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Hao Xing, Leipo Liu*, Xiangyang Cao, Zhumu Fu, and Shuzhong Song

Guaranteed cost finite-time control of positive switched nonlinear systems with D-perturbation

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Abstract: This paper considers the guaranteed cost finite-time boundedness of positive switched nonlinear systems with D-perturbation and time-varying delay. Firstly, the definition of guaranteed cost finite-time boundedness is introduced. By using the Lyapunov-Krasovskii functional and average dwell time (ADT) approach, an output feedback controller is designed and sufficient conditions are obtained to ensure the corresponding closed-loop systems to be guaranteed cost finite-time boundedness (GCFTB). Such conditions can be solved by linear programming. Finally, two examples are provided to show the effectiveness of the proposed method.

Keywords: Positive switched nonlinear systems, Average dwell time, D-perturbation, Time-varying delay, Output feedback control

MSC: 93C10

1 Introduction

The switched system, which comprises a set of subsystems and a switched controller that designating the switching between subsystems, has been studied very well. As a special kind of switched systems, the positive switched systems, whose output and state are nonnegative whenever the initial condition and input are nonnegative, have been applied in many practical systems, such as communication networks [1], viral mutation [2], formation flying [3], and so on. Many remarkable results have been presented, see [4-11] and references therein.

However, most results mentioned above focus on the Lyapunov asymptotic stability, which reflects the asymptotic behavior of the system within a sufficiently long (in principle infinite) time interval. In many practical circumstances, one is more interested in researching what happens in a finite time interval. [12] firstly defined the definition of finite-time stability (FTS) for linear deterministic systems. Recently, [13] firstly extended the concept of FTS to positive switched systems and gave some FTS conditions of positive switched systems. So far, there have been some meaningful results about FTS of positive switched systems, see [14-18]. In [14], the problem of finite-time L_1 control for a class of positive switched linear systems with time-varying delay was considered and the concept of finite-time L_1 boundedness was also proposed. In [15], the

Hao Xing: College of Information Engineering, Henan University of Science and Technology, Luoyang, 471023, China
***Corresponding Author: Leipo Liu:** College of Information Engineering, Henan University of Science and Technology, Luoyang, 471023, China, E-mail: liuleipo123@163.com
Xiangyang Cao: College of Information Engineering, Henan University of Science and Technology, Luoyang, 471023, China
Zhumu Fu: College of Information Engineering, Henan University of Science and Technology, Luoyang, 471023, China
Shuzhong Song: College of Information Engineering, Henan University of Science and Technology, Luoyang, 471023, China

problem of finite-time stability analysis and control synthesis of fractional order positive switched systems was considered. In [16], the finite-time stability problem of discrete switched singular positive systems was investigated. In [17], the robust finite-time stability and stabilisation of discrete-time switched positive systems was researched. In [18], the finite-time stability problem of switched positive linear systems was addressed. But, the above results are involved in positive switched linear systems. For positive switched nonlinear systems, only [10] considered local asymptotic stability and [19] studied absolute exponential L_1 stability analysis and control synthesis, respectively. Furthermore, there are no results about FTS of positive switched nonlinear systems, though many papers about FTS of switched nonlinear systems (non-positive) have been published [20-23]. On the other hand, in most practical systems, researchers desire to design the control system which is not only finite-time stable but also guarantees an adequate level of performance. One method to this problem is the so-called guaranteed cost control [24,25]. In [26], the problem of robust finite-time guaranteed cost control for a class of impulsive switched systems with time-varying delay was studied. Very recently, [27] and [28] considered the guaranteed cost finite-time control for positive switched linear systems, [29] studied the guaranteed cost finite-time control for fractional-order positive switched systems. However, the problem of guaranteed cost finite-time control for positive switched nonlinear systems is still open, because nonlinear system is more complex than the linear system and there is a lack of effective methods. These show that the topic addressed in the paper is interesting but full of challenge.

Moreover, there exists a hardware error, such as fluctuation and modeling error, which will lead to instability of the system and is called D-perturbation. Recently, [30] considered the robust stability of positive switched linear systems with D- perturbation and time-varying delay. Accordingly, for positive switched non-linear systems, the effect of D-perturbation must be also taken into account in analyzing and implementing guaranteed cost finite-time controller scheme.

Motivated by the above discussion, in this paper, we consider the problem of GCFTB for positive switched nonlinear systems with D-perturbation and time-varying delay. An output feedback controller is designed and some sufficient conditions are obtained to guarantee that the closed-loop system is GCFTB. The main contributions lie in two aspects: 1) This system model is more general, so that the systems dealt with in [2,4,5,7,14,19,27,28,30] can be regarded as especial forms of the system. 2) A new nonlinear Lyapunov-Krasovskii functional is constructed and the obtained conditions can be easily solved by linear programming. The remainder of the paper is organized as follows. Section 2 presents the preliminaries and problem statements. Main results are given in Section 3. In Section 4, two examples are provided. Section 5 concludes the paper.

Notations. The representation $A \succ 0\ (\succeq 0, \prec 0, \preceq 0)$ means that $a_{ij} > 0\ (\geq 0, < 0, \leq 0)$, which is also applying to a vector. $A \succ B\ (A \succeq B)$ means that $A - B \succ 0\ (A - B \succeq 0)$. R_+^n is the n-dimensional non-negative (positive) vector space. $R^{n\times n}$ denotes the space of $n \times n$ matrices with real entries. $\mathbf{1_n}$ represents the n-dimensional vector $[1, \ldots, 1]^T$. I represents the n-dimensional identity matric. A^T denotes the transpose of matrix A. 1-norm $\|x\|$ is defined by $\|x\| = \sum_{k=1}^{n} |x_k|$. Matrices are assumed to have compatible dimensions for calculating if their dimensions are not explicitly stated.

2 Preliminaries and problem statements

Consider the following positive switched nonlinear systems with D-perturbation and time-varying delay:

$$\begin{cases} \dot{x}(t) = D_1 A_{\sigma(t)} f(x(t)) + D_2 A_{d\sigma(t)} f(x(t-d(t))) + D_3 G_{\sigma(t)} u(t) + D_4 B_{\sigma(t)} w(t) \\ y(t) = C_{\sigma(t)} f(x(t)) \\ x(\theta) = \varphi(\theta), \theta \in [-\iota, 0] \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the system state, $u(t) \in R^m$ and $y(t) \in R^s$ represent the control input and output. $\sigma(t)$: $[0, \infty) \to \underline{N} = \{1, 2, \cdots, N\}$ is the system switching signal, where N is the number of subsystems; $\forall p \in \underline{N}$, A_p, A_{dp}, B_p, C_p and G_p are constant matrices with suitable dimensions, $d(t) \geq 0$ denotes time-varying delay, which satisfies $\dot{d}(t) \leq h < 1$, h is a known positive constants. $\varphi(\theta)$ is the initial condition on $[-\iota, 0]$, $f(x) =$

$(f_1(x_1), f_2(x_2), \dots, f_n(x_n))^T \in R^n$, perturbations $D_1 \in [\underline{D}_1, \bar{D}_1]$, $D_2 \in [\underline{D}_2, \bar{D}_2]$, $D_3 \in [\underline{D}_3, \bar{D}_3]$ and $D_4 \in [\underline{D}_4, \bar{D}_4]$ for $i \in \underline{N}$ with $\bar{D}_1 \geq \underline{D}_1 \geq 0$, $\underline{D}_2 \geq \bar{D}_2 \geq 0$, $\bar{D}_3 \geq \underline{D}_3 \geq 0$ and $\bar{D}_4 \geq \underline{D}_4 \geq 0$, where matrices $\underline{D}_1$, $\bar{D}_1$, $\underline{D}_2$, $\bar{D}_2$, $\underline{D}_3$, $\bar{D}_3$, $\underline{D}_4$, $\bar{D}_4$ are all diagonal, $w(t) \in R^l$ is the exogenous disturbance and is defined as

$$\exists \zeta > 0 : \int_0^T \|w(t)\| dt \leq \zeta \tag{2}$$

with a known scalar $\zeta > 0$.

Assumption 2.1. *The nonlinear function $f(x)$ lies in sector fields satisfying*

$$m_1 x_i^2 \leq f_i(x_i) x_i \leq m_2 x_i^2 \tag{3}$$

for $x_i \in R$ and $i = 1, 2, \dots, n$, where $0 \leq m_1 \leq m_2$, and $f_i(0) = 0$.

Remark 2.2. *The system model (1) is a more general form. Especially, if $m_1 = m_2 = 1$ (it means $f_i(x_i) = x_i$) and $D_1 = D_2 = D_3 = D_4 = I$, then the system (1) is transformed to positive switched linear systems, such as [2, 4, 5, 7, 14, 27, 28]; If $m_1 = m_2 = 1$, $u(t) = 0$ and $w(t) = 0$, then the system (1) is turned into the model in [30]; If $D_1 = D_2 = D_3 = D_4 = I$, then the system (1) is converted into the model in [19].*

Next, we will give some definitions and lemmas for the following positive switched nonlinear systems.

$$\begin{cases} \dot{x}(t) = D_1 A_{\sigma(t)} f(x(t)) + D_2 A_{d\sigma(t)} f(x(t - d(t))) + D_4 B_{\sigma(t)} w(t) \\ y(t) = C_{\sigma(t)} f(x(t)) \\ x(\theta) = \varphi(\theta), \theta \in [-\iota, 0] \end{cases} \tag{4}$$

Definition 2.3 ([4]). *System (4) is said to be positive if for any switching signals $\sigma(t)$, any initial conditions $\varphi(\theta) \geq 0$, $\theta \in [-\iota, 0]$, and any disturbance input $w(t) \geq 0$, the corresponding trajectory satisfies $x(t) \geq 0$ and $y(t) \geq 0$ for all $t \geq 0$.*

Definition 2.4 ([4]). *A is called a Metzler matrix if the off-diagonal entries of matrix A are non-negative.*

Definition 2.5 ([19]). *For any switching signal $\sigma(t)$ and any $t_2 \geq t_1 \geq 0$, let $N_\sigma(t_1, t_2)$ denote the switching numbers, over the interval $[t_1, t_2)$. For given $t_\alpha > 0$ and $n_0 > 0$, if the inequality*

$$N_\sigma(t_1, t_2) \leq n_0 + \tfrac{t_2 - t_1}{t_\alpha} \tag{5}$$

holds, then t_α is called an average dwell time, and n_0 is called a chatting bounding. Generally, we choose $n_0 = 0$.

Definition 2.6 ([14] Finite-Time Stability (FTS)). *For a given time T_f and two vectors $\varsigma > \rho > 0$, positive switched nonlinear system (4) is said to be FTS with respect to $(\varsigma, \rho, T_f, \sigma(t))$, if*

$$sup_{-\iota \leq t \leq 0} x^T(t) \varsigma \leq 1 \Rightarrow x^T(t) \rho < 1, \forall t \in [0, T_f]. \tag{6}$$

If the above condition is satisfied for any switching signals $\sigma(t)$, system (4) is said to be uniformly FTS with respect to(ς, ρ, T_f).

Definition 2.7 ([14] Finite-Time Boundedness (FTB)). *For a given constant T_f and two vectors $\varsigma > \rho > 0$, positive switched nonlinear system (4) is said to be FTB with respect to$(\varsigma, \rho, T_f, \zeta, \sigma(t))$, where $w(t)$ satisfies (2), if*

$$sup_{-\iota \leq t \leq 0} x^T(t) \varsigma \leq 1 \Rightarrow x^T(t) \rho < 1, \forall t \in [0, T_f]. \tag{7}$$

Lemma 2.8 ([4]). *A matrix $A \in R^{n \times n}$ is a Metzler matrix if and only if there exists a positive constant ϱ such that $A + \varrho I_n \geq 0$.*

Lemma 2.9 ([14]). *System (4) is positive if and only if A_p, $\forall p \in \underline{N}$ are Metzler matrices and $\forall p \in \underline{N}$, $A_{dp} \geq 0$, $B_p \geq 0$, $C_p \geq 0$ and $G_p \geq 0$.*

Definition 2.10 ([21]). *Define the cost function of positive switched nonlinear systems (1) as follows:*

$$J = \int_0^{T_f} [x^T(t)R_1 + u^T(t)R_2]dt \tag{8}$$

where $R_1 \succ 0$ and $R_2 \succ 0$ are two given vectors.

Definition 2.11 ([21]). *(GCFTB) For a given time constant T_f and two vectors $\varsigma \succ \rho \succ 0$, consider positive switched nonlinear systems (1) and cost function (8), if there exist a control law $u(t)$ and a positive scalar $J^\star$ such that the closed-loop system is FTB with respect to $(\varsigma, \rho, T_f, \zeta, \sigma(t))$ and the cost function satisfies $J \leq J^\star$, then the closed-loop system is called GCFTB, where $J^\star$ is a guaranteed cost value and $u(t)$ is a guaranteed cost finite-time controller.*

3 Main results

3.1 Guaranteed cost finite-time boundedness analysis

In this subsection, we will focus on the problem of GCFTB for positive switched nonlinear system (4). Firstly, we present the following lemma which is essential for our later development.

Lemma 3.1 ([14]). *Consider the positive switched nonlinear system (4), for a given time constant T_f and vectors $\varsigma \succ \rho \succ 0$, if there exist a set of positive vectors ν_p, υ_p, ϑ_p, $p \in \underline{N}$ and positive constants ϕ_1, ϕ_2, ϕ_3, ϕ_4, λ and γ, and such that the following inequalities hold:*

$$\Psi_p = diag\{\psi_{p1}, \psi_{p2}, \dots \psi_{pn}, \psi'_{p1}, \psi'_{p2}, \dots \psi'_{pn}\} \leq 0 \tag{9}$$

$$\phi_1\rho \prec \nu_p \prec \phi_2\varsigma, \upsilon_p \prec \phi_3\varsigma, \vartheta_p \prec \phi_4\varsigma \tag{10}$$

$$\phi_2 b_{pr}^T \bar{d}_{4r}\varsigma \prec \gamma \mathbf{1_n} \tag{11}$$

$$\phi_2 + m_2\iota e^{\lambda\iota}\phi_3 + m_2\iota^2 e^{\lambda\iota}\phi_4 + \lambda\zeta < \phi_1 e^{-\lambda T_f} \tag{12}$$

where
$\psi_{pr} = m_2 a_{pr}^T \bar{d}_{1r}\nu_{pr} + m_2\upsilon_{pr} + m_2\iota\vartheta_{pr} - \lambda_p\nu_{pr}$
$\psi'_{pr} = a_{dpr}^T \bar{d}_{2r}\nu_{pr} - (1-h)\upsilon_{pr}$
$\lambda = \max_{p \in \underline{N}}\{\lambda_p\}, r \in \underline{n} = \{1, 2, \dots, n\}.$
$a_{pr}(a_{dpr}, \bar{d}_{1r}, \bar{d}_{2r}, \bar{d}_{4r})$ represents the rth column vector of the matrix $A_p(A_{dp}, \bar{D}_1, \bar{D}_2, \bar{D}_4)$, $\nu_p = [\nu_{p1}, \nu_{p2}, \dots, \nu_{pn}]^T$, $\upsilon_p = [\upsilon_{p1}, \upsilon_{p2}, \dots, \upsilon_{pn}]^T$, and $\vartheta_p = [\vartheta_{p1}, \vartheta_{p2}, \dots, \vartheta_{pn}]^T$, ν_{pr}, υ_{pr} and ϑ_{pr} represent the rth elements of the vectors ν_p, υ_p and θ_p, respectively, then under the following ADT scheme

$$T_\alpha > T_\alpha^* = max\{\frac{T_f ln\mu}{ln(\phi_1 e^{-\lambda T_f}) - ln(\phi_2 + m_2\iota e^{\lambda\iota}\phi_3 + m_2\iota^2 e^{\lambda\iota}\phi_4 + \gamma\zeta)}, \frac{ln\mu}{\lambda}\}, \tag{13}$$

the system (4) is FTB with respect to $(\varsigma, \rho, T_f, \zeta, \sigma(t))$, where $\mu \geq 1$ satisfies

$$\nu_p \preceq \mu\nu_q, \upsilon_p \preceq \mu\upsilon_q, \vartheta_p \preceq \mu\vartheta_q, \forall p, q \in \underline{N}. \tag{14}$$

Proof. Construct the Lyapunov-Krasovskii functional for the system (4) as follows:

$$V_{\sigma(t)}(t) = V_{\sigma(t)}(t, x(t)) = x^T(t)\nu_p + \int_{t-d(t)}^{t} e^{\lambda_p(t-s)} f^T(x(s))\upsilon_p ds + \int_{-\iota}^{0}\int_{t+\theta}^{t} e^{\lambda_p(t-s)} f^T(x(s))\vartheta_p ds d\theta. \quad (15)$$

where ν_p, υ_p and $\vartheta_p \in R_+^n$, $\forall p \in \underline{N}$.

Along the trajectory of the system (4), we have

$$\begin{aligned}\dot{V}_{\sigma(t)}(t) =& f^T(x(t))A_p^T D_1^T \nu_p + f^T(x(t-d(t))A_{dp}^T D_2^T \nu_p + w^T(t)B_p^T D_4^T \nu_p \\ &+ \lambda_p \int_{t-d(t)}^{t} e^{\lambda_p(t-s)} f^T(x(s))\upsilon_p ds + f^T(x(t))\upsilon_p - (1-\dot{d}(t))e^{\lambda_p d(t)} f^T(x(t-d(t))\upsilon_p \\ &+ \lambda_p \int_{-\iota}^{0}\int_{t+\theta}^{t} e^{\lambda_p(t-s)} f^T(x(s))\vartheta_p ds d\theta + \iota f^T(x(t))\vartheta_p - \int_{-\iota}^{0} e^{-\lambda_p\theta} x^T(t+\theta)\vartheta_p d\theta \\ \le& f^T(x(t))A_p^T D_1^T \nu_p + f^T(x(t-d(t))A_{dp}^T D_2^T \nu_p + w^T(t)B_p^T D_4^T \nu_p \\ &+ \lambda_p \int_{t-d(t)}^{t} e^{\lambda_p(t-s)} f^T(x(s))\upsilon_p ds + f^T(x(t))\upsilon_p - (1-h)e^{\lambda_p d(t)} f^T(x(t-d(t))\upsilon_p \\ &+ \lambda_p \int_{-\iota}^{0}\int_{t+\theta}^{t} e^{\lambda_p(t-s)} f^T(x(s))\vartheta_p ds d\theta + \iota f^T(x(t))\vartheta_p - \int_{t-d(t)}^{t} f^T(x(s))\vartheta_p ds.\end{aligned} \quad (16)$$

From (3), (10), (11), (15) and (16), we have

$$\begin{aligned}\dot{V}_{\sigma(t)}(t) - \lambda_p V_{\sigma(t)}(t) \le& x^T(t)(m_2 A_p^T \bar{D}_1 \nu_p - \lambda_p \nu_p + m_2 \upsilon_p + m_2 \iota \vartheta_p) \\ &+ f^T(x(t-d(t))(A_{dp}^T \bar{D}_2 - (1-h)\upsilon_p) + \gamma\|w(t)\|\end{aligned} \quad (17)$$

Substituting (9) into (17) yields

$$\dot{V}_{\sigma(t)}(t) - \lambda_p V_{\sigma(t)}(t) - \gamma\|w(t)\| \le 0 \quad (18)$$

Integrating both sides of (18) during the period $[t_k, t)$ for $t \in [t_k, t_{k+1})$ leads to

$$V_{\sigma(t)}(t) \le e^{\lambda_{\sigma(t_k)}(t-t_k)} V_{\sigma(t_k)}(t_k) + \gamma \int_{t_k}^{t} e^{\lambda_{\sigma(t_k)}(t-s)} \|w^T(s)\| ds. \quad (19)$$

On the other hand, from (14) and (15), one can easily obtain

$$V_{\sigma(t_k)}(t_k) \le \mu_{\sigma(t_k)} V_{\sigma(t_k^-)}(t_k^-) \quad (20)$$

Let N be the switching number of $\sigma(t)$ over $[0, T_f)$, and denote $t_1, \cdots, t_k$ as the switching instants over the interval $[0, T_f)$. From (19), we have

$$\begin{aligned}V_{\sigma(t)}(t) \le & e^{\lambda_{\sigma(t_k)}(t-t_k)}V_{\sigma(t_k)}(t_k) + \gamma\int_{t_k}^{t} e^{\lambda_{\sigma(t_k)}(t-s)}\|w^T(s)\|ds \\ \le & \mu_{\sigma(t_k)}e^{\lambda_{\sigma(t_k)}(t-t_k)}V_{\sigma(t_k^-)}(t_k^-) + \gamma\int_{t_k}^{t} e^{\lambda_{\sigma(t_k)}(t-s)}\|w^T(s)\|ds \\ \le & \mu_{\sigma(t_k)}e^{\lambda_{\sigma(t_k)}(t-t_k)}\big[e^{\lambda_{\sigma(t_{k-1})}(t_k-t_{k-1})}V_{\sigma(t_{k-1})}(t_{k-1}) \\ & + \gamma\int_{t_{k-1}}^{t_k} e^{\lambda_{\sigma(t_{k-1})}(t_k-s)}\|w^T(s)\|ds\big] + \gamma\int_{t_k}^{t} e^{\lambda_{\sigma(t_k)}(t-s)}\|w^T(s)\|ds \le \cdots \\ \le & \mu^N e^{\lambda t}V_{\sigma(0)}(0) + \mu^N\gamma\int_0^{t_1} e^{\lambda(t-s)}\|w^T(s)\|ds + \mu^{N-1}\gamma\int_{t_1}^{t_2} e^{\lambda(t-s)}\|w^T(s)\|ds \\ & + \ldots + \gamma\int_{t_k}^{t} e^{\lambda(t-s)}\|w^T(s)\|ds \\ \le & \mu^N e^{\lambda T_f}V_{\sigma(0)}(0) + \gamma\int_0^{t} \mu^{N_{\sigma(t)}(s,t)}e^{\lambda(t-s)}\|w^T(s)\|ds \\ \le & \mu^N e^{\lambda T_f}V_{\sigma(0)}(0) + \gamma\mu^N\int_0^{t} e^{\lambda T_f}\|w^T(s)\|ds \\ \le & \mu^N e^{\lambda T_f}(V_{\sigma(0)}(0) + \gamma\zeta).\end{aligned} \tag{21}$$

From (10) and (15), we have

$$V_{\sigma(t)}(t) \ge \phi_1 x^T(t)\rho. \tag{22}$$

$$\begin{aligned}V_{\sigma(0)}(0) & \le \phi_2 x^T(0)\varsigma + \iota e^{\lambda\iota}\phi_3 \sup_{-\iota\le\theta\le 0}\{f^T(x(\theta))\varsigma\} + \iota^2 e^{\lambda\iota}\phi_4 \sup_{-\iota\le\theta\le 0}\{f^T(x(\theta))\varsigma\} \\ & \le (\phi_2 + m_2\iota e^{\lambda\iota}\phi_3 + m_2\iota^2 e^{\lambda\iota}\phi_4) \sup_{-\iota\le\theta\le 0}\{(x^T(\theta))\varsigma\} \\ & \le (\phi_2 + m_2 v e^{\lambda\iota}\phi_3 + m_2\iota^2 e^{\lambda\iota}\phi_4)\end{aligned} \tag{23}$$

From (21)-(23), we obtain

$$x^T(t)\rho \le \tfrac{1}{\phi_1}e^{(\lambda+\frac{ln\mu}{T_\alpha})T_f}(\phi_2 + m_2\iota e^{\lambda\iota}\phi_3 + m_2\iota^2 e^{\lambda\iota}\phi_4 + \lambda\zeta). \tag{24}$$

Substituting (13) into (24), one has

$$x^T(t)\rho < 1. \tag{25}$$

According to Definition 2.7, we conclude that the system (4) is FTB with respect to $(\varsigma, \rho, T_f, \zeta, \sigma(t))$. □

Remark 3.2. *For positive switched nonlinear system (4), a new nonlinear Lyapunov-Krasovskii functional (15) is constructed, which plays an important role in the proof procedure.*

The following theorem gives sufficient conditions of guaranteed cost finite-time boundedness for system (4) with ADT.

Theorem 3.3. *Consider the positive switched nonlinear system (4), for a given time constant T_f and vectors $\varsigma \succ \rho \succ 0$ and $R_1 \succ 0$, if there exist a set of positive vectors ν_p, υ_p, ϑ_p, $p \in \underline{N}$ and positive constants ϕ_1, ϕ_2, ϕ_3, ϕ_4, λ and γ, and such that the following inequalities hold:*

$$\Psi_p = diag\{\psi_{p1}, \psi_{p2}, \ldots \psi_{pn}, \psi'_{p1}, \psi'_{p2}, \ldots \psi'_{pn}\} \le 0 \tag{26}$$

$$\phi_1\rho < \nu_p < \phi_2\varsigma, \upsilon_p < \phi_3\varsigma, \vartheta_p < \phi_4\varsigma \tag{27}$$

$$\phi_2 b_{pr}^T \bar{d}_{4r}\varsigma < \gamma \mathbf{1_n} \tag{28}$$

$$\phi_2 + m_2\iota e^{\lambda\iota}\phi_3 + m_2\iota^2 e^{\lambda\iota}\phi_4 + \lambda\zeta < \phi_1 e^{-\lambda T_f} \tag{29}$$

where
$\psi_{pr} = m_2 a_{pr}^T \bar{d}_{1r}\nu_{pr} + m_2\upsilon_{pr} + m_2\iota\vartheta_{pr} - \lambda_p\nu_{pr} + R_{1r}$
$\psi'_{pr} = a_{dpr}^T \bar{d}_{2r}\nu_{pr} - (1-h)\upsilon_{pr}$
$\lambda = \max_{p\in \underline{N}}\{\lambda_p\}, r \in \underline{n} = \{1, 2, \ldots, n\}.$
$a_{pr}(a_{dpr}, \bar{d}_{1r}, \bar{d}_{2r}, \bar{d}_{4r})$ *represents the* r*th column vector of the matrix* $A_p(A_{dp}, \bar{D}_1, \bar{D}_2, \bar{D}_4)$, R_{1r} *denotes the rth element of the vector* R_1, $\nu_p = [\nu_{p1}, \nu_{p2}, \ldots, \nu_{pn}]^T$, $\upsilon_p = [\upsilon_{p1}, \upsilon_{p2}, \ldots, \upsilon_{pn}]^T$, *and* $\vartheta_p = [\vartheta_{p1}, \vartheta_{p2}, \ldots, \vartheta_{pn}]^T$, ν_{pr}, υ_{pr} *and* ϑ_{pr} *represent the* r*th elements of the vectors* ν_p, υ_p *and* θ_p*, respectively, then under the following ADT scheme*

$$T_\alpha > T_\alpha^* = max\{\frac{T_f ln\mu}{ln(\phi_1 e^{-\lambda T_f}) - ln(\phi_2 + m_2\iota e^{\lambda\iota}\phi_3 + m_2\iota^2 e^{\lambda\iota}\phi_4 + \gamma\zeta)}, \frac{ln\mu}{\lambda}\}, \tag{30}$$

the system (4) is GCFTB with respect to $(\varsigma, \rho, T_f, \zeta, \sigma(t))$*, where* $\mu \geq 1$ *satisfies*

$$\nu_p \leq \mu\nu_q, \upsilon_p \leq \mu\upsilon_q, \vartheta_p \leq \mu\vartheta_q, \forall p, q \in \underline{N}. \tag{31}$$

and the guaranteed cost value of system (4) is given by

$$J = \int_0^{T_f} x^T(s)R_1 ds \leq J^* = e^{2\lambda T_f}(\phi_2 + m_2\iota e^{\lambda\iota}\phi_3 + m_2\iota^2 e^{\lambda\iota}\phi_4 + \gamma\zeta). \tag{32}$$

Proof. Adding $x^T(t)R_1$ to both sides of (17), we get

$$\begin{aligned}\dot{V}_{\sigma(t)}(t) - \lambda_p V_{\sigma(t)}(t) + x^T(t)R_1 \leq x^T(t)(m_2 A_p^T \bar{D}_1\nu_p - \lambda_p\nu_p + m_2\upsilon_p + m_2\iota\vartheta_p + R_1) \\ +f^T(x(t-d(t))(A_{dp}^T\bar{D}_2 - (1-h)\upsilon_p) + \gamma\|w(t)\|\end{aligned} \tag{33}$$

Substituting (26) into (33) yields

$$\dot{V}_{\sigma(t)}(t) - \lambda_p V_{\sigma(t)}(t) + x^T(t)R_1 - \gamma\|w(t)\| \leq 0 \tag{34}$$

It implies

$$\dot{V}_{\sigma(t)}(t) - \lambda_p V_{\sigma(t)}(t) - \gamma\|w(t)\| \leq 0 \tag{35}$$

From lemma 3.1, we conclude the system (4) is FTB. Next, we will give the guaranteed cost value of system (4).

Denoting $\triangledown(t) = \gamma\|w(t)\| - x^T(t)R_1$ and integrating both sides of (34) from $[t_k, t)$, for $t \in [t_k, t_{k+1})$ it gives rise to

$$V_{\sigma(t)}(t) \leq e^{\lambda_p(t-t_k)}V_{\sigma(t_k)}(t_k) + \int_{t_k}^t e^{\lambda_p(t-s)} \triangledown(s)ds. \tag{36}$$

where $\lambda = \max_{p\in\underline{N}}\{\lambda_p\}$.

Similarly to the proof process of (21), for any $t \in [0, T_f]$, we can obtain

$$V_{\sigma(t)}(t) \leq \mu^{N_{\sigma(t)}(0,t)}e^{\lambda t}V_{\sigma(0)}(0) + \int_0^t e^{\lambda(t-s)} \triangledown(s)ds. \tag{37}$$

From (37), we can get

$$\int_0^t \mu^{N_{\sigma(t)}(s,t)}e^{\lambda(t-s)}x^T(s)R_1 ds \leq \mu^{N_{\sigma(t)}(0,t)}e^{\lambda t}V_{\sigma(0)}(0) + \gamma\int_0^t \mu^{N_{\sigma(t)}(s,t)}e^{\lambda(t-s)}\|w(s)\|ds \tag{38}$$

Multiplying both sides of (38) by $\mu^{-N_{\sigma(t)}(0,t)}$ leads to

$$\int_0^t \mu^{-N_{\sigma(t)}(0,s)}e^{\lambda(t-s)}x^T(s)R_1 ds \leq e^{\lambda T_f}V_{\sigma(0)}(0) + \gamma\int_0^t \mu^{-N_{\sigma(t)}(0,s)}e^{\lambda(t-s)}\|w(s)\|ds \tag{39}$$

Noting that $N_{\sigma(t)}(0,s) \le \frac{s}{T_f}$ and $T_\alpha > \frac{ln\mu}{\lambda}$, we obtain that $0 < N_{\sigma(t)}(0,s) \le \frac{s}{T_f} \le \frac{\lambda s}{ln\mu}$, that is $e^{-\lambda s} \le \mu^{-N_{\sigma(t)}(0,s)} < 1$. Then (39) can be turned into

$$\begin{aligned}\int_0^t e^{-\lambda s} e^{\lambda(t-s)} x^T(s)R_1 ds &\le \int_0^t \mu^{-N_{\sigma(t)}(0,s)} e^{\lambda(t-s)} x^T(s)R_1 ds \\ &\le e^{\lambda T_f} V_{\sigma(0)}(0) + \gamma \int_0^t e^{\lambda(t-s)} \|w(s)\| ds\end{aligned} \tag{40}$$

Let $t = T_f$, then multiplying both sides of (40) by $e^{-\lambda T_f}$ leads to

$$\begin{aligned}\int_0^t e^{-2\lambda s} x^T(s)R_1 ds &\le V_{\sigma(0)}(0) + \gamma \int_0^{T_f} e^{-\lambda s} \|w(s)\| ds \\ &\le V_{\sigma(0)}(0) + \gamma \int_0^{T_f} \|w(s)\| ds\end{aligned} \tag{41}$$

Substituting (2) into (41) yields

$$\int_0^t e^{-2\lambda T_f} x^T(s)R_1 ds \le V_{\sigma(0)}(0) + \gamma\zeta. \tag{42}$$

which can be rewritten as

$$\int_0^t x^T(s)R_1 ds \le e^{2\lambda T_f}(V_{\sigma(0)}(0) + \gamma\zeta). \tag{43}$$

Substituting (23) into (43), the guaranteed cost value of system (4) is given by

$$J = \int_0^{T_f} x^T(s)R_1 ds \le J^* = e^{2\lambda T_f}(\phi_2 + m_2 \iota e^{\lambda \iota}\phi_3 + m_2 \iota^2 e^{\lambda\iota}\phi_4 + \gamma\zeta). \tag{44}$$

Therefore, according to Definition 2.11, we can conclude that the system (4) is GCGTB. Thus, the proof is completed. □

Remark 3.4. *From $D_1 \in [\underline{D}_1, \bar{D}_1]$, $D_2 \in [\underline{D}_2, \bar{D}_2]$ and $D_4 \in [\underline{D}_4, \bar{D}_4]$, which means $D_1 A_p \in [\underline{D}_1 A_p, \bar{D}_1 A_p]$, $D_2 A_{dp} \in [\underline{D}_2 A_{dp}, \bar{D}_2 A_{dp}]$ and $D_4 B_p \in [\underline{D}_4 B_p, \bar{D}_4 B_p]$, the obtained results in Theorem 3.3 should be extended to positive switched nonlinear systems with interval uncertainties.*

3.2 Guaranteed cost finite-time controller design

In this subsection, we are concerned with the guaranteed cost finite-time controller design of positive switched nonlinear system (1). Under the controller $u(t) = K_{\sigma(t)} y(t)$ the corresponding closed-loop system is given by

$$\begin{cases}\dot{x}(t) = (D_1 A_{\sigma(t)} + D_3 G_{\sigma(t)} K_{\sigma(t)} C_{\sigma(t)}) f(x(t)) + D_2 A_{d\sigma(t)} f(x(t-d(t))) + D_4 B_{\sigma(t)} w(t) \\ x(\theta) = \varphi(\theta), \theta \in [-\iota, 0]\end{cases} \tag{45}$$

By Lemma 2.8, to guarantee the positivity of system (45), $D_1 A_p + D_3 G_p K_p C_p$ should be Metzler matrices, $\forall p \in \underline{N}$. The following Theorem 3.5 gives some sufficient conditions to guarantee that closed-loop system (45) is GCFTB.

Theorem 3.5 ([14]). *Consider the positive switched nonlinear system (45), for a given time constant T_f, vectors $\varsigma \succ \rho \succ 0$, $R_1 \succ 0$ and $R_2 \succ 0$, if there exist a set of positive vectors ν_p, υ_p, ϑ_p, $p \in \underline{N}$ and positive constants ϕ_1, ϕ_2, ϕ_3, ϕ_4, λ and γ, such that (27)-(29) and the following inequalities hold:*

$$\underline{D}_1 A_p + \underline{D}_3 G_p K_p C_p \text{ and } \bar{D}_1 A_p + \bar{D}_3 G_p K_p C_p \text{ are Metzler matrices}, \; K_p \ge 0. \tag{46}$$

$$\Psi_p = diag\{\psi_{p1}, \psi_{p2}, \ldots \psi_{pn}, \psi'_{p1}, \psi'_{p2}, \ldots \psi'_{pn}\} \le 0 \tag{47}$$

where

$\psi_{pr} = m_2 a_{pr}^T \bar{d}_{1r} \nu_{pr} + m_2 \upsilon_{pr} + m_2 \iota \vartheta_{pr} - \lambda_p \nu_{pr} + R_{1r} + f_{pr}$

$\psi'_{pr} = a_{dpr}^T \bar{d}_{2r} \nu_{pr} - (1-h)\upsilon_{pr}$

$\lambda = \max_{p \in \underline{N}} \{\lambda_p\}$, $r \in \underline{n} = \{1, 2, \ldots, n\}$.
$f_p = m_2 C_p^T K_p^T (G_p^T \bar{D}_3 v_p + R_2)$, $f_{pr}(R_{1r})$ *represents the rth element of vector* $f_p(R_1)$, $a_{pr}(a_{dpr}, \bar{d}_{1r}, \bar{d}_{2r})$ *represents the rth column vector of the matrix* $A_p(A_{dp}, \bar{D}_1, \bar{D}_2)$. $\nu_p = [\nu_{p1}, \nu_{p2}, \ldots, \nu_{pn}]^T$, $\upsilon_p = [\upsilon_{p1}, \upsilon_{p2}, \ldots, \upsilon_{pn}]^T$, *and* $\vartheta_p = [\vartheta_{p1}, \vartheta_{p2}, \ldots, \vartheta_{pn}]^T$, ν_{pr}, υ_{pr} *and* ϑ_{pr} *represent the rth elements of the vectors* ν_p, υ_p *and* θ_p, *respectively,* $\mu \geq 1$ *satisfies (31), then under the following ADT scheme (5), the resulting closed-loop system (45) is GCFTB with respect to* $(\varsigma, \rho, T_f, \zeta, \sigma(t))$ *and the guaranteed cost value of system (45) is given by*

$$\begin{aligned} J &= \int_0^{T_f} [x^T(t) R_1 + f^T(x(s)) C_{\sigma(t)}^T K_{\sigma(t)}^T R_2] ds \\ &\leq J^* = e^{2\lambda T_f} (\phi_2 + m_2 \iota e^{\lambda \iota} \phi_3 + m_2 \iota^2 e^{\lambda \iota} \phi_4 + \gamma \zeta). \end{aligned} \tag{48}$$

Proof. From Lemma 2.8 and (46), $\underline{D}_1 A_p + \underline{D}_3 G_p K_p C_p \leq D_1 A_p + D_3 G_p K_p C_p \leq \bar{D}_1 A_p + \bar{D}_3 G_p K_p C_p$ are held, for each $p \in \underline{N}$. It means that $D_1 A_p + D_3 G_p K_p C_p$ is Metzler matrice. According to Lemma 2.9, the system (45) is positive. Replacing $D_1 A_p$ in (26) with $D_1 A_p + D_3 G_p K_p C_p$ and letting $f_p = m_2 C_p^T K_p^T (G_p^T \bar{D}_3 v_p + R_2)$, similar to Theorem 3.3, we easily obtain that the resulting closed-loop system (45) is GCFTB with respect to $(\varsigma, \rho, T_f, \zeta, \sigma(t))$ and the guaranteed cost value is given by (48).

The proof is completed. □

Remark 3.6. *In Theorem 3.5, the gain matrix* $K_p \geq 0$, $\forall p \in \underline{N}$ *is used. Naturally, when* $K_p \leq 0$, *we only replace (46) by the following condition*

$$\underline{D}_1 A_p + \bar{D}_3 G_p K_p C_p \ and \ \bar{D}_1 A_p + \underline{D}_3 G_p K_p C_p \ are \ Metzler \ matrices, K_p \leq 0. \tag{49}$$

Following the proof line of Theorem 3.5, we can also conclude that the closed-loop system (45) is GCFTB with respect to $(\varsigma, \rho, T_f, \zeta, \sigma(t))$.

Next, an algorithm is presented to obtain the feedback gain matrices $K_p, p \in \underline{N}$.

Algorithm 3.7. **Step 1.** *By adjusting the parameters* λ_p *and solving (27)-(29), (31) and (47) via linear programming, positive vectors* ν_p, υ_p , ϑ_p *and* f_p *can be obtained.*
Step 2. *Substituting* v_p *and* f_p *into* $f_p = m_2 C_p^T K_p^T (G_p^T \bar{D}_3 v_p + R_2)$, K_p *can be obtained.*
Step 3. *If the gain* K_p *satisfy (46) or (49), then* K_p *are admissible. Otherwise, return to Step 1.*

4 Numerical example

Example 4.1. *Consider the positive switched nonlinear systems (1) with the parameters as follows:*

$$\underline{D}_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \bar{D}_1 = \begin{bmatrix} 0.12 & 0 \\ 0 & 0.15 \end{bmatrix}, \underline{D}_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.1 \end{bmatrix}, \bar{D}_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.8 \end{bmatrix},$$

$$\underline{D}_3 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \bar{D}_3 = \begin{bmatrix} 0.12 & 0 \\ 0 & 0.15 \end{bmatrix}, \underline{D}_4 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.1 \end{bmatrix}, \bar{D}_4 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.8 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} -4 & 1 \\ 2 & -3 \end{bmatrix}, A_{d1} = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.2 \end{bmatrix}, B_1 = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}, G_1 = \begin{bmatrix} 0.3 & 0.4 \\ 0.1 & 0.5 \end{bmatrix}, C_1 = \begin{bmatrix} 0.3 & 0.2 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -5 & 2 \\ 1 & -4 \end{bmatrix}, A_{d2} = \begin{bmatrix} 0.2 & 0.3 \\ 0.1 & 0.2 \end{bmatrix}, B_2 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, G_2 = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.3 \end{bmatrix}, C_2 = \begin{bmatrix} 0.1 & 0.3 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix}, R_2 = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, \rho = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, \varsigma = \begin{bmatrix} 2 \\ 3 \end{bmatrix},$$

Let $d(t) = 0.1 + 0.1cos(t)$, *then we get* $\iota = 0.2$, $h = 0.1$. $f_i(x_i(t)) = x_i(t) + \frac{x_i(t)}{x_i(t)+1}$, *we obtain* $m_1 = 1, m_2 = 2$. *Choosing* $\gamma = 1$, $\lambda = 0.1$, $\mu = 1.1$, $w(t) = 0.1e^{-0.4t}cos(0.3t)$, $\zeta = 0.02$. *Solving the inequalities in Theorem*

3.5by linear programming, we have

$$\nu_1 = \begin{bmatrix} 1.3524 \\ 1.1139 \end{bmatrix}, \upsilon_1 = \begin{bmatrix} 0.3122 \\ 0.2298 \end{bmatrix}, \vartheta_1 = \begin{bmatrix} 0.0054 \\ 0.0034 \end{bmatrix},$$

$$\nu_2 = \begin{bmatrix} 1.2452 \\ 1.1757 \end{bmatrix}, \upsilon_2 = \begin{bmatrix} 0.3104 \\ 0.2506 \end{bmatrix}, \vartheta_2 = \begin{bmatrix} 0.0057 \\ 0.0031 \end{bmatrix},$$

$$f_1 = \begin{bmatrix} 0.2234 \\ 0.2245 \end{bmatrix}, f_2 = \begin{bmatrix} 0.2688 \\ 0.2924 \end{bmatrix},$$

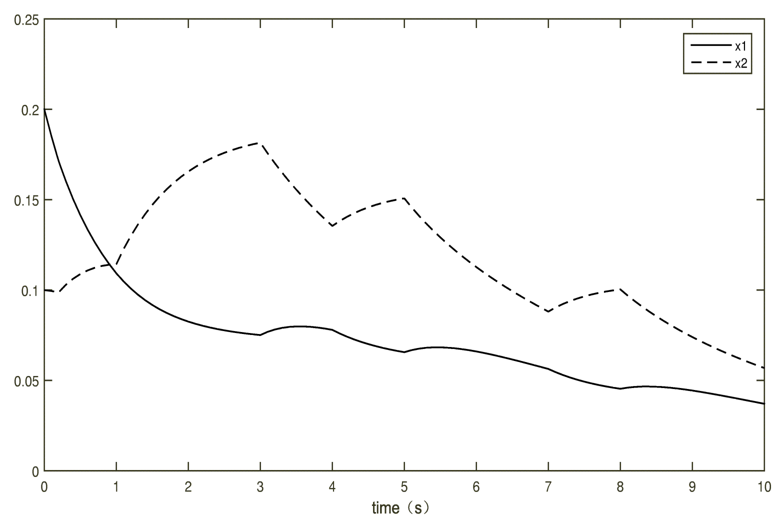
By $f_p = m_2 C_p^T K_p^T (G_p^T \bar{D}_3 v_p + R_2)$, *we obtain*

$$K_1 = \begin{bmatrix} 0.2202 \\ 0.2133 \end{bmatrix}, K_2 = \begin{bmatrix} 0.3766 \\ 0.5817 \end{bmatrix},$$

It is easy to confirm that (46) is satisfied. Then, according to (30), we get $T_\alpha^* = 1.4$.

The simulation results are shown in Figs. 1-3, where the initial conditions of system (1) are $x(0) = [0.2, 0.1]^T$, *which meet the condition* $x^T(t)\rho < 1$. *The state trajectory of the closed-loop system is shown in Fig. 1. The switching signal* $\sigma(t)$ *is depicted in Fig. 2. Fig. 3 plots the evolution of* $x(t)\rho$, *which implies that the corresponding closed-loop system is GCFTB with respect to* $(\varsigma, \rho, T_f, \zeta, \sigma(t))$, *and the cost value* $J^\star = 20.8$, *which can be obtained by (48).*

Fig. 1. State trajectories of closed-loop system (1).



Example 4.2. *In [30], a price dynamic model described by positive switched linear systems was presented. But, in fact, the demand and supply functions are nonlinear. So, it is more suitable to describe the price dynamic model by positive switched nonlinear systems. Consider the parameters as follows:*

$$\underline{D}_1 = \begin{bmatrix} 0.7 & 0 \\ 0 & 1.4 \end{bmatrix}, \underline{D}_2 = \begin{bmatrix} 0.8 & 0 \\ 0 & 1.0 \end{bmatrix}, \underline{D}_3 = \begin{bmatrix} 0.6 & 0 \\ 0 & 1.0 \end{bmatrix},$$

$$\bar{D}_1 = \begin{bmatrix} 0.9 & 0 \\ 0 & 1.6 \end{bmatrix}, \bar{D}_2 = \begin{bmatrix} 1.0 & 0 \\ 0 & 1.1 \end{bmatrix} \bar{D}_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1.2 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} 0.2 & 0.5 \\ 0.4 & -3.5 \end{bmatrix}, A_{d1} = \begin{bmatrix} 0.02 & 0.02 \\ 0.04 & 0.10 \end{bmatrix}, G_1 = \begin{bmatrix} 0.3 & 0.4 \\ 0.1 & 0.5 \end{bmatrix}, C_1 = \begin{bmatrix} 0.3 & 0.2 \end{bmatrix},$$

Fig. 2. Switching signal of system (1) with ADT.

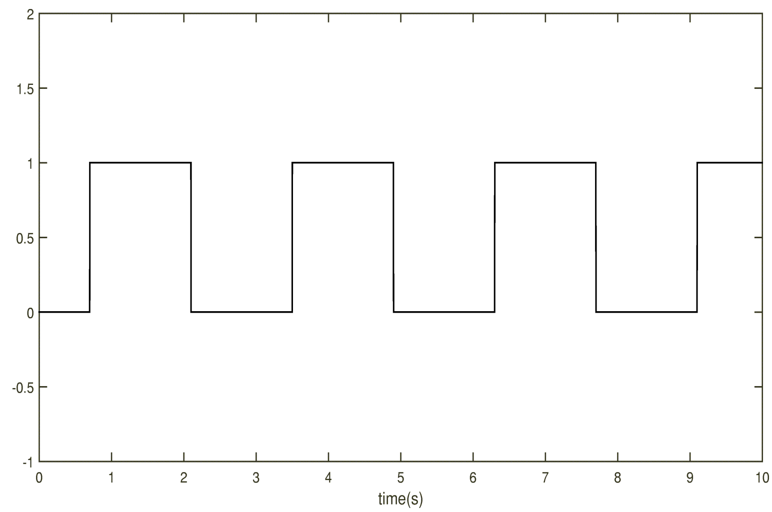
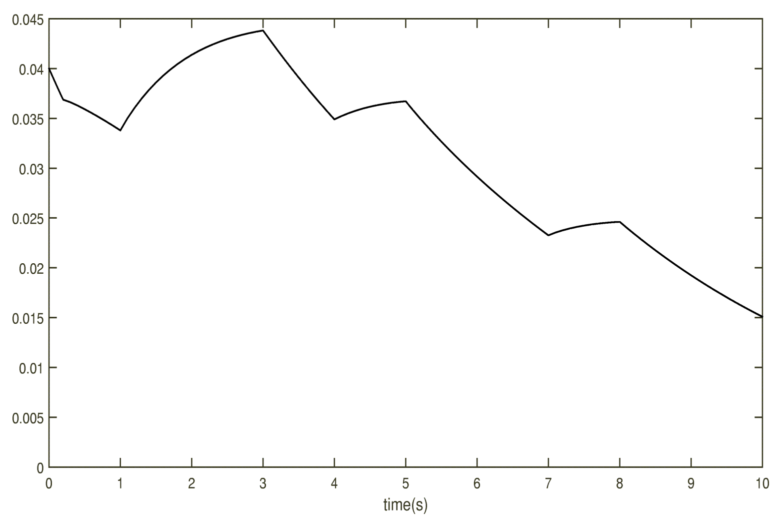


Fig. 3. The evolution of $x^T(t)\rho$ of system (1).



$$A_2 = \begin{bmatrix} -3.3 & 0 \\ 0.8 & 0.2 \end{bmatrix}, A_{d2} = \begin{bmatrix} 0.2 & 0.3 \\ 0.1 & 0.2 \end{bmatrix}, G_2 = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.3 \end{bmatrix}, C_2 = \begin{bmatrix} 0.1 & 0.3 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix}, R_2 = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, \rho = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, \varsigma = \begin{bmatrix} 2 \\ 3 \end{bmatrix},$$

Let $d(t) = 0.1 + 0.1sin^2t$, *then we get* $\iota = 0.2$, $h = 0.1$. $f_i(x_i(t)) = x_i(t) + \frac{x_i(t)}{x_i(t)+1}$, *we obtain* $m_1 = 1, m_2 = 2$. *Choosing* $\gamma = 1$, $\lambda = 0.1$, $\mu = 1.1$. *Solving the inequalities in Theorem 3.5 by linear programming, we have*

$$\nu_1 = \begin{bmatrix} 0.1517 \\ 1.1850 \end{bmatrix}, \upsilon_1 = \begin{bmatrix} 1.7248 \\ 0.7710 \end{bmatrix}, \vartheta_1 = \begin{bmatrix} 0.7634 \\ 0.0248 \end{bmatrix},$$

$$\nu_2 = \begin{bmatrix} 0.8870 \\ 0.0456 \end{bmatrix}, \upsilon_2 = \begin{bmatrix} 2.0427 \\ 2.0002 \end{bmatrix}, \vartheta_2 = \begin{bmatrix} 0.4074 \\ 0.0043 \end{bmatrix},$$

$$f_1 = \begin{bmatrix} 1.2769 \\ 0.6995 \end{bmatrix}, f_2 = \begin{bmatrix} 0.3383 \\ 2.0012 \end{bmatrix},$$

By $f_p = m_2 C_p^T K_p^T (G_p^T \bar{D}_3 v_p + R_2)$, *we obtain*

$$K_1 = \begin{bmatrix} 0.6543 \\ 0.5724 \end{bmatrix}, K_2 = \begin{bmatrix} 0.4652 \\ 0.3874 \end{bmatrix},$$

It is easy to confirm that (46) is satisfied. Then, according to (30), we get $T_\alpha^* = 1.2$.

The simulation results are shown in Figs. 4-6, where the initial conditions of system (1) are $x(0) = [0.2, 0.1]^T$, *which meet the condition* $x^T(t)\rho < 1$. *The state trajectory of the closed-loop system is shown in Fig. 4. The switching signal* $\sigma(t)$ *is depicted in Fig. 5. Fig. 6 plots the evolution of* $x(t)\rho$, *which implies that the corresponding closed-loop system is GCFTB with respect to* $(\varsigma, \rho, T_f, \zeta, \sigma(t))$, *and the cost value* $J^\star = 12.6$, *which can be obtained by (48).*

Fig. 4. State trajectories of closed-loop system (1).

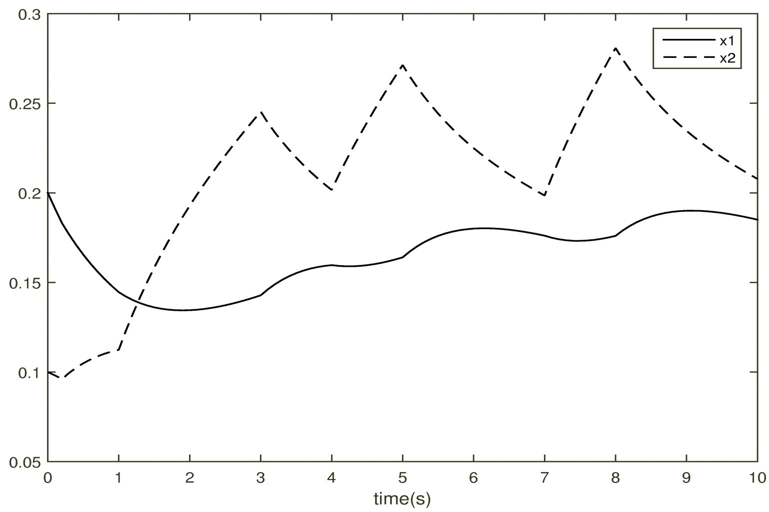


Fig. 5. Switching signal of system (1) with ADT.

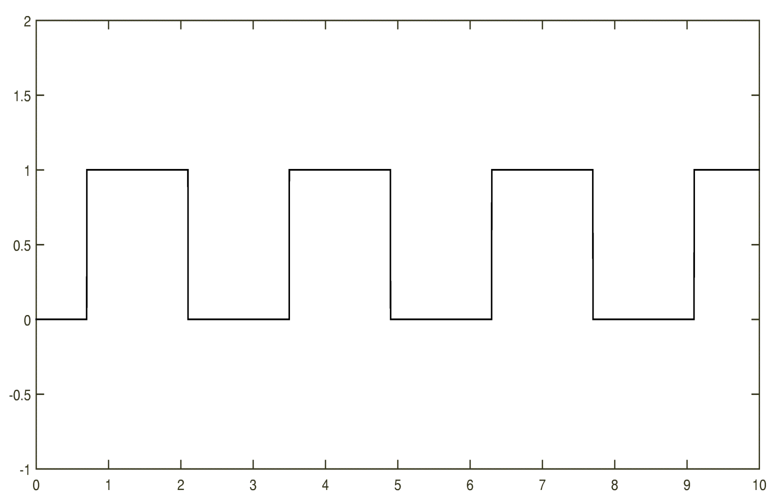
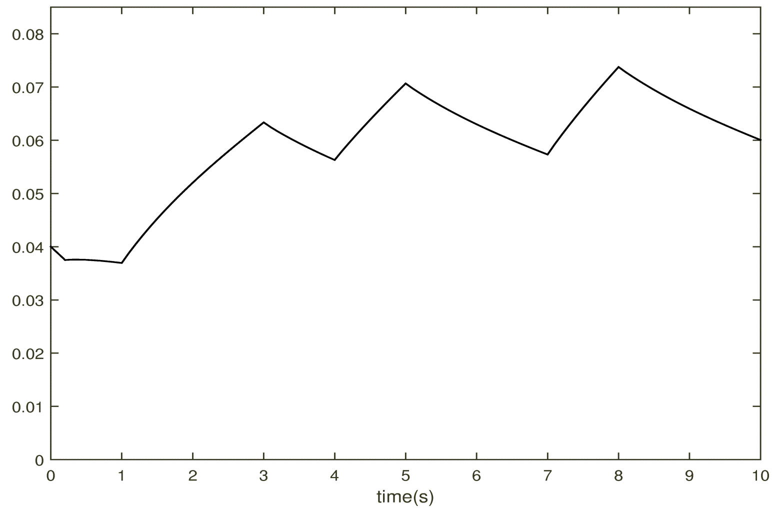


Fig. 6. The evolution of $x^T(t)\rho$ of system (1).



5 Conclusions

In this paper, we have considered the issue of guaranteed cost finite-time control for positive switched nonlinear systems with D-perturbation and time-varying delay. Based on the ADT approach, an output feedback controller is constructed to guarantee that the closed-loop system is GCFTB. Finally, two examples are given to illustrate the effectiveness of the proposed method.

It is worth noting that there are some interesting yet challenging issues like mode-dependent average dwell time approach (which is more applicable and less conservative than ADT), asynchronously switching approach and cyclic switching approach. So, how to apply these approaches to positive switched nonlinear systems is our further work.

Conflict of interests
The authors declare that there is no conflict of interests regarding the publication of this paper.

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