Zhang K., Optimal control of switching time in switched stochastic systems with multi-switching times and different costs, International Journal of Control, doi: 10.1080/00207179.2016.1214879

[4] Liu X., Zhang K., Li S.H., Fei S.H., Wei H., Time optimisation problem for switched stochastic systems with multi-switching times, IET Control Theory and Applications, 2014, 8(16): 1732-1740

[5] Zhao X., Liu X., Yin S., Improved results on stability of continuous-time switched positive linear systems, Automatica, 2014, 50(2): 614-621

[6] Liu S., Xiang Z., Exponential L_1 output tracking control for positive switched linear systems with time-varying delays, Nonlinear Analysis: Hybrid Systems, 2014, 11: 118-128

[7] Zong G., Ren H., Hou L., Finite-time stability of interconnected impulsive switched systems, IET Control Theory and Applications, 2016, 10(6): 648-654
[8] Zhang J., Han Z., Huang J., Stabilization of discrete-time positive switched systems Circuits Syst, Signal Process, 2013, 32(3), 1129-1145
[9] Fornasini E., Valcher M.E., Stability and stabilizability criteria for discrete-time positive switched systems, IEEE Transactions on Automatic Control , 2012, 57(5): 1208-1221
[10] Rami M.A., Tadeo F., Benzaouia A., Control of constrained positive discrete systems, American Control Conference, 2007: 5851-5856
[11] Guan Z.H., Hill D.J., Shen X., On hybrid impulsive and switching systems and application to nonlinear control, IEEE Transactions on Automatic Control, 2005, 50(7): 1058-1062
[12] Xiang M., Xiang Z., Exponential stability of discrete-time switched linear positive systems with time-delay, Applied Mathematics and Computation, 2014, 230(3): 193-199
[13] Liu T., Wu B., Liu L., Wang Y.E., Asynchronously finite-time control of discrete impulsive switched positive time-delay systems, Journal of the Franklin Institute, 2015, 352(10): 4503-4514
[14] Li X., Xiang Z., Observer design of discrete-time impulsive switched nonlinear systems with time-varying delays, Applied Mathematics and Computation, 2014, 229(6): 327-339
[15] Mahmoudi A., Momeni A., Aghdam A.G., Gohari P., On observer design for a class of impulsive switched systems, 2008 American Control Conference, 2008: 11-13
[16] Rami M.A., Helmke U., Tadeo F., Positive observation problem for linear time-delay positive systems, Conference on Control and Automation, 2008, 3(1): 1-6
[17] Xiang Z., Wang R., Jiang B., Nonfragile observer for discrete-time switched nonlinear systems with time delay, Circuits Systems and Signal Processing, 2011, 30(1): 73-87
[18] Huang S., Xiang Z., Karimi H.R., Input-output finite-time stability of discrete-time impulsive switched linear systems with state delays, Circuits Systems and Signal Processing, 2014, 33(1): 141-158
[19] Yang D., Cai K.Y., Finite-time quantized guaranteed cost fuzzy control for continuous-time nonlinear systems, Expert Systems with Applications, 2010 , 37(10): 6963-6967
[20] Niamsup P., Ratchagit K., Phat V.N., Novel criteria for finite-time stabilization and guaranteed cost control of delayed neural networks, Neurocomputing, 2015, 160: 281-286
[21] Zhang Y.Q., Liu L., Mu X.W, Stochastic finite-time guaranteed cost control of markovian jumping singular systems, Mathematical Problems in Engineering, 2011, 2011(11): 34-35
[22] Cai K.Y., Finite-time reliable guaranteed cost fuzzy control for discrete-time nonlinear systems, International Journal of Systems Science, 2011, 42(1): 121-128
[23] Yan Z., Zhang G., Wang J., Finite-time guaranteed cost control for linear stochastic systems, 30th Chinese Control Conference, 2011: 1389-1394
[24] Liu L., Cao X., Fu Z., Song S., Guaranteed cost finite-time control of fractional-order positive switched systems, Advances in Mathematical Physics, 2017, 2017(3): 1-11
[25] Cao X., Liu L., Fu Z., Song X., Song S., Guaranteed cost finite-time control for positive switched linear systems with time-varying delays, Journal of Control Science and Engineering, 2017, 1-10