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On the geoid and orthometric height vs. quasigeoid and normal height

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Abstract: The geoid, but not the quasigeoid, is an equipotential surface in the Earth's gravity field that can serve both as a geodetic datum and a reference surface in geophysics. It is also a natural zero-level surface, as it agrees with the undisturbed mean sea level. Orthometric heights are physical heights above the geoid, while normal heights are geometric heights (of the telluroid) above the reference ellipsoid. Normal heights and the quasigeoid can be determined without any information on the Earth's topographic density distribution, which is not the case for orthometric heights and geoid.

We show from various derivations that the difference between the geoid and the quasigeoid heights, being of the order of 5 m, can be expressed by the simple Bouguer gravity anomaly as the only term that includes the topographic density distribution. This implies that recent formulas, including the refined Bouguer anomaly and a difference between topographic gravity potentials, do not necessarily improve the result.

Intuitively one may assume that the quasigeoid, closely related with the Earth's surface, is rougher than the geoid. For numerical studies the topography is usually divided into blocks of mean elevations, excluding the problem with a non-star shaped Earth. In this case the smoothness of both types of geoid models are affected by the slope of the terrain, which shows that even at high resolutions with ultra-small blocks the geoid model is likely as rough as the quasigeoid model.

In case of the real Earth there are areas where the quasigeoid, but not the geoid, is ambiguous, and this problem increases with the numerical resolution of the requested solution. These ambiguities affect also normal and orthometric heights. However, this problem can be solved by using the mean quasigeoid model defined by using average topographic heights at any requested resolution. An exact solution of the ambiguity for the normal height/quasigeoid can be provided by GNSS-levelling.

Keywords: ambiguous quasigeoid, geoid, geoid-quasigeoid difference, resolution, vertical datum, quasigeoid

1 Introduction

The geoid is an important equipotential surface and vertical reference surface in geodesy and geophysics. The quasigeoid, introduced by M.S. Moldensky (Molodensky et al. 1962) is not an equipotential surface, and it has no special meaning in geophysics. The geoid serves as the ideal reference surface for height systems in all countries that adopt orthometric heigths, while the rest of the world uses the quasigeoid with normal height systems (or normal-orthometric heights with more or less unknown zero-levels). The great advantage of the quasigeoid to the geoid is that it can be determined without knowledge of the topographic density distribution. Foroughi et al. (2017), in a comparison of geoid and quasigeoid heights vs. GPS-levelling geoid/and quasigeoid heights used the test area/data in Auvergne, France (Duquenne 2007), to demonstrate that the uncertainty in topographic density is practically harmless in geoid estimation. However, as shown by Sjöberg (2018) GPS/levelling geoid heights cannot be used to validate the density model used in a gravimetric geoid model.

As the quasigeoid is closely related with the Earth's geometric surface, Vanicek et al. (2012) raised the question whether it can be practically determined according to Molodensky's proposed method by successive approximations. Sjöberg (2013) agreed that this computational technique will hardly be successful in a detailed modelling of the quasigeoid, but he instead suggested employing the extended Stokes' formula, which is closely related with the original Stokes' formula, the basis in most geoid determinations (e.g., Ellmann and Vanicek 2007). Nevertheless, as the quasigeoid is related with the Earth's irregular sur-

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face, one may still believe that it is much more irregular than the geoid. This problem will be analysed by comparing formulas for geoid and quasigeoid determination. Here the KTH-method, also named Least Squares Modification of Stokes formula with Additive corrections (LSMSA; e.g. Sjöberg 2003a, b; Sjöberg and Bagherbandi 2017, Chap. 6) will be used, as it in contrast to other methods (e.g., the UNB technique; Ellmann and Vanicek 2007) explicitly provides the corrections needed for topographic height and density. To simplify the discussion, the density is assumed to be constant, and the geoid and quasigeoid are determined only from surface gravity data by Stokes-types formulas. The minor effects of the Earth's atmosphere and ellipsoidal shape (being almost the same for both types of geoid modellings) are disregarded. The LSMSA technique is based on analytical continuation of the surface gravity anomaly to sea-level and surface level in geoid and quasigeoid determinations, respectively (see the references in Sect. 2).

2 Determination of the geoid

Using the KTH-method the geoid height is given by (cf. Sjöberg and Bagherbandi 2017, Sect. 6.2.2)

$$N = (N) + dN, \tag{1a}$$

where

$$(N) = \frac{T_0^\star + T_1^\star}{\gamma_0} + \frac{R}{4\pi\gamma_0}\Delta g d\sigma \tag{1b}$$

and

$$dN = dN_{dwc} + dN_{comb}^T \tag{1c}$$

are the additive corrections for the downward continuation (dwc) of the surface gravity anomaly (Δg) and the combined direct and indirect topographic effects. Moreover, R is the Mean Earth Radius, γ_0 is normal gravity at the reference ellipsoid, $S(\psi)$ is Stokes' kernel function with argument ψ being the geocentric distance between computation and integration points and σ is the unit sphere. $T_0^\star$ and $T_1^\star$ are the analytically continued zeroth- and first-degree disturbing potential harmonics.

The dwc effect becomes (Sjöberg 2003a, b and Sjöberg and Bagherbandi 2017, Sect.5.3.1)

$$\begin{aligned} dN_{dwc} &= \frac{R}{4\pi\gamma_0}\iint_\sigma S(\psi)\left[\Delta g_g^\star(Q) - \Delta g(Q)\right] d\sigma_Q \\ &= \frac{R}{4\pi\gamma_0}\iint_\sigma S(\psi)\left[\Delta g_g^\star(Q) - \Delta g^\star(r_P, Q)\right] d\sigma_Q \\ &+ \frac{R}{4\pi\gamma_0}\iint_\sigma S(\psi)\left[\Delta g^\star(r_P, Q) - \Delta g(Q)\right] d\sigma_Q, \end{aligned} \tag{2a}$$

where superscript * denotes harmonic analytical continuation (along the vertical) to geoid level in $\Delta g_g^\star$ and to computation point level of radius r_P in $\Delta g^\star(r_P, Q)$. After a few manipulations, including a Taylor expansion to first order in the last term, this formula can be approximated to

$$\begin{aligned} dN_{dwc} \approx &\frac{H_P \Delta g_P}{\gamma_0} + 3\zeta_P\frac{H_P}{R} - \frac{H_P^2}{2\gamma_0}\left(\frac{\partial \Delta g}{\partial H}\right)_P \\ &+ \frac{R}{4\pi\gamma_0}(H_P - H_Q)\left(\frac{\partial \Delta g}{\partial H}\right)_Q d\sigma_Q. \end{aligned} \tag{2b}$$

where H is the orthometric height.

The combined topographic effect is the same as the negative of the topographic bias (Sjöberg 2007 and 2009a, b; Sjöberg and Bagherbandi 2017, Sects. 5.2.3-5.2.4):

$$dN_{comb}^T = -\frac{2\pi G\rho}{\gamma_0}\left(H_P^2 + \frac{2H_P^3}{3R}\right) = -\frac{bias(T_g^\star)}{\gamma_0} \tag{3}$$

for the constant topographic density ρ and gravitational constant G. (For an arbitrary topographic density distribution, see Sjöberg 2007 and 2009b.)

3 Determination of the quasigeoid height

3.1 Gravimetric approach

The quasigeoid height (or height anomaly) at surface point P is given by (Ågren 2004, Sect. 9.5.1; Sjöberg and Bagherbandi 2017, Sect. 6.3.2)

$$\begin{aligned} \zeta_P &= \frac{T_P}{\gamma} = \frac{(T_0 + T_1)_P}{\gamma} + \frac{r_P}{4\pi\gamma}\Delta g^\star(r_P, Q) d\sigma_Q \\ &= \frac{R}{4\pi\gamma}\Delta g^\star(r_P, Q) d\sigma_Q + \frac{H_P}{r_P}\zeta_P \approx \frac{R}{4\pi\gamma_0}\Delta g^\star(r_P, Q) d\sigma_Q \\ &+ 3\frac{H_P}{r_P}\zeta_P, \end{aligned} \tag{4}$$

where γ is normal gravity at the telluroid, related to γ_0 by

$$\gamma \approx \gamma_0(1 - 2H/R). \tag{5}$$

The last step in Eq. (4) can also be decomposed into

$$\zeta_P \approx (\zeta_P) + d\zeta_P, \tag{6a}$$

where

$$(\zeta_P) = \frac{(T_0 + T_1)_P}{\gamma} + \frac{R}{4\pi\gamma}\Delta g d\sigma_Q, \tag{6b}$$

and

$$d\zeta_P = \frac{R}{4\pi\gamma_0}\left(\Delta g^{\star}(r_P, Q) - \Delta g(Q)\right) d\sigma_Q + 3\frac{H_P}{R}\zeta_P, \quad (6c)$$

or, after using a Taylor expansion of $\Delta g^{\star}$ to first order:

$$d\zeta_P \approx \frac{R}{4\pi\gamma_0}(H_P - H_Q)\left(\frac{\partial \Delta g}{\partial H}\right)_Q d\sigma_Q + 3\frac{\zeta_P}{R}H_P. \quad (6d)$$

3.2 GNSS-levelling approach

The normal height can be defined by the formula (Heiskanen and Moritz 1967, Sect. 4-5)

$$H^N = \frac{C}{\bar{\gamma}} \quad (7)$$

where C is the geopotential number (determined by precise levelling), and $\bar{\gamma}$ is the mean value of normal gravity (γ) between the reference ellipsoid and the telluroid. As γ decreases smoothly with height, H^N can easily be iterated from an approximate value (e.g., Sjöberg and Bagherbandi 2017, Sect. 3.5.3).

Finally, the quasigeoid height follows from the geodetic height (h) determined by GNSS:

$$\zeta = h - H^N \quad (8)$$

This shows that both normal height and height anomaly/quasigeoid can be determined from GNSS and levelling (alone).

Hence, the beauty of M. S. Molodensky's introduction of normal height and quasigeoid is that these components can be determined without any information about the Earth's density distribution.

4 The difference N – ζ

Taking the difference between Eqs. (1a)-(1c), (2b) and (3) on one hand and Eqs. (6a), (6b) and (6d) on the other and omitting some minor terms, one arrives at "the geoid-from-quasigeoid correction" (GQC):

$$GQC = N - \zeta \approx \frac{\Delta g_B}{\gamma_0}H_P - \frac{H_P^2}{2\gamma_0}\left(\frac{\partial \Delta g}{\partial H}\right)_P, \quad (9a)$$

where we have introduced the simple Bouguer gravity anomaly by

$$\Delta g_B = \Delta g - 2\pi G\rho H \quad (9b)$$

and omitted some minor terms.

Equation (9a) agrees with Sjöberg (1995), where the result was derived in two different ways, namely by external and internal harmonic expansions of the topographic potential and by using Stokes' original and extended formulas with Helmert gravity anomalies for the geoid and quasigeoid heights, respectively.

The first term in Eq. (9a) is the traditional representation for the GQC (Heiskanen and Moritz 1967, pp. 327-328), and the complete formula also agrees with the major terms of the refined formula of Sjöberg and Bagherbandi (2017, Eq. 7.31b), derived by employing analytical continuation of the external disturbing potential.

4.1 Other solutions to the GQC

Flury and Rummel (2009) suggested replacing the simple Bouguer anomaly by the refined one $\left(\Delta g^{BO}\right)$ as the basic contribution to the GQC, and they also refined the solution by the difference $\left(V_g^T - V_P^T\right)/\bar{\gamma}$, $\bar{\gamma}$ being the mean value of normal gravity at the reference ellipsoid and telluroid, between topographic potentials at the surface point P and the geoid along the plumb-line through P, yielding the expression:

$$GQC \approx \frac{\Delta g^{BO}}{\bar{\gamma}} + \frac{V_g^T - V_P^T}{\bar{\gamma}}. \quad (10)$$

Sjöberg (2010) showed that a minor term is missing in this equation. See also Sjöberg (2012).

The practical problem with Eq. (10) is to estimate the topographic potential difference. We will return to this issue later.

Another approach is as follows:

Proposition 1: For a constant topographic density ρ at the computation point

$$GQC = \frac{\Delta g_B}{\gamma_0}H_P - \frac{1}{\gamma_0}\sum_{k=1}^{\infty}\frac{(-H_P)^{k+1}}{(k+1)!}\left(\frac{\partial^k \delta g}{\partial H^k}\right)_P + \frac{4\pi G\rho}{3\gamma_0}\frac{H_P^3}{R} \quad (11)$$

Proof. The GQC can be found by Stokes' original and extended functions $S(\psi)$ and $S(r_P, \psi)$ as

$$GQC = \frac{T_g}{\gamma_0} - \frac{T_P}{\gamma} = \frac{R}{4\pi\gamma_0}\iint_\sigma [S(\psi) - S(r_P, \psi)]\Delta g_g^{\star} d\sigma - \frac{bias(T_g^{\star})}{\gamma_0} + \frac{T_P}{\gamma}\frac{\gamma - \gamma_0}{\gamma_0}, \quad (12)$$

where the bias term, given in Eq. (3), accounts for the bias in analytical continuation of the disturbing potential to the geoid. As Stokes' function can be developed into a Taylor series of its extended function:

$$S(\psi) = \sum_{k=0}^{\infty} \frac{(-H)^k}{k!} \frac{\partial^k S(R+H,\psi)}{\partial H^k}, \tag{13}$$

and also by considering

$$\delta g = -\frac{\partial T}{\partial H}, \Delta g = \delta g + \frac{T}{\gamma}\frac{\gamma-\gamma_0}{\gamma_0} \tag{14}$$

as well as Eqs. (7b) and (10), one finally arrives at the proposition, where the last term hardly exceeds one centimetre in the highest mountains. See also Sjöberg (2015a). □

Corollary 1: For a general topographic density distribution $\rho(r)$ along the vertical at the computation point the last term in Eq. (11) should be substituted by

$$-4\pi G \int_R^{R+H_P} \rho(r)\left(\frac{r^2}{R}-r\right)dr + 2\pi G\rho H_P^2.$$

Cf. Sjöberg (2007, Corollary 2) for this solution.

Note that the topographic bias does not include a terrain correction, because the topographic bias is not dependent on the mass distribution of the terrain. Actually, the terrain correction is already accounted for in the analytical continuation. However, for an accurate solution the bias should be corrected for a variable density distribution along the vertical (see Sjöberg 2007 and 2009 a, b).

The *GQC* can also be determined directly by comparing the quasigeoid and geoid heights determined by the external and internal Earth Gravitational Models at topographic and sea levels, respectively. The internal disturbing potential can either be determined by a remove-compute-restore technique or by analytical continuation (Sjöberg 2015b, Foroughi and Tenzer 2017). A numerical comparison can be found in the latter article.

5 Discussions

Formally, in view of that the topographic density distribution is not well known, the problem of determining the geoid is a gravimetric inverse problem. By assuming that the topography and its density distribution are known, the problem can be altered to solving a (free) boundary value problem. This approach is standard in one way or another in physical geodesy. One solution is shown in Eqs. (1a-c), (2a) and (3).

In contrast, the problem of determining the quasigeoid is a forward problem that does not rely on an estimated topographic density distribution model. Hence, if the Earth's surface is known, e.g. expressed by its laterally variable geocentric radius, the height anomaly can be determined by T_P/γ. As can be seen from Eqs. (6a) – (6d), this solution does not need knowledge of the topographic density, leading to a simpler solution vs. that for the geoid.

When comparing the above solutions for N and ζ one can see that both types of models are dependent on the (mean) topographic height variations. For the theoretician (but also for the practically minded surveyor requesting a very high resolution of the surface) it could be of interest to investigate which of the two surfaces is smoothest. Intuitively one would expect that the deflections of the vertical are smoother at the geoid than at the telluroid. On the other hand, all terms of the *GQC* of Eq. (9a) belong to the geoid formula, so the answer to this question is not obvious. To get some further insight to the problem we consider two points (denoted with subscripts 1 and 2) at the Earth's surface separated by a small lateral distance s and heights H_1 and H_2. Then the differential difference between the height anomalies can be deduced from Eq. (4) by

$$T_2(s,\Delta H) = T_1 - \delta g_1 \Delta H + \left.\frac{\partial T}{\partial s}\right|_1 s = T_1 - \delta g_1 \Delta H - \gamma_1\theta_1 s, \tag{15}$$

where $\Delta H = H_2 - H_1$, δg and θ are the gravity disturbance and deflection of the vertical in the direction s. Then Eq. (4) yields

$$\begin{aligned}\Delta\zeta = \zeta_2 - \zeta_1 &= \frac{T_2}{\gamma_2} - \frac{T_1}{\gamma_1} = \frac{T_2 - T_1}{\gamma_1} + \frac{T_2}{\gamma_2}\frac{\gamma_1-\gamma_2}{\gamma_1} \\ &= -\theta_1 s - \frac{\delta g_1 \Delta H}{\gamma_1} - \frac{T_2}{\gamma_2\gamma_1}\left.\frac{\partial\gamma}{\partial H}\right|_1 \Delta H \approx -\theta_1 s - \frac{\Delta g_1 \Delta H}{\gamma_1}.\end{aligned} \tag{16}$$

From Eq. (9a), approximated to first order of the potential, one obtains for the corresponding geoid difference

$$\begin{aligned}\Delta N = N_2 - N_1 &= \zeta_2 - \zeta_1 + \frac{\Delta g_{B2} H_2 - \Delta g_{B1} H_1}{\gamma_0} \\ &= \zeta_2 - \zeta_1 + \frac{\Delta g_2 H_2 - \Delta g_1 H_1}{\gamma_0} - 2\pi G\rho\left(H_2^2 - H_1^2\right)\end{aligned} \tag{17}$$

Considering Eq. (16) one finally obtains

$$\Delta N \approx -\theta_1 s + \frac{(\Delta g_2 - \Delta g_1) H_2}{\gamma_0} - \frac{4\pi G\rho}{\gamma_0}\bar{H}\Delta H, \tag{18}$$

where $\bar{H} = (H_1 + H_2)/2$.

As θ is the deflection of the vertical, it does not change much with the slope of the terrain. Hence, as the resolution of the solution increases (and s decreases), the first term in Eqs. (16) and (18) go towards zero with s. However, the last terms in each of the models are both related with the slope/roughness of the terrain. Obviously, one cannot generally state that one of the surfaces is smoother than the other at high resolution, at least not without further studies.

Foroughi and Tenzer (2017, Fig. 18) calculated the performances of the quasigeoid and geoid in the Himalayas, showing the details along a meridional profile near 88°E from latitude 25° N (at lower Ganges river close to sea level), passing the extreme elevations of more than 8 km around Mt. Everest, to 40 N (with topographic heights of less than 1 km). They concluded that the geoid geometry is modified at least 10 % less than the quasigeoid. However, their conclusion is based on the dominating very long-wavelength variation of the (quasi)geoid surfaces over the profile as part of the Earth's largest geoid low caused by the huge lower mantle density low south of India. In addition, as these geoid and quasigeoid extremes are negative, actually the geoid is more deformed than the quasigeoid. Generally, in accord with Newton's law, thegeoid is more sensitive to density structures below the crust than the quasigeoid (except for the limited regions with negative topographic heights).

If one, on the other hand, studies the short-wavelength variations of the two surfaces, which are related with smoothness, one can actually see from their Fig. 18 that the quasigeoid plot is smoother. However, one should bear in mind that this calculation is a single numerical result that cannot be used for drawing firm conclusion.

As stated earlier, this study is based on a star-shaped Earth model, which does not fully agree with reality as there are exceptional topographic areas with more than one point of intersection between the plumb-line and the surface, making the quasigeoid solution ambiguous. In this case a unique quasigeoid height can still be defined by the mean elevation even if the block size approaches zero. The geoid solution is still unique when replacing the combine topographic effect of Eq. (3) with the corresponding general formula that allows for a variable topographic density as described in Sjöberg (2007).

6 Conclusions

There is no doubt that the geoid but not the quasigeoid can serve both as a geodetic datum and a reference surface in geophysics. The geoid is also a natural surface in the sense that it is an equipotential surface in the Earth's gravity field that agrees with the undisturbed mean sea level.

One advantage of M. S. Molodensky's genius introduction of normal height and quasigeoid is that the former can always be determined from precise levelling, having its zero-level at the reference ellipsoid, and the latter by GNSS-levelling, both without any information about the Earth's density distribution, even if they are ambiguous. In contrast, the geoid datum is related with orthometric heights, which depend on topographic density and need a fixed geoid model as the zero-level, while the zero-level for the normal height is the well-defined surface of the reference ellipsoid.

For a star-shaped earth model we could not state that one of the two surfaces (geoid or quasigeoid) is smoother than the other at high resolution, as not only the quasigeoid but both surfaces vary with the slope of the topography. However, generally the geoid is more correlated with density variations in the Earth's interior than the quasigeoid.

In case of the real non-star shaped Earth the geoid is still unique, while the quasigeoid will be ambiguous in some limited areas. Although this problem might be rather academic and limited to a very high numerical resolution of the quasigeoid model, a unique model (the mean quasigeoid model) can still be defined by using mean topographic heights in ambiguous areas.

We have shown that recent formulas for estimating the *GQC*, using both the refined Bouguer gravity anomaly and a difference between topographic potentials at the geoid and surface, do not lead to improvements, but using the simple Bouguer anomaly (possibly corrected for variations of density along the vertical down to sea level) by the first term in Eq. (17), possibly refined by terms including vertical gradients of increasing order of the free-air gravity anomaly, should be preferred from a practical point of view. More convenient for regional and global studies *GQC* maps can be determined from the disturbing potential difference at the Earth's surface and sea level evaluated by an Earth Gravitational Model and a topographic potential model.

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References

Ågren J., 2004, Regional geoid determination methods for the era of satellite geodesy. PhD thesis in geodesy, Royal Institute of Technology, Stockholm

Duquenne H., 2007, A data set to test geoid computation methods. Proc. 1st Int. Symp. of the Int. Gravity Field Services, Istanbul, Harita Dergisi, Special Issue 18: 61-65

Ellmann A., Vanicek P., 2007, UNB application of Stokes's-Helmert's approach to geoid computation. J Geodyn 43:200-213

Heiskanen W. A., Moritz H., 1967, Physical Geodesy, W H Freeman and Co., San Francisco and London

Flury J., Rummel R., 2009, On the geoid-quasigeoid separation in mountainous areas. J Geod 83: 829-847.

Foroughi I., Tenzer R., 2017, Comparison of different methods for estimating the geoid-to-quasigeoid separation. Geophys J Int 2010: 1001-1020

Foroughi I., Vanicek P., Sheng M., Kingdon R. W., Santos M. C., 2017, In defence of the classical height system. Geophys J Int 211(2): 1154-61.

Molodensky M. S., Eremeev V. F., Yurkina M. I., 1962, Methods for study of the external gravitational field and figure of the earth, Transl. From Russian (1960), Israel program for Scientific Translations, Jerusalem, Israel

Sjöberg L.E., 1995, On the quasigeoid to geoid separation, Manuscr Geod 20: 182-192

Sjöberg L. E., 2003a, A computational scheme to model the geoid by the modified Stokes's formula without gravity reductions, J. Geod. 77: 423-432

Sjöberg L. E., 2003b, A general model of modifying Stokes' formula and its least-squares solution, J. Geod. 77(2003): 459-464

Sjöberg L. E., 2007, The topographic bias by analytical continuation in physical geodesy. J Geod 81: 345-350

Sjöberg L. E., 2009a, The terrain correction in gravimetric geoid determination - is it needed? Geophys J Int 176:14-18

Sjöberg L. E., 2009b, Solving the topographic bias as an Initial Value Problem.Art. Sat. 44(3): 77-84

Sjöberg L. E., 2010, A strict formula for geoid-to-quasigeoid separation. J Geod (2010) 84: 699-702

Sjöberg L. E. 2012, The geoid-to-quasigeoid difference using an arbitrary gravity reduction model. Stud Geophys Geod 56(2012): 929-933

Sjöberg L E, 2013. The geoid or quasigeoid- which reference surface should be preferred for a national height system? J Geod Sci 3: 103-109

Sjöberg, L. E., 2015a, Rigorous geoid-from-quasigeoid corrections using gravity disturbances. J Geod Sci 5:115-118

Sjöberg L. E., 2015b, The topographic bias in Stokes' formula vs. the error of analytical continuation by an Earth Gravitational Model- are they the same? J Geod Sci 5:171-179

Sjöberg L. E., 2018, On the topographic bias and density distribution in modelling the geoid and orthometric heights, J Geod Sci 8: 30-33

Sjöberg L. E., Bagherbandi M., 2017, Gravity Inversion and Integration- Theory and Applications in Geodesy and Geophysics, Springer Int. Publ. AG, Cham, Switzerland

Vanicek P., Kingdon R., Santos M., 2012, Geoid versus quasigeoid: a case of physics versus geometry, Contr. Geophys. & Geod. 42(1): 101-117