The STM R package

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Introduction

This package is meant to facilitate the simulation of vowel perception and the creation of related visualizations. First install the package if necessary, and load it. The finalized version of the package will be available on CRAN, it is only on GitHub for now.

Examples of Usage

Data setup

We load the STM package and the `h95_data` (Hillenbrand et al. 1995, henceforth H95) dataset. This is the complete data asociated with the H95 paper that was once hosted on James Hillenbrand's academic website, now down (and distributed with his permission). The data loaded below contains: formant data at a 'stable part' of the vowel, formant values at different time points, listener classifications for each judgement, and two estimates of $\hat{\psi}_s$: `psi_1` (based on the stable part of the formants) and `psi_2` (based on measurements at 20% and 80%).

```
library (STM)
data (h95_data)
head (h95_data)
```

```
     file dur  f0  f1   f2   f3 f1_1 f2_1 f3_1 f1_2 f2_2 f3_2 f1_3 f2_3 f3_3
1   b01ae 257 238 630 2423 3166  625 2388 3174  651 2413 3115  675 2463 3011
140 b01ah 212 241 831 1676 2602  820 1688 2511  845 1684 2583  845 1681 2606
279 b01aw 242 247 725 1384 2642  722 1411 2669  725 1384 2642  733 1384 2662
418 b01eh 184 214 713 2095 3129  611 2121 3242  643 2071 3078  672 2079 3120
557 b01ei 222 230 534 2690 3335  582 2454 3223  621 2573 3260  579 2632 3287
696 b01er 227 240 608 1733 2159  567 1599 2255  566 1664 2180  579 1709 2168
    f1_4 f2_4 f3_4 f1_5 f2_5 f3_5 f1_6 f2_6 f3_6 f1_7 f2_7 f3_7 f1_8 f2_8
f3_8
1    687 2391 2926  683 2295 2888  696 2187 2888  793 2074 2866  806 2049
2961
140  858 1690 2600  863 1696 2576  875 1745 2569  845 1858 2657  807 1980
2893
279  731 1385 2681  748 1388 2682  758 1458 2690  772 1579 2739  805 1699
2777
418  700 2109 3162  703 2079 3044  682 2133 3092  687 2124 3119  672 2140
3128
557  534 2690 3335  498 2797 3299  480 2883 3294  455 2956   NA  440 3012
```

```
3318
696  635 1746 2103  644 1678 2046  634 1672 2054  633 1778 2078  623 1781
2157
    type speaker vowel ipa correct    psi_1    psi_2 ae ah aw eh ei er ih iy
oa
1      b     b01    ae   æ      95 7.317269 7.315179 19  0  0  1  0  0  0  0
0
140    b     b01    ah   ɑ      95 7.317269 7.315179  0 19  0  0  0  0  0  0
0
279    b     b01    aw   ɔ      10 7.317269 7.315179  0  9  2  0  0  0  0  0
0
418    b     b01    eh   ɛ     100 7.317269 7.315179  0  0  0 20  0  0  0  0
0
557    b     b01    ei   e     100 7.317269 7.315179  0  0  0  0 20  0  0  0
0
696    b     b01    er   ɝ     100 7.317269 7.315179  0  0  0  0  0 20  0  0
0
    oo uh uw vote
1    0  0  0   ae
140  1  0  0   ah
279  0  9  0   ah
418  0  0  0   eh
557  0  0  0   ei
696  0  0  0   er
```

We first impute missing values in the formant data using the `impute_NA` function. This function uses a linear model to impute, and optionally adds error to estimates (not used here but useful when using resampling methods bootstrapping).

```
h95_data[,c(10:12,28:30)] =
  impute_NA (log(h95_data[,c(10:12,28:30)]), h95_data$speaker,
h95_data$vowel)
```

We will focus on the 20% and 80% time points, for the first 3 formants. We collect the formants, f0, psis, classification information, and category names.

```
ffs = log(h95_data[,c(10:12,28:30)])
f0s = log(h95_data[,3])
psis = h95_data$psi_2

data(h95_classifications)
classifications = h95_classifications
vs = colnames (classifications)
```

We normalize the log-transformed formant data using the `normalize` function, which employs the regression-based method of log-mean normalization outlined in Barreda and Nearey (2018).

```
nffs = normalize (h95_data[,c(10:12,28:30)],h95_data$speaker,h95_data$vowel)
```

And calculate templates for the dialect based on the available data. Templates contains category means, covariance matrices, and precision matrices.

```
template = create_template (ffs-psis, h95_data$vowel,shared_covar = FALSE)
```

Example Analysis

If ψ_s is known, formants may be normalized as usual and the `STM` function may be used. In addition, this function may be used on templates trained on unnormalized data for classification without normalization. The `STM` function calculates posterior probabilities for each category based on the template and token acoustic properties. It returns a matrix of posterior probabilities, one row for each observation and one column for each candidate category.

```
STM (nffs[1,1:6], template = template)
```

This function works by calculating the log density of the formant vector for each category, and then calculating posteriors based on these densities. `correctOUflow.internal` is a function that changes 0 and 1 to `.Machine$double.xmin` and .9999999 respectively, in order to avoid problems when calculating the log likelihood.

```
STM

function (formants, template, vowel_priors = NULL, correctOUflow = TRUE)
{
    n_samples = nrow(formants)
    n_classes = nrow(template$means)
    log_vowel_density = matrix(0, n_samples, n_classes)
    for (j in 1:n_classes) {
        log_vowel_density[, j] = dmvnorm_fast(formants, template$means[j,
            ], template$covariance[[j]], log = TRUE)
    }
    log_vowel_density = log_vowel_density - apply(log_vowel_density,
        1, function(x) mean(x))
    if (!is.null(vowel_priors))
        log_vowel_density = sweep(log_vowel_density, 2, log(vowel_priors),
            `+`)
    vowel_densities = exp(log_vowel_density)
    posterior_probabilities = vowel_densities/rowSums(vowel_densities)
    if (correctOUflow)
        posterior_probabilities =
correctOUflow_internal(posterior_probabilities)
    colnames(posterior_probabilities) = rownames(template$means)
    posterior_probabilities = STM_posteriors(posterior_probabilities)
    return(posterior_probabilities)
}
<bytecode: 0x000002c761219a48>
<environment: namespace:STM>
```

The BSTM function takes in a formant vector, an (optional depending on the method) f0 value, and a dialectal template. It returns information regarding the posterior probabilities for each category, the likelihoods, and the priors.

```
BSTM (ffs[1,],f0s[1], template = template)

   posterior_mu posterior_sd posterior_density posterior_probability
rounded_pp
ae        7.296        0.024             2.826                 0.996
0.996
ah        7.395        0.028           -24.474                 0.000
0.000
aw        7.489        0.030           -58.967                 0.000
0.000
eh        7.330        0.024            -2.693                 0.004
0.004
ei        7.354        0.026           -26.650                 0.000
0.000
er        7.541        0.031           -25.733                 0.000
0.000
ih        7.350        0.022            -9.490                 0.000
0.000
iy        7.262        0.030           -63.778                 0.000
0.000
oa        7.464        0.029           -33.754                 0.000
0.000
oo        7.406        0.026           -20.620                 0.000
0.000
uh        7.392        0.024           -21.023                 0.000
0.000
uw        7.457        0.031           -17.017                 0.000
0.000
   likelihood_mu likelihood_sd likelihood_density prior_mu prior_sd
ae         7.287         0.026              11.044    7.338    0.056
ah         7.415         0.033             -15.894    7.338    0.056
aw         7.549         0.035             -45.966    7.338    0.056
eh         7.328         0.027               5.197    7.338    0.056
ei         7.358         0.029             -18.722    7.338    0.056
er         7.634         0.038              -8.187    7.338    0.056
ih         7.352         0.024              -1.587    7.338    0.056
iy         7.231         0.036             -54.588    7.338    0.056
oa         7.512         0.035             -22.356    7.338    0.056
oo         7.425         0.029             -11.789    7.338    0.056
uh         7.404         0.027             -12.570    7.338    0.056
uw         7.508         0.037              -5.889    7.338    0.056
   prior_density
ae         1.822
ah         1.822
aw         1.822
eh         1.822
```

```
ei         1.822
er         1.822
ih         1.822
iy         1.822
oa         1.822
oo         1.822
uh         1.822
uw         1.822
```

Alternatively, the function can be run on a matrix of formant vectors, and a vector of f0 values. In this case, a list of results is returned as an `STM_output_list object`.

```
analysis = BSTM (ffs[1:5,],f0s[1:5], template = template)

analysis


STM_output_list object with the following elements:

$list: a list of 5 STM_output objects
$winners: a data frame of the winning psi and vowel for each token
$posterior: a data frame of the posterior probabilities for each token
```

Below, we show the results for the fifth observation in the dataset.

```
analysis[[5]]

   posterior_mu posterior_sd posterior_density posterior_probability
rounded_pp
ae        7.311        0.024           -31.228                 0.000
0.000
ah        7.357        0.028           -58.905                 0.000
0.000
aw        7.406        0.030           -75.977                 0.000
0.000
eh        7.290        0.024           -25.085                 0.000
0.000
ei        7.327        0.026             2.728                 1.000
1.000
er        7.489        0.031           -27.010                 0.000
0.000
ih        7.347        0.022           -19.607                 0.000
0.000
iy        7.338        0.030           -15.008                 0.000
0.000
oa        7.420        0.029           -68.827                 0.000
0.000
oo        7.420        0.026           -32.553                 0.000
0.000
uh        7.414        0.024           -35.136                 0.000
0.000
```

```
uw        7.460        0.031           -21.367                 0.000
0.000
   likelihood_mu likelihood_sd likelihood_density prior_mu prior_sd
ae         7.307         0.026            -25.799    7.325    0.056
ah         7.369         0.033            -53.289    7.325    0.056
aw         7.439         0.035            -69.108    7.325    0.056
eh         7.282         0.027            -19.458    7.325    0.056
ei         7.327         0.029              8.116    7.325    0.056
er         7.563         0.038            -15.366    7.325    0.056
ih         7.351         0.024            -14.131    7.325    0.056
iy         7.343         0.036             -9.583    7.325    0.056
oa         7.456         0.035            -61.450    7.325    0.056
oo         7.446         0.029            -25.315    7.325    0.056
uh         7.435         0.027            -28.174    7.325    0.056
uw         7.519         0.037            -11.774    7.325    0.056
   prior_density
ae         1.918
ah         1.918
aw         1.918
eh         1.918
ei         1.918
er         1.918
ih         1.918
iy         1.918
oa         1.918
oo         1.918
uh         1.918
uw         1.918
```

The `get_winners` function returns the category with the highest posterior probability for each observation, and the corresponding estimate of $\hat{\psi}_s$.

```
get_winners(analysis)
```

```
   psi_hat vowel_hat
1 7.296313        ae
2 7.302488        ah
3 7.329890        aw
4 7.284129        eh
5 7.326943        ei
```

And the `get_posterior` function returns the posterior probability for each category for each observation.

```
get_posterior (analysis)
```

```
     ae    ah    aw    eh ei er ih iy oa    oo    uh uw
1 0.996 0.000 0.000 0.004  0  0  0  0  0 0.000 0.000  0
2 0.000 0.977 0.016 0.000  0  0  0  0  0 0.000 0.007  0
3 0.000 0.292 0.422 0.000  0  0  0  0  0 0.000 0.286  0
```

```
4 0.008 0.000 0.000 0.991  0  0  0  0  0 0.001 0.000  0
5 0.000 0.000 0.000 0.000  1  0  0  0  0 0.000 0.000  0
```

To use a different estimation method we use the `method` parameter in `BSTM` as shown below.

```
BSTM (ffs[1,],f0s[1], template = template, method = method2)
```

```
   posterior_mu posterior_sd posterior_density posterior_probability
rounded_pp
ae        7.287        0.026             2.729                 0.997
0.997
ah        7.415        0.033           -24.208                 0.000
0.000
aw        7.549        0.035           -54.280                 0.000
0.000
eh        7.328        0.027            -3.117                 0.003
0.003
ei        7.358        0.029           -27.036                 0.000
0.000
er        7.634        0.038           -16.502                 0.000
0.000
ih        7.352        0.024            -9.902                 0.000
0.000
iy        7.231        0.036           -62.903                 0.000
0.000
oa        7.512        0.035           -30.671                 0.000
0.000
oo        7.425        0.029           -20.103                 0.000
0.000
uh        7.404        0.027           -20.884                 0.000
0.000
uw        7.508        0.037           -14.204                 0.000
0.000
   likelihood_mu likelihood_sd likelihood_density prior_mu prior_sd
ae         7.287         0.026             11.044       NA       NA
ah         7.415         0.033            -15.894       NA       NA
aw         7.549         0.035            -45.966       NA       NA
eh         7.328         0.027              5.197       NA       NA
ei         7.358         0.029            -18.722       NA       NA
er         7.634         0.038             -8.187       NA       NA
ih         7.352         0.024             -1.587       NA       NA
iy         7.231         0.036            -54.588       NA       NA
oa         7.512         0.035            -22.356       NA       NA
oo         7.425         0.029            -11.789       NA       NA
uh         7.404         0.027            -12.570       NA       NA
uw         7.508         0.037             -5.889       NA       NA
   prior_density
ae            NA
ah            NA
```

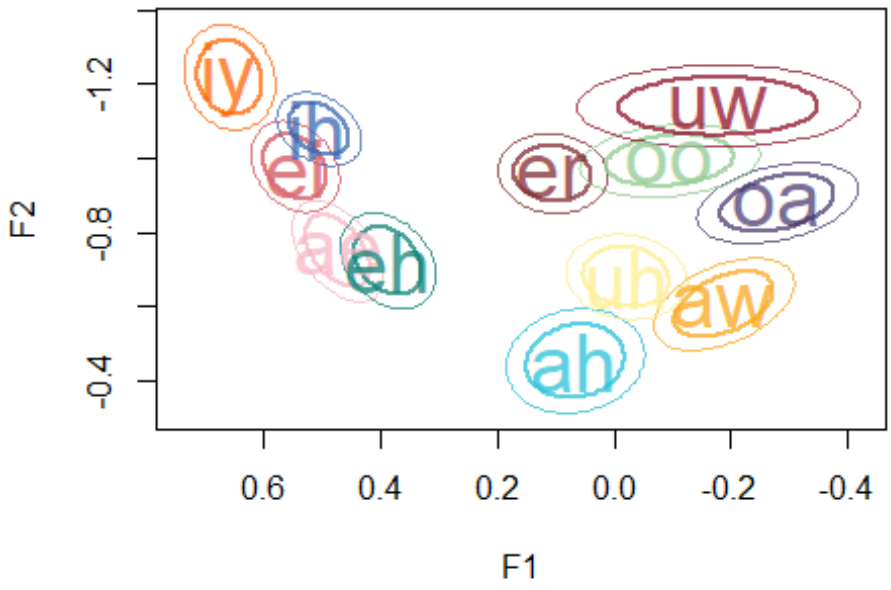
```
aw              NA
eh              NA
ei              NA
er              NA
ih              NA
iy              NA
oa              NA
oo              NA
uh              NA
uw              NA
```

Note that the prior information is set to `NA` because this method places no a priori constraints on values of $\hat{\psi}_s$.

Plotting

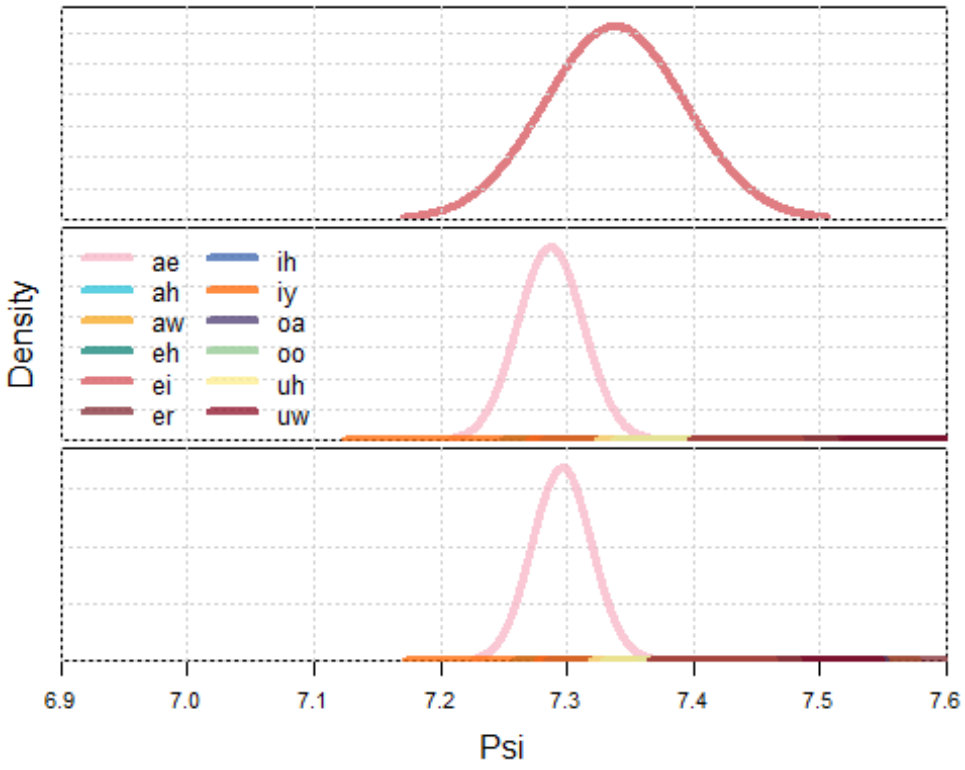
The package contains plotting functions defined for the `STM_template` and `STM_output_list` objects. The `plot` function for the `STM_template` object plots the means of the formant values for each category, and ellipses enclosing one and two standard deviations.

```
plot (template)
```



The `plot` function for `STM_output` objects compares the prior probabilities of ψ_s, the likelihood of of the formant pattern given different values of ψ_s and different vowel categories, and the posterior probabilities of ψ_s for each category.

```
plot (analysis[[1]])
```



Walkthrough of the the BSTM function

We will describe the sequence of steps underlying the `BSTM` function, relying on the steps carried out in section **?@sec-setup**.

The `BSTM` function takes in a formant vector, an (optional depending on the metho) f0 value, and a template. It returns information regarding the posterior probabilities for each category, the likelihoods, and the priors.

```
BSTM (ffs[1,],f0s[1], template = template)
```

I am going to walk through the way this function works. `BSTM` and `PSTM` are basically a wrapper for the `method` functions in the `estimation_methods.R` file. The `method` functions are the ones that actually calculate the likelihoods and priors. `method6` implements the method 6 algorithm as seen below:

```
method6
```

```
function (ffs, f0, template, PSI_prior_mean = 7.233, PSI_prior_sd = 0.1284,
    f0_hat_sd = 0.1327, f0_hat_intercept = -10.32, f0_hat_slope = 2.145,
    vowel_priors = vowel_priors, correctOUflow = TRUE, type = "BSTM",
    ...)
{
    n_classes = nrow(template$means)
```

```
    likelihoods = t(sapply(1:n_classes, function(j) {
        estimate_likelihood(ffs, template$means[j, ],
template$covariance[[j]],
            template$precision[[j]])
    }))
    rownames(likelihoods) = rownames(template$means)
    priors = estimate_f0_psi_prior(f0)
    priors = matrix(priors, n_classes, 3, byrow = TRUE)
    colnames(priors) = c("prior_mu", "prior_sd", "prior_density")
    if (!is.null(vowel_priors))
        priors$prior_density = priors$prior_density + log(vowel_priors)
    posterior = find_posterior(likelihoods, priors, type = type)
    if (correctOUflow)
        posterior[, 4] = correctOUflow_internal(posterior[, 4])
    stimulus = c(ffs, f0)
    parameters = c(PSI_prior_mean = PSI_prior_mean, PSI_prior_sd =
PSI_prior_sd,
        f0_hat_sd = f0_hat_sd, f0_hat_intercept = f0_hat_intercept,
        f0_hat_slope = f0_hat_slope)
    output = STM_output(cbind(posterior, likelihoods, priors),
        stimulus, template, parameters, vowel_priors)
    return(output)
}
<bytecode: 0x000002c76147e268>
<environment: namespace:STM>
```

The other methods (2 and 3) include subsets of the process above, so I will not go through method6. First, the `estimate_likelihood` function is called for each category. This function calculates the log likelihood of the formant vector given the category mean and covariance. This is done using the method in Terry's notes which yields the mean and standard deviation. The density at the mean is calculated using the density function. We will in general collect: the mu, the sd, the peak (log) density.

```
estimate_likelihood
```

```
function (ffs, means, covariance, precision = solve(covariance))
{
    intercept_template = sum(unlist(ffs - means) %*% precision)
    slope_template = -sum(precision)
    likelihood_mu = (-intercept_template/slope_template)
    likelihood_sd = sqrt(-1/slope_template)
    likelihood_density = dmvnorm_fast(ffs - likelihood_mu, means,
        covariance, log = TRUE)
    output = c(likelihood_mu = likelihood_mu, likelihood_sd = likelihood_sd,
        likelihood_density = likelihood_density)
    return(output)
}
<bytecode: 0x000002c761481f58>
<environment: namespace:STM>
```

We then calculate the joint prior pf ψ_s and f0 using the `estimate_f0_psi_prior` function. This function calculates the prior for the f0 value given the psi value. The mean and sd are calculated the way described by our derivation. The main addition is that we find the value of the density at the prior mean.

```
estimate_f0_psi_prior
```

```
function (f0, PSI_prior_mean = 7.233, PSI_prior_sd = 0.1284,
    f0_hat_sd = 0.1327, f0_hat_intercept = -10.32, f0_hat_slope = 2.145,
    ...)
{
    intercept_f0_given_psi = ((f0_hat_slope * f0)/(f0_hat_sd^2) -
        (f0_hat_intercept * f0_hat_slope)/(f0_hat_sd^2))
    slope_f0_given_psi = ((-f0_hat_slope^2/(f0_hat_sd^2)))
    f0_given_psi_mu = (-intercept_f0_given_psi/slope_f0_given_psi)
    f0_given_psi_sd = sqrt(-1/slope_f0_given_psi)
    f0_hat = f0_hat_intercept + f0_hat_slope * f0_given_psi_mu
    f0_given_psi_density = stats::dnorm(f0, f0_hat, f0_hat_sd,
        log = TRUE)
    psi_prior_density = stats::dnorm(PSI_prior_mean, PSI_prior_mean,
        PSI_prior_sd, log = TRUE)
    prior_density = f0_given_psi_density + psi_prior_density
    tmp = combine_gaussians(f0_given_psi_mu, f0_given_psi_sd,
        PSI_prior_mean, PSI_prior_sd, f0_given_psi_density,
psi_prior_density)
    output = c(prior_mu = tmp[1], prior_sd = tmp[2], prior_density = tmp[3])
    return(output)
}
<bytecode: 0x000002c76155f510>
<environment: namespace:STM>
```

We also call the `combine_gaussians` function which combines gaussian curves of any given peak density as follows:

```
combine_gaussians
```

```
function (mu_1, sd_1, mu_2, sd_2, d_1 = NULL, d_2 = NULL)
{
    if (is.null(d_1))
        d_1 = 1/(sqrt(2 * pi * sd_1^2))
    if (is.null(d_2))
        d_2 = 1/(sqrt(2 * pi * sd_2^2))
    mu = (mu_1/sd_1^2 + mu_2/sd_2^2)/(1/sd_1^2 + 1/sd_2^2)
    sd = sqrt(1/(1/sd_1^2 + 1/sd_2^2))
    d = dlike(mu, mu_1, sd_1, d_1, log = TRUE) + dlike(mu, mu_2,
        sd_2, d_2, log = TRUE)
    return(cbind(mu, sd, d))
}
<bytecode: 0x000002c761556b28>
<environment: namespace:STM>
```

Finally (in terms of important steps) we call the `find_posterior` function to combine our priors and likelihoods. This functions simply combines the priors and likelihoods and then scales the posterior probabilities. by calling `scale_posterior`.

```
find_posterior
```

```
function (likelihoods, priors, type = "BSTM")
{
    n_vs = nrow(likelihoods)
    posterior = matrix(0, n_vs, 5, byrow = TRUE)
    posterior[, 1:3] = combine_gaussians(likelihoods[, 1], likelihoods[,
        2], priors[, 1], priors[, 2], likelihoods[, 3], priors[,
        3])
    posterior = scale_posterior(posterior, type = type)
    colnames(posterior) = c("posterior_mu", "posterior_sd",
"posterior_density",
        "posterior_probability", "rounded_pp")
    rownames(posterior) = rownames(likelihoods)
    return(posterior)
}
<bytecode: 0x000002c761571698>
<environment: namespace:STM>
```

The `scale_posterior` function scales the posterior probabilities to sum to 1. This is done by integration of the posterior distributions of ψ_s for each category and summing across all categories (if `type=BSTM`), or by exponentiating and summing posterior densities (if `type=PSTM`).

```
scale_posterior
```

```
function (posterior, type = "BSTM")
{
    if (type == "BSTM") {
        vowel_integrals = posterior[, 3] + log(posterior[, 2]) +
            (log(pi) + log(2)) * (0.5)
        log_sum_vowel_integrals = log(sum(exp(vowel_integrals)))
        posterior[, 3] = posterior[, 3] - log_sum_vowel_integrals
        posterior[, 4] = exp(vowel_integrals - log_sum_vowel_integrals)
    }
    if (type == "PSTM") {
        posterior[, 4] = exp(posterior[, 3])/sum(exp(posterior[,
            3]))
    }
    posterior[, 5] = round(posterior[, 4], 4)
    return(posterior)
}
<bytecode: 0x000002c76157a5b0>
<environment: namespace:STM>
```

Comparison to Grid Search parameter estimation

As a sanity check we will compare the output of the `BSTM` function to the values we can calculate more directly using a grid search method. We use the same parameters as in methods 2, 3 and 6, and use the first observation from our data (any other can be used):

```
tmp_ffs = ffs[1,]
tmp_f0 = f0s[1]

psis = seq(6.8,7.6,.0001)
PSI_prior_mean = 7.233
PSI_prior_sd = 0.1284
f0_hat_sd = 0.1327
f0_hat_intercept = -10.32
f0_hat_slope = 2.145
```

To estimate the method 6 prior:

```
# psi prior normal density
psiprior = dnorm (psis,PSI_prior_mean,PSI_prior_sd)

# predicted f0 for each psi
f0_hat = f0_hat_intercept + f0_hat_slope*psis

# density of f0 given psi
f0_given_psi = dnorm (tmp_f0, f0_hat, f0_hat_sd)

# joint density of f0|psi and psi
prior = f0_given_psi * psiprior
```

And likelihood:

```
# matrix of formant vector, repeated
formant_matrix = matrix (unlist(tmp_ffs),length(psis),6,byrow=TRUE)

# matrix of candidate psis, repeated across columns
psi_matrix = matrix (unlist(psis),length(psis),6)

# for each category, density of pattern - psi candidate
v_likelihoods = matrix (0,length(psis),12)
for (i in 1:12)
  v_likelihoods[,i] =
  dmvnorm_fast(formant_matrix - psi_matrix,
               unlist(template$means[i,]),
               template$covariance[[i]])
```

The posterior (before scaling) is the product of the likelihood and the prior:

```
posterior = v_likelihoods * matrix (prior,length(psis),12)
```

We numerically estimate the integral of all posteriors and divide the densities by this number to scale the total integral across all categories to 1:

```
integrals = rep(0,12)
for (i in 1:12) integrals[i] = integrate_numerical (psis, posterior[,i])

posterior = posterior / sum(integrals)
```

And confirm that the total integral across categories is now 1:

```
integrals_posterior = rep(0,12)
for (i in 1:12) integrals_posterior[i] = integrate_numerical (psis,
posterior[,i])
sum (integrals_posterior)

[1] 1
```

We will compare to the output of the BSTM function:

```
BSTM_output = BSTM (tmp_ffs, tmp_f0, template = template)$df
t(BSTM_output[1,])

                              ae
posterior_mu           7.29631335
posterior_sd           0.02353500
posterior_density      2.82624049
posterior_probability  0.99592058
rounded_pp             0.99590000
likelihood_mu          7.28726847
likelihood_sd          0.02596349
likelihood_density    11.04363315
prior_mu               7.33799095
prior_sd               0.05573306
prior_density          1.82247956
```

The posterior mean corresponds to the MAP psi value:

```
BSTM_output[1,'posterior_mu']

[1] 7.296313

psis[which.max(posterior[,1])]

[1] 7.2963
```

the posterior probability corresponds well to the numerically estimated integral:

```
BSTM_output[1,'posterior_probability']

[1] 0.9959206

integrate_numerical(psis, posterior[,1])
```

```
[1] 0.9959206
```

The posterior sd corresponds to what we can estimate using the second derivative of the posterior density:

```
log_posterior <- log(posterior[,1])
log_posterior_dx = diff(log_posterior) / diff(psis)
log_posterior_dx_dx = mean (diff(log_posterior_dx) / diff(psis[-1]))


BSTM_output[1,'posterior_sd']

[1] 0.023535

sqrt (-1/log_posterior_dx_dx)

[1] 0.023535
```

and the maximum posterior log density also corresponds to our estimated value:

```
BSTM_output[1,'posterior_density']

[1] 2.82624

log(max(posterior[,1]))

[1] 2.82624
```

We can do the same estimates for our prior, seeing that these match our function outputs:

```
unlist(BSTM_output[1,9:11])

     prior_mu      prior_sd prior_density
   7.33799095    0.05573306    1.82247956

psis[which.max(prior)]

[1] 7.338

log_prior <- log(prior)
log_prior_dx = diff(log_prior) / diff(psis)
log_prior_dx_dx = mean (diff(log_prior_dx) / diff(psis[-1]))
sqrt (-1/log_prior_dx_dx)

[1] 0.05573306

log(max(prior))

[1] 1.82248
```

and for our likelihood:

```
unlist(BSTM_output[1,6:8])
```

```
     likelihood_mu      likelihood_sd likelihood_density 
        7.28726847         0.02596349        11.04363315 
```

```
psis[which.max(v_likelihoods[,1])]
```

```
[1] 7.2873
```

```
log_likelihood <- log(v_likelihoods[,1])
log_likelihood_dx = diff(log_likelihood) / diff(psis)
log_likelihood_dx_dx = mean (diff(log_likelihood_dx) / diff(psis[-1]))
sqrt (-1/log_likelihood_dx_dx)
```

```
[1] 0.02596349
```

```
log(max(v_likelihoods[,1]))
```

```
[1] 11.04363
```

Q.E.D.!