Review Article

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Local orthometric height based on a combination of GPS-derived ellipsoidal height and geoid model: A review paper

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Abstract: The use of orthometric height in geodetic applications provides elevations on the physical topographic surface of the earth rather than ellipsoidal heights that are not in conformity with the physical topography. Global positioning system (GPS)/levelling produces ellipsoidal heights that are not consistent with levelled heights above mean sea level. The study provides a practical solution of using the GPS levelling approach or the geoidal heights aimed at providing local orthometric height. Many research studies were conducted with a view of finding a viable solution to the derived orthometric heights. It was revealed that the research studies conducted were found lacking in the use of only lower order numerical solutions models, which limit the accuracy derived from the model, the use of online post-processing, RTKlib, and other non-precise software to obtain the coordinates of the stations used in the derivation of orthometric. Finally, the use of gravimetric data, with its temporal variation problem, poses a threat to the derivation of orthometric height, so also to the accuracy of the developed model. Considering factors while developing models for orthometric heights improves the accuracy in achieving required heights for geodetic applications and aids in fast-tracking mapping.

Keywords: ellipsoidal, geoid and orthometric

1 Introduction

Despite the presence of some error challenges in global positioning system (GPS) positioning, Yujun et al. (2021) showed numerous advantages of GPS techniques perspective in high-precision geodetic positioning, making use of satellite-based positioning systems in a very large spectrum of applications, including engineering surveys and scientific research studies. An orthometric height depicts the nature of the terrain (Tata and Matthew, 2018). The geoid being equipotential surface coincides with the mean ocean surface of the earth. It is the surface of the Earth's gravitation and rotation in the absence of other influences such as winds and tides. This surface extends through the continents and to all points on the geoid having the same gravity potential energy, and it is used as the reference surface for orthometric height measurement (Tata and Matthew, 2018). Farsat (2021) opined that heights obtained from GPS are typically heights above an ellipsoidal model of the Earth and are not consistent with levelled heights above mean sea level (MSL). Height determination has an increased importance in most of the practical applications of geodetic networks.

Ellipsoidal heights have to be converted to orthometric height to have physical meaning in application as heights referring to MSL (geoid). This transformation is applied using the geoidal heights (N) from a geoid model. The computation of geoid models is usually from the expansion of spherical harmonics and smoothing resulting from a data set suitable for interpolation. Exploiting such an approach for the determination of heights reduces the time and cost taken in ordinary levelling surveying. Transformation of ellipsoidal heights into local orthometric heights can be effective if the knowledge of the geoidal undulation between the surfaces is known (Kurotamuno and Elochukwu, 2020).

The accuracy values of global geoid models presented in local areas are not suitable for most geodetic applications at large scale but can perform better using accurate undulation values derived by GPS/levelling, resulting in minimal bias correction. The earth geopotential model (EGM) 2008 model, in combination GPS/levelling ellipsoidal heights, is a more suitable model than the orthometric heights obtained directly from global geoid models. Since the release of EGM2008 by the National Geospatial-Intelligence Agency (NGA), there exist several other developments of new global

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geoid models with the effort of improving the accuracy and reliability in its use. Levelling by the classical method requires a connection to an established and unified system that conforms to the physical topography of the earth surface. Determination of orthometric heights via GPS levelling requires high accuracy of a model enabling such, depending on the computational methodology and available data. The use of a global geoid model over a region introduces a lot of errors to the derived orthometric heights.

2 Review

2.1 Hybrid approach

Working with an ellipsoidal height from GPS levelling on the geoid surface (earth) is not a practical way due to the physical reality of the surface topography. Several techniques and methods are available for the integration of the two height systems; the work carried out by Eteje et al. (2018) used one of the techniques to develop a local height transformation model for Evboriaria, Benin City, using the geometric (GPS/levelling) method for the calculation of MSL heights. A differential global positioning system (DGPS) receiver was used to obtain Receiver Independent Exchange Format (RINEX) data, post-process with the necessary software to obtain the ellipsoidal height of 50 stations. The geometric method involves the use of GPS and levelling data, where both the ellipsoidal and orthometric heights are given. The authors deduced the separation of the stations from EGM08 and EGM96 using University NAVSTAR Consortium (UNAVCO) and GeoidEval software, respectively. The authors used the polynomial regression model to interpolate the orthometric heights for EGM08 and EGM96, termed models A and B. From the results, model A has an error range between −0.224 and +0.535 m, while B has −0.227 and 0.533 m. The authors calculated their mean change in orthometric height (ΔH) as 0.081 m from all the stations. The interpolated orthometric height for model A has an error range from 0.000900 to 0.286225 and that of model B from 0.000400 to 0.284089. The average residual of the model is between 8.888 and 8.839 cm, respectively. The computed mean standard deviation between the observed spirit levelling orthometric heights and that of GPS model interpolated orthometric heights was ±21 cm, and a mean geoidal undulation of 28.410 m was obtained. However, the model was limited by using the first-order linear regression model; it limits the accuracy derived from the model to determine orthometric heights. If higher models of the same linear regression are applied, a better result and accuracy will be produced. The points used are also limited by small spatial proximity and hence can only be used in a limited area.

The research conducted by Danar et al. (2018) showed that GPS/levelling was used for tide observation data between tidal stations and the survey area depth reduction. The authors utilise tidal zoning to provide a good transformation from the ellipsoidal to the orthometric system in the Java Sea between Indonesia and Borneo. Tide observation data, GPS/levelling height, geoid model data, earth gravity model (EGM2008), and coastline data on seawaters of over 25 stations were used. An average GPS observation time for each day was 5–6 h with an interval every 10 s sampling rate was used. In post-processing, real-time kinematic library (RTKLIB) was used to obtain the tide observation with co-tidal values. EGM2008 model data were used to obtain corrections for sea level, which still refers to the assumed geoid, and it does coincide with MSL. The sea level height from the GPS observation was processed using Pydro 18.4 in the form of contours of the amplitude and phase values of each constant (M2, S2, N2, K1, and O1). The authors obtained values of each amplitude and phase using the Matlab R2014a least square method. The largest amplitude for the K1 constant was at the Bangka tidal station, with a value of 0.811 m. Also, the largest amplitude for the M2 constant was recorded at Ketapang station with a value of 0.477 m. The largest amplitude for the N2 constant is at their Ketapang tidal station with a value of 0.094 m. The authors compare sea level height values from the tide pole and GPS levelling tidal results; the largest root mean square error (RMSE) value was on April 26, 2018, with a value of 0.246 m. The tide pole and co-tidal results gave the largest RMSE value on April 27, 2018, with a value of 0.286 m. While, on April 26 and 28, their resulting RMSE values were 0.237 and 0.109 m, respectively. From the comparison of sea level height values, the results of the tide pole with GPS and the results of the tide pole with co-tidal gave an average error value on the sea level of GPS results smaller than co-tidal average error results. The authors concluded that the value of the sea level of the GPS results is closer to the sea level height of the tide pole. The model is, however, limited by the use of lower geometrical equations that may hamper the accuracy of obtaining more accurate results. The use of RTKLIB is not best in obtaining a better result for the ellipsoidal height; precise point positioning and Bernese can provide a better ellipsoidal height, hence for a better ellipsoidal height value. The stations used are in maritime zones; the approach may be limited by application to the marine zones only.

Similarly, Lars (2018) showed how GPS levelling was used in obtaining orthometric height instead of quasi-geoid in determining the equipotential surface of the Earth's gravity field serving. The author approached normal height from GPS levelling defined by the formula Heiskanen and Moritz (1967). Both normal height and height anomaly/quasi-geoid can be determined from GPS levelling (alone). The beauty of Molodensky's introduction of normal height and quasi-geoid is that these components can be determined without any information about the Earth's density distribution. The author derived the method of least squares modification of the Stokes formula with additive corrections in contrast to other methods explicitly to provide the corrections needed for topographic height and density through his derivations. The topographic bias does not include a terrain correction (TC), because the topographic biases are not dependent on the mass distribution of the terrain. Also, the terrain correction was already accounted for in the analytical continuation. However, for an accurate solution in his derivations, the bias should be corrected for variable density distribution along the vertical. From the derivations, the topographic density distribution is a gravimetric inverse problem. The problem of determining the quasi-geoid is a forward problem that does not rely on an estimated topographic density distribution model; hence, if the Earth's surface is known, e.g. expressed by its laterally variable geocentric radius, the height anomaly can be determined from GPS levelling. The author concluded that the geoid geometry is modified at least 10% less than the quasi-geoid. However, from his derivations, no numerical solutions were made to confirm these equations made use; hence, there is a need to prove the workability of these equations.

Also, the research conducted by Daniel and Kevin (2019) to analyse the vertical component variation of sea surface showed that GPS levelling techniques are good for integrating systems of height by updating four terrestrial reference frames directly tied to the International Terrestrial Reference Frame (ITRF) and the North American-Pacific Geopotential Datum of 2022 (NAPGD2022) to fit out GPS observations on levelled benchmarks (GPS on BM) as part of the North American Vertical Datum (NAVD) 88 adjustment. The post-processed GPS data were obtained from Online Positioning User Service (OPUS), which is to warp the surface of gravimetric geoid to fit through the NAVD 88 and North American Datum (NAD) 83 surfaces at the benchmarks. Hence, the term "GPS-hybrid" is used to describe such models to distinguish them from geoid models based only on gravity field data (gravimetric geoid height models). To develop their GPS-hybrid, residuals were made first from the GPS-derived ellipsoid height (h), the levelling derived orthometric height (H), and the geoid height (N) from a gravimetric geoid model. To the authors, in a perfect world, the residuals would all be near zero; however, the residuals may be caused by datum defects, such as the established meter-level tilt in the NAVD 88 datum, local problems in the network, local gravity field problems, etc. The authors' residual values are formed as individual points, but they used least squares collocation to find the correlated signal between points and then added as a corrector surface to the gravimetric geoid height model to make it into a hybrid geoid height model. The performed analysis on the GPS-derived ellipsoid heights highlights potential outliers provided error assessments ranging between ±0.037 m; the strength and quality of the network ensure that the results were not overly optimistic, a known problem when performing a least squares adjustment. The authors also examined the gravimetric geoid height to determine if insufficient or poor quality gravity data may have impacted the local quality of the geoid height model, which made it possible for them to identify priority control data (GPS on BM) that were suspect and needed to be revisited. The established "GPS-hybrid" geoid models developed from an underlying gravimetric geoid and the GPS-derived ellipsoidal heights on the spirit-levelled benchmarks (GPS-BM) fit GPS-BM's with an accuracy range between ±0.451 m and as the standard deviation of ±0.213. From analysis and results, a GPS-hybrid geoid height model, GEOID18, was developed, refined, and improved that will serve as the final to such model until the eventual release of the North American Geopotential Datum of 2022 (NAPDG 2022) that will replace it as the defining vertical datum within the United States (US) national spatial reference System. The model provides continuity between the GPS-BM to develop a consistent transformation between GPS-derived ellipsoidal and NAVD 88 to facilitate work by surveyors around the country, giving a concerted effort to fill in gapped regions and provide a better spatial distribution for this model. However, the use of OPUS for post-processing the RINEX data for derivation of authors' ellipsoidal data poses a big threat to the accuracy of the NAPDG 2022 developed, to this, if better processing software was used, a better model can be derived from their model leading to increase in accuracy of heights derivation.

Finally, Mosbeh et al. (2019) use GPS/levelling measurements, four heuristic regression methods: least square to support vector regression (LSSVR), Gaussian process regression, kernel ridge regression (KRR), and multivariate adaptive regression SPlines for modelling local geoid undulation in Kuwait. It covers a total area of 17,818 km^2 located within latitudes 28.5°N and 30.1°N and longitudes 46.5°E and 48.5°E. The authors used dual-frequency GPS receivers for static and rapid-static measurement on ITRF 2008 datum. The approximate accuracies of GPS coordinates are ±1.0 and ±1.5 cm in the horizontal and vertical directions, respectively. The absolute accuracy of the orthometric heights is approximately ±1.0 cm. Local geoid undulation model measurements were done by integrating between land and ocean observation to improve the local geoid model of the region. The cross-validation method was applied to estimate the LSSVR parameters. The obtained parameters "γ" represent the biases of the linear model "b" and "σ" as the kernel function and bandwidth gave an error of 14.65, −0.83, and 0.08, respectively. The percentage of error accuracy of the KRR model was between 0.018 and 0.124% relative to gravity and GPS/levelling geoid models, respectively. In terms of standard deviation, the linear fitting equation of the models shows that RMSE, mean absolute error, mean bias error, agreement index (d), and Nash Sutcliffe efficiency (NSE) of the GPS-KRR model are 0.019, 0.024 m, and -5.19×10^{-4}, respectively. These values are the lowest for the tested models. In addition, the d and NSE of the KRR model are 0.992 and 0.999, respectively. These values are the highest for the other models. However, this model was generated from a second-order set of equations, and this minimises the accuracy of the model by not allowing the researcher to check the accuracy and precision that may be obtainable from a higher-degree model used over the same region. This led to proper analysis of various contexts in simulating the model to fit the study area.

2.2 Global geoid model approach

In a study by Oluyori et al. (2019) in developing geoid model used for transforming ellipsoidal to orthometric heights via GPS levelling, data were acquired using dual frequency GPS field observations and later post-processed with online processing software to determine the ellipsoidal heights of 25 stations in FCT Abuja. Using multi-quadratic interpolation to represent irregular surfaces, bi-cubic model, third-order polynomial, and multiquadratic equation were used while least squares equation was applied in solving the observation equation to determine the polynomial coefficients X_i. $H_{\text{Multiquadratic}}$, and H_{MSL} that were determined to be equal to 0. The computed "t" from the table is t = 1.717. This was compared to chi-squares table values with a decision rule: if $1.98/\sqrt{N} < \chi^2$, then the model is satisfactory at a 95% confidence level. In the study, $1.98/\sqrt{N} = 0.404$. Using the Chi squares (χ^2) test at the 95% degrees of freedom, the value is $\chi^2 = 24.996$ at 95%, bicubic model, degree of freedom $\chi^2 = 23.685$ at 95%. Since 0.404 < 24.996 or 23.685, the models proved satisfactory at 95% confidence limits for modelling orthometric heights. The modelled orthometric heights were then compared with their corresponding existing orthometric heights at the controls. The standard deviation of the multiquadratic gave ±11 cm and bi-cubic gave a value of ±14 cm. However, the model was generated from a second-order set of equation, and this limitation hampers the accuracy of the model. Higher order models if checked can produce a better accuracy and precision in height obtained when applied over the same region.

Oluyori et al. (2018) utilised online software Canadian Spatial Reference Service Precise Point Positioning software to post processing RINEX data acquired. This leads to obtaining primary data on latitudes, longitudes, and ellipsoidal height from the static observation. EGM2008 was used to compare the computations of orthometric height from GPS ellipsoidal height for geodetic applications in FCT Abuja with the use of a global geoid model. The use of All Trans 3.002 and EGM2008 geoid calculator to compute the relationship $N = h - H$ can be a factor that limits the greater part of accuracy in their model. The authors compared the differences in N_{GPS} and N_{EGM2008} with global models that do not satisfy the accuracy level of orthometric height desired for local applications, which showed a value between 1.256 and −0.313 m with a mean value of 0.836 m; from the result, a pointer to the value is not suitable for local applications; hence, the global model (EGM2008) alone is not an adequate source of orthometric height determination. The authors concluded that a geoid model for each state should be developed to encourage a geometric geoid model for local applications instead of adopting a model that is inadequate for practical geo-data acquisitions. And that efficient utilisation of GPS in almost all applications requires the development of an appropriate geoid model for transformation of ellipsoidal height to orthometric height. The use of online post processing to obtain the ellipsoidal height of the stations used in the derivation of orthometric height is a limiting factor in realising a better model from the start; however, better post-processing software is used, then a better improved ellipsoidal height can be obtained from the developed model. The maximum, minimum, and mean of the difference between the EGM2008 Alltrans calculator should not be a primary source of judgement of the model insufficiency. There has to be more mathematical equation to justify this limitation. The residual error and accuracy of the model on the researcher to check the accuracy and precision that the model provides should be incorporated to guard against biased judgement over the same region.

Also, Tran et al. (2019) presented a research article on developing methods for obtaining normal heights in Vietnam using the global geoid model EGM2008 and GNSS ellipsoidal height measurements. EGM2008, EIGEN-6C4, GECO, and GAO-2012 global geoid models were used for geopotential decomposition with a grid size of 1 × 1 or 2.5 min × 2.5 min of the NGA of the US. The adjustment algorithm combines geodesic height, normal height, and height anomaly for points in the network. The values of geodetic height, normal height, and height anomalies were obtained by a deviation vector $\Delta\zeta_i$ given by equation (1):

$$\Delta\zeta_i = H_i - h_i - \zeta_i^{2008} = \zeta_i^{\Gamma\mathrm{HCC/HuB}} - \zeta_i^{2008}, \tag{1}$$

the adjustment equations and amendments from

$$\Delta\zeta_i = f(\phi_i, \lambda_i) + v_i = a_i^T x_i - v_i. \tag{2}$$

The model parameters selected to describe the difference between the three elements of the height, derived from the EGM2008 model and the corresponding H_i, is given by equation (3):

$$H_i - h_i = \zeta_i^{\Gamma\mathrm{HCC/HuB}}. \tag{3}$$

The accuracy estimation of models includes four parameters, five parameters, a polynomial of first degree, a polynomial of second degree, and a polynomial of third degree. In the general case, they arrive at equation (4):

$$a_i^T x_i = \sum_{m=0}^{M}\sum_{n=0}^{N} x_q(\varphi_i - \varphi_0)^n(\lambda_i - \varphi_0)^m \cos^m \varphi_i. \tag{4}$$

The authors consider both cases with and without the use of the global EGM2008 model. The height anomaly of a point is interpolated by the points with the use of the SP-line function equation (5) according to the formula:

$$\Delta\zeta[p(\phi_i, \lambda_i)] = \sum_{i=1}^{n} a_i r_{ppi}^2 \mathrm{In}(r_{ppi}^2) + \tau_1 + \tau_2 x + \tau_3 y, \tag{5}$$

where

$$r_{ppi} = \sqrt{(x - x_i)^2 + (y - y_i)^2}.$$

By using the above mathematical models, the computed height anomalies of a point are moved to normal heights by the results of GNSS-positioning given by equation (6):

$$h = H - (\zeta^{2008} - \Delta\zeta). \tag{6}$$

According to the results, the accuracy assessment based on 17 test points, the EGM08_TN model yielded 100% accuracy corresponding to the fourth levelling class, for mountainous areas; 85% level corresponds to the third class. The error in determined normal height according to GNSS measurements was reduced from 0.0244 to 0.0086 m/km. About 80% of the routes have acceptable accuracy for the third-grade levelling under all four interpolation methods, even in the cases of mountain areas. The authors concluded that the developed calibration procedure for height anomalies of the EGM2008 is intended for use on a large scale with a large number of points in a network. The computation method is relatively simple and improves the accuracy of height determination based on GNSS measurements. The points are not distributed relatively uniformly and fairly densely, so, it cannot be used as the quasi-geoid model for the whole region. The model was also generated from a second-order set of equations, and this minimised the accuracy of the model by not allowing the researcher to check the accuracy and precision that may be obtainable from a higher-degree model used over the same region.

Another work on precise orthometric height determination required in field of construction, Geodesy and Geophysics to obtain geopotential model (GM) aimed at avoiding spirit levelling restrictions on long distances has been demonstrated by Alonso et al. (2019) in Costa Rica. Baseline was measured using GPS, spirit levelling and gravity measurements to validate the heights computed from EGM2008, EIGEN-6C4, GECO, EGM96, GGM-05C, and GOCO05C. The authors use GPS measurements obtained on 6 BMs set along 74 km of the Central Pacific of Costa Rica; post-processing took into account the IGS precise orbits (epoch: 2016.270) with respect to the WGS84 ellipsoid (National Imagery and Mapping Agency, 1997). The International Centre for Global Earth Models provided them with those GMs and used for height determination from the height anomaly plus spherical shell approximation of the topography. They designed a script programmed in inverse distance weighted interpolation on a grid. The observed gravity values were corrected to get the gravity value on the ground surface, and earth tides correction and instrument drift correction were applied after their measurement works. First, the authors did absolute height assessment, where geoid height from the GMs (N_{GM}) was directly compared to the geometric heights (N_{GPS}) obtained from GPS and spirit levelling. The obtained bias fit bias from their comparison to the GMs with respect to the local vertical reference surface. The results were substituted in to least-squares constant bias value for each GM that this vertical offset varies from model to model. The overall precision estimated by max standard deviation of ±0.32 m. The study concluded that an approximation of the bias fit for global geopotential models (GGMs) shows a correlation with a local reference geopotential value of old local Costa Rican reference datum. The bias fit was not constant among the models. Because of the subtraction of the geometric geoid separation to the GMs geoid separation is close to 0 and the standard deviation (S) is high for almost ±1.5 m for some models. Consequently, their fisher test on the variance (S_2), each tested geoid obtains a Fisher value 1.054 equal or less than the critical value. This means that all their tested geoids were equally precise, and thus, a two-sampled test could be applied and that each of their geoids are more suitable for use in local engineering or scientific projects such as levelling work to obtain orthometric height difference (ΔH) over long distances. Here, the techniques and post-processing strategy was reasonable, not much error is expected from their determined ellipsoidal height, but parameters and mathematical representation in deducing the derived model is limited by few coefficient making most generalisation of the numerical variables to conclude the accuracy derived from the model, if such system was given, the privilege of using multiple variable from few to large coefficient variables, a better suitable model can be derived for application in the region.

Ugo et al. (2018) work on global geoid adjustment to suite local area for geographic information system applications using GPS permanent station coordinates for Italy to harmonies the relationship between the two systems. The authors use of EGM2008, derived from satellite gravity measurements and 25 GPS Permanent Station (GPS), freely available on the web for orthometric and ellipsoidal heights, to calculate precise geoidal undulations in performing global geoid modelling on a local area located in North-Western Italy. The authors consider the differences between their GPS levelling geoidal heights and the corresponding EGM2008 1′ × 1′ ones as a starting dataset for ordinary Kriging applications. The model transforms an ellipsoidal height work by algebraically subtracting the WGS84 ellipsoid geoid separation using the simple mathematical relation $H = h - N$. The geoid undulation, N, and the horizontal coordinates, x and y, of some control points were used to determine the local geoid based on an interpolation method, with polynomial regression, inverse distance weighting, or kriging. Cross-validation of the result to assess the interpolation model performance and to define the accuracy level of predictive values. The geoidal heights were calculated by the authors using the accurate local geoid ItalGeo2005 whose differences compared to EGM2008. Differences between the ellipsoidal and orthometric heights are calculated to achieve geoid undulations. The results show the presence of a bias in EGM2008 due to the reference point considered for the definition of the zero level in the local area (datum inconsistency). The statistics on the resulting residuals shown in their table seem to prove a good performance, but further insights are needed. Considering all experiments, the differences between interpolated and measured values vary from ±0.177 to ±0.239 m, the standard deviation varies from 0.032 to 0.149 m, the mean values vary from ±0.006 to ±0.029 m, and the RMS varies from 0.033 to 0.149 m. From the result, EGM2008 localise for vertical translation performance to bring the model near the GPS levelling values; in other words, the bias was subtracted from their original value of each undulation to achieve an RMS value of ±0.112 m, while their residuals vary from ±0.251 to 0.265 m. The model results generated surface sample data by means of interpolation method results, which seem to attest to a good performance of the ordinary Kriging interpolator, but more reliable tests on check points are needed. The sample points are not fairly distributed for global geoid models presenting the local areas covered and are not suitable for applications at large scale and may not perform accurately to generalise the behaviour for undulation values over the area, even if improvements are made, it is limited to a few centimetres. By using the calculated geoid model to derive orthometric heights from ellipsoidal heights in the considered area, it is only possible to use this model for large-scale map applications to produce contours suitable for a scale of 1:2,000 with a contour interval of 0.40 m.

Hamdy and Shaheen (2020) worked on GGMs for assessing the accuracy of the use of DGPS/precise levelling observations to test the performance of GGMs in calculating geoid undulation for orthometric height determination along the Mediterranean western coastal line from El-Salloum to El-Alameen of Egypt. The orthometric heights of the stations were obtained through first-order levelling loops of the national vertical datum of Egypt that is based on the MSL at Alexandria tide gauge of 1906. The authors used online post-processing TBC planning software to obtain latitude, longitude, and ellipsoidal height data of each station from their DGPS measurement dual frequency Trimble 5700 GPS receivers in static mode for an average of 2 h session. The ellipsoidal heights have been computed for each station with a precision of ±0.003 m. The initial assessment conducted on four different methods of EGM96 and EGM08 (bi-linear interpolation, bi-quadratic interpolation, triangulation, and nearest neighbour) was compared with accurate DGPS/precise levelling derived undulations over 52 stations. From the results, the EGM08-bi-linear interpolation method is noted to be nearly consistent with an error range between −0.747 and 0.793 m, bi-quadratic interpolation gave an error range from −0.733 to 0.837 m, triangulation gave error range from −0.749 to 0.766 m and nearest neighbour gave error range from −0.504 to 1.506 m. EGM96 geoid undulations have also been computed and gave an error range from −2.659 to 2.739 m. The standard deviation of the undulation differences is estimated to be ±24 cm for EGM08 bi-linear interpolation to ±45 cm for EGM08-nearest neighbour and ±1.393 m for EGM96. The authors concluded that the release of the EGM08 GGM is a millstone step in improving geoidal modelling on a global scale. However, the use of online TBC for post-processing the RINEX data for derivation of their ellipsoidal data poses a big threat to the accuracy of the developed, to this, if better processing software was used, a better model can be derived from the model, leading to an increase in accuracy of height derivation. So also, lower geometric models were used.

2.3 Gravimetric approach

Another technique was use of gravity measurement on grid maps computed from high-resolution TCs and residual terrain model (RTM). The short-wavelengths of the gravity field and geoid were used by Salissou and Driss (2018) in Niger republic for orthometric height determination. The authors use gravity data from terrestrial gravity data covering 0° to 16° East and 11° to 24° North provided by the International Gravimetric Bureau (Bureau Gravimetrique International [BGI]). In 2015, after gross error removal, 8,393 gravity values were retained. The computations at gravity stations of 1.5 arc-minute regular grid, out to 10 and 200 km for inner and outer zones, respectively, and a standard density of 2,670 kg/m^{-3} were obtained. The amplitudes are 3–5 mm and 0.01–0.02 mgal at wavelengths of 15–20 km; in low-lying regions, at 0.02 mgal and 3 mm levels, the values are stable beyond R_2 = 100 km. In mountainous areas, the amplitudes are of 1–5 cm and 0.5 mgal at wavelengths of 15–20 km, and the values are stable beyond R_2 = 190 km. With 55 km as a reference value, the influence of R1 is negligible on the RTM height anomalies and TCs at the point of test. In mountainous areas, from R_1 = 5 km, the value of the differences is 0.035 mgal and 5 mm, respectively, for indirect effect and TCs. The low-lying area test result gave absolute values of differences of TCs lower than 0.5 mgal. Low resolutions are sufficient to have a good accuracy; the RMS of differences varies from 0.26 to 1.61 mgal. For 3 arc-seconds, resolution showed a value range of –1.78 to 2.67 mgal. Near the Niger River, the RMS of the differences of the direct effect varies from 0.08 to 0.83 mgal, and the bias is negligible. Values range from 0.39 to 5.70 mgal and –0.58 to 0.01 mgal, respectively, for RMS and mean. Three arc-second resolution showed the smallest difference values in both test areas, whereas the values exceed 10 mgal for lower resolutions in the mountainous area. However, from the result, the influence of GDEM resolutions on the indirect effect is negligible in low-lying regions; the expected improvement is of 10 mm order with respect to lower resolutions in mountainous areas. The accuracy of gravity data of 1 arcsecond, 2 arcseconds, and 3 arcseconds can be very disturbing and the accuracy obtained from the use of the three in a single model is a big challenge to the produced result. The model was also generated from a second order set of equations and terrain model, and the accuracy of the model surface and the frequency of the low and higher wavelength are directly affecting the accuracy and precision of the height derived from the model. Temporal variation in gravity measuring devices also sinks the result into jeopardy.

Similarly, Eteje et al. (2019) applied GPS levelling on gravity anomalies for a gravimetric geoid model in the computation of normal height for geodetic computation in Awka Anambra state, Nigeria. The authors used Somiglinana’s closed formula for normal gravity to derive the International Gravity Formula on the GPS levelling WGS 84 ellipsoid. There are two model forms, and each has its own computational procedures, parameters, and approach. Model A was obtained with equation (7), and model B was obtained with equation (8) as well as using a series of expansion formulas to determine the suitability and agreement of the model forms. The authors’ gravity anomaly was obtained by measuring the difference between point’s gravity reduced to the geoid at the latitude and the normal gravity computed on a specified ellipsoid corrected for free air and the effect of rock. From the result, the normal gravity difference obtained from the two model ranges between 0.000001303 and 0.000001352 degrees of error. This shows that model A is identical to those obtained from model B, which implies that there is no difference between the computation results of the two model forms. Thus, any of the model forms can be applied for accurate normal gravity computation in regions where Clarke's 1880 ellipsoid is adopted as a reference surface for geodetic computation. The authors concluded that, for precise practical local geoid model determination through GPS levelling, the normal gravity of the selected points is computed on the local ellipsoid adopted for geodetic computation in the region/area of study. However, the use of gravimetric data, with its temporal variation problem, poses a threat to the derivation of orthometric height from the ellipsoidal height; the accuracy of the developed model will be improved if necessary gravimetric adjustments were applied.

Model A

$$\gamma_{\text{Clarke1880}(B)} = g_{T\text{Clarke1880}(B)} = 9.780519381 * \left[\frac{(1 + 0.00182202113732435\sin^2\varphi)}{(1 - 0.00068035114546524\sin^2\varphi)^{1/2}}\right] \mathrm{m\,s^{-2}}$$

or

$$\begin{aligned}\gamma_{\text{Clarke1880}(B)} &= g_{T\text{Clarke1880}(B)}\\ &= 9.780519381(1 + 0.00524746\sin^2\varphi\\ &\quad - 0.0000087985\sin^2 2\varphi)\ \mathrm{ms^{-2}}.\end{aligned} \tag{7}$$

Model B

$$\gamma = g_T = \frac{a g_e \cos^2\varphi\cos 1 + b g_p \sin^2\varphi}{(a^2\cos^2\varphi + b^2\sin^2\varphi)^{1/2}}, \tag{8}$$

where $\gamma = g_T$ = is the point theoretical gravity, g_e is the theoretical gravity at the equator, g_p is the theoretical gravity at the pole, φ is the observed point latitude, a is the semi-major axis of specified ellipsoid, and b is the semi-minor axis of specified ellipsoid.

Also, Nestoras et al. (2019) showed the development of a geoid model for calculating orthometric height in Greece, consisting of over 2,000 islands of the Valcanian peninsula using GPS levelling. There is gravity database with 8,998 absolute gravity values in the whole Greek territory. These values had not been accurately surveyed on the ground, out of which 693 absolute gravity values were at triangulation points in central Greece. However, only 349 absolute gravity measurements at triangulation points were observed using mostly relative gravimeter SCINTREX CG5 with theoretical accuracy better than 0.2 mgal. The authors adopted the WGS84 coordinate system for compatibility with all system gravity and appropriate transformation from the epoch of the measurement. Digital terrain model and seafloor topography were used to derive the Hellenic Military Geographical Service (HMGS) originated from combined photogrammetric methods and height measurements. GGMs used are EGM2008, EIGEN-6C4, and GECO on all points for geoid undulations and free air gravity anomalies. The corrected geographic latitude is reduced to the ellipsoid of GRS 1980, while WGS84 adopted as the gravity datum so that it retained consistency with the relation. The Bouguer plate reductions were calculated, neglecting the effect of the Earth's curvature due to the limited area of interest. The TC calculated for onshore measurements was based on a process defined using correction from the 5 m grid of HMGS, and the elevation of the station was at a rectangular 10 km × 10 km around each point. The authors' calculation was based on the following formula:

$$t_c(X_pY_p) = \frac{\frac{1}{2} \times G\rho\iint s[h(X,Y) - h(X_p, Y_p)]^2}{(I^3(X_p - X, Y_p - Y))\frac{\mathrm{d}y}{\mathrm{d}x}}, \tag{9}$$

where t_c is the TC at the point$P(X_p, Y_p)$ with planar coordinates, $G = 6.6742 \times 10^{-11}\ \mathrm{m}^3\ \mathrm{kg}^{-1}\ \mathrm{s}^{-2}$ is the worldwide constant, $\rho = 2{,}670\ \mathrm{kg\ m}^{-3}$ or $\rho = 1{,}027\ \mathrm{kg\ m}^{-3}$ is the mean density of land and sea, respectively, and I is the distance from any point (X, Y) to point P, S the surface integral (10 km × 10 km, 100 km × 100 km, etc.) from station P.

The geoid undulation comparison from the datasets and the predicted values of the geoid models shows that ortho-biased has a mean of 0.008 m with a standard deviation of 0.048, while ortho-free has a mean of 0.203 m with a standard deviation of 0.668. Also, from the result, the GPS levelling geoid model gave a mean of 0.006 m with a standard deviation of 0.048, while ortho free has a mean of 0.020 m with a standard deviation of 0.242. However, the use of gravimetric data, with its temporal variation problem, poses a threat to the derivation of orthometric height from the ellipsoidal so also to the accuracy of the developed model, to this, if a better result is required, gravimetric refinement is necessary. This will provide reliable gravity data for better model production and increase the accuracy of orthometric height determination.

3 Conclusion

The transformation GPS derived ellipsoidal height into local orthometric height can be achieved through either of the methods discussed. However, the accuracy was limited to region and area of application, not on a global scale. Data from GPS levelling, satellite gravity (terrestrial gravity and airborne gravity), and the EGM model are very effective in achieving local orthometric heights. The use of OPUS, RTK-lib, BGI gravity data, gravimeter, and other non-refined software in the processing of GPS observational data in most of the studies limits the accuracy and the reliability of the derived results. Free software does not always provide the best result. Numerical solutions in both lower and higher ordered equations, procedure algorithms, and good statistical analysis will ensure better transformation of local orthometric heights. If higher modelling parameters are considered, the system will provide room for larger coefficients and parameterisation that accommodate a wide aspect of the modelling dynamism between meshing variables of the equation for local orthometric height determination.

4 Recommendation

The research recommends the use of both lower and higher-ordered equation numerical solutions and better ellipsoidal post-processing software in local orthometric height determination. Ensuring fairly uniform distributed reference points over the study area is also recommended.

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