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**Research Article**

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# Very true operators on MTL-algebras

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**Abstract:** The main goal of this paper is to investigate very true MTL-algebras and prove the completeness of the very true MTL-logic. In this paper, the concept of very true operators on MTL-algebras is introduced and some related properties are investigated. Also, conditions for an MTL-algebra to be an MV-algebra and a Gödel algebra are given via this operator. Moreover, very true filters on very true MTL-algebras are studied. In particular, subdirectly irreducible very true MTL-algebras are characterized and an analogous of representation theorem for very true MTL-algebras is proved. Then, the left and right stabilizers of very true MTL-algebras are introduced and some related properties are given. As applications of stabilizer of very true MTL-algebras, we produce a basis for a topology on very true MTL-algebras and show that the generated topology by this basis is Baire, connected, locally connected and separable. Finally, the corresponding logic very true MTL-logic is constructed and the soundness and completeness of this logic are proved based on very true MTL-algebras.

**Keywords:** Very true MTL-algebra, Subdirectly irreducible, Representation, Stabilizer topology, Very true MTL-logic

**MSC:** 03F50, 06F99

## 1 Introduction

Basic fuzzy logic (BL for short) is the many-valued residuated logic introduced by H$\acute{a}$jek [1] to handle continuous t-norms and their residua. The fuzzy logics such as Łukasiewicz, Gödel and Product logic can be regarded as schematic extensions of BL. It is a well-known result that a t-norm has a residuum if and only if it is left-continuous; so this shows that BL is not the most general t-norm based logic. In fact, a logic weaker than BL, called monoidal t-norm-based logic (MTL for short), was introduced by Esteva and Godo in [2] and Jenei and Montagna [3] proved that MTL is indeed the logic of all left-continuous t-norms and their residua. In connection with the MTL logic, a new class of algebras is defined, called MTL-algebras [2]. In the last few years, the theory of MTL-algebras has been enriched with structure theorems [4, 5]. Many of these results have a strong impact with its algebraic structure. For example, Vetterlein [4] proved that most of MTL-algebras can be embeddable into the positive cone of a partially ordered group. He also proved that an MTL-algebra is a bounded, commutative, integral, prelinear residuated lattice [5]. As a more general residuated structure based on left-continuous t-norm logic, an MTL-algebra is a BL-algebra without the identity $x \wedge y = x \odot (x \to y)$. Thus, MTL-algebras are the most fundamental residuated structures containing all algebras induced by (left) continuous t-norms and their residua. Therefore, MTL-algebra play an important role in studying fuzzy logics and their related structures. The filter theory of the MTL-algebras plays an important role in studying these algebras and the completeness of the MTL. From a logic point of view, various filters have natural interpretation as various sets of provable formulas. Recently, the filters on MTL-algebras have

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been widely studied and some important results have been obtained [2,6-8]. In particular, Esteva introduced the idea of filters and prime filters in MTL-algebras to prove the completeness and chain completeness of MTL [2]. After then, the concepts of implicative, positive and fantastic filters were defined in MTL-algebras in [6]. In [7], Borzooei was the first to systematically study filter theory in MTL-algebras, in which the relations between kinds of filters were obtained and some of their characterizations were presented. It was also proved that there exists at most one proper associative filter in any MTL-algebra ,which is composed of all non-zero elements in this MTL-algebra in [8].

The concept of "very true" was introduced by Hájek [9] as an answer for the question "whether any natural axiomatization is possible and how far can even this sort of fuzzy logic be captured by standard methods of mathematical logic?". In other words, very true operator as a tool for reducing the number of possible logical values in many-valued fuzzy logic. In fact, it is the same as the concept of hedge introduced by Zadeh [10], who gives some examples of handling these fuzzy truth values that seems uninterested in any sort of axiomatization. Apart from their important application in many valued fuzzy logic, very true operators were successfully used to formal concept analysis (FCA, in brief) (see [11]), which is another important branch of mathematics and becoming an popular method for analysis of object-attribute data. The main aim in FCA is to extract interesting clusters (called formal concepts) from tabular data, formal concepts correspond to maximal rectangles in a data table, hence the number of formal concepts in data can be extremely large. In order to reduce the number of formal concepts, B$\check{e}$lohl$\acute{a}$vek and Vyhodil [12] used the so called hedges, which are special cases of very true operators used in reducing the number of formal concepts in concept lattice. Since very true operator was successful in several distinct tasks in various branches of mathematics [10,12-14], it has been extended to other logical algebras such as MV-algebras [15], R$\ell$-monoids [16], commutative basic algebras [17], equality algebras [18], effect algebras [19] and so on.

As we have mentioned in the above paragraph, very true operators have been studied on MV-algebras, BL-algebras, R$\ell$-monoids and commutative basic algebras, etc. All the above-mentioned algebraic structures satisfy the divisibility condition $x \wedge y = x \odot (x \to y)$. In this case, the conjunction $\odot$ on the unit interval corresponds to a continuous t-norm. However, there are few research about the very true operators on residuated structures without the divisibility condition so far [19]. In fact, MTL-algebras are the more general residuated structure without the divisibility condition since it is an algebra induced by a left continuous t-norm and its corresponding residuum. Therefore, it is meaningful to study very true operators on MTL-algebras for treating a variant of the concept of very true operators within the framework of universal algebras and providing a solid algebraic foundation for reasoning about very true MTL logic. This is the motivation for us to investigate very true operators on MTL-algebras.

Based on the above considerations, we enrich the language of MTL by adding a very true operator to get algebras named very true MTL-algebras, which are the algebraic counterpart of very true MTL logic. This paper is structured in five sections. In order to make the paper as self-contained as possible, we recapitulate in Section 2 the definition of MTL-algebras, and review their basic properties that will be used in the remainder of the paper. In Section 3, we introduce very true operators on MTL-algebras and study some of their properties. Also, we give some characterizations of MV-algebra and Gödel algebra via such operator. In Section 4, we investigate very true filters of very true MTL-algebras and focus on an analogous of representation theorem for very true MTL-algebras and characterize subdirectly irreducible very true MTL-algebras by very true filters. Then, we introduce the left and right stabilizers of very true MTL-algebras and construct stabilizer topology via them. In Section 5, the corresponding very true MTL-logic is constructed, the soundness and completeness of this logic are proved based on the variety of very true MTL-algebras.

## 2 Preliminaries

In this section, we summarize some definitions and results about MTL-algebras, which will be used in the following and we shall not cite them every time they are used.

**Definition 2.1** ([2])**.** *An algebraic structure* $(L, \wedge, \vee, \odot, \to, 0, 1)$ *of type* $(2, 2, 2, 2, 0, 0)$ *is called an* MTL-algebra *if it satisfies the following conditions:*

(1) $(L, \wedge, \vee, 0, 1)$ *is a bounded lattice,*
(2) $(L, \odot, 1)$ *is a commutative monoid,*
(3) $x \odot y \leq z$ *if and only if* $x \leq y \to z$,
(4) $(x \to y) \vee (y \to x) = 1$,
*for any* $x, y, z \in L$.

In what follows, by $L$ we denote the universe of an MTL-algebra $(L, \wedge, \vee, \odot, \to, 0, 1)$. For any $x \in L$, we define $\neg x = x \to 0$.

**Proposition 2.2** ([5]). *In any MTL-algebra* $L$*, the following properties hold: for all* $x, y, z \in L$,
(1) $x \leq y$ *if and only if* $x \to y = 1$,
(2) $x \odot y \leq x \wedge y$,
(3) $1 \to x = x$,
(4) $x \to (y \wedge z) = (x \to y) \wedge (x \to z)$,
(5) $(x \vee y) \to z = (x \to z) \wedge (y \to z)$,
(6) $x \leq y$ *implies* $x \odot z \leq y \odot z$,
(7) $x \to y = x \to (x \wedge y)$,
(8) $x \to y = (x \vee y) \to y$,
(9) $x \wedge y \to z = (x \to z) \vee (y \to z)$,
(10) $\bigwedge_{i \in I}(x_i \to y) = \bigvee_{i \in I} x_i \to y$*, provided that both infimum as well as supremum exist,*
(11) $x \vee y = ((x \to y) \to y) \wedge ((y \to x) \to x)$,
(12) $(x \odot y) \to z = x \to (y \to z)$.

**Definition 2.3** ([5]). *Let* $L$ *be an MTL-algebra. Then* $L$ *is called:*
(1) *a* BL-algebra *if* $x \wedge y = x \odot (x \to y)$ *for any* $x, y \in L$.
(2) *an* MV-algebra *if* $(x \to y) \to y = (y \to x) \to x$ *for any* $x, y \in L$.
(3) *a* Gödel algebra *if* $x \odot x = x$ *for any* $x \in L$.

A nonempty subset $F$ of $L$ is called a *filter* of $L$ if it satisfies: (1) $x, y \in F$ implies $x \odot y \in F$; (2) $x \in F$,$y \in L$ and $x \leq y$ implies $y \in F$. We denote by $F[L]$ the set of all filers of $L$. A filter $F$ of $L$ is called a *proper filter* if $F \neq L$. A proper filter $F$ of $L$ is called a *prime filter* if for each $x, y \in F$ and $x \vee y \in F$, implies $x \in F$ or $y \in F$. For any filter $F$ of $L$ we can associate a *congruence* on $L$ defined by $x \sim_F y$ if and only if $(x \to y) \wedge (y \to x) \in F$. We denote by $L/F$ the set of congruence classes and $L/F$ becomes an MTL-algebra with the natural operations induced by those of $L$. Note that a filter $F$ of $L$ is prime iff $L/F$ is a linearly ordered MTL-algebra ([2, 6, 7]).

**Definition 2.4** ([20]). *A* Heyting algebra *is an algebra* $(H, \vee, \wedge, \to, 0, 1)$ *of type* $(2, 2, 2, 0, 0)$ *for which* $(H, \vee, \wedge, 0, 1)$ *is a bounded lattice and for* $a, b \in H$, $a \to b$ *is the relative pseudocomplement of* $a$ *with respect to* $b$*, i.e.,*$a \wedge c \leq b$ *if and only if* $c \leq a \to b$.

**Definition 2.5** ([20]). *Let* $L$ *be a complete lattice and* $a \in L$*. Then element* $a$ *is said to be* compact *if for every subset* $S$ *of* $L$, $a \leq \bigvee S$ *implies that* $a \leq \bigvee F$ *for some finite subset* $F$ *of* $S$.

**Definition 2.6** ([21]). *A topological space* $(X, \mathcal{T})$ *is called a* Baire space *if for each countable collection of open dense sets, their intersection is dense.*

At the end of this section, we review the known main results about representation theory of MTL-algebras, which is helpful for studying very true analogous representation theorem of MTL-algebras.

**Definition 2.7** ([22]). *An element* $b$ *of a lattice* $L$ *is* meet irreducible *if* $\bigwedge X = b$ *implies* $b \in X$*, for any finite subset* $X$ *of* $L$.

A filter $F$ of an MTL-algebra $L$ is called prime if $F$ is a finitely meet-irreducible element in the lattice $F[L]$. A prime filter $F$ is called minimal if $F$ is a minimal element in the set of prime filters of $L$ ordered by inclusion. By Zorn's lemma, every prime filter contains a minimal prime filter ([24]).

Let $L$ be a MTL-algebra, and $X \subseteq L$. The set

$$X^{\perp} = \{a \in L | a \vee x = 1, \text{ for each } x \in X\}$$

is called the *co-annihilator* of $X$ in $L$ [23]. For any $a \in L$, we write $a^{\perp}$ instead of $\{a\}^{\perp}$.

**Theorem 2.8** ([24]). *For $P \in F[L]$, the following conditions are equivalent:*
(1) *$P$ is a minimal prime,*
(2) *$P = \cup\{a^{\perp} | a \in P\}$.*

**Theorem 2.9** ([24]). *For an MTL-algebra L, the following conditions are equivalent:*
(1) *L is representable,*
(2) *There exists a set $S$ of prime filters such that $\bigcap S = \{1\}$.*

# 3 Very true operators on MTL-algebras

In this section, inspired by Hájek [9], we enlarge the language of MTL-algebra by introducing a very true operator, and investigate some related properties. As applications of very true operator, we discuss the structures of the fixed point set of a very true operator and give conditions for an MTL-algebra to be an MV-algebra and a Gödel algebra.

**Definition 3.1.** *Let L be an MTL-algebra. The mapping $\tau : L \longrightarrow L$ is called a* very true operator *if it satisfies the following conditions:*
(V1) $\tau(1) = 1$,
(V2) $\tau(x) \leq x$,
(V3) $\tau(x \to y) \leq \tau(x) \to \tau(y)$,
(V4) $\tau(x) \leq \tau\tau(x)$,
(V5) $\tau(x \to y) \vee \tau(y \to x) = 1$.

The pair $(L, \tau)$ is said to be a very true MTL-algebra.

Such a proliferation of conditions deserves some explanation. Then "1" seen in (V1) is considered as the logical value absolutely true. First note that (V1) means that absolutely true is very true, which is sound for each natural interpretation in many valued logic system. (V2) means that if $\varphi$ is very true then it is true. (V3) means that if both $\varphi$ and $\varphi \to \psi$ are very true then so is $\psi$, that means the connective $\tau$ preserve modus ponens. (V4) says that if $\varphi$ is very true then $\tau(\varphi)$ is very true, which is a kind of necessitation. To obtain very true MTL-algebras that are representable as subdirect products of very true MTL-chains, we using (V5).

**Example 3.2.**
(a) *Let L be an MTL-algebra. One can easily check that $id_L$ is a very true operator on L, that is to say, every MTL-algebra can be seen as a very true MTL-algebra.*
(b) *Any linearly ordered MTL-algebra L can admit a very true operator; i.e., $\tau(1) = 1$ and $\tau(x) = 0$ for any $x < 1$.*
(c) *Let $L = \{0, a, b, 1\}$ with $0 \leq a \leq b \leq 1$. Consider the operation $\odot$ and $\to$ given by the following tables:*

| $\odot$ | 0 | $a$ | $b$ | 1 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| $a$ | 0 | 0 | 0 | $a$ |
| $b$ | 0 | 0 | $b$ | $b$ |
| 1 | 0 | $a$ | $b$ | 1 |

| $\to$ | 0 | $a$ | $b$ | 1 |
|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 |
| $a$ | $b$ | 1 | 1 | 1 |
| $b$ | $a$ | $a$ | 1 | 1 |
| 1 | 0 | $a$ | $b$ | 1 |

*Then* $(L, \wedge, \vee, \odot, \rightarrow, 0, 1)$ *is an MTL-algebra. Now, we define* $\tau$ *as follows:* $\tau(0) = 0$, $\tau(a) = \tau(b) = a$, $\tau(1) = 1$. *One can easily check that* $\tau$ *is a very true operator on* $L$. *However,* $\tau$ *is not a homomorphisms on* $L$ *since* $\tau(b \odot b) = a \neq 0 = \tau(b) \odot \tau(b)$ *and* $\tau(b \rightarrow a) = a \neq 1 = \tau(b) \rightarrow \tau(a)$.

**Proposition 3.3.** *Let* $\tau$ *be a very true operator on* $L$. *Then for any* $x, y \in L$ *we have,*
(1) $\tau(0) = 0$,
(2) $\tau(x) = 1$ *if and only if* $x = 1$,
(3) $x \leq y$ *implies* $\tau(x) \leq \tau(y)$,
(4) $\tau(\neg x) \leq \neg(\tau x)$,
(5) $\tau(x) \odot \tau(y) \leq \tau(x \odot y)$,
(6) $\tau(x \wedge y) = \tau(x) \wedge \tau(y)$,
(7) $\tau(x \vee y) = \tau(x) \vee \tau(y)$,
(8) $\tau(x \rightarrow y) \vee (y \rightarrow x) = 1$,
(9) $\tau^2(x) = \tau(x)$,
(10) $\tau(x) \leq y$ *if and only if* $\tau(x) \leq \tau(y)$,
(11) $\tau(L) = Fix_\tau(L)$,*where* $Fix_\tau(L) = \{x \in L | \tau(x) = x\}$,
(12) $Fix_\tau(L)$ *is closed under* $\odot, \wedge, \vee$,
(13) *If* $\tau(L) = L$, *then* $\tau = id_L$,
(14) *Ker*$(\tau) = \{1\}$, *where Ker*$(\tau) = \{x \in L | \tau(x) = 1\}$,
(15) *Ker*$(\tau)$ *is a filter of* $L$.

*Proof.* (1) Applying (V2), we have $\tau(0) \leq 0$ and hence $\tau(0) = 0$.

(2) If $\tau(x) = 1$ for some $x \in L$ then by (V2), $1 = \tau(x) \leq x$ giving $x = 1$. The converse follows by (V1).

(3) If $x \leq y$, then $x \rightarrow y = 1$. It follows from (V1) and (V3) that $\tau(x) \rightarrow \tau(y) = 1$. Thus, $\tau(x) \leq \tau(y)$.

(4) It follows from (1) and (V3) that $\tau(\neg x) = \tau(x \rightarrow 0) \leq \tau(x) \rightarrow 0 = \neg\tau(x)$.

(5) From $x \odot y \leq x \odot y$, we get $y \leq x \rightarrow (x \odot y)$. By (V3) and (3), we have $\tau(y) \leq \tau(x \rightarrow (x \odot y)) \leq \tau(x) \rightarrow \tau(x \odot y)$ and hence $\tau(x) \odot \tau(y) \leq \tau(x \odot y)$.

(6) On one hand, it is easy to see that $\tau(x \wedge y) \leq \tau(x) \wedge \tau(y)$. On the other hand, by (V2),(V4) and Proposition 2.2 (4), we obtain, for all $x, y \in L$, $1 = \tau(x \rightarrow y) \vee \tau(y \rightarrow x) = \tau(x \rightarrow (x \wedge y)) \vee \tau(y \rightarrow (x \wedge y)) \leq (\tau(x) \rightarrow \tau(x \wedge y)) \vee (\tau(y) \rightarrow \tau(x \wedge y)$ and hence by Proposition 2.2 (9), we get $1 = (\tau(x) \wedge \tau(y)) \rightarrow \tau(x \wedge y)$. Therefore, $\tau(x \wedge y) = \tau(x) \wedge \tau(y)$.

(7) By(6) and Proposition 2.2 (11), we obtain, for all $x, y \in L$, $\tau(x \vee y) = \tau((x \rightarrow y) \rightarrow y) \wedge \tau((y \rightarrow x) \rightarrow x)$. By (3) and (V3), we get $\tau(x \vee y) \leq \tau((x \rightarrow y) \rightarrow (\tau(x) \vee \tau(y))) \wedge (\tau(y \rightarrow x) \rightarrow (\tau(x) \vee \tau(y)))$. Hence by Proposition 2.2 (10), we obtain $\tau(x \vee y) \leq (\tau(x \rightarrow y)) \vee (\tau(y \rightarrow x)) \rightarrow (\tau(x) \vee \tau(y))$ and hence $\tau(x \vee y) \leq \tau(x) \vee \tau(y)$. The other inequality follows easily from (3).

(8) Applying (V2) and (V5), we get $\tau(x \rightarrow y) \vee (y \rightarrow x) \geq \tau(x \rightarrow y) \vee \tau(y \rightarrow x) = 1$.

(9) By (V2) and (V4), we have $\tau^2(x) = \tau(x)$.

(10) For all $x, y \in L$, assume that $\tau(x) \leq y$, we have $\tau\tau(x) \leq \tau(y)$. By the (9), we get $\tau^2(x) = \tau(x)$. Thus $\tau(x) \leq \tau(y)$. Conversely, suppose that $\tau(x) \leq \tau(y)$, we have $\tau(x) \leq \tau(y) \leq y$.

(11) Let $y \in \tau(L)$, so there exists $x \in L$ such that $y = \tau(x)$. Hence $\tau(y) = \tau\tau(x) = \tau(x) = y$. This follows that $y \in Fix_\tau(L)$. Conversely, if $y \in Fix_\tau(L)$, we have $y \in \tau(L)$. Therefore, $\tau(L) = Fix_\tau(L)$.

(12) It follows from (5)-(7).

(13) For any $x \in L$, we have $x = \tau(x_0)$ for some $x_0 \in L$. By (9), we have $\tau(x) = \tau(\tau(x_0)) = \tau(x_0) = x$. Therefore, $\tau = id_L$.

(14) Assume that $x \in L$ but $x \neq 1$ such that $\tau(x) = 1$. Applying (V2), we have $1 = \tau(x) \leq x$ and hence $x = 1$, which is a contradiction. Therefore, Ker$(\tau) = \{1\}$.

(15) This is easy to check. Hence we omit the proof. □

The assertion (7) of above proposition gives us an idea of introducing a very true operator on an MTL-algebra in a different way. Namely, we can consider a mapping $\tau : L \longrightarrow L$ satisfying $(1) - (4)$ and the following axiom $(5')$ which replaces the axiom (5): $(5')$ $\tau(x \vee y) = \tau(x) \vee \tau(y)$. From this point, one can check that the very true MTL-algebra essentially generalize very true BL-algebra, which was introduced by Hájek in 2001. A very true operator $\tau$ on an MV-algebra $L$ was introduced in Leuştean (2006) as a mapping $\tau : L \to L$ satisfying conditions (V1)-(V3) and (8) in Proposition 3.3. From this point of view, the notion of very true MTL-algebra also generalizes that of very true MV-algebra.

Although the $Fix_\tau(L)$ is not necessary a subalgebra of an MTL-algebra in general (in Example 3.2 (c), one can check that $Fix_\tau(L)$ is not a subalgebra of $L$ since it is not closed under $\to$), while it forms an MTL-algebra after redefined its fuzzy implication.

**Theorem 3.4.** *Let $\tau$ be a very true operator on $L$. Then $(Fix_\tau(L), \wedge, \vee, \odot, \rightsquigarrow, 0, 1)$ is an MTL-algebra, where $x \rightsquigarrow y = \tau(x \to y)$ for all $x, y \in Fix_\tau(L)$.*

*Proof.* First, we show that $(Fix_\tau(L), \wedge, \vee, 0, 1)$ is a bounded lattice with 0 as the smallest element and 1 as the greatest element. From Proposition 3.3 (6),(7), we have that $Fix_\tau(L)$ is closed under $\vee$ and $\wedge$. Thus $(Fix_\tau(L), \wedge, \vee)$ is a lattice. For all $x \in \tau(L)$, one can easily check that $x \vee 1 = 1$ and $x \wedge 0 = 0$. Thus, 0 is the smallest element and 1 is the greatest element in $Fix_\tau(L)$, respectively. Therefore $(Fix_\tau(L), \wedge, \vee, 0, 1)$ is a bounded lattice.

Next, we prove that $(Fix_\tau(L), \odot, 1)$ is a commutative monoid with 1 as neutral element. By Proposition 3.3 (5), we have $Fix_\tau(L)$ is closed under $\odot$. It follows that $(Fix_\tau(L), \odot)$ is a commutative semigroup. For all $x \in Fix_\tau(L)$, we obtain that $x \odot 1 = x$, that is, 1 is a unital element.

Then, we prove that $\rightsquigarrow$ and $\odot$ form an adjoint pair. For all $x, y \in Fix_\tau(L)$, we define $x \rightsquigarrow y = \tau(x \to y)$. Now, we will show that $x \odot y \leq z$ if and only if $y \leq x \rightsquigarrow z$ for all $x, y, z \in Fix_\tau(L)$. From Proposition 3.3 (10), we have $\tau(x) \leq y$ if and only if $\tau(x) \leq \tau(y)$. Hence we have $x \odot y \leq z$ if and only if $y \leq x \to z$ if and only if $\tau(y) \leq x \to z$ if and only if $\tau(y) \leq \tau(x \to z)$ if and only if $\tau(y) \leq x \rightsquigarrow y$ if and only if $y \leq x \rightsquigarrow z$ for all $x, y, z \in Fix_\tau(L)$.

Finally, we prove that the prelinearity condition holds. For all $x, y \in Fix_\tau(L)$, by (V5), we have $(x \rightsquigarrow y) \vee (y \rightsquigarrow x) = \tau(x \to y) \vee \tau(y \to x) = 1$.

Therefore, we obtain that $(Fix_\tau(L), \wedge, \vee, \odot, \rightsquigarrow, 0, 1)$ is an MTL-algebra. □

The result of Theorem 3.4 shows that the fixed point set $Fix_\tau(L)$ of very true operator in an MTL-algebra $L$ has the same structure as $L$, which reveals the essence of the fixed point set.

In the following, using the properties of very true operators, we give some conditions for an MTL-algebra to be an MV-algebra and a Gödel algebra.

**Theorem 3.5.** *Let $(L, \wedge, \vee, \odot, \to, 0, 1)$ be an MTL-algebra and $\tau$ be a very true operator on $L$. Then the following conditions are equivalent:*
(1) *$(L, \wedge, \vee, \odot, \to, 0, 1)$ is an MV-algebra,*
(2) *every very true operator $\tau$ satisfies $\tau(x \vee y) = (\tau(x) \to \tau(y)) \to \tau(y) = (\tau(y) \to \tau(x)) \to \tau(x)$ for all $x, y \in L$.*

*Proof.* (1) $\Rightarrow$ (2) We note that an MV-algebra satisfies $(x \to y) \to y = (y \to x) \to x$ for all $x, y \in L$. By Proposition 2.2 (11) and 3.3(7), we have $\tau(x \vee y) = \tau(x) \vee \tau(y) = ((\tau(x) \to \tau(y)) \to \tau(y)$. In the similar way, we can prove $\tau(x \vee y) = \tau(x) \vee \tau(y) = ((\tau(y) \to \tau(x)) \to \tau(x)$. Thus $\tau(x \vee y) = (\tau(x) \to \tau(y)) \to \tau(y) = (\tau(y) \to \tau(x)) \to \tau(x)$.

(2) $\Rightarrow$ (1) Suppose that every very true operator $\tau$ satisfies $\tau(x \vee y) = (\tau(x) \to \tau(y)) \to \tau(y) = (\tau(y) \to \tau(x)) \to \tau(x)$ for all $x, y \in L$. Taking $\tau = id_L$, we have $x \vee y = (x \to y) \to y = (y \to x) \to x$ for all $x, y \in L$. Therefore, $L$ is an MV-algebra. □

**Theorem 3.6.** *Let $(L, \wedge, \vee, \odot, \to, 0, 1)$ be an MTL-algebra and $\tau$ be a very true operator on $L$. Then the following conditions are equivalent:*
(1) *$(L, \wedge, \vee, \odot, \to, 0, 1)$ is a Gödel algebra,*
(2) *every very true operator $\tau$ satisfies $\tau(x \wedge y) = \tau(x) \odot \tau(y)$ for all $x, y \in L$,*
(3) *every very true operator $\tau$ satisfies $\tau(x \odot y) = \tau(x) \odot \tau(x \to y)$ for all $x, y \in L$.*

*Proof.* (1) ⇒ (2) Suppose that $L$ is a Gödel algebra. Then one can obtain that $x \odot y = x \wedge y = x \odot (x \to y)$ for any $x, y \in L$. By Proposition 3.3 (6), we have $\tau(x \wedge y) = \tau(x) \wedge \tau(y) = \tau(x) \odot \tau(y)$. Thus $\tau(x \wedge y) = \tau(x) \odot \tau(y)$.

(2) ⇒ (1) Suppose that every very true operator $\tau$ satisfies $\tau(x \odot y) = \tau(x) \wedge \tau(y)$ for all $x, y \in L$. Taking $\tau = id_L$, we have $x \odot y = x \wedge y$ for any $x, y \in L$. Taking $x = y$, we get $x \odot x = x$ for all $x \in L$. Therefore, $L$ is a Gödel algebra.

(1) ⇒ (3) From (1) ⇒ (2), one can obtain that $\tau(x \odot y) = \tau(x) \odot \tau(y)$ for any $x, y \in L$. Hence $\tau(x \odot y) = \tau(x \odot (x \to y) = \tau(x) \odot \tau(x \to y)$.

(3) ⇒ (1) Suppose that every very true operator $\tau$ satisfies $\tau(x \odot y) = \tau(x) \odot \tau(x \to y)$ for all $x, y \in L$. Taking $\tau = id_L$, we have $x \odot y = x \odot (x \to y)$ for any $x, y \in L$. Taking $x = y$, we get $x \odot x = x$ for all $x \in L$. Therefore, $L$ is a Gödel algebra. □

## 4 Very true filters of very true MTL-algebras

In this section, we introduce very true filters of very true MTL-algebras. In particular, we focus on algebraic structures of $VF(L)$ of all very true filters in the very true MTL-algebras and obtain that $VF(L)$ forms a complete Heyting algebra. Moreover, we characterize subdirectly irreducible very true MTL-algebras and prove a representation theorem for very true MTL-algebras via very true filters.

**Definition 4.1.** *Let $(L, \tau)$ be a very true MTL-algebra and $F$ be a filter of $L$. Then $F$ is called* a very true filter *of $(L, \tau)$ if $x \in F$ implies $\tau(x) \in F$ for all $x \in L$.*

We will denote the set of all very true filters of $(L, \tau)$ by $VF[L]$.

**Example 4.2.** *Considering Example 3.2 (c), one can easily check that the very true filters of $(L, \tau)$ are $\{a, b, 1\}$ and $\{1\}$ and $L$. However, $\{b, 1\}$ is a filter of $L$ but not a very true filter of $(L, \tau)$.*

Let $(L, \tau)$ be a very true MTL-algebra. For any nonempty set $X$ of $L$, we denote by $\langle X\rangle_\tau$ the very true filter of $(L, \tau)$ generated by $X$, that is, $\langle X\rangle_\tau$ is the smallest very true filter of $(L, \tau)$ containing $X$. If $F$ is a very true filter of $(L, \tau)$ and $x \notin F$, we put $\langle F, x\rangle_\tau := \langle F \cup \{x\}\rangle_\tau$.

The next theorem gives a concrete description of the very true filter generated by a subset of very true MTL-algebra $(L, \tau)$.

**Theorem 4.3.** *Let $(L, \tau)$ be a very true MTL-algebra and $X$ be a nonempty set of $L$. Then $\langle X\rangle_\tau = \{x \in L | x \geq \tau(y_1) \odot \cdots \tau(y_n), y_i \in X, n \geq 1\}$.*

*Proof.* The proof is easy, and we hence omit the details. □

**Example 4.4.** *Considering Example 3.2 (c), one can easily obtain that $\langle 0, a\rangle_\tau = \langle 0, b\rangle_\tau = \langle 0, a, b\rangle_\tau = \{a, b, 1\}$, $\langle L\rangle_\tau = \langle a, b, 1\rangle_\tau = \langle 0, a, 1\rangle_\tau = \langle 0, b, 1\rangle_\tau = \{1\}$ and $\langle 0\rangle_\tau = L$.*

**Theorem 4.5.** *Let $F$, $F_1$, $F_2$ be very true filters of $(L, \tau)$ and $a \notin F$. Then:*
(1) $\langle a\rangle_\tau = \{x \in L | x \geq (\tau a)^n, n \geq 1\}$,
(2) $\langle F \cup a\rangle_\tau = \{x \in L | x \geq f \odot (\tau a)^n, f \in F\} = F \vee [\tau a)$,
(3) $\langle F_1 \cup F_2\rangle_\tau = \{x \in L | x \geq f_1 \odot f_2, f_1 \in F_1, f_2 \in F_2\}$,
(4) *if* $a \leq b$, *then* $\langle b\rangle_\tau \subseteq \langle a\rangle_\tau$,
(5) $\langle \tau(a)\rangle_\tau = \langle a\rangle_\tau$,
(6) $\langle a\rangle_\tau \vee \langle b\rangle_\tau = \langle a \wedge b\rangle_\tau = \langle a \odot b\rangle_\tau$,
(7) $\langle a\rangle_\tau \cap \langle b\rangle_\tau = \langle \tau(a) \vee \tau(b)\rangle_\tau$.

*Proof.* The proof of $(1) - (5)$ are obvious.

(6) Since $a \odot b \leq a \wedge b \leq a, b$, we deduce that $\langle a \rangle_\tau, \langle b \rangle_\tau \subseteq \langle a \wedge b \rangle_\tau \subseteq \langle a \odot b \rangle_\tau$. It follows from that $\langle a \rangle_\tau \vee \langle b \rangle_\tau \subseteq \langle a \wedge b \rangle_\tau \subseteq \langle a \odot b \rangle_\tau$. Conversely, let $a \in \langle a \odot b \rangle_\tau$. Then for some natural number $n \geq 1$, $a \geq (\tau(a \odot b))^n \geq (\tau a \odot \tau b)^n = (\tau a)^n \odot (\tau b)^n$. Hence $a \in \langle a \rangle_\tau \vee \langle b \rangle_\tau$, we deduce that $\langle a \odot b \rangle_\tau \subseteq \langle a \rangle_\tau \vee \langle b \rangle_\tau$. Therefore $\langle a \rangle_\tau \vee \langle b \rangle_\tau = \langle a \wedge b \rangle_\tau = \langle a \odot b \rangle_\tau$.

(7) Since $\tau(a) \leq \tau(a) \vee \tau(b)$, we deduce that $\langle \tau(a) \vee \tau(b) \rangle_\tau \subseteq \langle \tau(a) \rangle_\tau = \langle a \rangle_\tau$. Analogously, $\langle \tau(a) \vee \tau(b) \rangle_\tau \subseteq \langle \tau(b) \rangle_\tau = \langle b \rangle_\tau$. It follows that $\langle \tau(a) \vee \tau(b) \rangle_\tau \subseteq \langle a \rangle_\tau \cap \langle b \rangle_\tau$. Moreover, let $t \in \langle a \rangle_\tau \cap \langle b \rangle_\tau$. Then for some natural number $n, m \geq 1$, $t \geq (\tau(a))^m$ and $t \geq (\tau(b))^n$. Hence $t \geq (\tau(a))^m \vee (\tau(b))^n \geq (\tau(a) \vee \tau(b))^{mn} = (\tau(a \vee b))^{mn}$, we deduce that $a \in \langle \tau(a) \vee \tau(b) \rangle_\tau$, that is, $\langle a \rangle_\tau \cap \langle b \rangle_\tau \subseteq \langle \tau(a) \vee \tau(b) \rangle_\tau$. Therefore $\langle a \rangle_\tau \cap \langle b \rangle_\tau = \langle \tau(a) \vee \tau(b) \rangle_\tau$. □

The next results shows that the algebraic structure of $VF(L)$ of all very true filters in very true MTL-algebras forms a complete Heyting algebra.

**Theorem 4.6.** *Let $(L, \tau)$ be a very true MTL-algebra. Define binary operations $\wedge, \vee, \mapsto$ on $VF(L)$ as follows: for all $F_1, F_2 \in VF(L)$, $F_1 \wedge F_2 = F_1 \cap F_2$, $F_1 \vee F_2 = \langle F_1 \cup F_2 \rangle_\tau$, $F_1 \mapsto F_2 = \{x \in L | \tau(x) \vee f_1 \in F_2$ for any $f_1 \in F_1\}$. Then $(VF(L), \wedge, \vee, \mapsto, 1, L)$ is a complete Heyting algebra.*

*Proof.* Suppose that $\{F_i\}_{i \in I}$ is a family of very true filters of $(L, \tau)$. From Theorem 4.5, it is easy to check that the infimum of $\{F_i\}_{i \in I} = \cap_{i \in I} F_i$ and the supermum is $\vee_{i \in I} F_i = \{x \in L | x \geq f_{i_1} \odot f_{i_2} \odot \cdots f_{i_m}, f_{1_j} \in F_{i_j}, i_j \in I, 1 \leq j \leq m\}$. Therefore, $(VF(L), \wedge, \vee, 1, L)$ is a complete lattice under the inclusion order $\subseteq$. Next, we define $F_1 \mapsto F_2 = \{x \in L | \tau(x) \vee f_1 \in F_2$ for any $f_1 \in F_1\}$ for any $F_1, F_2 \in VF(L)$. And, we shall prove that $F_1 \cap F_2 \subseteq F_3$ if and only if $F_2 \subseteq F_1 \mapsto F_3$ for all $F_1, F_2, F_3 \in VF(L)$, that is, $(VF(L), \wedge, \vee, \mapsto, 1, L)$ is a complete Heyting algebra. In order to do this, we first show that $F_1 \mapsto F_2$ is a very true filter of $(L, \tau)$.

Now, we will show that $F_1 \mapsto F_2$ is a very true filter of $(L, \tau)$. Clearly $1 \in F_1 \mapsto F_2$. Let $x \in F_1 \mapsto F_2$ and $x \leq y$, then for any $f_1 \in F_1$ such that $\tau(x) \vee f_1 \in F_2$. Since $\tau(x) \vee f_1 \leq \tau(y) \vee f_1 \in F_2$ and hence $y \in F_1 \mapsto F_2$. Assume that $x, y \in F_1 \mapsto F_2$, then for any $f_1 \in F_1$, $\tau(x) \vee f_1, \tau(y) \vee f_1 \in F_2$ and hence $f_1 \vee \tau(x \odot y) \in F_2$. So $x \odot y \in F_1 \mapsto F_2$. Obviously, if $x \in F_1 \mapsto F_2$, then $\tau(x) \in F_1 \mapsto F_2$ and thus $F_1 \mapsto F_2$ is a very true filter of $(L, \tau)$.

Next, we will prove that $F_1 \wedge F_2 \leq F_3$ if and only if $F_1 \leq F_2 \mapsto F_3$. Assume that $F_1 \wedge F_2 \leq F_3$. Let $f_1 \in F_1$. Then $\tau(f_1) \in F_1$ and for any $f_2 \in F_2$, we have $\tau(f_2) \vee f_1 \geq f_1$, $\tau(f_2) \vee f_1 \geq \tau(f_2)$. Hence $\tau(f_2) \vee f_1 \in F_1 \wedge F_2 \leq F_3$ and hence $f_1 \in F_1 \wedge F_2$. Conversely, assume that $F_1 \leq F_2 \mapsto F_3$. Let $x \in F_2 \wedge F_3$, then $x \in F_2 \mapsto F_3$. For any $f_3 \in F_3$, we have $\tau(x) \vee f_3 \in F_3$. Taking $f_3 = x \in F_3$, we have $x \vee \tau(x) = x \in F_3$. Thus $F_1 \leq F_2 \mapsto F_3$.

Therefore, $(VF(L), \wedge, \vee, \mapsto, 1, L)$ is a complete Heyting algebra. □

**Theorem 4.7.** *Let $(L, \tau)$ be a very true MTL-algebra and $F \in VF(L)$. Then the following conditions are equivalent:*
(1) *$F$ is a compact element of $VF(L)$,*
(2) *$F$ is a principal very true filter of $(L, \tau)$.*

*Proof.* (1) $\Rightarrow$ (2) Suppose that $F$ is the compact element of $VF(L)$. Since $F = \vee_{x \in F} \langle x \rangle_\tau$, then there exist $x_1, x_2 \cdots x_n$ such that $F = \langle x_1 \rangle_\tau \vee \langle x_2 \rangle_\tau \vee \cdots \vee \langle x_n \rangle_\tau$. By Proposition 4.5 (6), we have $F = \langle x_1 \odot x_2 \odot \cdots \odot x_n \rangle_\tau$. Therefore, $F$ is a principal very true filter of $(L, \tau)$.

(2) $\Rightarrow$ (1) Let $F$ be a principal very true filter of $(L, \tau)$. Then there exists $x \in L$ such that $F = \langle x \rangle_\tau$. Suppose that $\{F_i\}_{i \in I} \subseteq VF(L)$ and $F = \langle x \rangle_\tau \subseteq \vee_{i \in I} \{F_i\}$. Then $x \in \vee_{i \in I} F_i = \langle \cup_{i \in I} F_i \rangle_\tau$. It follows that there exist $i_j \in I$, $f_{i_j} \in F_{i_j}$ for all $1 \leq j \leq m$ such that $x \geq f_{i_1} \odot f_{i_2} \odot \cdots f_{i_m}$, that is, $x \in \langle F_{i_1} \cup F_{i_2} \cup \cdots \cup F_{i_n} \rangle_\tau = F_{i_1} \vee F_{i_2} \vee \cdots \vee F_{i_n}$. Hence $F = \langle x \rangle_\tau \subseteq F_{i_1} \vee F_{i_2} \vee \cdots \vee F_{i_n}$. Therefore, $F$ is a compact element of $VF(L)$. □

**Definition 4.8.** *Let $(L, \tau)$ be a very true MTL-algebra and $\theta$ be a congruence on $L$. Then $\theta$ is called a* very true congruence *on $(L, \tau)$ if $(x, y) \in \theta$ implies $(\tau(x), \tau(y)) \in \theta$, for any $x, y \in L$.*

**Example 4.9.** *Considering Example 3.2 (c), one can see that $R = \{\{0, 0\}, \{a, a\}, \{b, b\}, \{1, 1\}, \{a, b\}, \{b, a\}, \{a, 1\}, \{1, a\}, \{b, 1\}, \{1, b\}\}$ is a very true congruence on $(L, \tau)$.*

**Theorem 4.10.** *For any very true MTL-algebra there exists a one to one correspondence between its very true filters and its very true congruences.*

*Proof.* The proof is easy, and we hence omit the details. □

Let $(L, \tau)$ be a very true MTL-algebra and $F$ be a very true filter. We define the mapping $\tau_F : L/F \to L/F$ such that $\tau_F([x]) = [\tau(x)]$ for any $x \in L$.

**Proposition 4.11.** *Let $(L, \tau)$ be a very true MTL-algebra and $F$ a very true filter of $(L, \tau)$. Then $(L/F, \tau_F)$ is a very true MTL-algebra.*

*Proof.* The proof is easy, and we hence omit the details. □

**Definition 4.12.** *Let $(L, \tau)$ be a very true MTL-algebra. A proper very true filter $F$ of $(L, \tau)$ is called a* prime very true filter *of $(L, \tau)$, if for all very true filter $F_1$, $F_2$ of $(L, \tau)$ such that $F_1 \cap F_2 \subseteq F$, then $F_1 \subseteq F$ or $F_2 \subseteq F$.*

**Example 4.13.** *Considering Example 3.2 (c), one can easily obtain that $\{a, b, 1\}$ is a prime very true filter of $(L, \tau)$.*

**Theorem 4.14.** *Let $(L, \tau)$ be a very true MTL-algebra and $F$ be a proper very true filter of $(L, \tau)$. Then the following are equivalent:*
(1) *$F$ is a prime very true filter of $(L, \tau)$,*
(2) *if $\tau(x) \vee \tau(y) \in F$ for some $x, y \in L$, then $x \in F$ or $y \in F$,*
(3) *$(L/F, \tau_F)$ is a chain.*

*Proof.* (1) $\Rightarrow$ (2) Let $\tau(x) \vee \tau(y) \in F$ for some $x, y \in L$. Then $\langle x\rangle_\tau \cap \langle y\rangle_\tau = \langle \tau(x) \vee \tau(y)\rangle_\tau \in F$. Since $F$ is a prime very true filter of $(L, \tau)$, then $\langle x\rangle_\tau \subseteq F$ or $\langle y\rangle_\tau \subseteq F$. Therefore, $x \in F$ or $y \in F$.

(2) $\Rightarrow$ (1) Suppose that $F_1$,$F_2 \in MF[L]$ such that $F_1 \cap F_2 \subseteq F$ and $F_1 \nsubseteq F$ and $F_2 \nsubseteq F$. Then there exist $x \in F_1$ and $y \in F_2$ such that $x, y \notin F$. Since $F_1$, $F_2$ are very true filters of $(L, \tau)$, then $\tau(x) \in F_1$ and $\tau(y) \in F_2$. From $\tau(x)$, $\tau(y) \leq \tau(x) \vee \tau(y)$, we obtain that $\tau(x) \vee \tau(y) \in F_1 \cap F_2 = F$. By (2), we get $x \in F$ or $y \in F$, which is a contradiction. Therefore, $F$ is a prime very true filter of $(L, \tau)$.

(1) $\Leftrightarrow$ (3) From (2), one can obtain that every prime very true filter of $(L, \tau)$ must be a prime filter of $L$. Based on this, the equivalence of (1) and (3) is clear. □

For proving the subdirect representation theorem of very true MTL-algebras we will need the following theorem.

**Theorem 4.15.** *Let $(L, \tau)$ be a very true MTL-algebra and $a \in L$. If $a \neq 1$, then there exists a prime very true filter $P$ of $(L, \tau)$ such that $a \notin P$.*

*Proof.* Denote $F_a = \{F' | F'$ is a proper very true filter of $(L, \tau)$ such that $F \subseteq F'$, $a \notin F'\}$. Then $F_a \neq \emptyset$ since $F$ is a very true filter not containing $a$ and $F_a$ is a partially set under inclusion relation. Suppose that $\{F_i | i \in I\}$ is a chain in $F_a$, then $\cup\{F_i | i \in I\}$ is a very true filter of $(L, \tau)$ and it is the upper bounded of this chain. By Zorn's Lemma, there exists a maximal element $P$ in $F_a$. Now, we shall prove that $P$ is the desire prime very true filter of ours. Since $P \in F_a$, then $P$ is a proper very true filter and $a \notin P$.

Let $x \vee y \in P$ for some $x, y \in L$. Suppose that $x \notin P$ and $y \notin P$. Since $P$ is strictly contained in $\langle P, x\rangle_\tau$ and $\langle P, y\rangle_\tau$ and by the maximality of $P$, we deduce that $\langle P, x\rangle_\tau \notin F_a$ and $\langle P, y\rangle_\tau \notin F_a$. Then $a \in \langle P, x\rangle_\tau = P \vee [\tau x)$ and $a \in \langle P, y\rangle_\tau = P \vee [\tau(y))$. Then we have $a \in (P \vee [\tau(x))) \wedge (P \vee [\tau(y))) = P \vee ([\tau(x)) \wedge [\tau(y))) = P \vee [\tau(x) \vee \tau(y)) = P \vee [\tau(x \vee y)) \in P$, which implies that $a \in P$, a contradiction. Therefore, $P$ is a prime very true filter such that $F \subseteq P$ and $a \notin P$. Put $F = \{1\}$, the result is easy to obtain. □

Now, we will prove that every very true MTL-algebra is a subdirect product of linearly ordered very true MTL-algebras.

**Theorem 4.16.** *Each very true MTL-algebra is a subalgebra of the direct product of a system of linearly ordered very true MTL-algebras.*

*Proof.* The proof of this theorem is as usual and the only critical point is the above Theorem 4.15. □

The next results shows that MTL-algebra is representable if and only if very true MTL-algebra is representable.

**Theorem 4.17.** *Let $(L,\tau)$ be a very true MTL-algebra. Then the following conditions are equivalent:*
(1) *$L$ is representable;*
(2) *$(L,\tau)$ is a subdirect product of linearly ordered very true MTL-algebras.*

*Proof.* (1) $\Rightarrow$ (2) Suppose that the MTL-algebra $L$ is representable. Then by Theorem 2.9, there exists a system $S$ of prime filter of $L$ such that $\bigcap S=\{1\}$. Since every prime filter of $L$ contains a minimal prime filter, we get that in our case the intersection of all minimal prime filter is equal to $\{1\}$. Moreover, we will show that every minimal prime filter in $(L,\tau)$. Let $P$ be a minimal prime filter of $L$. Then by Theorem 2.8, $P=\cup\{a^{\perp}|a\in P\}$. If $x\in P$, then there is $a\notin P$ such that $x\vee a=1$, hence $1=\tau(x\vee a)=\tau(x)\vee\tau(a)$. Since $a\notin P$, we get $\tau(a)\notin P$, therefore $\tau(x)\in P$, that means that $P$ is a very true filter in $(L,\tau)$. Therefore, $(L,\tau)$ is a subdirect product of linearly ordered very true NM-algebras.

(2) $\Rightarrow$ (1) The converse is trivial and we hence omit this. □

**Theorem 4.18.** *Let L be an MTL-algebra. Then the following condition are equivalent:*
(1) *$L$ is representable,*
(2) *$L$ can be embedded in a very true MTL-algebra $(L,\tau)$.*

*Proof.* (1) $\Rightarrow$ (2) If $L$ is representable, then $L$ is a subdirect product of MTL-chain. By Example 3.2 (b), any MTL-chain has a structure of very true MTL-algebra. Moreover, the class of very true MTL-algebras is a variety, so a direct product of very true MTL-algebra is still a very true MTL-algebra.

(2) $\Rightarrow$ (1) is straightforward, since any very true MTL-algebra is a subdirect product of very true MTL-chains. □

**Definition 4.19.** *A very true MTL-algebra $(L,\tau)$ is said to be a* subdirectly irreducible *if it has the least nontrivial very true congruence.*

Let $(L,\tau)$ be subdirectly irreducible. Then by Theorem 4.10, there is a very true filter $F$ of $(L,\tau)$ such that $\theta_F=F$, that means, $F$ is the least very true filter of $(L,\tau)$ such that $F\neq\{1\}$. Thus, we can conclude that a very true MTL-algebra $(L,\tau)$ is said to be subdirectly irreducible if among the nontrivial very true filters of $(L,\tau)$ there exists the least one, i.e., $\cap\{F\in VF(L)|F\neq\{1\}\}\neq\{1\}$.

**Example 4.20.** *Considering Example 3.2 (c), one can easily check that the very true MTL-algebra $(L,\tau)$ is subdirectly irreducible.*

Next, we will show that every subdirectly irreducible very true MTL-algebra is linearly ordered. To prove this important result, we need the following several propositions and theorems.

**Proposition 4.21.** *Let $(L,\tau)$ be a subdirectly irreducible very true MTL-algebra and $F_1, F_2\in VF(L)$. If $F_1\cap F_2=\{1\}$, then $F_1=\{1\}$ or $F_2=\{1\}$.*

*Proof.* Suppose $F_1\neq\{1\}$ and $F_2\neq\{1\}$, i.e., $F_1, F_2\in\cap\{F\in VF(L)|F\neq\{1\}\}\neq\{1\}$, then $\cap\{F\in VF(L)|F\neq\{1\}\}\neq\{1\}\subseteq F_1\cap F_2$. By $F_1\cap F_2=\{1\}$, we can get $\cap\{F\in VF(L)|F\neq\{1\}\}=\{1\}$, which contradicts the fact that $(L,\tau)$ is subdirectly irreducible. Hence, $F_1=\{1\}$ or $F_2=\{1\}$. □

**Theorem 4.22.** *Let $(L,\tau)$ be a very true MTL-algebra. Then the following conditions are equivalent:*
(1) *$(L,\tau)$ is a subdirectly irreducible very true MTL-algebra,*
(2) *there exists an element $a\in L$,$a<1$, such that for any $x\in L$, $x<1$ and $a\in\langle x\rangle_\tau$.*

*Proof.* (1) ⇒ (2) Suppose that $(L,\tau)$ is a subdirectly irreducible very true MTL-algebra, i.e.,$\cap\{F \in VF(L)|F \neq \{1\}\} \neq \{1\}$, then $\cap\{\langle x\rangle_\tau|x < 1\} \neq \{1\}$. Take $a \in \cap\{\langle x\rangle_\tau|x < 1\}$ satisfying $a \neq 1$, then for any $x \in L$, $x \neq 1$,$a \in \langle x\rangle_\tau$, by Theorem 4.5 (1), there exists $m \in N$, such that $a \geq (\tau(x))^m$. Clearly, $a$ is the element that we need.

(2) ⇒ (1) Conversely, we need to prove that for any $F \in VF(L)$, if $F \neq \{1\}$, then $a \in F$. In fact, by $F \neq \{1\}$, it follows that there exists $x \in F$, $x < 1$, and then, by the known condition, $a \in \langle x\rangle_\tau$, furthermore, $a \in F$, so $a \in \cap\{F \in VF(L)|F \neq \{1\}\}$. Hence, $\cap\{F \in VF(L)|F \neq \{1\}\} \neq \{1\}$, i.e., $(L,\tau)$ is a subdirectly irreducible very true MTL-algebra. □

We recall that a non-unit element $a \in L$ is said to be a *coatom* of $L$ if $a \leq b$, then $b \in \{a, 1\}$, i.e. $b = a$ or $b = 1$ ([25]). In the following proposition, we will show that every subdirectly irreducible very true MTL-algebra has at most one coatom.

**Proposition 4.23.** *Let $(L,\tau)$ be a subdirectly irreducible very true MTL-algebra. For any $x,y \in L$, if $x \vee y = 1$, then $x = 1$ or $y = 1$.*

*Proof.* For any $x, y \in L$, if $x\vee y = 1$, then by Theorem 4.5, we have $\langle x\rangle_\tau \cap \langle y\rangle_\tau = \langle \tau(x)\vee\tau(y)\rangle_\tau = \langle \tau(x\vee y)\rangle_\tau = \langle 1\rangle_\tau = \{1\}$. By Proposition 4.21, we have $\langle x\rangle_\tau = \{1\}$ or $\langle y\rangle_\tau = \{1\}$, hence $x = 1$ or $y = 1$. □

The following Theorem shows that the subdirectly irreducible very true MTL-algebra $(L,\tau)$ is linearly ordered, that is to say, the fuzzy truth value of all propositions in very true MTL logic are comparable. This is of key importance from the logical point of view.

**Theorem 4.24.** *Let $(L,\tau)$ be a very true MTL-algebra. Then the following conditions are equivalent:*
*(1) $(L,\tau)$ is a subdirectly irreducible very true MTL-algebra,*
*(2) $(L,\tau)$ is a chain.*

*Proof.* (1) ⇒ (2) Suppose $(L,\tau)$ is a subdirectly irreducible very true MTL-algebra. Applying Definition 2.1, we have $(x \to y) \vee (y \to x) = 1$ for any $x, y \in L$, then by Proposition 4.23, we have $x \to y = 1$ or $y \to x = 1$, i.e., $x \leq y$ or $y \leq x$, so $(L,\tau)$ is a chain.

(2) ⇒ (1) Since $(L,\tau)$ is a chain and nontrivial,there exists a unique dual atom, denoted as $a$. Suppose $F$ is any very true filter of $(L,\tau)$ satisfying $F \neq \{1\}$, then $a \in F$. Since $F$ is chosen arbitrarily from $VF(L)$, then $a \in \cap\{F \in VF(L)|F \neq \{1\}\}$. Hence $\cap\{F \in VF(L)|F \neq \{1\}\} \neq \{1\}$, i.e., $(L,\tau)$ is a subdirectly irreducible very true MTL-algebra. □

In what follows, we introduce the stabilizer of a nonempty subset set $X$ with respect to a very true operator $\tau$ and study some properties of them. Let $(L,\tau)$ be a very tue MTL-algebra. Given a nonempty subset $X$ of $L$, we put $R_\tau(X) = \{a \in L|\tau(a) \to x = x, \forall x \in X\}$, and $L_\tau(X) = \{a \in L|x \to \tau(a) = \tau(a), \forall x \in X\}$, which are called *right and left stabilizer* of $X$ with respect to $\tau$. Clearly, $R_\tau(X), L_\tau(X) \neq \emptyset$. In fact, $1 \in R_\tau(X) \cap L_\tau(X)$. In particular, if $\tau = id_L$, which is a right and left stabilizer of $X$ (see [26]).

**Example 4.25.** *Considering Example 3.2 (c). Let $X = \{a, b\}$. Then $L_\tau(X) = \{a, b, 1\}$ and $R_\tau(X) = \{1\}$.*

**Proposition 4.26.** *Let $L$ be a MTL-algebra and $X, Y \subseteq L$. Then the following conditions hold:*
*(1) $1 \in R_\tau(X) \cap L_\tau(X)$,*
*(2) If $X \subseteq Y$, then $L_\tau(X) \subseteq L_\tau(Y)$ and $R_\tau(X) \subseteq R_\tau(Y)$,*
*(3) $X \subseteq L_\tau(R_\tau(X)) \cap R_\tau(L_\tau(X))$,*
*(4) $R_\tau(X) = R_\tau(L_\tau(R_\tau(X)))$ and $L_\tau(X) = L_\tau(R_\tau(L_\tau(X)))$,*
*(5) $L_\tau(L) = R_\tau(L) = \{1\}$ and $L_\tau(1) = R_\tau(1) = L$,*
*(6) If $\emptyset \neq X \subseteq L$, then $R_\tau(X)$ is a very true filter of $(L,\tau)$.*

*Proof.* The proof of (1)-(5) is easy by Proposition 2.2.

(6) Let $a, b \in R_\tau(X)$. Then $\tau(a) \to x = x$ and $\tau(b) \to x = x$ for all $x \in X$. Hence by Proposition 2.2 (12), $(\tau(a) \odot \tau(b)) \to x = \tau(a) \to (\tau(b) \to x) = \tau(a) \to x = x$ for all $x \in X$ and so $\tau(a \odot b) \to x \leq (\tau(a) \odot \tau(b)) \to x = x$. On the other hand, we have $x \leq \tau(a \odot b) \to x$. Therefore $x = \tau(a \odot b) \to x$ for all $x \in X$ and hence $a \odot b \in R_\tau(X)$. Now, let $a \leq b$ and $a \in R_\tau(X)$. Then $\tau(a) \to x = x$, for all $x \in X$. Hence by Proposition 3.3, $\tau(b) \to x \leq \tau(a) \to x = x$. Since by $x \leq \tau(b) \to x$, then $\tau(b) \to x = x$ and $b \in R_\tau(X)$. Finally, one can easy check that if $a \in R_\tau(X)$ then $\tau(a) \in R_\tau(X)$. Therefore $R_\tau(X)$ is a very true filter of $(L, \tau)$. □

**Theorem 4.27.** *Let $F$ be a very true filter of $(L, \tau)$. Then $R_\tau(F)$ is a pseudocomplemented of $F$ in the complete Heyting algebra $(VF(L), \wedge, \vee, \mapsto, 1, L)$.*

*Proof.* First, we prove that $F \cap R_\tau(F) = \{1\}$. Let $x \in F \cap R_\tau(F)$. Since $x \in R_\tau(F)$, then for any $a \in F$, $\tau(x) \to a = a$. Now, since $x \in F$, put $a = x$ we have $x = 1$. Therefore, $F \cap R_\tau(F) = \{1\}$. Now, let $G$ be a very true filter of $(L, \tau)$ such that $F \cap G = \{1\}$. Let $a \in G$, then $\tau(a) \in G$. Then for any $x \in F$, since $\tau(a), x \leq \tau(a) \vee x$, $x \in F$, then $\tau(a) \vee x \in F$ and $\tau(a) \vee x \in G$ and so $\tau(a) \vee x \in \{1\}$. Hence $\tau(a) \vee x = 1$, that is $((\tau(a) \to x) \to x) \wedge ((x \to \tau(a)) \to \tau(a)) = 1$ and so $((\tau(a) \to x) \to x) = 1$. Hence, $\tau(a) \to x \leq x$. On the other hand, $x \leq \tau(a) \to x$, then $\tau(a) \to x = x$ and $a \in R_\tau(F)$. Thus $G \subseteq R_\tau(F)$. Therefore, $R_\tau(F)$ is a pseudocomplemented of $F$ in the complete Heyting algebra $(VF(L), \wedge, \vee, \mapsto, 1, L)$. □

**Theorem 4.28.** *Let $(L, \tau)$ be a very true MTL-algebra. Then $(VF(L), \wedge, \vee, \mapsto, 1, L)$ is a complete pseudocomplemented Heyting lattice.*

*Proof.* It follows from Theorem 4.6 and Theorem 4.27. □

Now, we use of the right and left stabilizers of a very true MTL-algebra to produce a basis for a topology on it. Then we show that the generated topology by this basis is Baire, connected, locally connected and separable.

**Theorem 4.29.** *Let $(L, \tau)$ be a very true MTL-algebra and $X \subseteq L$. Define a mapping $\alpha : \mathcal{P}(\mathcal{L}, \ll) \to \mathcal{P}(\mathcal{L}, \ll)$ such that $\alpha(X) = R_\tau(L_\tau(X))$ for all $X \in \mathcal{P}(\mathcal{L})$. Then the following conditions hold,*
*(1) $\alpha$ is a closure map on $(L, \tau)$,*
*(2) $X \subseteq \alpha(Y)$ if and only if $\alpha(X) \subseteq \alpha(Y)$, for all $Y \subseteq L$,*
*(3) $\beta_\alpha = \{X \subseteq \mathcal{P}(\mathcal{L}) | \alpha(X) = X\}$ is a basis for a topology on $(L, \tau)$.*

*Proof.* (1) It follows from Proposition 4.26 (2),(3),(4).

(2) The proof is easy.

(3) Let $\beta_\alpha = \{X \in \mathcal{P}(\mathcal{L}) | \alpha(X) = X\}$. It is clear that $\emptyset \in \beta_\alpha$. Also, by Proposition 4.26 (5), $\alpha(L) = R_\tau(L_\tau(L)) = R_\tau(\{1\})$. Thus, $\alpha(L) = L$, and $L \in \beta_\alpha$. Now, suppose that $X, Y \in \beta_\alpha$. Then $\alpha(X) = X$ and $\alpha(Y) = Y$. We prove that $X \cap Y \in \beta_\alpha$. Since $X \cap Y \subseteq X, Y$, by (1), $\alpha(X \cap Y) \subseteq \alpha(X), \alpha(Y)$. Thus, $\alpha(X \cap Y) \subseteq \alpha(X) \cap \alpha(Y)$. Also, since $X, Y \in \beta_\alpha$, we have $\alpha(X \cap Y) \subseteq X \cap Y$. Moreover, by Proposition 4.26 (3), $X \cap Y \subseteq \alpha(X \cap Y)$. Then $\alpha(X \cap Y) = X \cap Y$, and so $X \cap Y \in \beta_\alpha$. Therefore, $\beta_\alpha$ is a basis. □

**Definition 4.30.** *Based on Theorem 4.29, we introduce the topological space, $(L, \mathcal{T}_\oslash)$ is called* stabilizer topology *on very true MTL-algebras.*

From Proposition 4.26 (6), one can obtain that $R_\tau(X) \in VF(L)$, for any $X \subseteq L$ and hence every element of $\beta_\alpha$ is a very true filter of $(L, \tau)$.

**Theorem 4.31.** *The stabilizer topology $(L, \mathcal{T}_\oslash)$ is connected and locally connected.*

*Proof.* The proof is easy, and we hence omit the details. □

**Theorem 4.32.** *The stabilizer topology $(L, \mathcal{T}_\oslash)$ is Hausdorff space if and only if $L = \{1\}$.*

*Proof.* The proof is easy, so we hence omit the details. □

**Theorem 4.33.** *The stabilizer topology $(L, \mathcal{T}_{\oslash})$ is separable.*

*Proof.* First, we show that if $\emptyset \neq X \subseteq L$ such that $1 \in X$, then $\overline{X} = L$. Let $\emptyset \neq X \subseteq L$ such that $1 \in X$. We only show that $L \subseteq \overline{X}$. Let $x \in L$. If $x = 1$, then $x \in \overline{X}$. Hence $\overline{X} = L$. Now, suppose that $1 \neq x \in L$. Then there exists an open subset $U \in \beta_\alpha$ such that $x \in U$. Since $U \in VF(L)$ and $1 \in U$, we have $U \cap (X - \{x\}) \neq \emptyset$. Hence $x \in \overline{X}$, and so $\overline{X} = L$. Since $1 \in \beta_\alpha$, thus $\overline{1} = L$. Hence $(L, \mathcal{T}_{\oslash})$ is separable. □

**Theorem 4.34.** *The stabilizer topology $(L, \mathcal{T}_{\oslash})$ is a Baire space.*

*Proof.* Let $U \in \mathcal{T}_{\oslash}$. Since $U \in VF(L)$, we have $1 \in U$. Then by Theorem 4.32, $\overline{U} = L$. Thus, every open set of $(L, \mathcal{T}_{\oslash})$ is dense. On the other hand, for each collection of open set $U_n$, $\cap U_n \in VF(L)$, so $1 \in \cap U_n$. Thus, by Theorem 4.32, $\cap U_n$ is dense. Therefore, $(L, \mathcal{T}_{\oslash})$ is a Baire space. □

## 5 Very true MTL-logic

In this section, we translate the defining properties of very true MTL-algebras into logical axioms, and show that the resulting logic, i.e. very true MTL logic (MTL$_{vt}$, for short) is sound and complete with respect to the variety of very true MTL-algebras.

Now, we deal with propositional calculus and define the axioms of the logic MTL$_{vt}$ to be those of MTL [2] plus the following ones:

(vt1) $vt(\varphi) \Rightarrow \varphi$,
(vt2) $vt(\varphi \Rightarrow \psi) \Rightarrow (vt\varphi \Rightarrow vt\psi)$,
(vt3) $vt(\varphi) \Rightarrow vt(vt\varphi)$,
(vt4) $vt(\varphi \Rightarrow \psi) \sqcup vt(\psi \Rightarrow \varphi)$.

The deduction rules are modus ponens (MP, from $\phi$ and $\phi \Rightarrow \psi$ infer $\psi$), and Generalization(G,from $\phi$ infer $vt\phi$).

To prove the completeness theorem, we need some definitions and results about MTL$_{vt}$ logic.

The consequence relation $\vdash$ is defined in the usual way. Let $T$ be a theory,i.e.,a set of formulas in MTL$_{vt}$. A (formula) proof of a formula $\phi$ in $T$ is a finite sequence of formulas with $\phi$ at its end, such that every formula in the sequence is either an axiom of MTL$_{vt}$, a formula of $T$, or the result of an application of an deduction rule to previous formulas in the sequence. If a proof of $\phi$ exists in $T$, we say that $\phi$ can be deduced from $T$ and we denote this by $T \vdash \phi$. Moreover $T$ is complete if for each pair $\phi$,$\psi$, $T \vdash \phi \Rightarrow \psi$ or $T \vdash \psi \Rightarrow \phi$.

**Definition 5.1.** *Let $(L, \tau)$ be a very true MTL-algebra and $T$ be a theory. An $L$-evaluation is a mapping $e$ from the set of formulas of MTL$_{vt}$ to $L$ that satisfies, for each two formulas $\phi$ and $\psi$: $e(\phi \Rightarrow \psi) = e(\phi) \to e(\psi)$, $e(\phi \sqcup \psi) = e(\phi) \vee e(\psi)$, $e(\phi \sqcap \psi) = e(\phi) \wedge e(\psi)$, $e(\phi \& \psi) = e(\phi) \odot e(\psi)$, $e(vt\phi) = \tau e(\phi)$, $e(\bar{0})=0$ and $e(\bar{1})=1$. If an $L$-evaluation $e$ satisfies $e(\chi) = 1$ for every $\chi$ in $T$, it is called an $L$-model of $T$.*

Now, we stress our attention to the Lindenbaum-Tarski algebra of MTL$_{vt}$.

**Definition 5.2.** *Let $T$ be a fixed theory over MTL$_{vt}$. For each formula $\phi$, let $[\phi]_T$ be the set of all formulas $\psi$ such that $T \vdash \phi \equiv \psi$ (where $\phi \equiv \psi$ stands for $(\phi \to \psi)\&(\psi \to \varphi)$) and $L_T$ be the set of all the class $[\phi]_T$. We define: $0 = [0]_T$, $1 = [1]_T$, $[\phi]_T \Rightarrow [\psi]_T = [\phi \Rightarrow \psi]_T$, $[\phi]_T \& [\psi]_T = [\phi \& \psi]_T$, $[\phi]_T \sqcup [\psi]_T = [\phi \sqcup \psi]_T$, $[\phi]_T \sqcap [\psi]_T = [\phi \sqcap \psi]_T$, $\tau_T[\phi]_T = [vt\phi]_T$. This algebra is denoted by $(L_T, \tau_T)$.*

**Proposition 5.3.** *$(L_T, \tau_T)$ is a very true MTL-algebra.*

*Proof.* It follows similarly from the proof of Proposition 4.11. □

**Theorem 5.4.** *Let $T$ be a theory over MTL$_{vt}$. Then $T$ is complete if and only if the very true MTL-algebra $(L_T, \tau_T)$ is linearly ordered.*

*Proof.* It follows similarly from the proof of Theorem 4.14. □

It is easy to check that $\mathrm{MTL}_{vt}$ is sound with respect to the variety of very true MTL-algebras, i.e.,that if a formula $\phi$ can be deduced from a theory $T$ in $\mathrm{MTL}_{vt}$, then for every very true MTL-algebra $(L, \tau)$ and for every $L$-model $e$ of $T$, $e(\phi) = 1$. Indeed, we need to verify the soundness of the new axioms and deduction of $\mathrm{MTL}_{vt}$ (for the axioms and rules of MTL, the reader can check [2]). For the axioms this is easy, as they are straightforward generalizations of axioms of very true MTL-algebras. We will now verify the soundness of the new deduction rules.

**Proposition 5.5.** *The deduction rules of $MTL_{vt}$ are sound in the following sense, for any formula $\phi$ and $\psi$.*
(1) *If for all very true MTL-algebra $(L, \tau)$ and for all $L$-model e for $T$, $e(\phi) = 1$, then for all very true MTL-algebra $(L, \tau)$ and for all $L$-model e for $T$, $e(vt\phi) = 1$.*
(2) *If for all very true MTL-algebra $(L, \tau)$ and for all $L$-model e for $T$, $e(\phi) = 1$ and $e(\phi \to \psi) = 1$, then for all very true MTL-algebra $(L, \tau)$ and for all $L$-model e for $T$, $e(\psi) = 1$.*

*Proof.* (1) Take such a very true MTL-algebra $(L, \tau)$ and such a model $e$. Then $e(\phi) = 1$, and $e(vt\phi) = \tau e(\phi) = \tau(1) = 1$.

(2) Take such very true MTL-algebra and a model $e$. Then $e(\phi) \to e(\psi) = e(\phi) \to e(\psi) = 1$, which means $e(\phi) \leq e(\psi)$. If $e(\phi) = 1$, we have $1 = e(\phi) \leq e(\psi)$ and thus $e(\psi) = 1$. □

**Theorem 5.6.** *Let $T$ be a theory over $MTL_{vt}$. If $T$ is a theory and $T \nvdash \phi$, then there is a consistent complete supertheory $T' \supseteq T$ such that $T' \nvdash \phi$.*

*Proof.* It follows similarly with the proof of Theorem 4.15. □

The next result in the sequence is the completeness of very true MTL-logic is proved based on the variety of very true MTL-algebras.

**Theorem 5.7.** *Let $T$ be a theory over $MTL_{vt}$. For each formula $\varphi$, the following are equivalent:*
(1) *$T \vdash \phi$;*
(2) *for each very true MTL-algebra $(L, \tau)$ and for every $L$-model e of $T$, $e(\phi) = 1$;*
(3) *for each linearly ordered very true MTL-algebra $(L, \tau)$ and for every $L$-model e of $T$, $e(\phi) = 1$.*

*Proof.* It follows from Theorems 4.16, 5.4, 5.6 and Proposition 5.5. □

**Theorem 5.8.** *$MTL_{vt}$ is a conservative extension of MTL.*

*Proof.* Suppose $MTL \nvdash \varphi$ then there is a linearly ordered MTL-algebra $L$ such that $\varphi$ is not an $L$-model. Expand an MTL-algebra $L$ to a very true MTL-algebra follows from Example 3.2 (b). By Theorem 5.7, we have $MTL_{vt} \nvdash \varphi$. Therefore, $\mathrm{MTL}_{vt}$ is a conservative extension of MTL. □

# 6 Conclusion

In this paper, motivated by the previous research of very true operators on BL-algebras, we extended the concept of very true operators to MTL-algebras. Also, we gave s characterizations of MV-algebras and Gödel algebras via this operator. Moreover, we investigated very true filters of very true MTL-algebras and focus on an analogous of representation theorem for very true MTL-algebras and characterize subdirectly irreducible very true MTL-algebras by very true filters. Then, we introduced the left and right stabilizers of very true MTL-algebras and constructed stabilizer topology via it. Finally, the corresponding logic very true MTL-logic was constructed and the soundness and completeness of this logic were proved based on very true MTL-algebras. Our further work on this topic will focus on the varieties of very true MTL-algebras. In particular, we will investigate semisimple, locally finite, finitely approximated and splitting varieties of very true MTL-algebras as well as varieties with the disjunction and the existence properties.

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# References

[1] Hájek P., Metamathematics of Fuzzy logic, Kluwer Academic Publishers, Dordrech, 1998.
[2] Esteva F., Godo L., Monoidal t-norm based logic: towards a logic for left-continuoust-norms, Fuzzy Sets Syst., 2001, 124, 271-288.
[3] Jenei S., Montagna F., A proof of standard completeness for Esteva and Godo's logic MTL, Stud Log., 2002, 70, 183-192.
[4] Vetterlein T., MTL-algebras arising from partially ordered groups, Fuzzy Sets Syst., 2010, 161, 433-443.
[5] Noguera C., Esteva F., Gispert J., On Some Varieties of MTL-algebras, Log. J. IGPL., 2004, 13, 443-466.
[6] Haveshki M., Saeid A. B., Eslami E., Some type of filters in BL-algebras, Soft Comput., 2006, 10, 657-664.
[7] Borzooei R. A., Shoar S. Khosravi., Ameri. R., Some type of filters in MTL-algebras, Fuzzy Sets Syst., 2012, 187, 92-102.
[8] Haveshki M., A note on some types of filters in MTL-algebras, Fuzzy Sets Syst., 2014, 16, 135-137.
[9] Hájek P., On very true, Fuzzy Sets Syst., 2001, 124, 329-333.
[10] Zadeh L., Fuzzy logic and approximate reasoning, Synthese., 1975, 30, 407-428.
[11] Ganter B., Wille R., Formal concept analysis, Mathematical Foundations, Springer-Verlag, Berlin, 1999.
[12] Bělohlávek R., Vyhodil V., Reducing the size of fuzzy concept lattices by hedges, In: FUZZ-IEEE 2005, the IEEE internationnal conference on fuzzy systems (25-25 May 2005), Reno, NV, USA, 2005, 65-74.
[13] Chajda I., Hedges and successors in basic algebras. Soft Comput., 2011, 15, 613-618.
[14] Hájek P., Harmancová D., A hedge for Gödel fuzzy logic, International Journal of Uncertainly, Fuzziness and Knowledge-Based Systems., 2000, 8, 495-498.
[15] Leuştean I., Non-commutative Łukasiewicz propositional logic, Arch Math Logic., 2006, 45, 191-213.
[16] Rachunek J., Šalounová D., Truth values on generalizations of some commutative fuzzy structures, Fuzzy Sets Syst., 2006,157,3159-3168.
[17] Botur M., Švrček F., Very true on CBA fuzzy logic, Math Slovaca., 2010, 60, 435-446.
[18] Wang J T., Xin X L., Young B J., Very true operators on equality algebras, submitted for publication.
[19] Chajda I., Kolařík M., Very true operators in effect algebras, Soft Comput., 2012, 16, 1213-1218.
[20] Grätzer G., Lattice theory, W H Freeman and Company, San Franciso, 1979.
[21] Joshi K. D., Introduction to general topology, New Age International Publisher, India, 1983.
[22] Blyth T. S., Lattices and Ordered Algebraic Structures, Springer, London, 2005.
[23] Turunen E., BL-algebras of basic fuzzy logic, Mathware Soft Comput., 1998, 6, 49-61.
[24] Zhang J. L., Topological properties of prime filters in MTL-algebras and fuzzy set representations for MTL-algebras, Fuzzy Sets Syst., 2011, 178, 38-53.
[25] Abbasloo M., Saeid A. B., Coatoms and comolecules of BL-algebras, J Intell Fuzzy Syst., 2014, 27, 351-360.
[26] Saeid A. B., Mohtashamnia N., Stabilizer in residuated lattices, Politehn. Univ. Bucharest Sci. Bull. Ser. A Appl. Math. Phys., 2012, 74, 65-74.