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Almost periodic solution of a discrete competitive system with delays and feedback controls

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Abstract: A discrete nonlinear almost periodic multispecies competitive system with delays and feedback controls is proposed and investigated. We obtain sufficient conditions to ensure the permanence of the system. Also, we establish a criterion for the existence and uniformly asymptotic stability of unique positive almost periodic solution of the system. In additional, an example together with its numerical simulation are presented to illustrate the feasibility of the main result.

Keywords: Almost periodic solution, Competitive system, Lyapunov functional, Delays

MSC: 34C25, 92D25, 34D20, 34D40

1 Introduction

The importance of species competition in nature is obvious. For example, competition may be territory which is directly related to food resources. The widely used Lotka-Volterra system is considered as a disadvantage and that is the linearity. Ayala et al. [1] presented the following competitive system:

$$\begin{cases} \dot{x}_1(t) = x_1(t)\Big[r_1 - x_1(t) - a_1 x_2(t) - c_1 x_2^2(t)\Big], \\ \dot{x}_2(t) = x_2(t)\Big[r_2 - x_2(t) - a_2 x_1(t) - c_2 x_1^2(t)\Big]. \end{cases} \tag{1}$$

Besides, Gopalsamy [2] discussed the continuous version with discrete delays, Tan and Liao [3] established the discrete time version with discrete delays, Xue et al. [4] proposed the discrete time version with infinite delays and single feedback control. Recently, the almost periodic solutions of discrete system with feedback controls has more extensively investigated (see [5–10]). Motivated by above, we study the following system with delays and feedback controls:

$$\begin{cases} x_i(k+1) = x_i(k)\exp\Big[r_i(k) - a_i(k)x_i(k) - \sum\limits_{j=1,j\neq i}^{n} b_{ij}(k)x_j(k-\tau_j) - \sum\limits_{j=1,j\neq i}^{n} d_{ij}(k)x_j^2(k-\tau_j) - e_i(k)u_i(k)\Big], \\ u_i(k+1) = \Big(1 - f_i(k)\Big)u_i(k) + \sum\limits_{j=1}^{n} g_{ij}(k)x_j(k), \quad i,j = 1,2,\cdots,n, \end{cases} \tag{2}$$

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where $\{r_i(k)\}, \{a_i(k)\}, \{b_{ij}(k)\}, \{d_{ij}(k)\}, \{e_i(k)\}, \{f_i(k)\}$ and $\{g_{ij}(k)\}$ are bounded nonnegative almost periodic sequences such that

$$\begin{aligned} &0 < r_i^l \le r_i(k) \le r_i^u, \quad 0 < a_i^l \le a_i(n) \le a_i^u, \quad 0 < b_{ij}^l \le b_{ij}(n) \le b_{ij}^u \\ &0 < d_{ij}^l \le d_{ij}(n) \le d_{ij}^u, \quad 0 < e_i^l \le e_i(n) \le e_i^u, \quad 0 < f_i^l \le f_i(n) \le f_i^u < 1, \\ &0 < g_{ij}^l \le g_{ij}(n) \le g_{ij}^u. \end{aligned} \tag{3}$$

For any almost periodic sequence $\{f(k)\}$ defined on Z, we use the notations $f^l = \inf_{k\in Z} f(k)$ and $f^u = \sup_{k\in Z} f(k)$.

We consider the solution of system (2) with the following initial conditions:

$$\begin{aligned} &x_i(\theta) = \varphi_i(\theta) \ge 0, \quad x_i(0) = \varphi_i(0) > 0, \\ &u_i(\theta) = \psi_i(\theta) \ge 0, \quad u_i(0) = \psi_i(0) > 0, \\ &\tau = \max\{\tau_i : i = 1, 2, \cdots, n\}, \quad \theta \in \{-\tau, -\tau+1, \cdots, 0\}. \end{aligned} \tag{4}$$

The main objective of this paper is to investigate the existence of the almost periodic solutions of system (2). The set-up of this paper is as follows. In the coming section, we present some useful definitions and lemmas. In the rest of this paper, we systematically explore the existence of a unique positive almost periodic solution, which is uniformly asymptotically stable. An example together with its numerical simulation are presented to show the feasibility of the main results. This study reveals that the feedback controls, to some extent, will destroy the stability of the system.

2 Preliminaries

In this section, first we will mention several foundational definitions and lemmas. Denote $[a, b]_Z = [a, b] \bigcap Z$ and $K = [-\tau, +\infty)_Z$, where τ is defined as before.

Definition 1 (see [11]). A sequence $x : Z \to R^k$ is called an almost periodic sequence if the ε-translation set of x

$$E\{\varepsilon, x\} = \{\tau \in Z : |x(n+\tau) - x(n)| < \varepsilon, \forall n \in Z\} \tag{5}$$

is a relatively dense set in Z for all $\varepsilon > 0$; that is, for any given $\varepsilon > 0$, there exists an integer $l(\varepsilon) > 0$ such that each discrete interval of length $l(\varepsilon)$ contains an integer $\tau = \tau(\varepsilon) \in E\{\varepsilon, x\}$ such that

$$|x(n+\tau) - x(n)| < \varepsilon, \forall n \in Z. \tag{6}$$

Definition 2 (see [11]). Let $f : Z \times D \to R^k$, where D is an open set in $C = \{\varphi : [-\tau, 0]_Z \to R^k\}$. $f(n, \varphi)$ is said to be almost periodic in n uniformly for $\varphi \in D$, if for any $\varepsilon > 0$ and any compact set $S \subset D$, there exists a positive integer $l = l(\varepsilon, S)$, such that any interval of length $l = l(\varepsilon, S)$ contains an integer τ, for which

$$|f(n+\tau, \varphi) - f(n, \varphi)| < \varepsilon, \forall (n, \varphi) \in Z \times S. \tag{7}$$

Definition 3 (see [12]). The hull of f, denoted by $H(f)$, is defined by

$$H(f) = \{g(n, x) : \lim_{k\to\infty} f(n+\tau_k, x) = g(n, x), \textit{uniformly on } Z \times S\}, \tag{8}$$

for some sequence τ_k, where S is any compact set in D.

Lemma 4 (see [13]). $\{x(n)\}$ is an almost periodic sequence if and only if for any integer sequence $\{k_i'\}$, there exists a subsequence $\{k_i\} \subset \{k_i'\}$ such that $x(n+k_i)$ converges uniformly on $n \in Z$ as $i \to \infty$. Furthermore, the limit sequence is also an almost periodic sequence.

Lemma 5 (see [14]). Assume that $r(n) > 0$, $\{x(n)\}$ satisfies $x(n) > 0$, and

$$x(n+1) \le x(n) \exp\left\{r(n)(1 - ax(n))\right\}, \tag{9}$$

for $n \in [n_1, +\infty)$, where a is a positive constant. Then

$$\limsup_{n\to+\infty} x(n) \le \frac{1}{ar^u}\exp(r^u - 1). \tag{10}$$

Lemma 6 (see [14]). Assume that $r(n) > 0$, $\{x(n)\}$ satisfies $x(n) > 0$, and

$$x(n+1) \ge x(n)\exp\left\{r(n)(1 - ax(n))\right\}, \tag{11}$$

for $n \in [n_1, +\infty)$, $\limsup_{n\to+\infty} x(n) \le x^*$, and $x(n_1) > 0$, where a and x^* are positive constants such that $ax^* > 1$. Then

$$\liminf_{n\to+\infty} x(n) \ge \frac{1}{a}\exp\left(r^u(1 - ax^*)\right). \tag{12}$$

Lemma 7 (see [15]). Assume that $A > 0$ and $x(0) > 0$, and further suppose that

$$x(n+1) \le Ax(n) + B(n), \tag{13}$$

then, for any integer $k \le n$,

$$x(n) \le A^k x(n-k) + \sum_{i=0}^{k-1} A^i B(n-i-1). \tag{14}$$

Specifically, if $A < 1$ and $B(n)$ is bounded above with respect to M, then

$$\limsup_{n\to\infty} x(n) \le \frac{M}{1-A}. \tag{15}$$

Lemma 8 (see [15]). Assume that $A > 0$ and $x(0) > 0$, and further suppose that

$$x(n+1) \ge Ax(n) + B(n), \tag{16}$$

then, for any integer $k \le n$,

$$x(n) \ge A^k x(n-k) + \sum_{i=0}^{k-1} A^i B(n-i-1). \tag{17}$$

Specifically, if $A < 1$ and $B(n)$ is bounded below with respect to m, then

$$\liminf_{n\to\infty} x(n) \le \frac{m}{1-A}. \tag{18}$$

Lemma 9 (see [11]). Suppose that there exists a Lyapunov function $V(n, \varphi, \psi)$ satisfying the following conditions:
(i) $a(|\varphi(0) - \psi(0)|) \le V(n, \varphi, \psi) \le b(\|\varphi - \psi\|)$, where $a, b \in P$ with $P = \{a : [0, +\infty) \to [0, +\infty), a(0) = 0$ and $a(u)$ is continuous, increasing in $u\}$.
(ii) $V(n, \varphi_1, \psi_1 - V(n, \varphi_2, \psi_2) \le L(\|\varphi_1 - \varphi_2\| + \|\psi_1 - \psi_2\|)$, where $L > 0$ is a constant.
(iii) $\triangle V(n, \varphi, \psi) \le -\alpha V(n, \varphi, \psi)$, where $0 < \alpha < 1$ is a constant.
Remark 10 (see [16]). Condition (iii) of Lemma 9 can be replaced by:
(iii)' $\triangle V(n, \varphi, \psi) \le -\beta V(n, \varphi, \psi)$, where $\beta \in \{c : [0, +\infty) \to [0, +\infty)$, c is continuous , $c(0) = 0$ and $c(s) > 0$ for $s > 0\}$.

3 Permanence

Theorem 11. Assume that

$$\triangle_i \overset{\text{def}}{=} r_i^l - \sum_{j=1, j\ne i}^{n} b_{ij}^u M_j - \sum_{j=1, j\ne i}^{n} d_{ij}^u M_j^2 - e_i^u H_i > 0 \tag{19}$$

hold, then the system (2) is permanent. i.e., there exist positive constants m_i, M_i, h_i and H_i, such that for any positive solution $(x_1(k), \cdots, x_n(k), u_1(k), \cdots, u_n(k))$ of system (2), one has

$$\begin{aligned} m_i &\le \liminf_{k\to+\infty} x_i(k) \le \limsup_{k\to+\infty} x_i(k) \le M_i, \\ h_i &\le \liminf_{k\to+\infty} u_i(k) \le \limsup_{k\to+\infty} u_i(k) \le H_i, \quad i = 1, 2, \cdots, n. \end{aligned} \tag{20}$$

Proof. From system (2),

$$x_i(k+1) \le x_i(k) \exp\left[r_i(k)\left(1 - \tfrac{a_i^l}{r_i^u} x_i(k)\right)\right]. \tag{21}$$

By Lemma 5, we know that

$$\limsup_{k\to+\infty} x_i(k) \le \frac{1}{a_i^l} \exp(r_i^u - 1) \overset{\text{def}}{=} M_i. \tag{22}$$

We can choose a sufficiently small ε such that for large enough $K_1 > 0$, we have

$$x_i(k) \le M_i + \varepsilon, \quad \forall k > K_1. \tag{23}$$

For $k > K_1$, we have

$$u_i(k+1) \le \left(1 - f_i^l\right) u_i(k) + \sum_{j=1}^{n} g_{ij}(k)(M_j + \varepsilon). \tag{24}$$

As a direction corollary of Lemma 7, one has

$$\limsup_{k\to+\infty} u_i(k) \le \frac{1}{f_i^l} \sum_{j=1}^{n} g_{ij}^u (M_j + \varepsilon). \tag{25}$$

Setting $\varepsilon \to 0$,

$$\limsup_{k\to+\infty} u_i(k) \le \frac{1}{f_i^l} \sum_{j=1}^{n} g_{ij}^u M_j \overset{\text{def}}{=} H_i. \tag{26}$$

For above ε, there exists an integer $K_2 > K_1$ such that

$$u_i(k) \le H_i + \varepsilon, \quad \forall k > K_2. \tag{27}$$

From (23), (27) and system (2),

$$x_i(k+1) \ge x_i(k) \exp\left[\triangle_i^\varepsilon - a_i^u x_i(k)\right], \quad k > K_2 + \tau, \tag{28}$$

where $\triangle_i^\varepsilon = r_i^l - \sum_{j=1, j\neq i}^{n} b_{ij}^u (M_j + \varepsilon) - \sum_{j=1, j\neq i}^{n} d_{ij}^u (M_j + \varepsilon)^2 - e_i^u (H_i + \varepsilon) > 0$. Noting the fact that $\exp(x - 1) > x$, for $x > 0$, we have

$$\frac{a_i^u}{\Delta_i^\varepsilon} M_i = \frac{a_i^u}{a_i^l} \cdot \frac{\exp(r_i^u - 1)}{r_i^l - \sum\limits_{j=1, j\neq i}^{n} b_{ij}^u (M_j + \varepsilon) - \sum\limits_{j=1, j\neq i}^{n} d_{ij}^u (M_j + \varepsilon)^2 - e_i^u (H_i + \varepsilon)} > 1. \tag{29}$$

Then by Lemma 6, one has

$$\liminf_{k\to+\infty} \ge x_i(k) \frac{\Delta_i^\varepsilon}{a_i^u} \exp[\Delta_i^\varepsilon - a_i^u M_i]. \tag{30}$$

Setting $\varepsilon \to 0$,

$$\liminf_{k\to+\infty} x_i(k) \ge \frac{\Delta_i}{a_i^u} \exp[\Delta_i - a_i^u M_i] \overset{\text{def}}{=} m_i. \tag{31}$$

There exists a positive integer $K_3 > K_2 + \tau$ such that

$$x_i(k) \ge m_i - \varepsilon, \ \forall k > K_3. \tag{32}$$

From (32) and system (2), we have

$$u_i(k+1) \geq \left(1 - f_i^u\right) u_i(k) + \sum_{j=1}^{n} g_{ij}(k)(m_j - \varepsilon). \tag{33}$$

By Lemma 8, one has

$$\liminf_{k \to +\infty} u_i(k) \geq \frac{1}{f_i^u} \sum_{j=1}^{n} g_{ij}^l (m_j - \varepsilon). \tag{34}$$

Setting $\varepsilon \to 0$, it follows that

$$\liminf_{k \to +\infty} u_i(k) \geq \frac{1}{f_i^u} \sum_{j=1}^{n} g_{ij}^l m_j \overset{\text{def}}{=} h_i. \tag{35}$$

Denoting

$$\Omega = \{\text{every solution of system (2) satisfying}$$
$$m_i \leq x_i(k) \leq M_i, h_i \leq u_i(k) \leq H_i, \forall k \in Z^+\}.$$

Theorem 12. Assume that the condition (19) holds, then $\Omega \neq \Phi$.

Proof. Since the coefficients are almost periodic sequences, there exists an integer valued sequence $\{t_p\}$ with $t_p \to \infty$ as $p \to \infty$ such that

$$\begin{aligned} &r_i(k+t_p) \to r_i(k), \quad a_i(k+t_p) \to a_i(k), \quad b_{ij}(k+t_p) \to b_{ij}(k), \\ &d_{ij}(k+t_p) \to d_{ij}(k), \quad e_i(k+t_p) \to e_i(k), \quad f_i(k+t_p) \to f_i(k), \\ &g_{ij}(k+t_p) \to g_{ij}(k). \end{aligned} \tag{36}$$

We can choose a sufficiently small ε. From Theorem 11, there exists a positive integer N_0 such that

$$m_i - \varepsilon \leq x_i(k) \leq M_i + \varepsilon, \quad h_i - \varepsilon \leq u_i(k) \leq H_i + \varepsilon, \quad k > N_0. \tag{37}$$

Denoting $x_{ip}(k) = x_i(k+t_p)$, $u_{ip}(k) = u_i(k+t_p)$ for $k > N_0 - t_p$ and $p = 1, 2, \cdots$. For any positive integer q, it is easy to see that there exists a sequence $\{x_{ip}(k) : p \geq q\}$ such that the sequence $\{x_{ip}(k)\}$ has a subsequence, also denoted by $\{x_{ip}(k)\}$, converging on any finite interval A of Z^+ as $p \to \infty$.

In fact, for any finite subset $A = \{l_1, l_2, \cdots, l_m\} \subseteq Z^+$, where m is a finite number, $t_p + l_j > N_0 (j = 1, 2, \cdots, m)$, when p is large enough. Therefore $m_i - \varepsilon \leq x_i(k+t_p) \leq M_i + \varepsilon (i = 1, 2, \cdots, n)$; that is, $x_i(k+t_p)$ are uniformly bounded when p is sufficiently large. Next, for $l_1 \in A$, we choose a subsequence $\{t_p^{(1)}\}$ of $\{t_p\}$ such that $x_i(k + t_p^{(1)})$ uniformly converge on Z^+ for p sufficiently large. Similar to the arguments of l_1, for $l_2 \in A$, one can select a subsequence $\{t_p^{(2)}\}$ of $\{t_p^{(1)}\}$ such that $x_i(k + t_p^{(2)})$ uniformly converge on Z^+ for p sufficiently large. Repeating above-mentioned process, for $l_m \in A$, one obtains a subsequence $\{t_p^{(m)}\}$ of $\{t_p^{(m-1)}\}$ such that $x_i(k + t_p^{(m)})$ uniformly converge on Z^+ for p sufficiently large.

Based on the above, one selects the sequence $\{t_p^{(m)}\}$ which is a subsequence of $\{t_p\}$, still denoted by $\{t_p\}$, then one gets $x_i(k + t_p) \to x_i^*$ uniformly in $k \in A$ as $k \to \infty$. So the conclusion holds truly due to the arbitrariness of A. Thus we have a sequence $\{y_i(k)\}$ such that for $k \in Z^+$,

$$x_{ip}(k) \to y_i(k), \quad u_{ip}(k) \to v_i(k), \quad as \ p \to \infty. \tag{38}$$

which, together with (36), yields that

$$\begin{cases} x_{ip}(k+1) = x_{ip}(k) \exp\Big[r_i(k+t_p) - a_i(k+t_p)x_{ip}(k) - \sum\limits_{j=1, j\neq i}^{n} b_{ij}(k+t_p)x_{jp}(k-\tau_j) \\ \qquad - \sum\limits_{j=1, j\neq i}^{n} d_{ij}(k+t_p)x_{jp}^2(k-\tau_j) - e_i(k+t_p)u_{ip}(k)\Big], \\ u_{ip}(k+1) = \Big(1 - f_i(k+t_p)\Big)u_{ip}(k) + \sum\limits_{j=1}^{n} g_{ij}(k+t_p)x_{jp}(k), \end{cases} \tag{39}$$

It follows from (36), (38) and (39) that

$$\begin{cases} y_i(k+1) = y_i(k)\exp\left[r_i(k) - a_i(k)y_i(k) - \sum\limits_{j=1,j\neq i}^{n} b_{ij}(k)y_j(k-\tau_j) - \sum\limits_{j=1,j\neq i}^{n} d_{ij}(k)y_j^2(k-\tau_j) - e_i(k)v_i(k)\right], \\ v_i(k+1) = \left(1 - f_i(k)\right)v_i(k) + \sum\limits_{j=1}^{n} g_{ij}(k)y_j(k), \end{cases} \tag{40}$$

It is easy to see that $(y_1(k), \cdots, y_n(k), v_1(k), \cdots, v_n(k))$ is a solution of system (2) and $m_i - \varepsilon \le y_i(k) \le M_i + \varepsilon$, $h_i - \varepsilon \le v_i(k) \le H_i + \varepsilon$ for $k \in [-\tau, 0]_Z$.

4 Stability of almost periodic solution

Theorem 13. Assume that the condition (19) and

$$\begin{aligned} \lambda_i^{(1)} &= 1 - \max\{|1 - a_i^u M_i|, |1 - a_i^l m_i|\} - \sum_{j=1,j\neq i}^{n} (b_{ji}^u M_i + 2d_{ji}^u M_i^2) - \sum_{j=1}^{n} g_{ji}^u M_i > 0, \\ \lambda_i^{(2)} &= f_i^l - e_i^u. \end{aligned} \tag{41}$$

hold, where $i, j = 1, 2, \cdots, n$. Then there exists a unique uniformly asymptotically stable almost periodic solution $(x_1(k), \cdots, x_n(k), u_1(k), \cdots, u_n(k))$ of system (2) which is bounded by Ω for all $k \in N^+$.

Proof. Let $\omega_i(k) = \ln x_i(k)$, $i = 1, 2, \cdots, n$, then system (2) can be rewritten as

$$\begin{cases} \omega_i(k+1) = \omega_i(k) + r_i(k) - a_i(k)e^{\omega_i(k)} - e_i(k)u_i(k) - \sum\limits_{j=1,j\neq i}^{n} b_{ij}(k)e^{\omega_j(k-\tau_j)} - \sum\limits_{j=1,j\neq i}^{n} d_{ij}(k)e^{2\omega_j(k-\tau_j)}, \\ u_i(k+1) = (1 - f_i(k))u_i(k) + \sum\limits_{j=1}^{n} g_{ij}(k)e^{\omega_j(k)}. \end{cases} \tag{42}$$

From Theorem 12, there exists a solution $(\omega_1(k), \cdots, \omega_n(k), u_1(k), \cdots, u_n(k))$ of system (42) such that

$$\ln m_i \le \omega_i(k) \le \ln M_i, \quad h_i \le u_i(k) \le H_i, \quad \forall k \in K, \tag{43}$$

which implies that $|\omega_i(k)| \le B_i = \max\{|\ln m_i|, |\ln M_i|\}$ and $|u_i(k)| \le C_i = \max\{h_i, H_i\}$, $i = 1, 2, \cdots, n$. For $k \in Z^+$ and $s \in [-\tau, 0]_Z$, assign

$$\begin{aligned} W_k(s) &= (\omega_1(k+s), \cdots, \omega_n(k+s), u_1(k), \cdots, u_n(k)), \\ Z_k(s) &= (z_1(k+s), \cdots, z_n(k+s), v_1(k), \cdots, v_n(k)). \end{aligned} \tag{44}$$

are two solutions of system (42) defined on D,

$$\begin{aligned} D = \{&(\omega_1(k), \cdots, \omega_n(k), u_1(k), u_n(k)) : \\ &m_i \le \omega_i(k) \le \ln M_i, h_i \le u_i(k) \le H_i, i = 1, 2, \cdots, n, \forall k \in K\}. \end{aligned} \tag{45}$$

Defining

$$\|W_k(s)\| = \sup_{s\in[-\tau,0]_Z} \sum_{i=1}^{n} [|\omega_i(k+s)| + |u_i(k)|], \tag{46}$$

then $\|W_k(s)\| \le E = \sum_{i=1}^{n}[B_i + C_i]$.

Consider the product system of (42)

$$\begin{cases} \omega_i(k+1) = \omega_i(k) + r_i(k) - a_i(k)e^{\omega_i(k)} - e_i(k)u_i(k) - \sum\limits_{j=1,j\neq i}^{n} b_{ij}(k)e^{\omega_j(k-\tau_j)} - \sum\limits_{j=1,j\neq i}^{n} d_{ij}(k)e^{2\omega_j(k-\tau_j)}, \\ u_i(k+1) = (1-f_i(k))u_i(k) + \sum\limits_{j=1}^{n} g_{ij}(k)e^{\omega_j(k)}. \\ z_i(k+1) = z_i(k) + r_i(k) - a_i(k)e^{z_i(k)} - e_i(k)v_i(k) - \sum\limits_{j=1,j\neq i}^{n} b_{ij}(k)e^{z_j(k-\tau_j)} - \sum\limits_{j=1,j\neq i}^{n} d_{ij}(k)e^{2z_j(k-\tau_j)}, \\ v_i(k+1) = (1-f_i(k))v_i(k) + \sum\limits_{j=1}^{n} g_{ij}(k)e^{z_j(k)}. \end{cases} \tag{47}$$

Construct the Lyapunov functional $V(k)$,

$$V(k) = V(k, W_k, Z_k) = \sum_{i=1}^{n}[|\omega_i(k) - z_i(k)| + |u_i(k) - v_i(k)|] + \sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}\sum_{m=k-\tau_j}^{k-1}(b_{ij}^u M_j + 2d_{ij}^u M_j^2)|\omega_j(m) - z_j(m)|. \tag{48}$$

Apparently,

$$|W_k(0) - Z_k(0)| = \left(\sum_{i=1}^{n}(\omega_i(k) - z_i(k))^2 + (u_i(k) - v_i(k))^2\right)^{\frac{1}{2}} \le \left(\sum_{i=1}^{n}|\omega_i(k) - z_i(k)| - |u_i(k) - v_i(k)|\right) \le V(k). \tag{49}$$

Denoting

$$\delta_i = \max\left\{1, \sum_{j=1,j\neq i}^{n}(b_{ij}^u M_i + 2d_{ij}^u M_i^2)\right\}, \quad \delta = \max\{\delta_i\}, \quad \rho = \delta(\tau+1). \tag{50}$$

On the other side,

$$\begin{aligned} V(k) &\le \sum_{i=1}^{n}\left[|\omega_i(k) - z_i(k)| + |u_i(k) - v_i(k)| + \sum_{j=1,j\neq i}^{n}(b_{ij}^u M_j + 2d_{ij}^u M_j^2)\sum_{m=k-\tau}^{k-1}|\omega_j(m) - z_j(m)|\right] \\ &= \sum_{i=1}^{n}\left[|\omega_i(k) - z_i(k)| + |u_i(k) - v_i(k)| + \sum_{j=1,j\neq i}^{n}(b_{ji}^u M_i + 2d_{ji}^u M_i^2)\sum_{m=k-\tau}^{k-1}|\omega_i(m) - z_i(m)|\right] \\ &= \sum_{i=1}^{n}\left[|\omega_i(k) - z_i(k)| + |u_i(k) - v_i(k)| + \sum_{j=1,j\neq i}^{n}(b_{ji}^u M_i + 2d_{ji}^u M_i^2)\sum_{s=-\tau}^{-1}|\omega_i(k+s) - z_i(k+s)|\right] \\ &\le \sum_{i=1}^{n}\delta_i\sum_{s=-\tau}^{0}|\omega_i(k+s) - z_i(k+s)| + \sum_{i=1}^{n}|u_i(k) - v_i(k)| \\ &\le \delta\sum_{s=-\tau}^{0}\sum_{i=1}^{n}|\omega_i(k+s) - z_i(k+s)| + \sum_{i=1}^{n}|u_i(k) - v_i(k)| \\ &\le \delta(\tau+1)\sup_{s\in[-\tau,0]_Z}\sum_{i=1}^{n}\left[|\omega_i(k+s) - z_i(k+s)| + |u_i(k) - v_i(k)|\right] \\ &= \rho\|W_k - Z_k\|. \end{aligned} \tag{51}$$

Thus condition (i) in Lemma 9 is satisfied if we take $a(x) = x$ and $b(x) = \rho x$, where $a, b \in C(R^+, R^+)$.
By using (3.2) in Ref. [16], for any $W_k, Z_k, \bar{W}_k, \bar{Z}_k \in D$, we have

$$\begin{aligned}
&|V(k, W_k, Z_k) - V(k, \bar{W}_k, \bar{Z}_k| \\
&\leq \sum_{i=1}^{n}\Big\{\Big||\omega_i(k) - z_i(k)| - |\bar{\omega}_i(k) - \bar{z}_i(k)|\Big| + \Big||u_i(k) - v_i(k)| - |\bar{u}_i(k) - \bar{v}_i(k)|\Big| \\
&\quad + \sum_{j=1, j\neq i}^{n}\sum_{m=k-\tau_j}^{k-1}(b_{ij}^u M_j + 2d_{ij}^u M_j^2)\Big||\omega_j(m) - z_j(m)| - |\bar{\omega}_j(m) - \bar{z}_j(m)|\Big|\Big\} \\
&\leq \sum_{i=1}^{n}\Big\{\Big(|\omega_i(k) - \bar{\omega}_i(k)| + |z_i(k) - \bar{z}_i(k)|\Big) + \Big(|u_i(k) - \bar{u}_i(k)| + |v_i(k) - \bar{v}_i(k)|\Big) \\
&\quad + \sum_{j=1, j\neq i}^{n}\sum_{m=k-\tau_j}^{k-1}(b_{ij}^u M_j + 2d_{ij}^u M_j^2)\Big(|\omega_j(m) - \bar{\omega}_j(m)| + |z_j(m) - \bar{z}_j(m)|\Big)\Big\} \\
&\leq \rho\Big(\|W_k - \bar{W}_k\| + \|Z_k - \bar{Z}_k\|\Big),
\end{aligned} \tag{52}$$

so, condition (ii) in Lemma 9 is also satisfied.

By the mean value theorem, it derives that

$$\begin{aligned}
&e^{\omega_i(k)} - e^{z_i(k)} = e^{\theta_i(k)}(\omega_i(k) - z_i(k)), \\
&e^{\omega_i(k-\tau_i)} - e^{z_i(k-\tau_i)} = e^{\eta_i(k-\tau_i)}(\omega_i(k-\tau_i) - z_i(k-\tau_i)), \\
&e^{2\omega_i(k-\tau_i)} - e^{2z_i(k-\tau_i)} = 2e^{2\xi_i(k-\tau_i)}(\omega_i(k-\tau_i) - z_i(k-\tau_i)),
\end{aligned} \tag{53}$$

where $\theta_i(k)$ lie between $\omega_i(k)$ and $z_i(k)$, and $\eta_i(k-\tau_i)$, $\xi_i(k-\tau_i)$ all lie between $\omega_i(k-\tau_i)$ and $z_i(k-\tau_i)$, respectively. Then

$$\ln m_i \leq \omega_i(k), \eta_i(k-\tau_i), \xi_i(k-\tau_i) \leq \ln M_i, \quad k \in Z^+. \tag{54}$$

Calculating the $\triangle V(k)$ along with the solution of system (47), we have

$$\begin{aligned}
&\triangle V_{(47)}(k) = \sum_{i=1}^{n}\Big\{\Big|\omega_i(k) - z_i(k) - a_i(k)e^{\theta_i(k)}(\omega_i(k) - z_i(k)) - e_i(k)(u_i(k) - v_i(k)) \\
&- \sum_{j=1, j\neq i}^{n}\Big(b_{ij}(k)e^{\eta_j(k-\tau_j)} + 2d_{ij}(k)e^{2\xi_j(k-\tau_j)}\Big)(\omega_j(k-\tau_j) - z_j(k-\tau_j))\Big| \\
&+\Big|(1 - f_i(k))(u_i(k) - v_i(k)) + \sum_{j=1}^{n} g_{ij}(k)e^{\theta_j(k)}(\omega_j(k) - z_j(k))\Big|\Big\} - \sum_{i=1}^{n}\Big\{|\omega_i(k) - z_i(k)| + |u_i(k) - v_i(k)|\Big\} \\
&+ \sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n}(b_{ij}^u M_j + 2d_{ij}^u M_j^2)\Big(|\omega_j(k) - z_j(k)| - |\omega_j(k-\tau_j) - z_j(k-\tau_j)|\Big) \\
&\leq \sum_{i=1}^{n}\Big(|1 - a_i(k)e^{\theta_i(k)}| - 1 + \sum_{j=1, j\neq i}^{n}(b_{ji}^u M_i + 2d_{ji}^u M_i^2) + \sum_{j=1}^{n} g_{ji}(k)e^{\theta_i(k)}\Big)|\omega_i(k) - z_i(k)| \\
&+ \sum_{i=1}^{n}\Big(e_i(k) - f_i(k)\Big)|u_i(k) - v_i(k)| \\
&\leq -\sum_{i=1}^{n}\Big\{\lambda_i^{(1)}|\omega_i(k) - z_i(k)| + \lambda_i^{(2)}|u_i(k) - v_i(k)|\Big\} \\
&\leq -\lambda\sum_{i=1}^{n}\Big\{|\omega_i(k) - z_i(k)| + |u_i(k) - v_i(k)|\Big\} \\
&\leq -\lambda\sum_{i=1}^{n}\Big((\omega_i(k) - z_i(k))^2 + (u_i(k) - v_i(k))^2\Big)^{\frac{1}{2}} \\
&\leq -\lambda\sum_{i=1}^{n}\Big(|W_k(0) - Z_k(0)|\Big),
\end{aligned} \tag{55}$$

where $\lambda = \min_{1\le i\le n}\{\min\{\lambda_i^{(1)}, \lambda_i^{(2)}\}\} > 0$. Denoting $c(x) = \lambda x \in C(R^+, R^+)$, also, the condition in Remark 10 is satisfied. Therefore, system (42) has a unique uniformly asymptotically stable almost periodic solution denoted by $(\omega_1^\star(k), \cdots, \omega_n^\star(k), u_1^\star(k), \cdots, u_n^\star(k))$, which is equivalent to saying that the system (2) has a unique uniformly asymptotically stable almost periodic solution denoted by $(x_1^\star(k), \cdots, x_n^\star(k), u_1^\star(k), \cdots, u_n^\star(k))$.

If the coefficients are bounded positive periodic sequences, we have the following corollary.

Corollary 14. System (2) shows a unique positive periodic solution which is uniformly asymptotically stable under the same assumptions of Theorem 13.

5 Numerical Simulations

We give an example to check the feasibility of our results.

Example 15. Consider the following system:

$$\begin{cases} x_1(k+1) = x_1(k)\exp\Big[0.78 + 0.02\sin(\sqrt{2}k\pi) - x_1(k) \\ \quad -(0.011 - 0.001\cos(\sqrt{2}k\pi))x_2(k-2) - (0.012 - 0.002\sin(\sqrt{2}k\pi))x_2^2(k-2) \\ \quad -(0.025 - 0.005\cos(\sqrt{2}k\pi))u_1(k), \\ x_2(k+1) = x_2(k)\exp\Big[0.93 - 0.03\cos(\sqrt{3}k\pi) - x_2(k) \\ \quad -(0.021 - 0.001\sin(\sqrt{3}k\pi))x_1(k-1) - (0.022 - 0.002\sin(\sqrt{3}k\pi))x_1^2(k-1) \\ \quad -(0.015 - 0.005\sin(\sqrt{2}k\pi))u_2(k), \\ u_1(k+1) = (0.073 + 0.007\sin(\sqrt{2}k\pi))u_1(k) \\ \quad +(0.013 - 0.003\sin(\sqrt{2}k\pi))x_1(k) + (0.015 - 0.005\cos(\sqrt{2}k\pi))x_2(k), \\ u_2(k+1) = (0.035 + 0.005\cos(\sqrt{3}k\pi))u_2(k) \\ \quad +(0.017 - 0.007\sin(\sqrt{3}k\pi))x_1(k) + (0.014 - 0.004\cos(\sqrt{3}k\pi))x_2(k). \end{cases} \tag{56}$$

By calculating, one has

$$\begin{aligned} &M_1 = 0.8187, \quad m_1 = 0.675, \quad M_2 = 0.9608, \quad m_2 = 0.7868, \\ &H_1 = 0.0351, \quad h_1 = 0.0157, \quad H_2 = 0.0385, \quad h_2 = 0.0151, \\ &\lambda_1^{(1)} = 0.5921, \quad \lambda_1^{(2)} = 0.89, \quad \lambda_2^{(1)} = 0.7129, \quad \lambda_2^{(2)} = 0.95, \\ &\lambda = 0.5921 > 0, \quad \Delta_1 = 0.744 > 0, \quad \Delta_2 = 0.8659 > 0. \end{aligned} \tag{57}$$

Clearly, the assumption of Theorem 13 is satisfied, i.e., system (56) admits a unique uniformly asymptotically stable positive almost periodic solution. The numerical simulations support our results (see Figure 1).

6 Discussion

In this paper we consider a discrete competitive system with delays and feedback controls. By constructing Lyapunov functional and using mean value theorem, the conditions on the asymptotical stability of the

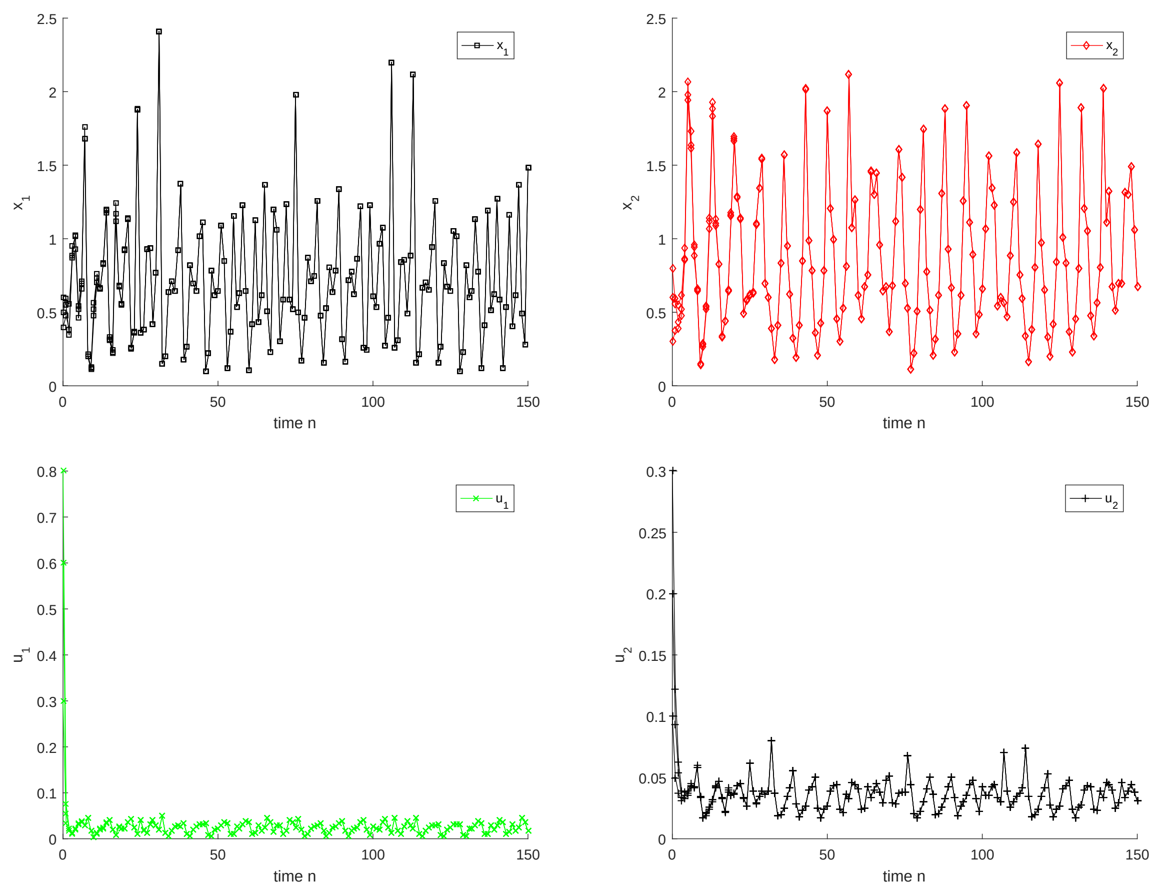

Figure 1: Dynamic behaviors of the solutions $(x_1(k), x_2(k), u_1(k), u_2(k))$ of system (56) with the initial conditions $(x_1(\theta), x_2(\theta), u_1(\theta), u_2(\theta)) = (0.5, 0.3, 0.2, 0.1), (0.4, 0.6, 0.1, 0.3)$ and $(0.6, 0.8, 0.3, 0.2)$ for $\theta = -2, -1, 0$, respectively.

positive almost periodic solution are established. Compared with the Theorem 10 in [3], it is easy to see that the feedback controls, to some extent, destroy the stability of the system.

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