Appendix to
"On Trend Breaks and Initial Condition in Unit Root Testing"

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1 Asymptotic local power for OLS-based tests

In this section we provide additional simulation results for asymptotic local power functions of OLS-based tests by using teh same notation as in the main text (see Figures 5 and 6 in the main text). Additional notation is used for break date estimators for $ADF\text{-}OLS(\lambda)$, where λ is a generic break fraction: $\hat{\lambda}^{D_m}$ is the break fraction estimator proposed by Harvey and Leybourne (2013) and $\tilde{\lambda}$ is the break fraction estimator based on first difference regression proposed by Harris *et al.* (2009).

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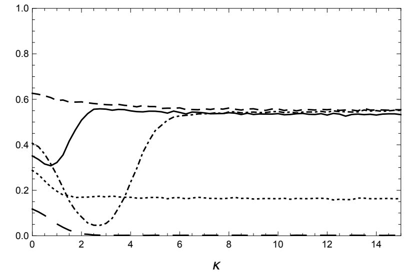
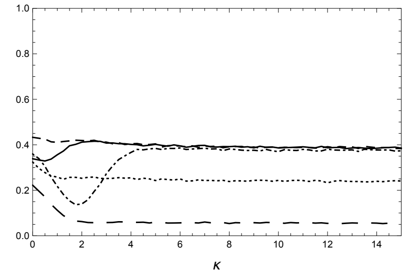
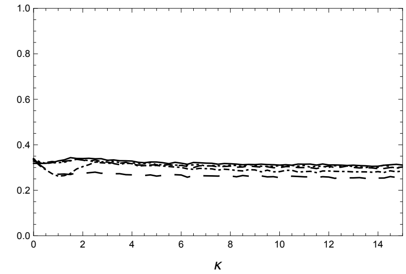


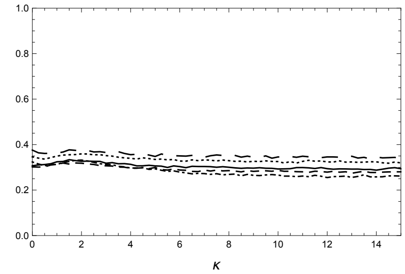
(a) $\alpha = -6$



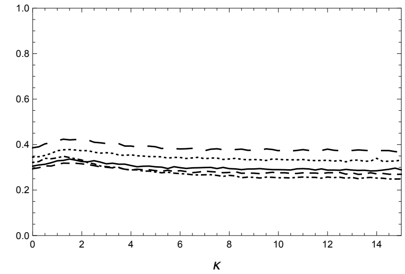
(b) $\alpha = -4$



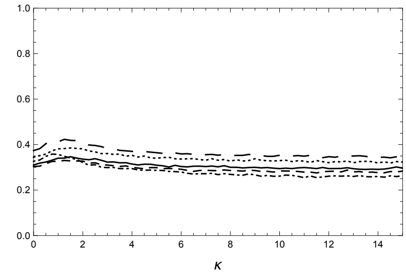
(c) $\alpha = -2$



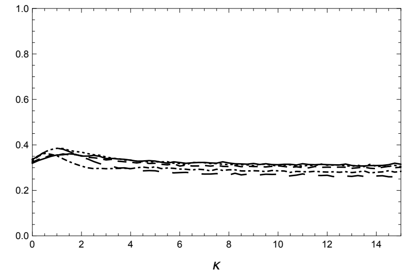
(d) $\alpha = -1$



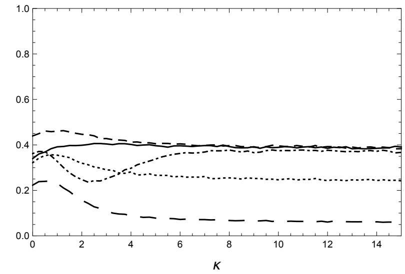
(e) $\alpha = 0$



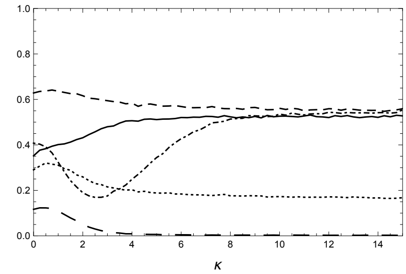
(f) $\alpha = 1$



(g) $\alpha = 2$

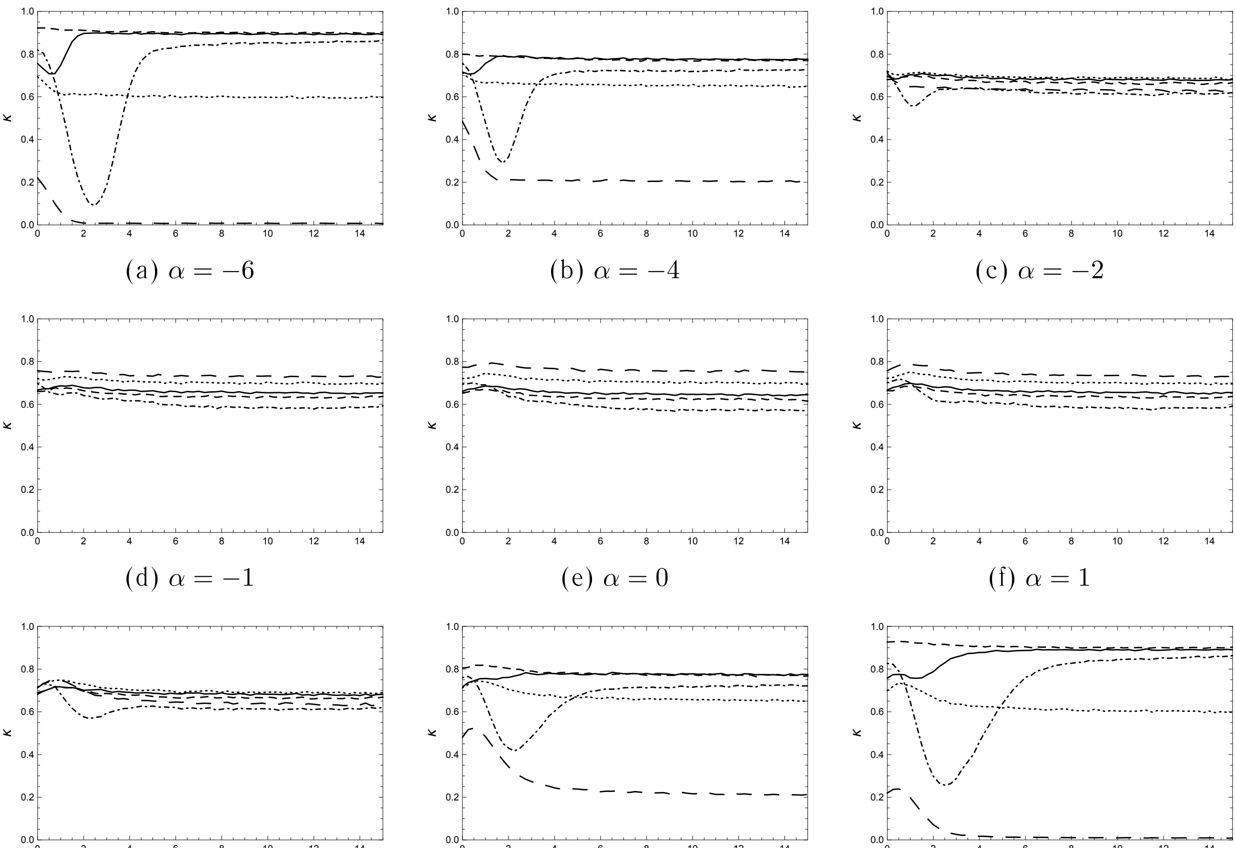


(h) $\alpha = 4$



(i) $\alpha = 6$

Figure 1. Асимптотическая локальная мощность, $c = 20$
$ADF\text{-}OLS(\hat{\lambda}^{D_m})$: ——, $ADF\text{-}OLS^{\max}$: — —, $MDF\text{-}OLS$: - -, $MDF\text{-}OLS_{\rho}$: · · ·, $ADF\text{-}OLS(\tilde{\lambda})$: —·—



(a) $\alpha = -6$ (b) $\alpha = -4$ (c) $\alpha = -2$

(d) $\alpha = -1$ (e) $\alpha = 0$ (f) $\alpha = 1$

(g) $\alpha = 2$ (h) $\alpha = 4$ (i) $\alpha = 6$

Figure 2. Асимптотическая локальная мощность, $c = 30$
$ADF\text{-}OLS(\hat{\lambda}^{D_m})$: ——— , $ADF\text{-}OLS^{\max}$: — — , $MDF\text{-}OLS$: - - , $MDF\text{-}OLS_{\rho}$: · · · ,
$ADF\text{-}OLS(\tilde{\lambda})$: — · —

2 Finite sample size (for Zivot-Andrews MDF-OLS test) and finite sample power (for robust test for a break)

In this section we provide finite sample size of Zivot-Andrews MDF-OLS test for various κ. We also provide the finite sample size and power of robust tests for break, t_{HLT} and t_{PY}, proposed by Perron and Yabu (2009) and Harvey *et al.* (2009), respectively. The DGP is

$$y_t = \mu + \beta t + \gamma_T DT_t(\lambda_0) + u_t,\ t = 1, \ldots, T, \quad (1)$$
$$u_t = \rho_T u_{t-1} + \varepsilon_t,\ t = 2, \ldots, T, \quad (2)$$

where $\gamma_T = \kappa T^{-1/2}$ and the error term ε_t is generated according to either an $i.i.d$ sequence or the AR(1) and MA(1) processes according to $\varepsilon_t \sim i.i.d.N(0,1)$ for the case of $i.i.d.$, $\varepsilon_t = \phi\varepsilon_{t-1} + e_t$ for the case of AR(1), and $\varepsilon_t = e_t - \phi e_{t-1}$ for the case of MA(1), where $e_t \sim i.i.d.N(0,1)$. The sample size $T \in \{100, 200\}$, the break magnitude $\kappa \in \{0, 2, \ldots, 40\}$, the parameter $\phi \in \{-0.8, -0.5, -0.2, 0, 0.2, 0.5, 0.8\}$, the number of replications is 10,000.

Table 1. Finite sample size (for Zivot-Andrews test) and finite sample power (for robust test for a break), AR(1) case, $T = 100$, $\lambda = 0.15$

κ	$\phi=-0.8$			$\phi=-0.5$			$\phi=-0.2$			$\phi=0$			$\phi=0.2$			$\phi=0.5$			$\phi=0.8$		
	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}
0	0.04	0.11	0.14	0.04	0.12	0.15	0.07	0.15	0.11	0.05	0.17	0.08	0.02	0.18	0.13	0.04	0.24	0.13	0.06	0.44	0.19
2	0.04	0.14	0.24	0.04	0.18	0.20	0.07	0.19	0.13	0.05	0.20	0.11	0.02	0.20	0.15	0.04	0.24	0.13	0.07	0.44	0.19
4	0.04	0.28	0.54	0.04	0.36	0.39	0.06	0.33	0.20	0.05	0.29	0.18	0.02	0.28	0.21	0.03	0.27	0.15	0.07	0.45	0.20
6	0.06	0.59	0.86	0.05	0.66	0.68	0.06	0.55	0.37	0.06	0.47	0.34	0.03	0.41	0.35	0.04	0.33	0.19	0.07	0.47	0.20
8	0.13	0.87	0.98	0.07	0.90	0.90	0.06	0.79	0.60	0.06	0.67	0.55	0.04	0.57	0.51	0.04	0.40	0.23	0.07	0.48	0.21
10	0.25	0.98	1.00	0.11	0.98	0.98	0.08	0.93	0.82	0.09	0.84	0.77	0.06	0.74	0.68	0.04	0.49	0.31	0.07	0.50	0.22
12	**0.38**	**1.00**	**1.00**	**0.18**	**1.00**	**1.00**	0.11	0.99	0.95	0.12	0.94	0.91	0.09	0.88	0.83	0.04	0.59	0.39	0.07	0.52	0.24
14	0.47	1.00	1.00	0.26	1.00	1.00	0.18	1.00	0.99	0.19	0.99	0.97	0.13	0.95	0.92	0.04	0.70	0.49	0.07	0.56	0.26
16	0.43	1.00	1.00	0.32	1.00	1.00	**0.26**	**1.00**	**1.00**	0.29	1.00	0.99	0.19	0.98	0.97	0.04	0.79	0.59	0.07	0.58	0.28
18	0.34	1.00	1.00	0.35	1.00	1.00	0.37	1.00	1.00	**0.39**	**1.00**	**1.00**	0.24	1.00	0.99	0.04	0.86	0.68	0.07	0.61	0.30
20	0.22	1.00	1.00	0.32	1.00	1.00	0.49	1.00	1.00	0.50	1.00	1.00	**0.28**	**1.00**	**1.00**	0.04	0.92	0.77	0.08	0.66	0.33
22	0.14	1.00	1.00	0.30	1.00	1.00	0.58	1.00	1.00	0.56	1.00	1.00	0.29	1.00	1.00	0.04	0.96	0.85	0.07	0.69	0.35
24	0.07	1.00	1.00	0.24	1.00	1.00	0.65	1.00	1.00	0.61	1.00	1.00	0.29	1.00	1.00	0.04	0.98	0.90	0.08	0.72	0.38
26	0.04	1.00	1.00	0.19	1.00	1.00	0.67	1.00	1.00	0.62	1.00	1.00	0.28	1.00	1.00	0.04	0.99	0.94	0.08	0.76	0.43
28	0.02	1.00	1.00	0.15	1.00	1.00	0.68	1.00	1.00	0.57	1.00	1.00	0.25	1.00	1.00	0.04	1.00	0.96	0.08	0.79	0.45
30	0.01	1.00	1.00	0.12	1.00	1.00	0.66	1.00	1.00	0.53	1.00	1.00	0.22	1.00	1.00	0.04	1.00	0.98	0.08	0.83	0.49
32	0.01	1.00	1.00	0.09	1.00	1.00	0.62	1.00	1.00	0.46	1.00	1.00	0.18	1.00	1.00	0.04	1.00	0.99	0.08	0.86	0.53
34	0.01	1.00	1.00	0.09	1.00	1.00	0.59	1.00	1.00	0.39	1.00	1.00	0.14	1.00	1.00	0.04	1.00	0.99	0.08	0.89	0.58
36	0.01	1.00	1.00	0.08	1.00	1.00	0.53	1.00	1.00	0.32	1.00	1.00	0.10	1.00	1.00	**0.04**	**1.00**	**1.00**	0.09	0.91	0.61
38	0.02	1.00	1.00	0.07	1.00	1.00	0.45	1.00	1.00	0.26	1.00	1.00	0.08	1.00	1.00	0.04	1.00	1.00	0.08	0.93	0.65
40	0.02	1.00	1.00	0.07	1.00	1.00	0.39	1.00	1.00	0.20	1.00	1.00	0.06	1.00	1.00	0.04	1.00	1.00	0.09	0.94	0.69

Table 2. Finite sample size (for Zivot-Andrews test) and finite sample power (for robust test for a break), MA(1) case, $T = 100$, $\lambda = 0.15$

κ	$\phi = -0.8$			$\phi = -0.5$			$\phi = -0.2$			$\phi = 0$			$\phi = 0.2$			$\phi = 0.5$			$\phi = 0.8$		
	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}
0	0.44	0.08	0.59	0.13	0.10	0.42	0.09	0.14	0.12	0.05	0.17	0.08	0.02	0.18	0.12	0.02	0.19	0.12	0.01	0.19	0.17
2	0.31	0.18	0.81	0.12	0.16	0.44	0.09	0.19	0.14	0.05	0.20	0.11	0.02	0.22	0.16	0.02	0.20	0.14	0.01	0.20	0.18
4	0.23	0.54	0.96	0.09	0.40	0.54	0.08	0.33	0.19	0.05	0.29	0.18	0.02	0.33	0.27	0.02	0.24	0.16	0.01	0.23	0.21
6	0.23	0.95	0.99	0.08	0.78	0.73	0.07	0.57	0.36	0.06	0.47	0.34	0.02	0.52	0.46	0.02	0.33	0.22	0.01	0.29	0.25
8	**0.25**	**1.00**	**1.00**	0.08	0.97	0.91	0.07	0.81	0.59	0.06	0.67	0.55	0.02	0.72	0.67	0.02	0.43	0.29	0.01	0.36	0.31
10	0.29	1.00	1.00	0.11	1.00	0.98	0.09	0.94	0.82	0.09	0.84	0.77	0.03	0.87	0.84	0.02	0.57	0.40	0.01	0.46	0.38
12	0.32	1.00	1.00	**0.15**	**1.00**	**1.00**	0.11	0.99	0.95	0.12	0.94	0.91	0.03	0.96	0.94	0.02	0.71	0.52	0.01	0.57	0.47
14	0.37	1.00	1.00	0.22	1.00	1.00	0.19	1.00	0.99	0.19	0.99	0.97	0.04	0.99	0.98	0.03	0.83	0.65	0.01	0.69	0.58
16	0.43	1.00	1.00	0.30	1.00	1.00	**0.28**	**1.00**	**1.00**	0.29	1.00	0.99	**0.04**	**1.00**	**1.00**	0.03	0.91	0.76	0.01	0.80	0.66
18	0.50	1.00	1.00	0.39	1.00	1.00	0.40	1.00	1.00	**0.39**	**1.00**	**1.00**	0.05	1.00	1.00	0.03	0.95	0.85	0.01	0.87	0.75
20	0.57	1.00	1.00	0.44	1.00	1.00	0.53	1.00	1.00	0.50	1.00	1.00	0.03	1.00	1.00	0.03	0.98	0.92	0.01	0.93	0.83
22	0.61	1.00	1.00	0.49	1.00	1.00	0.63	1.00	1.00	0.56	1.00	1.00	0.03	1.00	1.00	0.03	0.99	0.96	0.01	0.97	0.89
24	0.64	1.00	1.00	0.52	1.00	1.00	0.71	1.00	1.00	0.61	1.00	1.00	0.02	1.00	1.00	0.02	1.00	0.98	0.01	0.98	0.93
26	0.65	1.00	1.00	0.52	1.00	1.00	0.74	1.00	1.00	0.62	1.00	1.00	0.02	1.00	1.00	0.02	1.00	0.99	0.01	0.99	0.96
28	0.62	1.00	1.00	0.53	1.00	1.00	0.75	1.00	1.00	0.57	1.00	1.00	0.01	1.00	1.00	**0.03**	**1.00**	**1.00**	0.01	1.00	0.98
30	0.59	1.00	1.00	0.52	1.00	1.00	0.73	1.00	1.00	0.53	1.00	1.00	0.01	1.00	1.00	0.02	1.00	1.00	0.02	1.00	0.99
32	0.55	1.00	1.00	0.52	1.00	1.00	0.70	1.00	1.00	0.46	1.00	1.00	0.01	1.00	1.00	0.03	1.00	1.00	0.02	1.00	0.99
34	0.52	1.00	1.00	0.52	1.00	1.00	0.67	1.00	1.00	0.39	1.00	1.00	0.01	1.00	1.00	0.03	1.00	1.00	**0.02**	**1.00**	**1.00**
36	0.49	1.00	1.00	0.54	1.00	1.00	0.60	1.00	1.00	0.32	1.00	1.00	0.01	1.00	1.00	0.03	1.00	1.00	0.02	1.00	1.00
38	0.48	1.00	1.00	0.53	1.00	1.00	0.53	1.00	1.00	0.26	1.00	1.00	0.01	1.00	1.00	0.02	1.00	1.00	0.02	1.00	1.00
40	0.48	1.00	1.00	0.52	1.00	1.00	0.46	1.00	1.00	0.20	1.00	1.00	0.01	1.00	1.00	0.03	1.00	1.00	0.02	1.00	1.00

Table 3. Finite sample size (for Zivot-Andrews test) and finite sample power (for robust test for a break), AR(1) case, $T = 200$, $\lambda = 0.15$

κ	$\phi = -0.8$			$\phi = -0.5$			$\phi = -0.2$			$\phi = 0$			$\phi = 0.2$			$\phi = 0.5$			$\phi = 0.8$		
	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}
0	0.03	0.08	0.09	0.04	0.08	0.08	0.05	0.10	0.09	0.04	0.12	0.06	0.02	0.13	0.09	0.04	0.18	0.08	0.05	0.42	0.12
2	0.04	0.10	0.18	0.04	0.13	0.14	0.04	0.14	0.12	0.04	0.14	0.09	0.02	0.15	0.11	0.04	0.19	0.09	0.05	0.42	0.12
4	0.04	0.22	0.50	0.04	0.31	0.34	0.04	0.28	0.21	0.04	0.24	0.17	0.02	0.22	0.16	0.04	0.22	0.10	0.05	0.43	0.12
6	0.08	0.54	0.86	0.05	0.64	0.68	0.04	0.52	0.40	0.04	0.42	0.33	0.02	0.34	0.26	0.04	0.27	0.14	0.05	0.44	0.13
8	0.15	0.88	0.98	0.09	0.90	0.92	0.06	0.78	0.66	0.06	0.65	0.56	0.03	0.51	0.42	0.05	0.36	0.19	0.05	0.46	0.14
10	**0.22**	0.99	1.00	0.15	0.99	0.99	0.09	0.94	0.86	0.09	0.84	0.77	0.04	0.69	0.59	0.04	0.46	0.28	0.05	0.48	0.15
12	**0.19**	**1.00**	**1.00**	**0.19**	**1.00**	**1.00**	0.13	0.99	0.96	0.13	0.95	0.91	0.05	0.83	0.74	0.04	0.56	0.36	0.05	0.50	0.16
14	0.12	1.00	1.00	0.19	1.00	1.00	0.17	1.00	0.99	0.17	0.99	0.97	0.06	0.93	0.87	0.04	0.67	0.45	0.05	0.53	0.18
16	0.06	1.00	1.00	0.13	1.00	1.00	**0.20**	**1.00**	**1.00**	**0.22**	**1.00**	**1.00**	0.07	0.98	0.95	0.04	0.77	0.56	0.05	0.56	0.19
18	0.02	1.00	1.00	0.08	1.00	1.00	0.22	1.00	1.00	0.26	1.00	1.00	0.09	0.99	0.98	0.04	0.86	0.67	0.05	0.60	0.22
20	0.02	1.00	1.00	0.05	1.00	1.00	0.20	1.00	1.00	0.26	1.00	1.00	0.09	1.00	0.99	0.04	0.91	0.77	0.05	0.63	0.24
22	0.01	1.00	1.00	0.02	1.00	1.00	0.15	1.00	1.00	0.22	1.00	1.00	**0.10**	**1.00**	**1.00**	0.04	0.96	0.85	0.05	0.67	0.28
24	0.01	1.00	1.00	0.01	1.00	1.00	0.11	1.00	1.00	0.18	1.00	1.00	0.10	1.00	1.00	0.04	0.98	0.90	0.05	0.71	0.30
26	0.01	1.00	1.00	0.01	1.00	1.00	0.07	1.00	1.00	0.13	1.00	1.00	0.10	1.00	1.00	0.05	0.99	0.95	0.04	0.74	0.33
28	0.01	1.00	1.00	0.01	1.00	1.00	0.05	1.00	1.00	0.09	1.00	1.00	0.09	1.00	1.00	0.05	1.00	0.97	0.05	0.78	0.37
30	0.01	1.00	1.00	0.01	1.00	1.00	0.03	1.00	1.00	0.06	1.00	1.00	0.08	1.00	1.00	0.05	1.00	0.99	0.05	0.81	0.40
32	0.01	1.00	1.00	0.01	1.00	1.00	0.02	1.00	1.00	0.04	1.00	1.00	0.07	1.00	1.00	0.05	1.00	0.99	0.05	0.85	0.45
34	0.01	1.00	1.00	0.01	1.00	1.00	0.02	1.00	1.00	0.03	1.00	1.00	0.06	1.00	1.00	**0.05**	**1.00**	**1.00**	0.05	0.87	0.49
36	0.01	1.00	1.00	0.01	1.00	1.00	0.02	1.00	1.00	0.02	1.00	1.00	0.05	1.00	1.00	0.05	1.00	1.00	0.05	0.90	0.53
38	0.01	1.00	1.00	0.01	1.00	1.00	0.01	1.00	1.00	0.01	1.00	1.00	0.04	1.00	1.00	0.05	1.00	1.00	0.05	0.92	0.57
40	0.01	1.00	1.00	0.01	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00	0.03	1.00	1.00	0.05	1.00	1.00	0.05	0.93	0.60

Table 4. Finite sample size (for Zivot-Andrews test) and finite sample power (for robust test for a break), MA(1) case, $T = 200$, $\lambda = 0.15$

κ	$\phi = -0.8$			$\phi = -0.5$			$\phi = -0.2$			$\phi = 0$			$\phi = 0.2$			$\phi = 0.5$			$\phi = 0.8$		
	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}
0	0.19	0.15	0.73	0.07	0.07	0.23	0.06	0.10	0.10	0.04	0.12	0.06	0.02	0.12	0.09	0.03	0.13	0.09	0.01	0.13	0.11
2	0.11	0.19	0.82	0.06	0.11	0.25	0.05	0.13	0.12	0.04	0.14	0.09	0.02	0.14	0.11	0.02	0.14	0.10	0.01	0.14	0.11
4	0.10	0.49	0.93	0.06	0.35	0.41	0.05	0.28	0.21	0.04	0.24	0.17	0.02	0.22	0.18	0.03	0.19	0.14	0.01	0.18	0.14
6	0.14	0.98	1.00	0.06	0.78	0.73	0.05	0.54	0.39	0.04	0.42	0.33	0.02	0.36	0.30	0.02	0.27	0.19	0.01	0.23	0.18
8	**0.22**	**1.00**	**1.00**	0.10	0.98	0.95	0.06	0.80	0.65	0.06	0.65	0.56	0.04	0.56	0.48	0.03	0.40	0.29	0.01	0.32	0.25
10	0.25	1.00	1.00	**0.16**	**1.00**	**1.00**	0.10	0.95	0.86	0.09	0.84	0.77	0.05	0.74	0.67	0.03	0.55	0.41	0.01	0.43	0.34
12	0.22	1.00	1.00	0.22	1.00	1.00	0.14	0.99	0.96	0.13	0.95	0.91	0.06	0.88	0.82	0.03	0.68	0.54	0.01	0.54	0.43
14	0.16	1.00	1.00	0.24	1.00	1.00	0.19	1.00	0.99	0.17	0.99	0.97	0.08	0.95	0.92	0.03	0.81	0.67	0.01	0.66	0.53
16	0.12	1.00	1.00	0.20	1.00	1.00	**0.24**	**1.00**	**1.00**	**0.22**	**1.00**	**1.00**	0.10	0.99	0.98	0.03	0.91	0.79	0.01	0.78	0.64
18	0.09	1.00	1.00	0.14	1.00	1.00	0.26	1.00	1.00	0.26	1.00	1.00	0.12	1.00	0.99	0.04	0.96	0.88	0.01	0.87	0.75
20	0.09	1.00	1.00	0.09	1.00	1.00	0.25	1.00	1.00	0.26	1.00	1.00	**0.12**	**1.00**	**1.00**	0.04	0.99	0.94	0.02	0.93	0.83
22	0.09	1.00	1.00	0.05	1.00	1.00	0.19	1.00	1.00	0.22	1.00	1.00	0.13	1.00	1.00	0.04	1.00	0.98	0.01	0.97	0.90
24	0.09	1.00	1.00	0.04	1.00	1.00	0.14	1.00	1.00	0.18	1.00	1.00	0.12	1.00	1.00	0.04	1.00	0.99	0.01	0.99	0.94
26	0.08	1.00	1.00	0.03	1.00	1.00	0.10	1.00	1.00	0.13	1.00	1.00	0.12	1.00	1.00	**0.04**	**1.00**	**1.00**	0.01	1.00	0.97
28	0.09	1.00	1.00	0.03	1.00	1.00	0.07	1.00	1.00	0.09	1.00	1.00	0.10	1.00	1.00	0.04	1.00	1.00	0.01	1.00	0.98
30	0.09	1.00	1.00	0.03	1.00	1.00	0.04	1.00	1.00	0.06	1.00	1.00	0.08	1.00	1.00	0.03	1.00	1.00	0.01	1.00	0.99
32	0.09	1.00	1.00	0.03	1.00	1.00	0.03	1.00	1.00	0.04	1.00	1.00	0.07	1.00	1.00	0.04	1.00	1.00	**0.01**	**1.00**	**1.00**
34	0.09	1.00	1.00	0.03	1.00	1.00	0.02	1.00	1.00	0.03	1.00	1.00	0.05	1.00	1.00	0.03	1.00	1.00	0.01	1.00	1.00
36	0.09	1.00	1.00	0.03	1.00	1.00	0.02	1.00	1.00	0.02	1.00	1.00	0.04	1.00	1.00	0.03	1.00	1.00	0.01	1.00	1.00
38	0.09	1.00	1.00	0.03	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00	0.03	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00
40	0.09	1.00	1.00	0.03	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00	0.02	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00

Table 5. Finite sample size (for Zivot-Andrews test) and finite sample power (for robust test for a break), AR(1) case, $T = 100$, $\lambda = 0.2$

κ	$\phi=-0.8$			$\phi=-0.5$			$\phi=-0.2$			$\phi=0$			$\phi=0.2$			$\phi=0.5$			$\phi=0.8$		
	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}
0	0.04	0.11	0.14	0.04	0.12	0.15	0.07	0.15	0.11	0.05	0.17	0.08	0.02	0.18	0.13	0.04	0.24	0.13	0.06	0.44	0.19
2	0.04	0.17	0.30	0.04	0.22	0.24	0.07	0.22	0.15	0.05	0.22	0.12	0.02	0.22	0.16	0.04	0.25	0.13	0.06	0.44	0.19
4	0.03	0.40	0.70	0.03	0.49	0.53	0.06	0.42	0.27	0.05	0.37	0.26	0.02	0.33	0.27	0.04	0.29	0.17	0.06	0.46	0.20
6	0.03	0.76	0.95	0.03	0.82	0.84	0.05	0.70	0.53	0.04	0.59	0.49	0.02	0.52	0.46	0.03	0.38	0.23	0.07	0.48	0.21
8	0.04	0.96	1.00	0.03	0.97	0.97	0.05	0.90	0.79	0.04	0.80	0.74	0.02	0.71	0.67	0.03	0.48	0.31	0.06	0.50	0.23
10	**0.05**	**1.00**	**1.00**	**0.03**	**1.00**	**1.00**	0.05	0.98	0.95	0.05	0.94	0.90	0.03	0.87	0.83	0.03	0.59	0.42	0.06	0.53	0.25
12	0.04	1.00	1.00	0.03	1.00	1.00	0.05	1.00	0.99	0.05	0.98	0.98	0.03	0.95	0.93	0.03	0.71	0.54	0.06	0.55	0.27
14	0.03	1.00	1.00	0.03	1.00	1.00	**0.07**	**1.00**	**1.00**	**0.07**	**1.00**	**1.00**	0.04	0.99	0.98	0.03	0.81	0.66	0.06	0.60	0.31
16	0.02	1.00	1.00	0.03	1.00	1.00	0.09	1.00	1.00	0.09	1.00	1.00	**0.04**	**1.00**	**1.00**	0.03	0.89	0.77	0.06	0.64	0.34
18	0.02	1.00	1.00	0.03	1.00	1.00	0.11	1.00	1.00	0.10	1.00	1.00	0.04	1.00	1.00	0.03	0.93	0.84	0.06	0.67	0.38
20	0.02	1.00	1.00	0.02	1.00	1.00	0.11	1.00	1.00	0.09	1.00	1.00	0.03	1.00	1.00	0.03	0.97	0.91	0.06	0.72	0.42
22	0.01	1.00	1.00	0.02	1.00	1.00	0.10	1.00	1.00	0.08	1.00	1.00	0.03	1.00	1.00	0.03	0.99	0.95	0.06	0.76	0.47
24	0.02	1.00	1.00	0.02	1.00	1.00	0.08	1.00	1.00	0.06	1.00	1.00	0.02	1.00	1.00	0.02	1.00	0.98	0.06	0.79	0.50
26	0.01	1.00	1.00	0.02	1.00	1.00	0.07	1.00	1.00	0.05	1.00	1.00	0.01	1.00	1.00	0.02	1.00	0.99	0.06	0.83	0.55
28	0.01	1.00	1.00	0.02	1.00	1.00	0.05	1.00	1.00	0.03	1.00	1.00	0.01	1.00	1.00	**0.03**	**1.00**	**1.00**	0.06	0.86	0.60
30	0.01	1.00	1.00	0.02	1.00	1.00	0.04	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00	0.03	1.00	1.00	0.05	0.89	0.65
32	0.01	1.00	1.00	0.02	1.00	1.00	0.04	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00	0.03	1.00	1.00	0.06	0.92	0.69
34	0.01	1.00	1.00	0.02	1.00	1.00	0.04	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00	0.02	1.00	1.00	0.06	0.94	0.73
36	0.01	1.00	1.00	0.02	1.00	1.00	0.04	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00	0.03	1.00	1.00	0.06	0.95	0.77
38	0.01	1.00	1.00	0.02	1.00	1.00	0.03	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00	0.03	1.00	1.00	0.05	0.97	0.81
40	0.01	1.00	1.00	0.02	1.00	1.00	0.04	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00	0.03	1.00	1.00	0.06	0.97	0.84

Table 6. Finite sample size (for Zivot-Andrews test) and finite sample power (for robust test for a break), MA(1) case, $T = 100$, $\lambda = 0.2$

κ	$\phi=-0.8$			$\phi=-0.5$			$\phi=-0.2$			$\phi=0$			$\phi=0.2$			$\phi=0.5$			$\phi=0.8$		
	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}
0	0.44	0.08	0.59	0.13	0.10	0.42	0.09	0.14	0.12	0.05	0.17	0.08	0.02	0.18	0.12	0.02	0.19	0.12	0.01	0.19	0.17
2	0.31	0.27	0.92	0.11	0.21	0.47	0.08	0.22	0.16	0.05	0.22	0.12	0.02	0.22	0.16	0.02	0.21	0.14	0.01	0.21	0.19
4	0.29	0.78	0.98	0.08	0.57	0.65	0.07	0.43	0.27	0.05	0.37	0.26	0.01	0.32	0.25	0.02	0.28	0.19	0.01	0.25	0.23
6	**0.30**	**1.00**	**1.00**	0.07	0.91	0.88	0.06	0.72	0.52	0.04	0.59	0.49	0.02	0.50	0.42	0.02	0.40	0.28	0.01	0.34	0.30
8	0.30	1.00	1.00	0.08	1.00	0.98	0.06	0.92	0.79	0.04	0.80	0.74	0.02	0.69	0.62	0.01	0.55	0.40	0.01	0.45	0.39
10	0.30	1.00	1.00	**0.08**	**1.00**	**1.00**	0.06	0.99	0.95	0.05	0.94	0.90	0.03	0.85	0.79	0.02	0.70	0.54	0.01	0.58	0.50
12	0.30	1.00	1.00	0.08	1.00	1.00	0.06	1.00	0.99	0.05	0.98	0.98	0.03	0.95	0.91	0.01	0.83	0.70	0.01	0.71	0.61
14	0.30	1.00	1.00	0.09	1.00	1.00	**0.08**	**1.00**	**1.00**	**0.07**	**1.00**	**1.00**	0.03	0.98	0.97	0.01	0.92	0.82	0.01	0.82	0.72
16	0.29	1.00	1.00	0.09	1.00	1.00	0.10	1.00	1.00	0.09	1.00	1.00	0.03	1.00	0.99	0.01	0.97	0.91	0.01	0.90	0.82
18	0.29	1.00	1.00	0.10	1.00	1.00	0.13	1.00	1.00	0.10	1.00	1.00	**0.03**	**1.00**	**1.00**	0.01	0.99	0.96	0.01	0.95	0.89
20	0.29	1.00	1.00	0.11	1.00	1.00	0.14	1.00	1.00	0.09	1.00	1.00	0.02	1.00	1.00	0.01	1.00	0.98	0.01	0.98	0.94
22	0.28	1.00	1.00	0.11	1.00	1.00	0.13	1.00	1.00	0.08	1.00	1.00	0.02	1.00	1.00	**0.01**	**1.00**	**1.00**	0.01	0.99	0.97
24	0.26	1.00	1.00	0.10	1.00	1.00	0.10	1.00	1.00	0.06	1.00	1.00	0.01	1.00	1.00	0.01	1.00	1.00	0.01	1.00	0.98
26	0.26	1.00	1.00	0.09	1.00	1.00	0.08	1.00	1.00	0.05	1.00	1.00	0.01	1.00	1.00	0.01	1.00	1.00	0.01	1.00	0.99
28	0.25	1.00	1.00	0.08	1.00	1.00	0.07	1.00	1.00	0.03	1.00	1.00	0.01	1.00	1.00	0.01	1.00	1.00	**0.01**	**1.00**	**1.00**
30	0.25	1.00	1.00	0.08	1.00	1.00	0.06	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00
32	0.24	1.00	1.00	0.08	1.00	1.00	0.05	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00
34	0.23	1.00	1.00	0.07	1.00	1.00	0.05	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00	0.01	1.00	1.00	0.01	1.00	1.00
36	0.22	1.00	1.00	0.08	1.00	1.00	0.05	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00
38	0.22	1.00	1.00	0.08	1.00	1.00	0.05	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00
40	0.22	1.00	1.00	0.07	1.00	1.00	0.05	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00

Table 7. Finite sample size (for Zivot-Andrews test) and finite sample power (for robust test for a break), AR(1) case, $T = 200$, $\lambda = 0.2$

κ	$\phi = -0.8$			$\phi = -0.5$			$\phi = -0.2$			$\phi = 0$			$\phi = 0.2$			$\phi = 0.5$			$\phi = 0.8$		
	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}
0	0.03	0.08	0.09	0.04	0.08	0.08	0.05	0.10	0.09	0.04	0.12	0.06	0.02	0.13	0.09	0.04	0.18	0.08	0.05	0.42	0.12
2	0.03	0.12	0.24	0.04	0.15	0.17	0.04	0.16	0.13	0.04	0.16	0.10	0.02	0.16	0.12	0.04	0.19	0.09	0.05	0.42	0.12
4	0.03	0.33	0.68	0.03	0.44	0.50	0.04	0.37	0.30	0.04	0.31	0.24	0.02	0.26	0.21	0.04	0.25	0.12	0.05	0.43	0.13
6	0.03	0.74	0.96	0.03	0.80	0.85	0.03	0.67	0.58	0.03	0.55	0.48	0.02	0.44	0.37	0.04	0.32	0.18	0.05	0.45	0.14
8	0.04	0.97	1.00	0.03	0.97	0.98	0.03	0.90	0.84	0.04	0.80	0.75	0.02	0.65	0.58	0.04	0.44	0.27	0.05	0.47	0.15
10	**0.04**	**1.00**	**1.00**	**0.04**	**1.00**	**1.00**	0.04	0.98	0.96	0.04	0.93	0.92	0.02	0.82	0.77	0.04	0.56	0.39	0.05	0.51	0.18
12	0.02	1.00	1.00	0.04	1.00	1.00	**0.04**	**1.00**	**1.00**	0.05	0.99	0.98	0.02	0.93	0.90	0.03	0.68	0.50	0.05	0.54	0.20
14	0.02	1.00	1.00	0.02	1.00	1.00	0.04	1.00	1.00	**0.06**	**1.00**	**1.00**	0.02	0.98	0.96	0.03	0.79	0.63	0.04	0.57	0.22
16	0.01	1.00	1.00	0.02	1.00	1.00	0.04	1.00	1.00	0.06	1.00	1.00	0.02	1.00	0.99	0.03	0.88	0.75	0.05	0.61	0.25
18	0.01	1.00	1.00	0.01	1.00	1.00	0.03	1.00	1.00	0.05	1.00	1.00	**0.02**	**1.00**	**1.00**	0.03	0.94	0.85	0.04	0.65	0.29
20	0.01	1.00	1.00	0.02	1.00	1.00	0.03	1.00	1.00	0.04	1.00	1.00	0.02	1.00	1.00	0.03	0.97	0.91	0.04	0.69	0.32
22	0.01	1.00	1.00	0.01	1.00	1.00	0.02	1.00	1.00	0.03	1.00	1.00	0.01	1.00	1.00	0.02	0.99	0.96	0.04	0.74	0.38
24	0.01	1.00	1.00	0.01	1.00	1.00	0.02	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00	0.02	1.00	0.98	0.04	0.78	0.41
26	0.01	1.00	1.00	0.02	1.00	1.00	0.02	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00	0.02	1.00	0.99	0.04	0.82	0.46
28	0.01	1.00	1.00	0.02	1.00	1.00	0.02	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00	**0.02**	**1.00**	**1.00**	0.04	0.85	0.52
30	0.01	1.00	1.00	0.01	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00	0.01	1.00	1.00	0.02	1.00	1.00	0.04	0.87	0.56
32	0.01	1.00	1.00	0.02	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00	0.01	1.00	1.00	0.03	1.00	1.00	0.04	0.90	0.61
34	0.01	1.00	1.00	0.01	1.00	1.00	0.02	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00	0.02	1.00	1.00	0.04	0.93	0.66
36	0.01	1.00	1.00	0.02	1.00	1.00	0.02	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00	0.03	1.00	1.00	0.04	0.95	0.71
38	0.01	1.00	1.00	0.02	1.00	1.00	0.02	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00	0.02	1.00	1.00	0.03	0.96	0.75
40	0.01	1.00	1.00	0.02	1.00	1.00	0.02	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00	0.03	1.00	1.00	0.04	0.97	0.78

Table 8. Finite sample size (for Zivot-Andrews test) and finite sample power (for robust test for a break), MA(1) case, $T = 200$, $\lambda = 0.2$

κ	$\phi = -0.8$			$\phi = -0.5$			$\phi = -0.2$			$\phi = 0$			$\phi = 0.2$			$\phi = 0.5$			$\phi = 0.8$		
	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}	ZA	t_{HLT}	t_{PY}
0	0.19	0.15	0.73	0.07	0.07	0.23	0.06	0.10	0.10	0.04	0.12	0.06	0.02	0.13	0.08	0.03	0.13	0.09	0.01	0.13	0.11
2	0.11	0.26	0.87	0.06	0.15	0.28	0.05	0.15	0.14	0.04	0.16	0.10	0.02	0.16	0.11	0.03	0.15	0.11	0.01	0.14	0.12
4	0.10	0.74	0.98	0.04	0.51	0.55	0.04	0.37	0.29	0.04	0.31	0.24	0.02	0.27	0.21	0.02	0.23	0.17	0.01	0.20	0.16
6	**0.11**	**1.00**	**1.00**	0.04	0.92	0.90	0.04	0.68	0.56	0.03	0.55	0.48	0.02	0.45	0.37	0.02	0.34	0.26	0.01	0.28	0.23
8	0.12	1.00	1.00	**0.04**	**1.00**	**1.00**	0.04	0.92	0.84	0.04	0.80	0.75	0.02	0.67	0.58	0.02	0.51	0.40	0.01	0.41	0.34
10	0.12	1.00	1.00	0.05	1.00	1.00	0.04	0.99	0.96	0.04	0.93	0.92	0.02	0.84	0.78	0.02	0.68	0.58	0.01	0.55	0.47
12	0.11	1.00	1.00	0.05	1.00	1.00	**0.05**	**1.00**	**1.00**	0.05	0.99	0.98	0.02	0.94	0.91	0.02	0.82	0.72	0.01	0.68	0.59
14	0.11	1.00	1.00	0.04	1.00	1.00	0.05	1.00	1.00	**0.06**	**1.00**	**1.00**	0.02	0.98	0.97	0.01	0.92	0.84	0.01	0.80	0.71
16	0.12	1.00	1.00	0.04	1.00	1.00	0.05	1.00	1.00	0.06	1.00	1.00	0.02	1.00	0.99	0.02	0.97	0.93	0.01	0.89	0.82
18	0.11	1.00	1.00	0.04	1.00	1.00	0.04	1.00	1.00	0.05	1.00	1.00	**0.02**	**1.00**	**1.00**	0.02	0.99	0.97	0.01	0.95	0.90
20	0.11	1.00	1.00	0.04	1.00	1.00	0.04	1.00	1.00	0.04	1.00	1.00	0.02	1.00	1.00	0.01	1.00	0.99	0.00	0.98	0.95
22	0.10	1.00	1.00	0.04	1.00	1.00	0.03	1.00	1.00	0.03	1.00	1.00	0.01	1.00	1.00	**0.01**	**1.00**	**1.00**	0.00	0.99	0.97
24	0.10	1.00	1.00	0.04	1.00	1.00	0.03	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00	0.01	1.00	1.00	0.01	1.00	0.99
26	0.10	1.00	1.00	0.04	1.00	1.00	0.03	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00	0.01	1.00	1.00	**0.00**	**1.00**	**1.00**
28	0.10	1.00	1.00	0.04	1.00	1.00	0.03	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00	0.01	1.00	1.00	0.01	1.00	1.00
30	0.10	1.00	1.00	0.04	1.00	1.00	0.03	1.00	1.00	0.01	1.00	1.00	0.01	1.00	1.00	0.01	1.00	1.00	0.01	1.00	1.00
32	0.10	1.00	1.00	0.04	1.00	1.00	0.03	1.00	1.00	0.01	1.00	1.00	0.01	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00
34	0.09	1.00	1.00	0.04	1.00	1.00	0.03	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00	0.01	1.00	1.00	0.01	1.00	1.00
36	0.10	1.00	1.00	0.04	1.00	1.00	0.03	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00	0.01	1.00	1.00	0.01	1.00	1.00
38	0.09	1.00	1.00	0.04	1.00	1.00	0.03	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00	0.01	1.00	1.00	0.01	1.00	1.00
40	0.09	1.00	1.00	0.04	1.00	1.00	0.03	1.00	1.00	0.02	1.00	1.00	0.01	1.00	1.00	0.01	1.00	1.00	0.01	1.00	1.00

3 Further extensions

Based on the properties of our proposed modifications described in the previous section, these procedures can be expanded in several directions. Furthermore, we briefly consider the following generalizations indicating the associated limitations: the presence of more than one structural break, and the presence of *a priori* information about the location of the break.

3.1 Multiple structural breaks

The test procedures discussed above can be used in a case of multiple structural breaks, because if the number of breaks is more than taken into account when constructing the test, the power will drop to zero. For the unit root tests we can consider two types of tests based on OLS- or GLS-detrending, respectively, similar to the previous section. The infimum test based on GLS-detrending was proposed by HLT2013 in the context of a case of multiple breaks, and this test was effective under small initial conditions. The extensions of the OLS-based tests are constructed in a similar way.

However, the problem of the construction of the effective combination of OLS-based tests under large initial conditions has arisen. The behavior of the $MDF\text{-}OLS$ with multiple breaks is not investigated under fixed break magnitudes and unclear how large the magnitudes of breaks should be in order experience serious size distortions with $MDF\text{-}OLS$. If we find an effective combination of OLS-based tests, it could be used in similar strategies to those described in Section 4, with the Kejriwal and Perron (2010) and Sobreira and Nunes (2012) procedures used as pre-tests to determine the number of breaks (see also Sobreira *et al.* (2014)). However, the power "valleys" for these strategies can be much more severe and can increase with the number of tests. One might ask whether it makes sense in the case of, for example, three breaks, to use a fairly complex combination of tests, and the best solution is to use only one test (with the maximum number of breaks). We have left all these questions open for future research. Currently, the test based on (augmented) Dickey-Fuller regression with with break dates estimators according Harvey and Leybourne (2013) (the $ADF\text{-}OLS^{tb}(\hat{\lambda}^{D_m})$ in case of multiple breaks) seems to be the most robust.

3.2 *A priori* information on the break date

One modification when testing with the allowance for one structural break was proposed in Harvey *et al.* (2014). Many unit root tests with a break use the fact that the break can occur in any location, except for some values at the beginning and end of the series. However, the researcher may know some *a priori* information about the location of the break without knowing its exact location. This approach was first considered by Andrews (1993) when testing for general structural instability by motivating it using two examples: a significant political event (economic reform, war, etc.) can occur in a certain period of time, but it is unknown exactly when the effects begin; the event may occur on a certain date, but the effect occurs with some delay.

A priori information about the date of the break implies that the true break fraction is $\lambda_0 \in \Lambda(\tau_{mid}, \delta)$, where $\Lambda(\tau_{mid}, \delta)$ is a window in which the break occurs. This window is defined as $\Lambda(\tau_{mid}, \delta) = [\tau_{mid} - \delta/2, \tau_{mid} + \delta/2]$, where $\delta > 0$ denotes the width of the window containing all permissible break fractions, and τ_{mid} denotes the mid-point of the window. $\tau_{mid} - \delta/2 > 0$ and $\tau_{mid} + \delta/2 < 1$ is a requirement.

Therefore, all minimizations considered in this paper are performed not on the whole set $\Lambda = [\lambda_L, \lambda_U]$, but on the set $\Lambda(\tau_{mid}, \delta)$. It is clear that the power of the test increases when there is more known *a priori* information about the location of the true break date, i.e. with a decreasing value of δ. The location of the break within the selected window has a small effect on the asymptotic size and power. However, the size and power will be seriously downward biased if the information about the true break date location is wrong, and the distortion increases as the error of this information increases. Thus, all procedures considered in this paper can be easily expanded to cases of partial information about the break date location.[1] The ox-code for calculating the critical values and scaling constants for different δ and τ_{mid} is available on the author's web-page.

[1]Note, that the $MDF\text{-}OLS$ test can be used for all κ if the break is occurred in the second half of sample.

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