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The well-posedness of solution to a compressible non-Newtonian fluid with self-gravitational potential

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Abstract: We study the initial boundary value problem of a compressible non-Newtonian fluid. The system describes the motion of the compressible viscous isentropic gas flow driven by the non-Newtonian self-gravitational force. The existence of strong solutions are derived in one dimensional bounded intervals by constructing a semi-discrete Galerkin scheme. Moreover, the uniqueness of solutions are also investigated. The main point of the study is that the viscosity term and potential term are fully nonlinear, and the initial vacuum is allowed.

Keywords: Well-posedness, Non-Newtonian, Self-gravitational potential, Vacuum

MSC: 76N10, 76A05

1 Introduction

In mathematical physics, the Navier-Stokes equation is known as one of the most fundamental equations in fluid mechanics. The compressible isentropic Navier-Stokes equations, which are the basic models describing the evolution of a viscous compressible fluid in a domain $x \in \Omega$, read as follows:

$$\begin{cases} \rho_t + \operatorname{div}(\rho u) = 0, \\ (\rho u)_t + \operatorname{div}(\rho u \otimes u) - 2\operatorname{div}(\mu D(u)) - \nabla(\lambda \operatorname{div} u) + \nabla P(\rho) = 0, \end{cases} \tag{1}$$

where the unknowns ρ, u represent the density and the velocity of the fluid, respectively. Here $D(u) = (\nabla u + (\nabla u)^T)/2$ is the strain tensor and $P(\rho) = a\rho^\gamma (a > 0, \gamma > 1)$ is the pressure, μ and λ are the viscosity constants which satisfy the physical requirements $\mu \geq 0$ and $2\mu + 3\lambda \geq 0$. The Navier-Stokes equations are the equations governing the motion of usual fluids like water, air, oil etc., under quite general conditions, and they appear in the study of many important phenomena, either alone or coupled with other equations. For instance, they are used in theoretical studies in aeronautical sciences, in meteorology, in thermo-hydraulics, in the petroleum industry, in plasma physics, etc. From the point of view of continuum mechanics the Navier-Stokes equations are essentially the simplest equations describing the motion of a fluid, and they are derived under a quite simple physical assumption, namely, the existence of a linear local relation between stresses and strain rates. The compressible isentropic Navier-Stokes equations (1.1) are derived from the conservation laws of mass and the balance of momentum, (for details see [1, 2]). While the physical model leading to the Navier-Stokes equations is simple, the situation is quite different from the mathematical point of view. In

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particular, because of the nonlinearity, the mathematical study of these equations is difficult and requires the full power of modern functional analysis. A major question is whether the solution remains smooth all the time. These and other related questions are interesting not only for mathematical understanding of the equations but also for understanding the phenomenon of turbulence.

There are huge literatures on the study of the existence and behavior of solutions to Navier-Stokes equations. Some of the previous relevant works in this direction can be summarized as follows. For instance, the 1D version of (1) were addressed by Kazhikhov et al. in [3] for sufficiently smooth data, and by Hoff [4] for discontinuous initial data, where the data is uniformly away from the vacuum; the existence of global weak solutions for isentropic flow were investigated by Lions in [5] by using the weak convergence method. In [6], the authors employed a new method to prove the existence and uniqueness of local strong solutions in the case where the initial data satisfies some compatibility conditions. The dynamics of weak solutions and vacuum states were investigated in [7] for the 1D compressible Navier-Stokes equations with density-dependent viscosity in bounded spatial domains or periodic domains. For other results we refer the reader to [8-13] and the references cited therein.

The above references mainly concerned the fluid which the relation between the stress and rate of strain is linear, namely, the Newtonian fluid. The study of non-Newtonian fluids (the relation between the stess and rate of strain is not linear) mechanics is of great significance because such fluids describe more realistic phenomenon. These flows are frequently encountered in many physical and industrial processes [14], such as porous flows of oils and gases [15], biological fluid flows of blood [16], saliva and mucus, penetration grouting of cement mortar and mixing of massive particles and fluids in drug production [17]. Many studies are based on the field of non-Newtonian flows, both theoretically and experimentally. In [18], Ladyzhenskaya first proposed a special form for the incompressible model, namely that the viscous stress tensor $\Gamma_{ij} = (\mu_0 + \mu_1|E(\nabla u)^{p-2}|)E_{ij}(\nabla u)$. For $\mu_0 = 0$, if $p < 2$, it is a pseudo-plastic fluid, and when $p > 2$, it is a dilatant fluid (see also [19]). From a Physics point of view, the model captures the shear thinning fluid for the case of $1 < p < 2$, and captures the shear thickening fluid for the case of $p > 2$. In [20], the trajectory attractor and global attractor for an autonomous non-Newtonian fluid in dimension two was studied. The existence and uniqueness of solutions for non-Newtonian fluids were established in [21] by applying the Ladyzhenskaya's viscous stress tensor model. Then the global existence and exponential stability of solutions to the one-dimensional full non-Newtonian fluids were investigated in [22]. Recently, in [23], the authors present a decoupling multiple-relaxation-time lattice Boltzmann flux solver for simulating non-Newtonian power-law fluid flows.

On the other hand, another basiclly important case is when the motion of compressible viscous isentropic flow is driven by a self-gravitational force. However, at the present, little is known yet on the strong solutions to system (1) with non-Newtonian self-gravitational potential on bounded domain, even in one dimensional case. In this paper we focus on the following 1D system of compressible non-Newtonian equations

$$\begin{cases} \rho_t + (\rho u)_x = 0, \\ (\rho u)_t + (\rho u^2)_x + \rho\Phi_x - \lambda(|u_x|^{p-2}u_x)_x + P_x = 0, \\ ((\Phi_x^2 + \mu_0)^{\frac{q-2}{2}}\Phi_x)_x = 4\pi g(\rho - \dfrac{1}{|\Omega|}\displaystyle\int_\Omega \rho \mathrm{d}x) \end{cases} \tag{2}$$

in $\Omega_T = \Omega \times (0, T)$ with the initial and boundary conditions

$$(\rho, u)|_{t=0} = (\rho_0, u_0), \qquad x \in \Omega, \tag{3}$$

$$u(x, t)|_{\partial\Omega} = \Phi(x, t)|_{\partial\Omega} = 0, \qquad t \in [0, T]. \tag{4}$$

Here, ρ, u, Φ denote the density, velocity and the non-Newtonian gravitational potential, respectively. $P = a\rho^\gamma (a > 0, \gamma > 1)$ is the pressure, the initial density $\rho_0 \geq 0$, $\frac{4}{3} < p < 2, q > 2$ are given constants. Our purpose is to some further light on problem (2)-(4). When $1 < p < 2$, the second equation of (2) always has singularity. Secondly, we emphasize that the vacuum of initial density may exist. In the presence of vacuum, the parabolicity is lost. Moreover, the equations are strongly coupled with each other. We will investigate the existence and uniqueness of local strong solutions of (2)-(4) by overcoming the above difficulties, and the proof is inspired by the previous work in [6] and [21].

We state the main results as follows:

Theorem 1.1. *Assume that* (ρ_0, u_0) *satisfies the following conditions*

$$0 \le \rho_0 \in H^1(\Omega), u_0 \in H_0^1(\Omega) \cap H^2(\Omega).$$

If there is a function $g \in L^2(\Omega)$*, such that the initial data satisfy the following compatibility condition:*

$$-[|u_{0x}|^{p-2}u_{0x}]_x + P_x(\rho_0) = \rho_0^{\frac{1}{2}} g, \qquad \text{for a.e. } x \in \Omega,$$

then there exist a time $T_* \in (0, +\infty)$ *and a unique strong solution* (ρ, u, Φ) *to (2)-(4) such that*

$$\begin{cases} \rho \in L^\infty([0, T_*]; H^1(\Omega)), \rho_t \in L^\infty([0, T_*]; L^2(\Omega)), u_t \in L^2(0, T_*; H_0^1(\Omega)), \\ \Phi \in L^\infty(0, T_*; H^2(\Omega)), \Phi_t \in L^\infty(0, T_*; H^1(\Omega)), \sqrt{\rho} u_t \in L^\infty(0, T_*; L^2(\Omega)) \\ u \in \text{ffj}^\infty(0, T_*; W_0^{1,p}(\Omega) \cap H^2(\Omega)), (|u_x|^{p-2}u_x)_x \in L^2([0, T_*]; L^2(\Omega)). \end{cases}$$

The rest of the paper is organized as follows. After stating the notations, in Section 2, we first present some useful lemmas, then the analysis of a priori estimates for smooth solutions are derived. In Section 3, we give the proof of existence, and finally complete the proof of uniqueness of the main theorem in Section 4.

In what follows, we use the following abbreviations for simplicity of notation:

$$H^1 = H^1(\Omega), \quad L^p = L^p(\Omega), \quad |\cdot|_{L^2} = \|\cdot\|_{L^2(\Omega)}, \quad |\cdot|_{L^\infty} = \|\cdot\|_{L^\infty(\Omega)}.$$

Throughout this paper, we will omit the variables t, x of functions if it does not cause any confusion. We use C to denote a generic constant that may vary in different estimates.

2 A priori estimates for smooth solutions

In this section, we provide some known facts that will be used in the proof of the main result.

Lemma 2.1 ([24])**.** *Let* Ω *be bounded set in* R^1*, and* $1 \le q \le p \le +\infty$*. Then*

$$\|u\|_{L^q(\Omega)} \le |\Omega|^{\frac{1}{q}-\frac{1}{p}} \|u\|_{L^p(\Omega)}.$$

Lemma 2.2 ([21])**.** *Let* $\rho_0 \in H^1(\Omega), u_0 \in H_0^1(\Omega) \cap H^2(\Omega), g \in L^2(\Omega)$ *and* $u_0^\varepsilon \in H_0^1(\Omega) \cap H^2(\Omega)$ *be a solution of the boundary value problem*

$$\begin{cases} -\left[\left(\frac{\varepsilon(u_{0x})^2+1}{(u_{0x})^2+\varepsilon}\right)^{\frac{2-p}{2}} u_{0x}\right]_x + P_x(\rho_0) = (\rho_0)^{\frac{1}{2}} g \\ u_0|_{\partial\Omega} = 0. \end{cases}$$

Then there is a subsequences $\{u_0^{\varepsilon_j}\}, j = 1, 2, 3\cdots$*, of* $\{u_0^\varepsilon\}$*, as* $\varepsilon_j \to 0$*,*

$$u_0^{\varepsilon_j} \to u_0 \quad in \quad H_0^1(\Omega) \cap H^2(\Omega),$$

$$\left[\left(\frac{\varepsilon_j(u_{0x}^{\varepsilon_j})^2 + 1}{(u_{0x}^{\varepsilon_j})^2 + \varepsilon_j}\right)^{\frac{2-p}{2}} u_{0x}^{\varepsilon_j}\right]_x \to (|u_{0x}|^{p-2}u_{0x})_x \quad in \quad L^2(\Omega).$$

To prove the existence of strong solutions, we require some more regular estimates. Next, we derive some a priori estimates for smooth solutions which are crucial to prove the local existence of strong solutions.

Let (ρ, u, Φ) be a smooth solution of (2)-(4) and $\rho_0 \ge \delta$, where $0 < \delta \ll 1$ is a positive number, $m_0 := \int_\Omega \rho_0(x)dx$ be initial mass and $m_0 > 0$. Throughout the paper, we will denote

$$M_0 = 1 + \mu_0 + \mu_0^{-1} + |\rho_0|_{H^1} + |g|_{L^2}.$$

As stated above, we need to estimate the uniform bound of the approximate solutions. Now, we consider the following linearized problem

$$\rho_t + (\rho u)_x = 0, \tag{5}$$

$$(\rho u)_t + (\rho u^2)_x + \rho \Phi_x + L_p u + P_x = 0, \tag{6}$$

$$L_q \Phi = 4\pi g(\rho - \frac{1}{|\Omega|}\int_{\Omega} \rho \mathrm{d}x) \tag{7}$$

with the initial and boundary conditions

$$(\rho, u)|_{t=0} = (\rho_0, u_0), x \in \Omega \tag{8}$$

$$u(x,t)|_{\partial\Omega} = \Phi(x,t)|_{\partial\Omega} = 0, t \in [0,T] \tag{9}$$

where

$$L_p u = -\Big[\Big(\frac{\varepsilon u_x^2 + 1}{u_x^2 + \varepsilon}\Big)^{\frac{2-p}{2}} u_x\Big]_x, \quad L_q \Phi = \big((\Phi_x)^2 + \mu_0\big)^{\frac{q-2}{2}} \Phi_x)_x$$

and $u_0 \in H_0^1 \cap H^2$ is the smooth solution of the boundary value problem

$$\begin{cases} -\Big[\Big(\frac{\varepsilon u_{0x}^2 + 1}{u_{0x}^2 + \varepsilon}\Big)^{\frac{2-p}{2}} u_{0x}\Big]_x + P_x(\rho_0) = \rho_0^{\frac{1}{2}} g \\ u_0|_{\partial\Omega} = 0. \end{cases} \tag{10}$$

We will prove the existence of solutions for (5)-(9) by virtue of the uniform estimates which do not depend on ε, depending only on M_0, and prove the limit of the approximate solutions is the solution of problem (2)-(4) with vacuum.

It follows from (10) and Young's inequality that there exists a constant C depending only on M_0, such that

$$|u_{0xx}|_{L^2} \le C(1 + |\rho_0|_{L^\infty}^{\frac{1}{2}} |g|_{L^2} + |P_x(\rho_0)|_{L^2})^{\frac{1}{p-1}} \le C.$$

We construct an auxiliary function

$$\Psi(t) = 1 + |\rho(s)|_{H^1(\Omega)} + |u(s)|_{W_0^{1,p}(\Omega)} + |\sqrt{\rho} u_t(s)|_{L^2(\Omega)})$$

Our derivation will be based on the local boundedness of $\Psi(t)$. Before we estimate each term of Ψ, we need to do the estimate of $|u_{xx}|_{L^2}$. Firstly, multiplying (7) by Φ and integrating over Ω and using Young's inequality,

$$\int_{\Omega} |\Phi_x|^q \mathrm{d}x \le C(m_0). \tag{11}$$

Then, it follows from equation (6) and (5) that

$$|u_{xx}| \le \frac{1}{p-1}(|u_x|^{2-p} + 1)|\rho u_t + \rho u u_x + \rho \Phi_x + P_x|.$$

Taking it by L^2-norm, using Young's inequality, we obtain

$$\begin{aligned} |u_{xx}|_{L^2}^{p-1} &\le C[1 + |\rho u_t|_{L^2} + |\rho u u_x|_{L^2} + |\rho \Phi_x|_{L^2} + |P_x|_{L^2}] \\ &\le C[1 + |\rho|_{L^\infty}^{\frac{1}{2}} |\sqrt{\rho} u_t|_{L^2} + (|\rho|_{L^\infty} |u_x|_{L^p}^{\frac{p}{2}+1})^{\frac{2(p-1)}{3p-4}} + |\rho|_{L^\infty} |\Phi_x|_{L^q} + |P_x|_{L^2} + \frac{1}{2}|u_{xx}|_{L^2}^{p-1} \end{aligned}$$

which along with (11), implies that

$$|u_{xx}(t)|_{L^2} \le C\Psi^{\frac{6\gamma}{3p-4}}(t). \tag{12}$$

Estimate for $|\rho|_{H^1}$

We are going to estimate the first term of $\Psi(t)$. Multiplying (5) by ρ and integrating over Ω with respect to x, we obtain from Sobolev inequality

$$\frac{d}{dt}|\rho(t)|_{L^2}^2 \le |u_{xx}|_{L^2}|\rho|_{L^2}^2. \tag{13}$$

Differentiating (5) with respect to x, multiplying it by ρ_x and integrating over Ω on x, and using Sobolev inequality, we have

$$\frac{d}{dt}\int_\Omega |\rho_x|^2 \mathrm{d}x = -\int_\Omega [\frac{3}{2}u_x(\rho_x)^2 + \rho\rho_x u_{xx}](t)\mathrm{d}x \le 3|\rho_x|_{L^2}^2|u_{xx}|_{L^2}. \tag{14}$$

Together with (13),(14) and Gronwall's inequality, it follows that

$$\sup_{0\le t\le T}|\rho(t)|_{H^1}^2 \le C\exp(C\int_0^t \Psi^{\frac{6\gamma}{3p-4}}(s)\mathrm{d}s). \tag{15}$$

Using (5) we obtain

$$|\rho_t(t)|_{L^2} \le |\rho_x(t)|_{L^2}|u(t)|_{L^\infty} + |\rho(t)|_{L^\infty}|u_{xx}(t)|_{L^2} \le C\Psi^{\frac{6\gamma+2}{3p-4}}(t). \tag{16}$$

Besides, differentiating (7) with respect to time t, multiplying it by Φ_t and integrating over Ω with respect to x and using Young's inequality, we have

$$|\Phi_{xt}|_{L^2}^2 \le C\Psi^4(t), \tag{17}$$

where C is a positive constant depending only on M_0.

Estimate for $|u|_{W_0^{1,p}}$

We turn to the second term of $\Psi(t)$. Multiplying (6) by u_t, integrating over Ω_T, together with (5), (10), Sobolev inequality and Young's inequality, we obtain

$$\begin{aligned}\int_0^t |\sqrt{\rho}u_t(s)|_{L^2}^2(s)\mathrm{d}s + |u_x(t)|_{L^p}^p &\le \iint_{\Omega_T}(|\rho u u_x u_t| + |\rho\Phi_x u_t| + |P_x u u_x| + \gamma|Pu_x^2|)\mathrm{d}x\mathrm{d}s \\ &\quad + \int_\Omega (|Pu_x|\mathrm{d}x + \gamma|Pu_x^2|)\mathrm{d}x\mathrm{d}s + C \\ &\le C(1 + \int_\Omega \Psi^{\frac{24\gamma}{3p-4}}(s)\mathrm{d}s).\end{aligned} \tag{18}$$

In the second inequality we have used

$$|\rho(t)|_{L^\infty} + |P(t)|_{H^1} \le |\rho(t)|_{H^1} + C|\rho(t)|_{L^\infty}^{\gamma-1}|\rho(t)|_{H^1} \le C\Psi^\gamma(t).$$

$$\begin{aligned}\int_\Omega |P(t)|^{\frac{p}{p-1}}\mathrm{d}x &= \int_\Omega |P(0)|^{\frac{p}{p-1}}\mathrm{d}x + \int_0^t \frac{\partial}{\partial s}\Big(\int_\Omega P(s)^{\frac{p}{p-1}}\mathrm{d}x\Big)\mathrm{d}s \\ &\le C + C\int_\Omega |\rho(s)|_{L^\infty}^{\gamma-1}|P(s)|_{L^\infty}^{\frac{1}{p-1}}|\rho(s)|_{H^1}|u_x(s)|_{L^p}\mathrm{d}s \\ &\le C(1 + \int_\Omega \Psi^{\frac{2\gamma+1}{p-1}}(s)\mathrm{d}s),\end{aligned}$$

where C is a positive constant, depending only on M_0.

Estimate for $|\sqrt{\rho}u_t|_{L^2}$

We estimate the last term of $\Psi(t)$. Differentiating (6) with respect to t yields

$$\rho u_{tt} + \rho u u_{xt} - \Big[\Big(\frac{\varepsilon u_x^2 + 1}{u_x^2 + \varepsilon}\Big)^{\frac{2-p}{2}} u_x\Big]_{xt} = (-u_t - u u_x - \Phi_x)\rho_t - \rho u_t u_x - \rho \Phi_{xt} - P_{xt}.$$

Multiplying it by u_t and integrating over Ω with respect to x, we derive

$$\begin{aligned}
&\frac{1}{2}\frac{d}{dt}\int_\Omega \rho|u_t|^2 \mathrm{d}x + \int_\Omega \Big[\Big(\frac{\varepsilon u_x^2 + 1}{u_x^2 + \varepsilon}\Big)^{\frac{2-p}{2}} u_x\Big]_t u_{xt} \mathrm{d}x \\
&= \int_\Omega [\rho_t(-u_t - u u_x - \Phi_x) - \rho u_t u_x - \rho \Phi_{xt}] u_t \mathrm{d}x + \int_\Omega P_t u_{xt} \mathrm{d}x.
\end{aligned} \tag{19}$$

Note that

$$\int_\Omega \Big[\Big(\frac{\varepsilon u_x^2 + 1}{u_x^2 + \varepsilon}\Big)^{\frac{2-p}{2}} u_x\Big]_t u_{xt} \mathrm{d}x \geq (p-1)\int_\Omega (u_x^2 + 1)^{\frac{p-2}{2}} |u_{xt}|^2 \mathrm{d}x. \tag{20}$$

Let

$$\beta = (u_x^2 + 1)^{\frac{p-2}{4}}.$$

From (12), it follows that

$$|\beta^{-1}|_{L^\infty} = |(u_x^2 + 1)^{\frac{2-p}{4}}|_{L^\infty} \leq C(|u_{xx}|_{L^2}^{\frac{2-p}{2}} + 1) \leq C\Psi^{\frac{(p+4)(p-1)(2-p)\gamma}{2(3p-4)}} \leq C\Psi^{\frac{3\gamma}{3p-4}}.$$

Then, from (20) and (5), (19) can be rewritten as

$$\begin{aligned}
&\frac{1}{2}\frac{d}{dt}\int_\Omega \rho|u_t|^2 \mathrm{d}x + (p-1)\int_\Omega |\beta u_{xt}|^2 \mathrm{d}x \\
&\leq \int_\Omega 2\rho|u||u_t||u_{xt}| \mathrm{d}x + \int_\Omega |\rho_x||u|^2|u_x||u_t| \mathrm{d}x + \int_\Omega \rho|u||u_x|^2|u_t| \mathrm{d}x \\
&+ \int_\Omega |P_x||u||u_{xt}| \mathrm{d}x + \int_\Omega \gamma P|u_x||u_{xt}| \mathrm{d}x + \int_\Omega |\rho_x||u||\Phi_x||u_t| \mathrm{d}x \\
&+ \int_\Omega \rho|u_x||\Phi_x||u_t| \mathrm{d}x + \int_\Omega \rho|u_t||u_x||u_t| \mathrm{d}x + \int_\Omega \rho|\Phi_{xt}||u_t| \mathrm{d}x = \sum_{j=1}^{9} I_j.
\end{aligned} \tag{21}$$

Using Sobolev inequality, Young's inequality, (6) and (12), we obtain

$$\begin{aligned}
&I_1 \leq 2|\rho|_{L^\infty}^{\frac{1}{2}}|u|_{L^\infty}|\sqrt{\rho}u_t|_{L^2}|\beta u_{xt}|_{L^2}|\beta^{-1}|_{L^\infty} \leq C\Psi^{\frac{16\gamma}{3p-4}}(t) + \frac{p-1}{6}|\beta u_{xt}|_{L^2}^2, \\
&I_2 \leq |\rho_x|_{L^2}|u|_{L^\infty}^2|u_x|_{L^p}^p|u_x|_{L^\infty}^{1-\frac{p}{2}}|u_t|_{L^\infty} \leq |\rho_x|_{L^2}|u_x|_{L^p}^{2+p}|u_{xx}|_{L^2}^{1-\frac{p}{2}}|\beta u_{xt}|_{L^2}|\beta^{-1}|_{L^\infty} \\
&\quad \leq C\Psi^{\frac{23\gamma}{3p-4}}(t) + \frac{p-1}{6}|\beta u_{xt}|_{L^2}^2, \\
&I_3 \leq |\rho|_{L^2}^{\frac{1}{2}}|u|_{L^\infty}||u_x|_{L^\infty}^2|\sqrt{\rho}u_t|_{L^2} \leq C\Psi^{\frac{22\gamma}{3p-4}}(t), \\
&I_4 \leq |P_x|_{L^2}|u|_{L^\infty}|\beta u_{xt}|_{L^2}|\beta^{-1}|_{L^\infty} \leq C\Psi^{\frac{14\gamma}{3p-4}}(t) + \frac{p-1}{6}|\beta u_{xt}|_{L^2}^2, \\
&I_5 \leq C|P|_{L^2}|u_x|_{L^\infty}|\beta u_{xt}|_{L^2}|\beta^{-1}|_{L^\infty} \leq C\Psi^{\frac{22\gamma}{3p-4}}(t) + \frac{p-1}{6}|\beta u_{xt}|_{L^2}^2, \\
&I_6 \leq |\rho_x|_{L^2}|u|_{L^\infty}|\Phi_x|_{L^2}|u_t|_{L^\infty} \leq |\rho_x|_{L^2}|u_x|_{L^p}|\Phi_x|_{L^q}|\beta u_{xt}|_{L^2}|\beta^{-1}|_{L^\infty} \\
&\quad \leq C\Psi^{\frac{14\gamma}{3p-4}}(t) + \frac{p-1}{6}|\beta u_{xt}|_{L^2}^2, \\
&I_7 \leq |\rho|_{L^\infty}^{\frac{1}{2}}|u_x|_{L^\infty}|\Phi_x|_{L^2}|\sqrt{\rho}u_t|_{L^2} \leq |\rho|_{L^\infty}^{\frac{1}{2}}|u_{xx}|_{L^2}|\Phi_x|_{L^q}|\sqrt{\rho}u_t|_{L^2} \leq C\Psi^{\frac{18\gamma}{3p-4}}(t), \\
&I_8 \leq |\sqrt{\rho}u_t|_{L^2}^2|u_x|_{L^\infty}| \leq C\Psi^{\frac{16\gamma}{3p-4}}(t),
\end{aligned}$$

$$I_9 \le |\rho|_{L^2}|\Phi_{xt}|_{L^2}|u_t|_{L^\infty} \le |\rho|_{L^2}|\Phi_{xt}|_{L^2}|\beta u_{xt}|\beta^{-1}|_{L^\infty} \le C\Psi^{\frac{18\gamma}{3p-4}} + \frac{p-1}{6}|\beta u_{xt}|^2_{L^2}.$$

Substituting $I_j (j = 1, 2, \dots, 9)$ into (21) and integrating over $(\tau, t) \subset (0, T)$, we have

$$|\sqrt{\rho}u_t(t)|^2_{L^2} + (p-1)\int_\tau^t |\beta u_{xt}|^2_{L^2}(s)\mathrm{d}s \le C\int_\tau^t \Psi^{\frac{23\gamma}{3p-4}}(s)\mathrm{d}s + |\sqrt{\rho}u_t(\tau)|^2_{L^2}. \tag{22}$$

Next, we estimate $\lim_{\tau\to 0}|\sqrt{\rho^k}u^k_t(\tau)|^2_{L^2}$.

Multiplying (6) by u_t and integrating over Ω, we obtain

$$\int_\Omega \rho|u_t|^2\mathrm{d}x \le 2\int_\Omega (\rho|u|^2|u_x|^2 + \rho|\Phi_x|^2 + \rho^{-1}|L_p u + P_x|^2)\mathrm{d}x.$$

Since (ρ, u, Φ) is a smooth solution, we have

$$\begin{aligned}\lim_{\tau\to 0}\int_\Omega &(\rho|u|^2|u_x|^2 + \rho|\Phi_x|^2 + \rho^{-1}|L_p u + P_x|^2)\mathrm{d}x\\ &= \int_\Omega (\rho_0|u_0|^2|u_{0x}|^2 + \rho_0|\Phi_{0x}|^2 + |g|^2)\mathrm{d}x\\ &\le |\rho_0|_{L^\infty}|u_0|^2_{L^\infty}|u_{0x}|^2_{L^2} + |\rho_0|_{L^\infty}|\Phi_{0x}|^2_{L^2} + |g|^2_{L^2}.\end{aligned}$$

Then taking a limit on τ for (22) as $\tau \to 0$, we get

$$|\sqrt{\rho}u_t(t)|^2_{L^2} + \int_0^t |\beta u_{xt}|^2_{L^2}(s)\mathrm{d}s \le C(1 + \int_0^t \Psi^{\frac{23\gamma}{3p-4}}(s)\mathrm{d}s), \tag{23}$$

which combined with (12), (15)-(17), (18) and the definition of $\Psi(t)$ leads to

$$\Psi(t) \le C\exp(\tilde{C}\int_0^t \Psi^{\frac{6\gamma}{3p-4}}(s)\mathrm{d}s),$$

where C, $\tilde{C}$ are positive constants, depending only on M_0.

In view of this inequality, we can find a time $T_* > 0$ and a constant C, such that

$$\sup_{0\le t\le T_1}(|\rho|_{H^1} + |u|_{W^{1,p}_0\cap H^2} + |\sqrt{\rho}u_t|_{L^2} + |\rho_t|_{L^2} + \int_0^{T_*}(|u_{xt}(s)|^2_{L^2})\mathrm{d}s \le C,$$

where C is a positive constant depending only on M_0.

3 Proof of the existence

Since a priori estimates for higher regularity have been derived, the existence of strong solutions can be established by a standard argument, in the case of bounded domains, we construct approximate solutions via a semi-discrete Galerkin scheme, derive uniform bounds and thus obtain solutions by passing to the limit. Our method that constructed approximate systems is similar to that in [6]. To implement a semi-discrete Galerkin scheme, we take our basic function space as $X = H^1_0(\Omega) \cap H^2(\Omega)$ and its finite-dimensional subspaces as $X = \mathrm{span}\{\text{'}^1, \text{'}^2, \Pi, \text{'}^m\} \subset \mathrm{X}\cap\mathrm{C}^2(\Omega)$. Hence φ^m is the mth eigenfunction of the general elliptic operator defined on X.

Let ρ_0, u_0, Φ_0, and f be functions satisfying the hypothes of theorem, assume for the moment that $\rho^\delta_0 \in C(\Omega)$ and $\rho_0\delta \ge \delta$ in Ω for some constant $\delta > 0$. We can construct an approximate solution for any $v \in X^m$, $\varphi \in C^2(\Omega)$, such that

$$\rho^m_t + (\rho^m u^m)_x = 0,$$

$$\int_\Omega (\rho^m u_t^m + \rho^m u^m u_x^m + \rho^m \Phi_x^m + L_p u^m + P_x^m) v \mathrm{d}x = 0,$$
$$\int_\Omega L_q \Phi^m \phi \mathrm{d}x = 4\pi g \int_\Omega (\rho^m - \frac{m_0}{|\Omega|}) \phi \mathrm{d}x.$$

The initial and boundary conditions are

$$u_0^m \equiv \sum_{k=1}^m (u_0, \varphi^k)_{L^2(\Omega)} \varphi^k, \quad \rho^m(0) = \rho_0^\delta, \quad \rho^\delta < |\rho_0|_{L^\infty(\Omega)} + 1,$$
$$|\rho_0^\delta - \rho_0|_{H^1(\Omega)} \to 0, \quad u^m|_{\partial\Omega} = \Phi^m|_{\partial\Omega} = 0.$$

Under the hypotheses of the theorem, similarly, for any fixed $\delta > 0$, we may get the similar estimate

$$\sup_{0\le t\le T_1} (|\rho^m|_{H^1} + |u^m|_{W_0^{1,p}\cap H^2} + |\sqrt{\rho}^m u_t^m|_{L^2} + |\rho_t^m|_{L^2} + \int_0^{T_*} (|u_{xt}^m(s)|_{L^2}^2) \mathrm{d}s \le C. \tag{24}$$

Since C does not depend on ε, δ and m (for any $m > M$, M is dependent on the approximate velocity of the initial condition). We can deduce from the uniform estimate(24) that (ρ^m, u^m, Φ^m) converges, up to an extraction of subsequences. Let $m \to \infty$. We obtain the following estimates in the obvious weak sense

$$\sup_{0\le t\le T_1} (|\rho^\delta|_{H^1} + |u^\delta|_{W_0^{1,p}\cap H^2} + |\sqrt{\rho}^\delta u_t^\delta|_{L^2} + |\rho_t^\delta|_{L^2} + \int_0^{T_*} (|u_{xt}^\delta(s)|_{L^2}^2) \mathrm{d}s \le C.$$

For each small $\delta > 0$, $\rho_0^\delta = J_\delta * \rho_0 + \delta$ is a mollifier on Ω, and $u_0^\delta \in H_0^1(\Omega) \cap H^2(\Omega)$ is the unique solution of the boundary value problem

$$\begin{cases} -\Big[\Big(\frac{\varepsilon(u_{0x}^\delta)^2+1}{(u_{0x}^\delta)^2+\varepsilon}\Big)^{\frac{2-p}{2}} u_{0x}^\delta\Big]_x + P_x(\rho_0^\delta) = (\rho_0^\delta)^{\frac{1}{2}} g_\delta \\ u_0^\delta|_{\partial\Omega} = 0, \end{cases} \tag{25}$$

where $g_\delta \in C_0^\infty$, and $|g_\delta|_{L^2(\Omega)} \le |g|_{L^2(\Omega)}$, $\lim\limits_{\delta\to 0^+} |g_\delta - g|_{L^2} = 0$.

We deduce that $(\rho^\delta, u^\delta, \Phi^\delta)$ is a solution of the following initial and boundary value problem

$$\begin{cases} \rho_t + (\rho u)_x = 0, \\ (\rho u)_t + (\rho u^2)_x + \rho\Phi_x - \lambda(|u_x|^{p-2}u_x)_x + P_x = 0, \\ ((\Phi_x^2 + \mu_0)^{\frac{q-2}{2}} \Phi_x)_x = 4\pi g(\rho - \frac{1}{|\Omega|}\int_\Omega \rho \mathrm{d}x) \\ (\rho, u)|_{t=0} = (\rho_0^\delta, u_0^\delta), \\ u(x,t)|_{\partial\Omega} = \Phi(x,t)|_{\partial\Omega} = 0, \end{cases}$$

where $\frac{4}{3} < p < 2, q > 2$.

Together with Lemma 2.2, there is s subsequence $\{u^{\varepsilon_j,\delta}\}$ of $\{u^{\varepsilon,\delta}\}$, as $\varepsilon_j \to 0$, $\{u_0^{\varepsilon_j,\delta}\} \to u_0^\delta$ in $H_0^1(\Omega) \cap H^2(\Omega)$. Also there is a subsequence $\{u_0^{\delta_j}\}$ of $\{u_0^\delta\}$, such that as $\delta_j \to 0^+$, $u_0^{\delta_j} \to u_0$ in $H_0^1(\Omega) \cap H^2(\Omega)$. With $\rho_0^\delta = J_\delta * \rho_0 + \delta$, we have, as $\delta_j \to 0^+$, $P_x(\rho_0^{\delta_j}) - (\rho_0^{\delta_j})^{\frac{1}{2}} g_{\delta_j} \to P_x(\rho_0) - (\rho_0)^{\frac{1}{2}} g$ in $L^2(\Omega)$. By (25), there exists a subsequence $\{u_0^{\delta_j}\}$ of $\{u_0^\delta\}$, such that as $\delta_j \to 0^+$, $(|u_{0x}^{\delta_j}|^{p-2}u_{0x}^{\delta_j})_x \to (|u_{0x}|^{p-2}u_{0x})_x$ in $L^2(\Omega)$. Hence, $u_0 \in H_0^1(\Omega) \cap H^2(\Omega)$ satisfies equation $-(|u_{0x}|^{p-2}u_{0x})_x + P_x(\rho_0) = (\rho_0)^{\frac{1}{2}} g$ for a.e. $x \in \Omega$ and is a unique solution. According to above uniform estimates, by the lower semi-continuity of various norms, we have

$$\sup_{0\le t\le T_1} (|\rho|_{H^1} + |u|_{W_0^{1,p}\cap H^2} + |\sqrt{\rho} u_t|_{L^2} + |\rho_t|_{L^2} + \int_0^{T_*} (|u_{xt}(s)|_{L^2}^2) \mathrm{d}s \le C$$

where C is a positive constant depending only on M_0.

4 Proof of the uniqueness

We now prove the uniqueness results. Let (ρ, u, Φ), $(\bar{\rho}, \bar{u}, \bar{\Phi})$ be two solutions of the problem (2)-(4). Combining $(2)_1$ and $(2)_2$,

$$(\rho u)_t + (\rho u^2)_x + \rho\Phi_x + (|u_x|^{p-2}u_x)_x + P_x = 0,$$
$$(\bar{\rho}\bar{u})_t + (\bar{\rho}\bar{u}^2)_x + \bar{\rho}\bar{\Phi}_x + (|\bar{u}_x|^{p-2}\bar{u}_x)_x + \bar{P}_x = 0,$$

we show that

$$\begin{aligned}&\rho(u-\bar{u})_t + \rho u(u-\bar{u})_x - [(|u_x|^{p-2}u_x)_x - (|\bar{u}_x|^{p-2}\bar{u}_x)_x]\\ &= (\rho-\bar{\rho})(-\bar{u}_t - \bar{u}\bar{u}_x - \bar{\Phi}_x) - (P-\bar{P})_x - \rho(\Phi-\bar{\Phi})_x - \rho(u-\bar{u})\bar{u}_x.\end{aligned}$$

Multiplying the above equation by $(u-\bar{u})$ and integrating over Ω, we obtain

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\int_\Omega \rho(u-\bar{u})^2 \mathrm{d}x + \int_\Omega (|u_x|^{p-2}u_x - |\bar{u}_x|^{p-2}\bar{u}_x)(u-\bar{u})_x \mathrm{d}x\\ \le &\int_\Omega \{|\rho-\bar{\rho}||-\bar{u}_t - \bar{u}\bar{u}_x - \bar{\Phi}_x||u-\bar{u}| + |P-\bar{P}||(u-\bar{u})_x|\\ &\quad + \rho|(\Phi-\bar{\Phi})_x||u-\bar{u}| + \rho|u-\bar{u}|^2|\bar{u}_x|\}\mathrm{d}x\\ \le &|\rho-\bar{\rho}|_{L^2}|-\bar{u}_t - \bar{u}\bar{u}_x - \bar{\Phi}_x|_{L^2}|u-\bar{u}|_{L^\infty} + |P-\bar{P}|_{L^2}|(u-\bar{u})_x|_{L^2}\\ &\quad + |\rho|_{L^\infty}|(\Phi-\bar{\Phi})_x|_{L^2}|u-\bar{u}|_{L^2} + |\sqrt{\rho}(u-\bar{u})|^2_{L^2}|\bar{u}_x|_{L^\infty}\\ \le &|\rho-\bar{\rho}|^2_{L^2}(C + C|\bar{u}_t|^2_{L^2}) + C|P-\bar{P}|^2_{L^2} + |\sqrt{\rho}(u-\bar{u})|^2_{L^2} + \varepsilon|(u-\bar{u})_x|^2_{L^2}.\end{aligned} \tag{26}$$

Since

$$\int_\Omega (|u_x|^{p-2}u_x - |\bar{u}_x|^{p-2}\bar{u}_x)(u-\bar{u})_x \mathrm{d}x = \frac{1}{p-1}\int_\Omega \Big(\int_0^1 |\theta u_x + (1-\theta)\bar{u}_x|^{p-2}\mathrm{d}\theta\Big)(u-\bar{u})_x^2 \mathrm{d}x$$

$$\int_0^1 |\theta u_x + (1-\theta)\bar{u}_x|^{p-2}\mathrm{d}\theta \ge \int_0^1 \frac{1}{(|u_x| + |\bar{u}_x|)^{2-p}}\mathrm{d}\theta = \frac{1}{(|u_x| + |\bar{u}_x|)^{2-p}}.$$

Thus

$$\begin{aligned}&\int_\Omega (|u_x|^{p-2}u_x - |\bar{u}_x|^{p-2}\bar{u}_x)(u-\bar{u})_x \mathrm{d}x\\ &\ge \frac{1}{(|u_x|_{L^\infty(0,t;L^\infty(I))} + |\bar{u}_x|_{L^\infty(0,t;L^\infty(I))})^{2-p}}\int_\Omega (u-\bar{u})_x^2 \mathrm{d}x\\ &\ge \frac{1}{C}\int_\Omega (u-\bar{u})_x^2 \mathrm{d}x.\end{aligned}$$

Moreover, from $(2)_3$, by a direct calculation, we have

$$\Big((\Phi_x^2 + \mu_0)^{\frac{q-2}{2}}\Phi_x\Big)_x - \Big((\bar{\Phi}_x^2 + \mu_0)^{\frac{q-2}{2}}\bar{\Phi}_x\Big)_x = 4\pi g(\rho-\bar{\rho}).$$

Multiplying it by $(\Phi-\bar{\Phi})$ and integrating over Ω, we obtain

$$\begin{aligned}&\int_\Omega \Big[(\Phi_x^2 + \mu_0)^{\frac{q-2}{2}}\Phi_x - (\bar{\Phi}_x^2 + \mu_0)^{\frac{q-2}{2}}\bar{\Phi}_x\Big](\Phi-\bar{\Phi})_x \mathrm{d}x\\ &= -\int_\Omega 4\pi g(\rho-\bar{\rho})(\Phi-\bar{\Phi})_x \mathrm{d}x \le C|\rho-\bar{\rho}|_{L^2} + \varepsilon|(\Phi-\bar{\Phi})_x|_{L^2}.\end{aligned} \tag{27}$$

Note that

$$\int_\Omega \left[(\Phi_x^2+\mu_0)^{\frac{q-2}{2}}\Phi_x-(\bar\Phi_x^2+\mu_0)^{\frac{q-2}{2}}\bar\Phi_x\right](\Phi-\bar\Phi)_x\mathrm{d}x$$
$$=\int_\Omega\Big[\int_0^1\omega'(\theta\Phi_x+(1-\theta)\bar\Phi_x)\mathrm{d}\theta\Big](\Phi_x-\bar\Phi_x)^2\mathrm{d}x, \tag{28}$$

$$\omega'(s)=[(s^2+\mu_0)^{\frac{q-2}{2}}s]'=(s^2+\mu_0)^{\frac{p-4}{2}}((p-1)s^2+\mu_0)\geq\mu_0^{\frac{p-2}{2}}.$$

Together with (27) and (28), we arrive at $|(\Phi-\bar\Phi)_x|_{L^2}^2\leq C|\rho-\bar\rho|_{L^2}^2$.

Consequently, (26) can be rewritten as

$$\begin{aligned}&\frac12\frac{d}{dt}|\sqrt{\rho}(u-\bar u)(t)|_{L^2}^2+\frac1C|u_x-\bar u_x|_{L^2}^2\mathrm{d}s\\ \leq&|\rho-\bar\rho|_{L^2}^2(C+C|\bar u_t|_{L^2}^2)+C|P-\bar P|_{L^2}^2+|\sqrt{\rho}(u-\bar u)|_{L^2}^2.\end{aligned} \tag{29}$$

On the other hand, from the conservation of mass equation $(2)_1$, using the identity

$$(\rho-\bar\rho)_t+(\rho-\bar\rho)_xu+\bar\rho_x(u-\bar u)+(\rho-\bar\rho)u_x+\bar\rho(u_x-\bar u_x)=0. \tag{30}$$

Multiplying the above equation by $(\rho-\bar\rho)$ and integrating it over Ω, we obtain

$$\begin{aligned}\frac12\frac{d}{dt}\int_\Omega(\rho-\bar\rho)^2\mathrm{d}x+\int_\Omega\frac12(\rho-\bar\rho)^2u_x\mathrm{d}x+\int_\Omega\bar\rho_x(u-\bar u)(\rho-\bar\rho)\mathrm{d}x\\ +\int_\Omega(\rho-\bar\rho)^2u_x\mathrm{d}x+\int_\Omega\bar\rho(u-\bar u)_x(\rho-\bar\rho)\mathrm{d}x=0.\end{aligned}$$

Thus

$$\begin{aligned}\frac12\frac{d}{dt}\int_\Omega(\rho-\bar\rho)^2\mathrm{d}x&\leq C(|u_x|_{L^\infty}|\rho-\bar\rho|_{L^2}^2+|\rho_x|_{L^2}|u-\bar u|_{L^\infty}|\rho-\bar\rho|_{L^2}\\ &\quad+|\bar\rho|_{L^\infty}|(u-\bar u)_x|_{L^2}|\rho-\bar\rho|_{L^2})\\ &\leq C|\rho-\bar\rho|_{L^2}^2+C(\epsilon)|(u-\bar u)_x|_{L^2}^2.\end{aligned} \tag{31}$$

Furthermore, $(5)_1$ implies

$$(P-\bar P)_t+(P-\bar P)_x\bar u+\bar P_x(u-\bar u)+\gamma(P-\bar P)u_x+\gamma\bar P(u-\bar u)_x=0.$$

Multiplying it by $(P-\bar P)$ and integrating over Ω, we get

$$\begin{aligned}\frac12\frac{d}{dt}\int_\Omega(P-\bar P)^2\mathrm{d}x&=\frac12\int_\Omega(P-\bar P)^2u_x\mathrm{d}x+\int_\Omega\bar P_x(u-\bar u)(P-\bar P)\mathrm{d}x\\ &\quad-\gamma\int_\Omega(P-\bar P)^2u_x\mathrm{d}x+\gamma\int_\Omega\bar P(u-\bar u)_x(P-\bar P)\mathrm{d}x\\ &\leq(C|u_x|_{L^\infty}|P-\bar P|_{L^2}^2+C|\bar P|_{H^1}|u-\bar u|_{L^\infty}|P-\bar P|_{L^2})\\ &\leq(C(|u_x|_{L^\infty}+|\bar P|_{H^1}+1)|P-\bar P|_{L^2}^2)\mathrm{d}s+\varepsilon|(u-\bar u)_x|_{L^2}^2.\end{aligned} \tag{32}$$

From(29)-(32), we obtain

$$\begin{aligned}&\frac{d}{dt}\int_\Omega(\rho(u-\bar u)^2+(\rho-\bar\rho)^2+(P-\bar P)^2)\mathrm{d}x+\int_\Omega(u-\bar u)_x^2\mathrm{d}x\\ \leq&C((1+|\bar u_t|_{L^2}^2+|\bar u_x|_{L^\infty}^2+|\rho|_{H^1}^2+|u_x|_{L^\infty}+|\bar P_x|_{L^2}^2)\end{aligned}$$

$$(|\sqrt{\rho}(u-\bar{u})|_{L^2}^2 + |(\rho-\bar{\rho})|_{L^2}^2 + |(P-\bar{P})|_{L^2}^2).$$

From this and the Grownwall's inequality, yields

$$|\sqrt{\rho}(u-\bar{u})|_{L^2}^2 + |\rho-\bar{\rho}|_{L^2}^2 + |P-\bar{P}|_{L^2}^2 = 0$$

which means

$$u = \bar{u}, \rho = \bar{\rho}, \Phi = \bar{\Phi}.$$

This complete the proof of the main theorem.

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