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Extinction of a two species non-autonomous competitive system with Beddington-DeAngelis functional response and the effect of toxic substances

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Abstract: A two species non-autonomous competitive phytoplankton system with Beddington-DeAngelis functional response and the effect of toxic substances is proposed and studied in this paper. Sufficient conditions which guarantee the extinction of a species and global attractivity of the other one are obtained. The results obtained here generalize the main results of Li and Chen [Extinction in two dimensional nonautonomous Lotka-Volterra systems with the effect of toxic substances, Appl. Math. Comput. 182(2006)684-690]. Numeric simulations are carried out to show the feasibility of our results.

Keywords: Extinction, Competition, Functional response, Phytoplankton system

MSC: 34D23, 92D25, 34D20, 34D40

1 Introduction

Given a function $g(t)$, let g_L and g_M denote $\inf_{-\infty < t < \infty} g(t)$ and $\sup_{-\infty < t < \infty} g(t)$, respectively.

The aim of this paper is to investigate the extinction property of the following two species non-autonomous competitive system with Beddington-DeAngelis functional response and the effect of toxic substances

$$\begin{aligned}\dot{x}_1(t) &= x_1(t) \left[r_1(t) - a_1(t)x_1(t) - \frac{b_1(t)x_2(t)}{d_1(t) + e_1(t)x_1(t) + f_1(t)x_2(t)} - c_1(t)x_1(t)x_2(t) \right], \\ \dot{x}_2(t) &= x_2(t) \left[r_2(t) - \frac{b_2(t)x_1(t)}{d_2(t) + e_2(t)x_1(t) + f_2(t)x_2(t)} - a_2(t)x_2(t) - c_2(t)x_1(t)x_2(t) \right],\end{aligned}\quad (1)$$

where $r_i(t)$, $a_i(t)$, $b_i(t)$, $d_i(t)$, $i = 1, 2$, $c_i(t)$ are assumed to be continuous and bounded above and below by positive constants, $e_i(t)$, $f_i(t)$, $i = 1, 2$ are all non-negative continuous functions bounded above by positive constants. $x_1(t)$, $x_2(t)$ are population density of species x_1 and x_2 at time t , respectively. $r_i(t)$, $i = 1, 2$ are the intrinsic growth rates of species; a_i ($i = 1, 2$) are the rates of intraspecific competition of the first and second species, respectively. Here we make the following assumptions:

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(1) The interspecific competition between two species takes the Beddington-DeAngelis functional response type

$$\frac{b_1(t)x_2(t)}{d_1(t) + e_1(t)x_1(t) + f_1(t)x_2(t)}, \quad \frac{b_2(t)x_1(t)}{d_2(t) + e_2(t)x_1(t) + f_2(t)x_2(t)},$$

respectively;

(2) The terms $c_1(t)x_1(t)x_2(t)$ and $c_2x_1(t)x_2(t)$ denote the effect of toxic substances, each species produces a substance toxic to the other, only when the other is present.

We also consider the extinction property of the following two species non-autonomous competitive phytoplankton system with Beddington-DeAngelis functional response

$$\begin{aligned}\dot{x}_1(t) &= x_1(t) \left[r_1(t) - a_1(t)x_1(t) - \frac{b_1(t)x_2(t)}{d_1(t) + e_1(t)x_1(t) + f_1(t)x_2(t)} - c_1(t)x_1(t)x_2(t) \right], \\ \dot{x}_2(t) &= x_2(t) \left[r_2(t) - \frac{b_2(t)x_1(t)}{d_2(t) + e_2(t)x_1(t) + f_2(t)x_2(t)} - a_2(t)x_2(t) \right],\end{aligned}\quad (2)$$

where all the coefficients have the same meaning as that of system (1). However, we assume that the second species could produce toxic, while the first one is non-toxic produce.

Traditional two species Lotka-Volterra competition model takes the form:

$$\begin{aligned}\dot{x}_1(t) &= x_1(t)[r_1 - a_1x_1(t) - b_1x_2(t)], \\ \dot{x}_2(t) &= x_2(t)[r_2 - a_2x_2(t) - b_2x_1(t)],\end{aligned}\quad (3)$$

where $r_i, a_i, b_i, i = 1, 2$ are all positive constants, and $x_1(t), x_2(t)$ are population density of species x_1 and x_2 at time t , respectively. $r_i, i = 1, 2$ are the intrinsic growth rates of species; $a_i, i = 1, 2$ are the rates of intraspecific competition of the first and second species, respectively; $b_i, i = 1, 2$ are the rates of interspecific competition of the first and second species, respectively. This model is the foundation stone in the study of competition model. Depending on the relationship of the coefficients, the system could have three different kinds of dynamics: (1) a unique positive equilibrium which is globally attractive; (2) bistable; the positive equilibrium is unstable, and the stability of the boundary equilibrium is dependent on the initial conditions; (3) the boundary equilibrium is globally stable, which means the extinction of the partial species.

Based on the Lotka-Volterra model (3), Chattopadhyay [2] proposed a two species competition model, each species produces a substance toxic to the other only when the other is present. The model takes the form:

$$\begin{aligned}\dot{x}_1(t) &= x_1(t)[r_1 - a_1x_1(t) - a_2x_2(t) - d_1x_1(t)x_2(t)], \\ \dot{x}_2(t) &= x_2(t)[r_2 - b_1x_2(t) - b_2x_2(t) - d_2x_1(t)x_2(t)].\end{aligned}\quad (4)$$

By constructing some suitable Lyapunov function, he obtained sufficient conditions which ensure the global stability of the unique positive equilibrium. By using the iterative method, Li and Chen [6] showed that if the system without toxic substance admits the unique positive equilibrium, then system (4) also admits a unique positive equilibrium, in this case, the toxic substance term has no influence on the stability of the positive equilibrium.

Li and Chen [4] argued that with the change of the circumstance, the coefficients of the system should be time-varying, and they studied the nonautonomous case of system (4), i.e.,

$$\begin{aligned}\dot{x}_1(t) &= x_1(t)[r_1(t) - a_1(t)x_1(t) - b_2(t)x_2(t) - c_1(t)x_1(t)x_2(t)], \\ \dot{x}_2(t) &= x_2(t)[r_2(t) - b_2(t)x_2(t) - a_2(t)x_2(t) - c_2(t)x_1(t)x_2(t)].\end{aligned}\quad (5)$$

By applying the fluctuation theorem, they obtained a set of sufficient conditions which guarantee the extinction of the second species and the globally attractive of the first species.

Solé et al [16] and Bandyopadhyay [14] considered a Lotka-Volterra type of model for two interacting phytoplankton species, where one species could produce toxic, while the other one is non-toxic produce. The model takes the form

$$\begin{aligned}\dot{x}_1(t) &= x_1(t)[r_1 - a_1x_1(t) - a_2x_2(t) - d_1x_1(t)x_2(t)], \\ \dot{x}_2(t) &= x_2(t)[r_2 - b_1x_2(t) - b_2x_2(t)].\end{aligned}\quad (6)$$

By constructing some suitable Lyapunov function, Bandyopadhyay [14] obtained a set of sufficient conditions which ensure the global attractivity of the positive equilibrium. For more work on competitive system with toxic substance, one could refer to [1-20,35-38] and the references cited therein.

On the other hand, based on the traditional Lotka-Volterra competition model, some scholars argued that the more appropriate competition model should with nonlinear inter-inhibition terms. Wang, Liu and Li [23] proposed the following two species competition model,

$$\begin{aligned}\dot{x}_1(t) &= x_1(t) \left[r_1(t) - a_1(t)x_1(t) - \frac{b_1(t)x_2(t)}{1+x_2(t)} \right], \\ \dot{x}_2(t) &= x_2(t) \left[r_2(t) - \frac{b_2(t)x_1(t)}{1+x_1(t)} - a_2(t)x_2(t) \right].\end{aligned}\quad (7)$$

In this system, the inter-inhibition terms take the form $\frac{b_1(t)x_2(t)}{1+x_2(t)}$ and $\frac{b_2(t)x_1(t)}{1+x_1(t)}$, respectively, which is of Holling II type. By using differential inequality, the module containment theorem and the Lyapunov function, the authors obtained sufficient conditions which ensure the existence and global asymptotic stability of positive almost-periodic solutions of system (7).

Corresponding to system (7), several scholars [24, 25] investigated the dynamic behaviors of the discrete type two species competition system with nonlinear inter-inhibition terms.

$$\begin{aligned}x_1(k+1) &= x_1(k) \exp \left\{ r_1(k) - a_1(k)x_1(k) - \frac{b_1(k)x_2(k)}{1+x_2(k)} \right\}, \\ x_2(k+1) &= x_2(k) \exp \left\{ r_2(k) - \frac{b_2(k)x_1(k)}{1+x_1(k)} - a_2(k)x_2(k) \right\}.\end{aligned}\quad (8)$$

Wang and Liu [24] studied the almost-periodic solution of the system (8) and Yu [25] further incorporated the feedback control variables to the system (8) and investigated the persistent property of the system.

Recently, combining with the effect of toxic substance and the nonlinear inter-inhibition term, Yue [1] proposed the following two species discrete competitive system

$$\begin{aligned}x_1(k+1) &= x_1(k) \exp \left\{ r_1(k) - a_1(k)x_1(k) - \frac{b_1(k)x_2(k)}{1+x_2(k)} - c_1(k)x_1(k)x_2(k) \right\}, \\ x_2(k+1) &= x_2(k) \exp \left\{ r_2(k) - \frac{b_2(k)x_1(k)}{1+x_1(k)} - a_2(k)x_2(k) \right\}.\end{aligned}\quad (9)$$

By constructing some suitable Lyapunov type extinction function, the author obtained some sufficient conditions which guarantee the extinction of one of the components and the global attractivity of the other one.

It is well known that the functional response plays important role in the predator-prey model, and during the past two decades, the Beddington-DeAngelis functional response, which is a combination of the famous Holling II functional response and ratio-dependent functional response, having overcome the defect of the both functional response, is studied by many scholars, see [26-30] and the references therein.

The success of [26-30] motivated us to propose the competition system with Beddington-DeAngelis functional response, also, if we further assume that each species produces a substance toxic to the other only when the other is present, or assume that one species is toxic produce while the other one is non-toxic producing, then, we could establish the model (1) and (2), respectively. It is, to the best of the knowledge of the authors, the first time such kind of model proposed. During the last decade, many scholars ([3-5], [8], [11-13], [31-38]) investigated the extinction property of the competition system. In this paper, we still focus our attention to the extinction property of the system (1) and (2).

The aim of this paper is, by developing the analysis technique of [1, 8, 9], to investigate the extinction property of the system (1) and (2). The remaining part of this paper is organized as follows. In Section 2, we state several useful Lemmas and we state the main results in Section 3. These results are then proved in Section 4. Some examples together with their numerical simulations are presented in Section 5 to show the feasibility of our results. We give a brief discussion in the last section.

2 Lemmas

Following Lemma 2.1 is a direct corollary of Lemma 2.2 of F. Chen [10].

Lemma 2.1. *If $a > 0, b > 0$ and $\dot{x} \geq x(b - ax)$, when $t \geq 0$ and $x(0) > 0$, we have*

$$\liminf_{t \rightarrow +\infty} x(t) \geq \frac{b}{a}.$$

If $a > 0, b > 0$ and $\dot{x} \leq x(b - ax)$, when $t \geq 0$ and $x(0) > 0$, we have

$$\limsup_{t \rightarrow +\infty} x(t) \leq \frac{b}{a}.$$

Lemma 2.2. *Let $x(t) = (x_1(t), x_2(t))^T$ be any solution of system (1) (system (2)) with $x_i(t_0) > 0, i = 1, 2$, then $x_i(t) > 0, t \geq t_0$ and there exists a positive constant M_0 such that*

$$\limsup_{t \rightarrow +\infty} x_i(t) \leq M_0, \quad i = 1, 2,$$

i.e., any positive solution of system (1) are ultimately bounded above by some positive constant.

Proof. Let $x(t) = (x_1(t), x_2(t))^T$ be any solution of system (1) with $x_i(t_0) > 0, i = 1, 2$, then

$$\begin{aligned} x_1(t) &= x_1(t_0) \exp \left\{ \int_{t_0}^t \Delta_1(s) ds \right\} > 0, \\ x_2(t) &= x_2(t_0) \exp \left\{ \int_{t_0}^t \Delta_2(s) ds \right\} > 0. \end{aligned} \tag{10}$$

where

$$\begin{aligned} \Delta_1(s) &= r_1(s) - a_1(s)x_1(s) - \frac{b_1(s)x_2(s)}{d_1(s) + e_1(s)x_1(s) + f_1(s)x_2(s)} - c_1(s)x_1(s)x_2(s), \\ \Delta_2(s) &= r_2(s) - \frac{b_2(s)x_1(s)}{d_2(s) + e_2(s)x_1(s) + f_2(s)x_2(s)} - a_2(s)x_2(s) - c_2(s)x_1(s)x_2(s). \end{aligned}$$

From the first equation of system (1), we have

$$\dot{x}_1(t) \leq x_1(t)[r_1(t) - a_1(t)x_1(t)] \leq x_1(t)[r_{1M} - a_{1L}x_1(t)]. \tag{11}$$

By applying Lemma 2.1 to differential inequality (11), it follows that

$$\limsup_{t \rightarrow +\infty} x_1(t) \leq \frac{r_{1M}}{a_{1L}} \stackrel{\text{def}}{=} M_1. \tag{12}$$

Similarly to the analysis of (11) and (12), from the second equation of system (1), we have

$$\limsup_{t \rightarrow +\infty} x_2(t) \leq \frac{r_{2M}}{a_{2L}} \stackrel{\text{def}}{=} M_2. \tag{13}$$

Set $M_0 = \max\{M_1, M_2\}$, then the conclusion of Lemma 2.2 follows.

The proof for system (2) is similar to the above proof, with some minor revision, we omit the detail here. This ends the proof of Lemma 2.2. $\square$

Lemma 2.3 ([21], Fluctuation lemma). *Let $x(t)$ be a bounded differentiable function on (α, ∞) , then there exist sequences $\tau_n \rightarrow \infty, \sigma_n \rightarrow \infty$ such that*

$$\begin{aligned} (a) \quad & \dot{x}(\tau_n) \rightarrow 0 \text{ and } x(\tau_n) \rightarrow \limsup_{t \rightarrow \infty} x(t) = \bar{x} \text{ as } n \rightarrow \infty, \\ (b) \quad & \dot{x}(\sigma_n) \rightarrow 0 \text{ and } x(\sigma_n) \rightarrow \liminf_{t \rightarrow \infty} x(t) = \underline{x} \text{ as } n \rightarrow \infty. \end{aligned}$$

For the Logistic equation

$$\dot{x}_1(t) = x_1(t)(r_1(t) - a_1(t)x_1(t)). \quad (14)$$

From Lemma 2.1 of Zhao and Chen [22], we have

Lemma 2.4. Suppose that $r_1(t)$ and $a_1(t)$ are continuous functions bounded above and below by positive constants, then any positive solutions of Eq. (14) are defined on $[0, +\infty)$, bounded above and below by positive constants and globally attractive.

3 Main results

Our main results are the following Theorem 3.1-3.9.

Theorem 3.1. Assume that

$$\frac{r_{1L}}{r_{2M}} > \max \left\{ \frac{a_{1M}(d_{2M} + e_{2M}M_1 + f_{2M}M_2)}{b_{2L}}, \frac{c_{1M}}{c_{2L}}, \frac{b_{1M}}{d_{1L}a_{2L}} \right\} \quad (15)$$

holds, then the species x_2 will be driven to extinction, that is, for any positive solution $(x_1(t), x_2(t))^T$ of system (1), $x_2(t) \rightarrow 0$ as $t \rightarrow +\infty$.

Remark 3.2. The main result of Li and Chen [4] is the special case of Theorem 3.1. If we take $d_i(t) \equiv 1$, $e_i(t) = f_i(t) = 0$, $i = 1, 2$ in system (1), then system (1) is degenerate to system (5), and Theorem 3.1 is degenerate to the main result in [4]. Hence we generalize the main result of [4].

Theorem 3.3. Assume that

$$\frac{a_{1M}(d_{2M} + e_{2M}M_1 + f_{2M}M_2)}{b_{2L}} < \frac{r_{1L} - c_{1M}M_1M_2}{r_{2M}} \quad (16)$$

and

$$\frac{b_{1M}}{d_{1L}a_{2L}} < \frac{r_{1L} - c_{1M}M_1M_2}{r_{2M}} \quad (17)$$

hold, then the species x_2 will be driven to extinction, that is, for any positive solution $(x_1(t), x_2(t))^T$ of system (1), $x_2(t) \rightarrow 0$ as $t \rightarrow +\infty$.

Theorem 3.4. Assume that

$$\frac{(a_{1M} + c_{1M}M_2)(d_{2M} + e_{2M}M_1 + f_{2M}M_2)}{b_{2L}} < \frac{r_{1L}}{r_{2M}} \quad (18)$$

and

$$\frac{b_{1M}}{d_{1L}a_{2L}} < \frac{r_{1L}}{r_{2M}} \quad (19)$$

hold, then the species x_2 will be driven to extinction, that is, for any positive solution $(x_1(t), x_2(t))^T$ of system (1), $x_2(t) \rightarrow 0$ as $t \rightarrow +\infty$.

Theorem 3.5. Assume that

$$\frac{a_{1M}(d_{2M} + e_{2M}M_1 + f_{2M}M_2)}{b_{2L}} < \frac{r_{1L}}{r_{2M}} \quad (20)$$

and

$$\frac{\frac{b_{1M}}{d_{1L}} + c_{1M}M_1}{a_{2L}} < \frac{r_{1L}}{r_{2M}} \quad (21)$$

hold, then the species x_2 will be driven to extinction, that is, for any positive solution $(x_1(t), x_2(t))^T$ of system (1), $x_2(t) \rightarrow 0$ as $t \rightarrow +\infty$.

Remark 3.6. From the proof of Theorem 3.3-3.5 in Section 4, one could easily see that under the assumption of Theorem 3.3-3.5, the conclusion also holds for system (2). i.e., under the assumption of Theorem 3.3, 3.4 or 3.5, the species x_2 in system (2) will be driven to extinction.

Remark 3.7. Another interesting thing is to investigate the extinction property of species x_1 in system (1). One could easily establish some parallel results as that of Theorem 3.1-3.5 for the extinction of species x_1 , and we omit the detail here.

Theorem 3.8. Assume that the conditions of Theorem 3.1 or 3.3 or 3.4 or 3.5 hold, let $x(t) = (x_1(t), x_2(t))^T$ be any positive solution of system (1), then the species x_2 will be driven to extinction, that is, $x_2(t) \rightarrow 0$ as $t \rightarrow +\infty$, and $x_1(t) \rightarrow x_1^*(t)$ as $t \rightarrow +\infty$, where $x_1^*(t)$ is any positive solution of the system

$$\dot{x}_1(t) = x_1(t)(r_1(t) - a_1(t)x_1(t)).$$

Theorem 3.9. Assume that

$$\frac{r_{1M}}{r_{2L}} < \frac{b_{1L}}{a_{2M}(d_{1M} + e_{1M}M_1 + f_{1M}M_2)} \quad (22)$$

and

$$\frac{r_{1M}}{r_{2L}} < \frac{a_{1L}d_{2L}}{b_{2M}} \quad (23)$$

hold, then the species x_1 will be driven to extinction, that is, for any positive solution $(x_1(t), x_2(t))^T$ of system (1), $x_1(t) \rightarrow 0$ as $t \rightarrow +\infty$ and $x_2(t) \rightarrow x_2^*(t)$ as $t \rightarrow +\infty$, where $x_2^*(t)$ is any positive solution of system $\dot{x}_2(t) = x_2(t)(r_2(t) - b_2(t)x_2(t))$.

4 Proof of the main results

Proof of Theorem 3.1. It follows from (15) that one could choose enough small positive constant $\varepsilon_1 > 0$ such that

$$\frac{r_{1L}}{r_{2M}} > \max \left\{ \frac{a_{1M}(d_{2M} + e_{2M}(M_1 + \varepsilon_1) + f_{2M}(M_2 + \varepsilon_1))}{b_{2L}}, \frac{c_{1M}}{c_{2L}}, \frac{b_{1M}}{d_{1L}a_{2L}} \right\}. \quad (24)$$

(24) is equivalent to

$$\begin{aligned} \frac{\frac{a_{1M}}{b_{2L}}}{d_{2M} + e_{2M}(M_1 + \varepsilon_1) + f_{2M}(M_2 + \varepsilon_1)} &< \frac{r_{1L}}{r_{2M}}, \\ \frac{\frac{b_{1M}}{d_{1L}}}{a_{2L}} &< \frac{r_{1L}}{r_{2M}}, \quad \frac{c_{1M}}{c_{2L}} < \frac{r_{1L}}{r_{2M}}. \end{aligned} \quad (25)$$

Therefore, there exist two constants α, β such that

$$\begin{aligned} \frac{\frac{a_{1M}}{b_{2L}}}{d_{2M} + e_{2M}(M_1 + \varepsilon_1) + f_{2M}(M_2 + \varepsilon_1)} &< \frac{\beta}{\alpha} < \frac{r_{1L}}{r_{2M}}, \\ \frac{\frac{b_{1M}}{d_{1L}}}{a_{2L}} &< \frac{\beta}{\alpha} < \frac{r_{1L}}{r_{2M}}, \quad \frac{c_{1M}}{c_{2L}} < \frac{\beta}{\alpha} < \frac{r_{1L}}{r_{2M}}. \end{aligned} \quad (26)$$

That is

$$\begin{aligned} \alpha c_{1M} - \beta c_{2L} &< 0, \quad \alpha a_{1M} - \frac{\beta b_{2L}}{d_{2M} + e_{2M}(M_1 + \varepsilon_1) + f_{2M}(M_2 + \varepsilon_1)} < 0, \\ \alpha \frac{b_{1M}}{d_{1L}} - \beta a_{2L} &< 0, \quad -\alpha r_{1L} + \beta r_{2M} \stackrel{\text{def}}{=} -\delta_1 < 0. \end{aligned} \quad (27)$$

Let $x(t) = (x_1(t), x_2(t))^T$ be a solution of system (1) with $x_i(0) > 0, i = 1, 2$. For above $\varepsilon_1 > 0$, from Lemma 2.2 there exists T_1 large enough such that

$$x_1(t) < M_1 + \varepsilon_1, \quad x_2(t) < M_2 + \varepsilon_1 \quad \text{for all } t \geq T_1. \quad (28)$$

From (1) we have

$$\begin{aligned} \frac{\dot{x}_1(t)}{x_1(t)} &= r_1(t) - a_1(t)x_1(t) - \frac{b_1(t)x_2(t)}{d_1(t) + e_1(t)x_1(t) + f_1(t)x_2(t)} - c_1(t)x_1(t)x_2(t), \\ \frac{\dot{x}_2(t)}{x_2(t)} &= r_2(t) - \frac{b_2(t)x_1(t)}{d_2(t) + e_2(t)x_1(t) + f_2(t)x_2(t)} - a_2(t)x_2(t) - c_2(t)x_1(t)x_2(t). \end{aligned} \quad (29)$$

Let

$$V(t) = x_1^{-\alpha}(t)x_2^\beta(t).$$

From (27)-(29), for $t \geq T_1$, it follows that

$$\begin{aligned} \dot{V}(t) &= V(t) \left[-\alpha \left(r_1(t) - a_1(t)x_1(t) - \frac{b_1(t)x_2(t)}{d_1(t) + e_1(t)x_1(t) + f_1(t)x_2(t)} - c_1(t)x_1(t)x_2(t) \right) \right. \\ &\quad \left. + \beta \left(r_2(t) - a_2(t)x_2(t) - \frac{b_2(t)x_1(t)}{d_2(t) + e_2(t)x_1(t) + f_2(t)x_2(t)} - c_2(t)x_1(t)x_2(t) \right) \right] \\ &= V(t) \left[(-\alpha r_1(t) + \beta r_2(t)) + (\alpha c_1(t) - \beta c_2(t))x_1(t)x_2(t) \right. \\ &\quad \left. + \left(\alpha a_1(t) - \frac{\beta b_2(t)}{d_2(t) + e_2(t)x_1(t) + f_2(t)x_2(t)} \right) x_1(t) \right. \\ &\quad \left. + \left(\alpha \frac{b_1(t)}{d_1(t) + e_1(t)x_1(t) + f_1(t)x_2(t)} - \beta a_2(t) \right) x_2(t) \right] \\ &\leq V(t) \left[(-\alpha r_{1L} + \beta r_{2M}) + (\alpha c_{1M} - \beta c_{2L})x_1(t)x_2(t) \right. \\ &\quad \left. + \left(\alpha a_{1M} - \frac{\beta b_{2L}}{d_{2M} + e_{2M}(M_1 + \varepsilon_1) + f_{2M}(M_2 + \varepsilon_1)} \right) x_1(t) \right. \\ &\quad \left. + \left(\alpha \frac{b_{1M}}{d_{1L}} - \beta a_{2L} \right) x_2(t) \right] \\ &\leq -\delta_1 V(t), \quad t \geq T_1. \end{aligned}$$

Integrating this inequality from T_1 to $t (\geq T_1)$, it follows

$$V(t) \leq V(T_1) \exp(-\delta_1(t - T_1)). \quad (30)$$

By Lemma 2.2 we know that there exists $M > M_0 > 0$ such that

$$x_i(t) < M \quad \text{for all } i = 1, 2 \text{ and } t \geq T_1. \quad (31)$$

Therefore, (30) implies that

$$x_2(t) < C \exp\left(-\frac{\delta_1}{\beta}(t - T_1)\right), \quad (32)$$

where

$$C = M^{\alpha/\beta} (x_1(T_1))^{-\alpha/\beta} x_2(T_1) > 0. \quad (33)$$

Consequently, we have $x_2(t) \rightarrow 0$ exponentially as $t \rightarrow +\infty$. This ends the proof of Theorem 3.1. $\square$

Proof of Theorem 3.3. It follows from (16) and (17) that one could choose a positive constant $\varepsilon_2 > 0$ small enough such that

$$\frac{\frac{a_{1M}}{b_{2L}}}{d_{2M} + e_{2M}(M_1 + \varepsilon_2) + f_{2M}(M_2 + \varepsilon_2)} < \frac{r_{1L} - c_{1M}(M_1 + \varepsilon_2)(M_2 + \varepsilon_2)}{r_{2M}} \quad (34)$$

and

$$\frac{b_{1M}}{d_{1L}a_{2L}} < \frac{r_{1L} - c_{1M}(M_1 + \varepsilon_2)(M_2 + \varepsilon_2)}{r_{2M}} \quad (35)$$

hold. Therefore, there exist two constants α, β such that

$$\frac{\frac{a_{1M}}{b_{2L}}}{d_{2M} + e_{2M}(M_1 + \varepsilon_2) + f_{2M}(M_2 + \varepsilon_2)} < \frac{\beta}{\alpha} < \frac{r_{1L} - c_{1M}(M_1 + \varepsilon_2)(M_2 + \varepsilon_2)}{r_{2M}} \quad (36)$$

and

$$\frac{b_{1M}}{d_{1L}a_{2L}} < \frac{\beta}{\alpha} < \frac{r_{1L} - c_{1M}(M_1 + \varepsilon_2)(M_2 + \varepsilon_2)}{r_{2M}} \quad (37)$$

hold. That is

$$\begin{aligned} \alpha a_{1M} - \frac{\beta b_{2L}}{d_{2M} + e_{2M}(M_1 + \varepsilon_2) + f_{2M}(M_2 + \varepsilon_2)} &< 0, \\ \alpha \frac{b_{1M}}{d_{1L}} - \beta a_{2L} &< 0, \\ -\alpha r_{1L} + \beta r_{2M} + \alpha c_{1M}(M_1 + \varepsilon_2)(M_2 + \varepsilon_2) &\stackrel{\text{def}}{=} -\delta_2 < 0. \end{aligned} \quad (38)$$

Let $x(t) = (x_1(t), x_2(t))^T$ be a solution of system (1) with $x_i(0) > 0, i = 1, 2$. For above $\varepsilon_2 > 0$, from Lemma 2.2 there exists T_2 large enough such that

$$x_1(t) < M_1 + \varepsilon_2, \quad x_2(t) < M_2 + \varepsilon_2 \quad \text{for all } t \geq T_2. \quad (39)$$

Let

$$V(t) = x_1^{-\alpha}(t)x_2^\beta(t).$$

From (29), (38) and (39), for $t \geq T_2$, it follows that

$$\begin{aligned} \dot{V}(t) &= V(t) \left[-\alpha(r_1(t) - a_1(t)x_1(t) - \frac{b_1(t)x_2(t)}{d_1(t) + e_1(t)x_1(t) + f_1(t)x_2(t)} \right. \\ &\quad \left. - c_1(t)x_1(t)x_2(t)) + \beta(r_2(t) - a_2(t)x_2(t) \right. \\ &\quad \left. - \frac{b_2(t)x_1(t)}{d_2(t) + e_2(t)x_1(t) + f_2(t)x_2(t)} - c_2(t)x_1(t)x_2(t)) \right] \\ &\leq V(t) \left[(-\alpha r_1(t) + \beta r_2(t)) + \alpha c_1(t)x_1(t)x_2(t) \right. \\ &\quad \left. + (\alpha a_1(t) - \frac{\beta b_2(t)}{d_2(t) + e_2(t)x_1(t) + f_2(t)x_2(t)})x_1(t) \right. \\ &\quad \left. + (\alpha \frac{b_1(t)}{d_1(t) + e_1(t)x_1(t) + f_1(t)x_2(t)} - \beta a_2(t))x_2(t) \right] \\ &\leq V(t) \left[-\alpha r_{1L} + \beta r_{2M} + \alpha c_{1M}(M_1 + \varepsilon_2)(M_2 + \varepsilon_2) \right. \\ &\quad \left. + (\alpha a_{1M} - \frac{\beta b_{2L}}{d_{2M} + e_{2M}(M_1 + \varepsilon_2) + f_{2M}(M_2 + \varepsilon_2)})x_1(t) \right. \\ &\quad \left. + (\alpha \frac{b_{1M}}{d_{1L}} - \beta a_{2L})x_2(t) \right] \\ &\leq -\delta_2 V(t), \quad t \geq T_2. \end{aligned}$$

Integrating this inequality from T_2 to $t (\geq T_2)$, it follows

$$V(t) \leq V(T_2) \exp(-\delta_2(t - T_2)). \quad (40)$$

From (40), similarly to the analysis of (30)–(33), we have $x_2(t) \rightarrow 0$ exponentially as $t \rightarrow +\infty$. This ends the proof of Theorem 3.3. $\square$

Proof of Theorem 3.4. It follows from (18) and (19) that one could choose a positive constant $\varepsilon_3 > 0$ small enough such that

$$\frac{\frac{a_{1M} + c_{1M}(M_2 + \varepsilon_3)}{b_{2L}}}{d_{2M} + e_{2M}(M_1 + \varepsilon_3) + f_{2M}(M_2 + \varepsilon_3)} < \frac{r_{1L}}{r_{2M}} \quad (41)$$

and

$$\frac{b_{1M}}{d_{1L}a_{2L}} < \frac{r_{1L}}{r_{2M}}. \quad (42)$$

hold. Therefore, there exist two constants α, β such that

$$\frac{\frac{a_{1M} + c_{1M}(M_2 + \varepsilon_3)}{b_{2L}}}{d_{2M} + e_{2M}(M_1 + \varepsilon_3) + f_{2M}(M_2 + \varepsilon_3)} < \frac{\beta}{\alpha} < \frac{r_{1L}}{r_{2M}} \quad (43)$$

and

$$\frac{b_{1M}}{d_{1L}a_{2L}} < \frac{\beta}{\alpha} < \frac{r_{1L}}{r_{2M}} \quad (44)$$

hold. That is

$$\begin{aligned} \alpha a_{1M} + \alpha c_{1M}(M_2 + \varepsilon_3) - \frac{\beta b_{2L}}{d_{2M} + e_{2M}(M_1 + \varepsilon_3) + f_{2M}(M_2 + \varepsilon_3)} &< 0, \\ \alpha \frac{b_{1M}}{d_{1L}} - \beta a_{2L} < 0, \quad -\alpha r_{1L} + \beta r_{2M} \stackrel{\text{def}}{=} -\delta_3 < 0. \end{aligned} \quad (45)$$

Let $x(t) = (x_1(t), x_2(t))^T$ be a solution of system (1) with $x_i(0) > 0, i = 1, 2$. For above $\varepsilon_3 > 0$, from Lemma 2.2 there exists T_3 large enough such that

$$x_1(t) < M_1 + \varepsilon_3, \quad x_2(t) < M_2 + \varepsilon_3 \quad \text{for all } t \geq T_3. \quad (46)$$

Let

$$V(t) = x_1^{-\alpha}(t)x_2^\beta(t).$$

From (29), (45) and (46), for $t \geq T_3$, it follows that

$$\begin{aligned} \dot{V}(t) &= V(t) \left[-\alpha(r_1(t) - a_1(t)x_1(t) - \frac{b_1(t)x_2(t)}{d_1(t) + e_1(t)x_1(t) + f_1(t)x_2(t)} \right. \\ &\quad \left. - c_1(t)x_1(t)x_2(t)) + \beta(r_2(t) - a_2(t)x_2(t) \right. \\ &\quad \left. - \frac{b_2(t)x_1(t)}{d_2(t) + e_2(t)x_1(t) + f_2(t)x_2(t)} - c_2(t)x_1(t)x_2(t)) \right] \\ &\leq V(t) \left[(-\alpha r_1(t) + \beta r_2(t)) + (\alpha a_1(t) + \alpha c_1(t)x_2(t) \right. \\ &\quad \left. - \frac{\beta b_2(t)}{d_2(t) + e_2(t)x_1(t) + f_2(t)x_2(t)})x_1(t) \right. \\ &\quad \left. + (\alpha \frac{b_1(t)}{d_1(t) + e_1(t)x_1(t) + f_1(t)x_2(t)} - \beta a_2(t))x_2(t) \right] \end{aligned}$$

$$\begin{aligned}
&\leq V(t) \left[-\alpha r_{1L} + \beta r_{2M} + (\alpha a_{1M} + \alpha c_{1M}(M_2 + \varepsilon_3) \right. \\
&\quad \left. - \frac{\beta b_{2L}}{d_{2M} + e_{2M}(M_1 + \varepsilon_3) + f_{2M}(M_2 + \varepsilon_3)}) x_1(t) \right. \\
&\quad \left. + (\alpha \frac{b_{1M}}{d_{1L}} - \beta a_{2L}) x_2(t) \right] \\
&\leq -\delta_3 V(t), \quad t \geq T_3.
\end{aligned}$$

Integrating this inequality from T_3 to $t (\geq T_3)$, it follows

$$V(t) \leq V(T_3) \exp(-\delta_3(t - T_3)). \quad (47)$$

From (47), similarly to the analysis of (30)-(33), we have $x_2(t) \rightarrow 0$ exponentially as $t \rightarrow +\infty$. This ends the proof of Theorem 3.4. $\square$

Proof of Theorem 3.5. It follows from (20) and (21) that one could choose a positive constant $\varepsilon_4 > 0$ small enough such that

$$\frac{\frac{a_{1M}}{b_{2L}}}{d_{2M} + e_{2M}(M_1 + \varepsilon_4) + f_{2M}(M_2 + \varepsilon_4)} < \frac{r_{1L}}{r_{2M}} \quad (48)$$

and

$$\frac{\frac{b_{1M}}{d_{1L}} + c_{1M}(M_1 + \varepsilon_4)}{d_{1L}a_{2L}} < \frac{r_{1L}}{r_{2M}}. \quad (49)$$

hold. Therefore, there exist two constants α, β such that

$$\frac{\frac{a_{1M}}{b_{2L}}}{d_{2M} + e_{2M}(M_1 + \varepsilon_4) + f_{2M}(M_2 + \varepsilon_4)} < \frac{\beta}{\alpha} < \frac{r_{1L}}{r_{2M}} \quad (50)$$

and

$$\frac{\frac{b_{1M}}{d_{1L}} + c_{1M}(M_1 + \varepsilon_4)}{d_{1L}a_{2L}} < \frac{\beta}{\alpha} < \frac{r_{1L}}{r_{2M}} \quad (51)$$

hold. That is

$$\begin{aligned}
&\alpha a_{1M} - \frac{\beta b_{2L}}{d_{2M} + e_{2M}(M_1 + \varepsilon_4) + f_{2M}(M_2 + \varepsilon_4)} < 0, \\
&\alpha \frac{b_{1M}}{d_{1L}} + \alpha c_{1M}(M_2 + \varepsilon_4) - \beta a_{2L} < 0, \\
&-\alpha r_{1L} + \beta r_{2M} \stackrel{\text{def}}{=} -\delta_3 < 0.
\end{aligned} \quad (52)$$

Let $x(t) = (x_1(t), x_2(t))^T$ be a solution of system (1) with $x_i(0) > 0, i = 1, 2$. For above $\varepsilon_4 > 0$, from Lemma 2.2 there exists T_4 large enough such that

$$x_1(t) < M_1 + \varepsilon_4, \quad x_2(t) < M_2 + \varepsilon_4 \quad \text{for all } t \geq T_4. \quad (53)$$

Let

$$V(t) = x_1^{-\alpha}(t) x_2^\beta(t).$$

From (29), (52) and (53), for $t \geq T_4$, it follows that

$$\begin{aligned}
 \dot{V}(t) &= V(t) \left[-\alpha(r_1(t) - a_1(t)x_1(t) - \frac{b_1(t)x_2(t)}{d_1(t) + e_1(t)x_1(t) + f_1(t)x_2(t)} \right. \\
 &\quad \left. - c_1(t)x_1(t)x_2(t)) + \beta(r_2(t) - a_2(t)x_2(t) \right. \\
 &\quad \left. - \frac{b_2(t)x_1(t)}{d_2(t) + e_2(t)x_1(t) + f_2(t)x_2(t)} - c_2(t)x_1(t)x_2(t)) \right] \\
 &\leq V(t) \left[(-\alpha r_{1L} + \beta r_{2M}) + (\alpha c_1(t)x_1(t) - \beta a_2(t) \right. \\
 &\quad \left. + \alpha \frac{b_1(t)}{d_1(t) + e_1(t)x_1(t) + f_1(t)x_2(t)})x_2(t) \right. \\
 &\quad \left. + (\alpha a_1(t) - \frac{\beta b_2(t)}{d_2(t) + e_2(t)x_1(t) + f_2(t)x_2(t)})x_1(t) \right] \\
 &\leq V(t) \left[(-\alpha r_{1L} + \beta r_{2M}) \right. \\
 &\quad \left. + (\alpha a_{1M} - \frac{\beta b_{2L}}{d_{2M} + e_{2M}(M_1 + \varepsilon_4) + f_{2M}(M_2 + \varepsilon_4)})x_1(t) \right. \\
 &\quad \left. + (\alpha \frac{b_{1M}}{d_{1L}} + \alpha c_{1M}(M_1 + \varepsilon_4) - \beta a_{2L})x_2(t) \right] \\
 &\leq -\delta_4 V(t), \quad t \geq T_4.
 \end{aligned}$$

Integrating this inequality from T_4 to $t (\geq T_4)$, it follows

$$V(t) \leq V(T_4) \exp(-\delta_4(t - T_4)). \quad (54)$$

From (54), similarly to the analysis of (30)-(33), we have $x_2(t) \rightarrow 0$ exponentially as $t \rightarrow +\infty$. This ends the proof of Theorem 3.5. $\square$

Proof of Theorem 3.8. By applying Lemma 2.3 and 2.4, the proof of Theorem 3.8 is similar to that of the proof of Theorem in [4]. We omit the detail here. $\square$

Proof of Theorem 3.9. Conditions (22) and (23) imply that there exist two constants α, β and a positive constant ε_5 small enough, such that

$$\begin{aligned}
 \frac{r_{1M}}{r_{2L}} &< \frac{\beta}{\alpha} < \frac{\frac{b_{1L}}{d_{1M} + e_{1M}(M_1 + \varepsilon_5) + f_{1M}(M_2 + \varepsilon_5)}}{a_{2M}}, \\
 \frac{r_{1M}}{r_{2L}} &< \frac{\beta}{\alpha} < \frac{a_{1L}d_{2L}}{b_{2M}}.
 \end{aligned} \quad (55)$$

That is

$$\begin{aligned}
 -\alpha a_{1L} + \beta \frac{b_{2M}}{d_{2L}} &< 0, \quad \alpha r_{1M} - \beta r_{2L} \stackrel{\text{def}}{=} -\delta_4 < 0, \\
 -\frac{\alpha b_{1L}}{d_{1M} + e_{1M}(M_1 + \varepsilon_5) + f_{1M}(M_2 + \varepsilon_5)} + \beta a_{2M} &< 0.
 \end{aligned} \quad (56)$$

For above $\varepsilon_5 > 0$, from Lemma 2.2 there exists T_5 large enough such that

$$x_2(t) < M_2 + \varepsilon_5 \quad \text{for all } t \geq T_5. \quad (57)$$

Let

$$V_1(t) = x_1^\alpha(t) x_2^{-\beta}(t).$$

It follows from (56) and (57) that

$$\begin{aligned}
 \dot{V}_1(t) &= V_1(t) \left[\alpha(r_1(t) - a_1(t)x_1(t) - \frac{b_1(t)x_2(t)}{d_1(t) + e_1(t)x_1(t) + f_1(t)x_2(t)} \right. \\
 &\quad \left. - c_1(t)x_1(t)x_2(t)) \right. \\
 &\quad \left. - \beta(r_2(t) - \frac{b_2(t)x_1(t)}{d_2(t) + e_2(t)x_1(t) + f_2(t)x_2(t)} - a_2(t)x_2(t)) \right] \\
 &= V_1(t) \left[(\alpha r_1(t) - \beta r_2(t)) \right. \\
 &\quad \left. + \left(-\alpha a_1(t) + \frac{\beta b_2(t)}{d_2(t) + e_2(t)x_1(t) + f_2(t)x_2(t)} \right) x_1(t) \right. \\
 &\quad \left. + \left(-\frac{\alpha b_1(t)}{d_1(t) + e_1(t)x_1(t) + f_1(t)x_2(t)} + \beta a_2(t) \right) x_2(t) \right] \\
 &\leq V_1(t) \left[(\alpha r_{1M} - \beta r_{2L}) + \left(-\alpha a_{1L} + \beta \frac{b_{2M}}{d_{2L}} \right) x_1(t) \right. \\
 &\quad \left. + \left(-\frac{\alpha b_{1L}}{d_{1M} + e_{1M}(M_1 + \varepsilon_5) + f_{1M}(M_2 + \varepsilon_5)} + \beta a_{2M} \right) x_2(t) \right] \\
 &\leq -\delta_5 V_1(t).
 \end{aligned}$$

Integrating this inequality from T_5 to $t (\geq T_5)$, it follows

$$V_1(t) \leq V_1(T_5) \exp(-\delta_5(t - T_5)). \quad (58)$$

From this, similarly to the analysis of (30)-(33), we have $x_1(t) \rightarrow 0$ exponentially as $t \rightarrow +\infty$. The rest of the proof of Theorem 3.9 is similar to that of the proof of Theorem in [4]. We omit the detail here. $\square$

5 Numeric examples

Now let us consider the following examples.

Example 5.1.

$$\begin{aligned}
 \dot{x}(t) &= x \left(10 - (2.3 + 0.3 \sin(4t))x - \frac{(3 + \frac{1}{2} \cos(4t))y}{1 + 0.1y + 0.1x} - 6xy \right), \\
 \dot{y}(t) &= y \left(2 - \frac{(6 + \sin(4t))x}{1 + 0.1x + 0.1y} - 2y - (4 + \cos(4t))xy \right).
 \end{aligned} \quad (59)$$

Corresponding to system (1), one has

$$\begin{aligned}
 r_1(t) &= 10, \quad a_1(t) = 2.3 + 0.3 \sin(2t), \quad b_1(t) = 3 + \frac{1}{2} \cos(2t), \quad c_1(t) = 6, \\
 r_2(t) &= 2, \quad a_2(t) = 2, \quad b_2(t) = 6 + \sin(2t), \quad c_2(t) = 4 + \cos(2t). \\
 d_2(t) &= d_1(t) = 1, \quad e_1(t) = e_2(t) = f_1(t) = f_2(t) = 0.1.
 \end{aligned}$$

And so,

$$M_1 = \frac{r_{1M}}{a_{1L}} = 5, \quad M_2 = \frac{r_{2M}}{a_{2L}} = 1. \quad (60)$$

Consequently

$$\frac{r_{1L}}{r_{2M}} = 5, \quad \frac{a_{1M}(d_{2M} + e_{2M}M_1 + f_{2M}M_2)}{b_{2L}} = \frac{104}{125}, \quad \frac{c_{1M}}{c_{2L}} = 2, \quad \frac{b_{1M}}{d_{1L}a_{2L}} = \frac{7}{4}.$$

Fig. 1. Dynamic behavior of the first component $x(t)$ of the solution $(x(t), y(t))$ of system (59) with the initial conditions $(x(0), y(0)) = (1, 1), (2, 2), (0.2, 3)$ and $(7, 4)$, respectively.

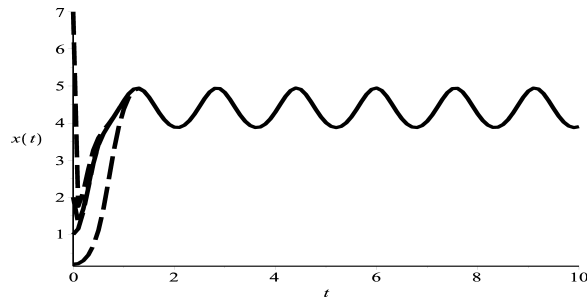
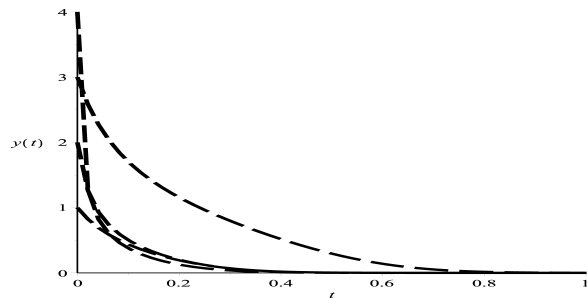


Fig. 2. Dynamic behavior of the second component $y(t)$ of the solution $(x(t), y(t))$ of system (59) with the initial conditions $(x(0), y(0)) = (1, 1), (2, 2), (0.2, 3)$ and $(7, 4)$, respectively.



Since

$$5 > \max \left\{ \frac{104}{125}, 2, \frac{7}{4} \right\},$$

it follows from Theorem 3.1 that the first species of the system (59) is globally attractive, and the second species will be driven to extinction. Numeric simulations (Fig. 1, 2) also support this findings.

Example 5.2.

$$\begin{aligned} \dot{x}(t) &= x \left(10 - (2.3 + 0.3 \sin(4t))x - \frac{(3 + \frac{1}{2} \cos(4t))y}{1 + 0.1y + 0.1x} - 1xy \right), \\ \dot{y}(t) &= y \left(2 - \frac{(6 + \sin(4t))x}{1 + 0.1x + 0.1y} - 2y - 0.1xy \right). \end{aligned} \quad (61)$$

In system (61), we let $c_1(t) = 1, c_2(t) = 0.1$, and all the other coefficients are the same as that of system (59). In this case, since

$$\frac{r_{1L}}{r_{2M}} = 5 < 10 = \frac{c_{1M}}{c_{2L}},$$

the conditions of Theorem 3.1 could not satisfied, however,

$$\frac{a_{1M}(d_{2M} + e_{2M}M_1 + f_{2M}M_2)}{b_{2L}} = \frac{104}{125} < \frac{5}{2} = \frac{r_{1L} - c_{1M}M_1M_2}{r_{2M}} \quad (62)$$

and

$$\frac{b_{1M}}{d_{1L}a_{2L}} = \frac{7}{4} < \frac{5}{2} = \frac{r_{1L} - c_{1M}M_1M_2}{r_{2M}}. \quad (63)$$

(62) and (63) show that all the conditions of Theorem 3.3 are satisfied, and so, the first species in system (61) is globally attractive, and the second species will be driven to extinction. Numeric simulations (Fig. 3, 4) also support this findings.

Fig. 3. Dynamic behavior of the first component $x(t)$ of the solution $(x(t), y(t))$ of system (61) with the initial conditions $(x(0), y(0)) = (1, 1), (2, 2), (0.2, 3)$ and $(7, 4)$, respectively.

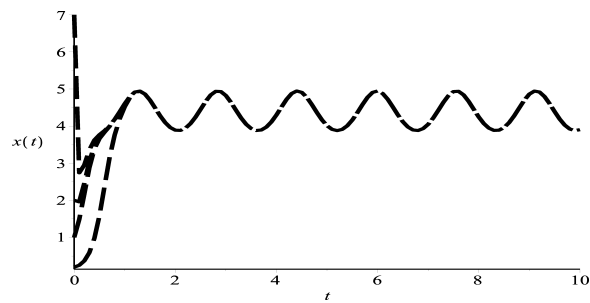
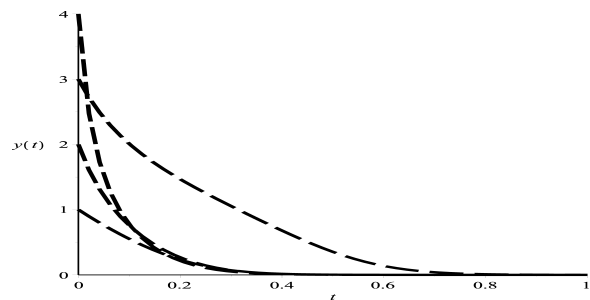


Fig. 4. Dynamic behavior of the second component $y(t)$ of the solution $(x(t), y(t))$ of system (61) with the initial conditions $(x(0), y(0)) = (1, 1), (2, 2), (0.2, 3)$ and $(7, 4)$, respectively.



Example 5.3.

$$\begin{aligned}\dot{x}(t) &= x \left(2 - \frac{(6 + \sin(4t))y}{1 + 0.1x + 0.1y} - 2x - 0.1xy \right), \\ \dot{y}(t) &= y \left(10 - (2.3 + 0.3 \sin(4t))y - \frac{(3 + \frac{1}{2} \cos(4t))x}{1 + 0.1y + 0.1x} \right).\end{aligned}\quad (64)$$

Corresponding to system (1), one has

$$\begin{aligned}r_2(t) &= 10, \quad a_2(t) = 2.3 + 0.3 \sin(2t), \quad b_2(t) = 3 + \frac{1}{2} \cos(2t), \\ r_1(t) &= 2, \quad a_1(t) = 2, \quad b_1(t) = 6 + \sin(2t), \quad c_1(t) = 0.1, \\ d_2(t) &= d_1(t) = 1, \quad e_1(t) = e_2(t) = f_1(t) = f_2(t) = 0.1.\end{aligned}$$

And so,

$$M_1 = \frac{r_{1M}}{a_{1L}} = 1, \quad M_2 = \frac{r_{2M}}{a_{2L}} = 5. \quad (65)$$

Consequently

$$\begin{aligned}\frac{r_{1M}}{r_{2L}} &= \frac{1}{5} < \frac{125}{104} = \frac{b_{1L}}{a_{2M}(d_{1M} + e_{1M}M_1 + f_{1M}M_2)}, \\ \frac{r_{1M}}{r_{2L}} &= \frac{1}{5} < \frac{a_{1L}d_{2L}}{b_{2M}} = \frac{4}{7}.\end{aligned}$$

Hence, all the conditions of Theorem 3.9 hold. It follows from Theorem 3.9 that the first species of the system (64) will be driven to extinction, and the second species is globally attractive, numeric simulations (Fig. 5, 6) also support this findings.

Fig. 5. Dynamic behavior of the first component $x(t)$ of the solution $(x(t), y(t))$ of system (64) with the initial conditions $(x(0), y(0)) = (1, 1), (2, 2), (0.2, 3)$ and $(7, 4)$, respectively.

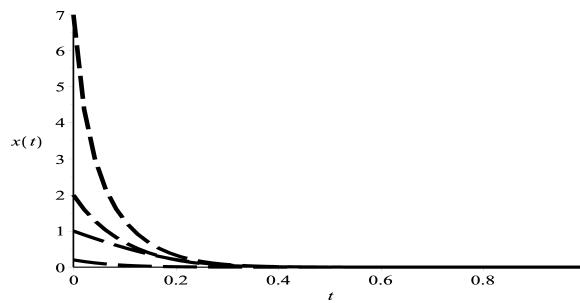
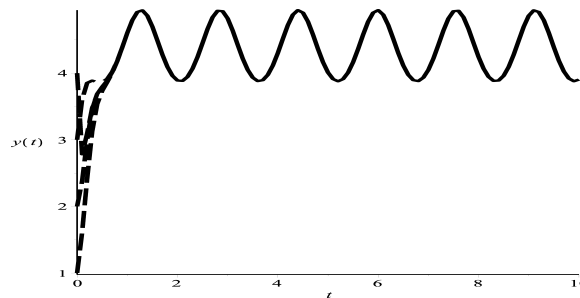


Fig. 6. Dynamic behavior of the second component $y(t)$ of the solution $(x(t), y(t))$ of system (64) with the initial conditions $(x(0), y(0)) = (1, 1), (2, 2), (0.2, 3)$ and $(7, 4)$, respectively.



6 Discussion

During the last decade, many scholars paid attention to the extinction property of the competition system, in their series work, Li and Chen [3-7], Chen et al [8, 9] studied the extinction property of the competition system with toxic substance. He et al [31], Chen et al [34, 35] studied the extinction property of the Gilpin-Ayala competition model. Recently, Yue [1] proposed a competitive system with both toxic substance and nonlinear inter-inhibition terms, i.e., system (9), she also investigated the extinction property of the system. Noting that the functional response used in [1] is of Holling II type, in this paper, we consider a more plausible one, i.e., the Beddington-DeAngelis functional response. By constructing some suitable Lyapunov type extinction function, several set of sufficient conditions which ensure the extinction of a species are obtained. Our results generalize the main result of Li and Chen [6].

We mention here that in system (1) and (2), we did not consider the influence of delay, we leave this for future investigation.

Competing interests

The authors declare that there is no conflict of interests.

Authors' contributions

All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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