J. Geod. Sci. 2018; 8:72–82 DE GRUYTER

Research Article Open Access

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Evaluation of recent combined global geopotential models in Brazil

https://doi.org/10.1515/jogs-2018-0008
Received January 22, 2017; accepted April 16, 2018

Abstract: The aim of this paper is to present a quantitative analysis of the adequacy of the main currently existing combined Global Geopotential Models (GGMs) for modeling normal-geoid heights throughout Brazil. As major advances have been reached since mid-2016 in the combined GGMs elaboration and development, the main objective of this analysis is to verify if, in fact, the most recent models present superior or equivalent performance to the most performant previous models. The analysis was based on comparisons between normal-geoid height values obtained from GNSS/leveling solutions and values calculated from GGMs XGM2016, GOCO05C, EIGEN-6C4 and EGM2008, according to different geopotential functionals – geoid height and height anomaly – and in different degrees of development, always through the relative method. This procedure was applied to 997 stations which carry information of both ellipsoidal and normal-orthometric heights, located all over Brazil. As a main result, it was observed the superior performance of the recent combined GGMs, GOCO05C and XGM2016, when compared to the older models, EIGEN-6C4 and EGM2008, when all of them are developed up to degree 720, the maximum degree of the recent models; and a approximate equality of results when all of the models are used in their individual maximum degrees.

Keywords: Brazil, Combined Global Geopotential Models, Geopotential functionals, Normal-geoid height modeling

1 Introduction

The terrestrial gravitational field modeling has always been one of the main objectives of Geodesy. Since the 17th century, with the first Normal Earth model, scientists in this field have been searching ways to understand, model and represent gravity itself and other quantities which arise or depend on it.

Furthermore, according to a general point of view and concerning important milestones in the evolution of these studies, the differentiated manner in which scientists have come to observe and analyze data from the aforementioned gravity field over the last few years must be mentioned. In particular, the popularization of space techniques has revolutionized the activities that need positioning due to their speed and precision in obtaining coordinates. This fact prompted interest and need for suitable and reliable global models to, for example, provide the determination of a more precise and accurate gravity field equipotential surface for mapping and engineering applications. In addition, new techniques for obtaining gravity field information from satellite missions were developed, popularizing the knowledge and use of Global Geopotential Models (GGMs).

To sum up, GGMs consist of a set of numeric values for some parameters, the error statistics associated and a collection of mathematical expressions, numeric values and particular algorithms, as well as the providential application of these data when developing the geopotential in spherical harmonics, as shown in Eq. 1 [1].

$$W(r,\varphi,\lambda)=\frac{GM}{r}\left[1+\sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{a}{r}\right)^{n}(C_{nm}cos(m\lambda)+ S_{nm}sen(m\lambda))P_{nm}(sen\varphi)\right]+\frac{1}{3}\omega^{2}r^{2}\left[1-P_{20}(sen\varphi)\right] \quad (1)$$

In such equation, a is the major semi-axis of the level ellipsoid associated to the model, ω is the rotation speed of the model, C_{nm} and S_{nm} are the series development Stokes coefficients, GM is the geocentric gravitational constant associated to the model, (r,φ,λ) are the geocentric coordinates, as follows: r is the distance of the calculation point to the centre of the adopted model, φ is the calculation point geodetic latitude and λ is the calculation point geodetic longitude, and P_{nm} represent the associated Legendre functions with n degree and m order.

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Still according to the up-mentioned reference, a GGM must be able to support such calculation in any arbitrary points, located over or above the terrestrial surface, characterizing though it globality. These aspects, joined to internal information consistence and coherent physical modeling, provide accuracy and reliability of the results.

However, hereupon, different GGMs have been developed through the last five decades, each one with data sources and sets coherent to the purpose for which it is intended. Nowadays, due to the complexity for obtaining and maintaining these models, as well as the need to control and standardize all related variables, their management is performed globally by ICGEM - International Centre for Global Earth Models. Its database sums 163 static models, from which we highlight in this paper the combined models, which gather gravity information upcoming from satellite orbit observations, terrestrial gravimetry and altimetry (both digital elevation model data, for land and coastal regions, and satellite altimetry, for oceans) in order to solve the geopotential modeling [2].

Thus, the present paper intends to evaluate the adequacy of the main currently existing combined GGMs - i.e., GGMs XGM2016, GOCO05C, EIGEN-6C4 and EGM2008, according to different geopotential functionals and in different degrees of development. This evaluation is based on comparisons of their solutions with those obtained from GNSS and levelling techniques throughout the study area according to a relative approach.

2 Methods

2.1 Terrestrial Data

In order to perform the quantitative evaluation proposed, terrestrial data were selected according to the information needed to perform comparisons with GNSS/levelling solutions, which are ellipsoidal heights (h) and normal-orthometric heights (H^{NOrt}). The first ones can be related to previously observed points with GPS/GNSS techniques and concern to the Brazilian SAT-GPS Network, and the last ones are the closest approach to heights with physical meaning, produced by applying gravity field equipotential surfaces non-parallelism corrections δH^{NOrt} to leveled heights [3] and concern to the benchmarks of Brazilian Vertical Reference Network (BVRN). Both of these networks are provided and maintained by the Brazilian Institute of Geography and Statistics (IBGE), and are freely accessible to the community.

So, benchmarks from BVRN with connection to the SAT-GPS Network were selected, since they gather the needed information, as explained on section 3.1 of the present paper. A number of 997 stations distributed all along Brazil were selected, as presented in Fig. 1, being henceforth named GNSS/BM stations.

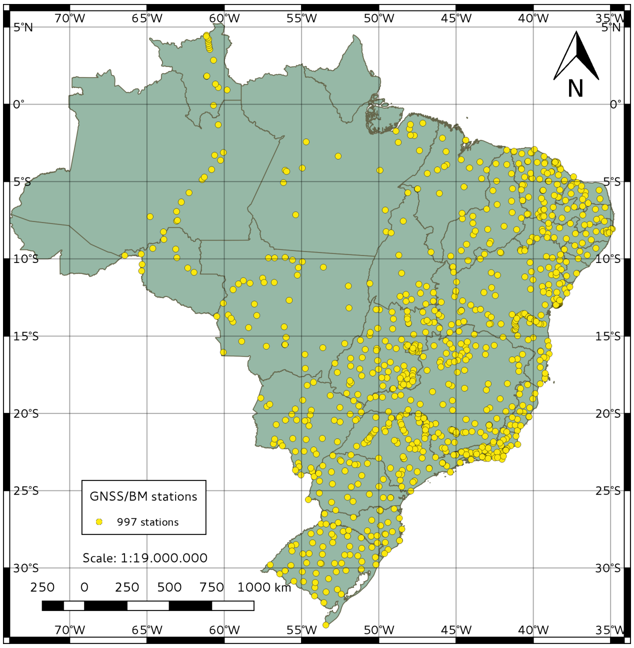

Figure 1: 997 GNSS/BM stations used along Brazil.

Observing Fig. 1, it is remarkable a non-uniform distribution of the stations through the surface. According to the aim of this paper, it was considered that such characteristic is acceptable for a primary evaluation of the GGMs. However, in order to perform improved studies, it would be necessary to perform selection and prioritization of these stations, under coherence with GGMs' spatial resolution and, consequently, their degree of development.

Another important observation concerns the accuracy of interest quantities, which are prompted in Fig. 2 and 3. The intended comparison is only as good as the entry data applied to the formulation shown in item 3.2 of the present paper. Furthermore, with convenient analysis, one can notice that there are two relevant issues concerning the accuracy of both the quantities handled: the first one is that, as stated in Fig. 2, there is a total of 77 (seventy seven) GNSS/BM stations with ellipsoidal heights precision over 6 cm - 8 (eight) of them are even superior to 19 cm; these precisions are under current needs and requirements for geodetic applications, but they are used and considered as provided by the responsible organization.

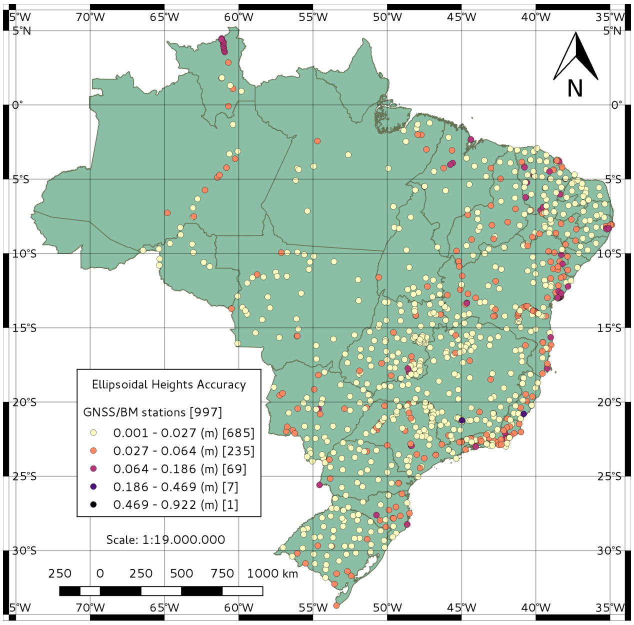

Figure 2: Precision of ellipsoidal heights in the 997 GNSS/BM stations.

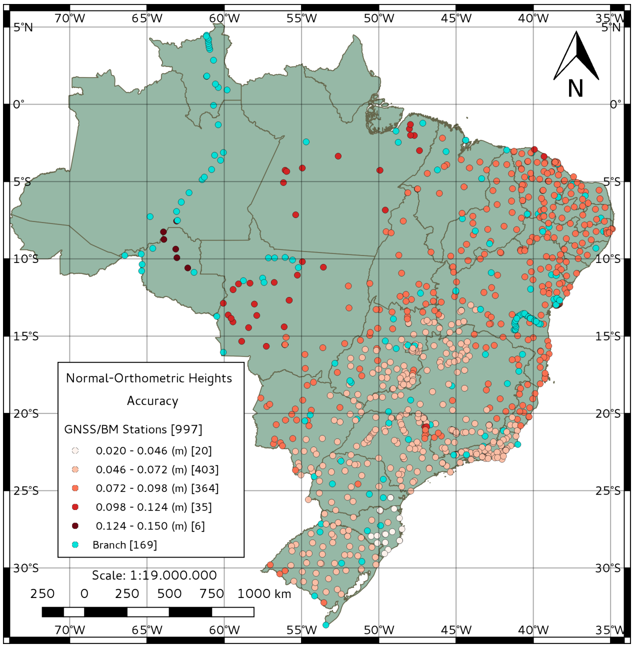

Figure 3: Precision of normal-orthometric heights in the 997 GNSS/BM stations.

The second issue, as stated in Fig. 3, is related to the normal-orthometric heights precisions: there are, in such figure, a total of 169 (one hundred sixty-nine) stations which belong to *"branches"*, which are segments of BVRN that were not adjusted and which heights are provided only from coordinate transportation operations; these stations are taken in consideration due to their large number, specially in the north portion of the study area and due to previous studies results which show that there is no correlation between branch stations and misleading altimetric determinations [4].

2.2 GGMs

As stated in the Introduction, the usage of combined GGMs was adopted, since they gather information from different sources in order to solve the geopotential modeling. In this context, the first model based not only in the solutions of estimates of a gravity anomaly set which came from satellite orbit observations, but also using gravimetric, topographic and satellite altimetry information was EGM2008 [5].

However, since its deployment, different techniques and processing approaches have been developed and applied in benefit of a better comprehension and information extraction of the gravity field from suited sources. For brevity issues, the present paper omits concept analysis on combined GGMs evolution through last decades, which can be consulted in references [5-8].

From the analysis presented by these authors, the models EGM2008, EIGEN-6C4, GOCO05C and XGM2016 were selected for the present evaluation. The first two ones are considered well established combined GGMs, as well as their positive results for modeling geopotential functionals - such as geoid heights, height anomalies, and others - are already wide spread in the scientific community. The last two ones are modern combined GGMs, which have been provided since mid-2016 and integrate gravimetric data from different sources, by weighting the best individual solutions and by applying them regionally, among other major advances. Thus, they present themselves as milestones in the history of GGMs development [7,8].

The acquirement of GGMs extracts was performed pointwisely, in each of the 997 GNSS/BM stations, in order to preserve characteristics and precision of each model. This procedure was applied in order to avoid further precision losses with the application any interpolation methods [9]. For such task, we used the standalone application SPGG v2.0 [10], which provide point-wise extracts of GGMs according to defining characteristics inserted by the user, and through direct interaction with ICGEM web service. A visualization of its interface is prompted in Fig. 4.

Table 1 presents a list of models and development degrees of the extracts employed in this paper, obtained with the up-mentioned application. The geopotential functionals used are discussed in the following item. The specific choice for the development degrees used (2190, 720/719,

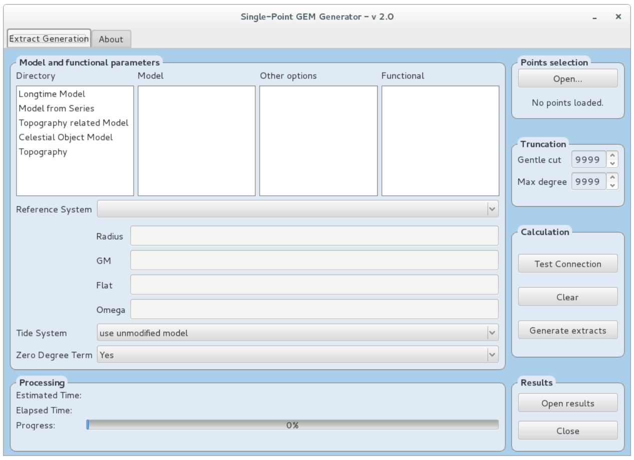

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Figure 4: Standalone application SPGG v2.0.

360), as well as the theoretical equality for comparison reasons between extracts developed up to degrees 719 and 720, is based on successful previour work developed in similar regions [4, 13].

Table 1: GGM extracts obtained for the research

Model	Development Degrees			Functionals
XGM2016	-	719	360	Geoid (N) and Height anomaly (ζ)
GOCO05C	-	720	360	
EIGEN-6C4	2190	720	360	
EGM2008	2190	720	360	

2.3 Comparison criteria

2.3.1 Normal-geoid heights modeling

Normal-geoid heights modeling through GNSS/levelling solutions as presented in this paper considers the fact that, in Brazil, the heights provided by BVRN are, as already discussed, normal-orthometric. That means they are produced by applying gravity field equipotential surfaces non-paralelism corrections δH^{NOrt} to leveled heights [3].

In this sense, reference [11] discuss the fact that, since normal-orthometric heights behavior do not present a complete physical meaning, they are not referred or related to a classical reference surface, such as the geoid or the quasi-geoid. Thus, a regular mathematical equation that relates ellipsoidal heights (h) and orthometric heights (H^{Ort}), for example, by means of geoid heights N ($h = H^{Ort} + N$) may not be freely used as if BVRN benchmarks heights were, in fact, orthometric heights.

Consequently, the up-mentioned reference describes, still, the dependency of ellipsoidal heights (h) and normal-orthometric heights (H^{NOrt}) to the models of Eq. 2, in which η is by them named normal-geoid heights, as an analogy to normal-orthometric heights and the naming used in this paper.

$$\eta \cong h - H^{NOrt} \tag{2}$$

Since normal-geoid heights are not geopotential functionals, they may, still, be better modelled by geoid heights N themselves or by height anomalies ζ, depending on the study area. Reference [12], for instance, verified that, once taken different approximations to the separation between geoid and quasi-geoid, normal-geoid heights are slightly better modelled by height anomalies than by geoid heights in a study conducted in Brazilian southern region. Furthermore, Nicacio and Dalazoana (presented at the SIRGAS 2017 Symposium, Mendoza, AR, 27–30 November 2017) indicated that there is not a standard behavior of this variable in dependency of the functionals considering the whole Brazilian territory.

However, in order to mitigate aditive errors inherent to the processing approach and to the obtaintion method of the GGMs, references [13, 14] propose the use of the relative method instead of the absolute one described in Eq. 2, which is based, in the context of this work, on the use of an origin point P_0 as a reference, with known normal-orthometric height H_0, ellipsoidal height h_0 and normal-geoid height η_0 and through the mathematical approach presented in Eq. 3. The described formulation has already proved being more efficient when handling with GGMs in previous studies [4, 13].

$$\begin{cases} H_0^{NOrt} = h_0 - {\eta_0}^{model} = h_0 - {\eta_0}^{real} + \varepsilon \\ H_P^{NOrt} = h_P - {\eta_P}^{model} = h_P - {\eta_P}^{real} + \varepsilon \end{cases} \Rightarrow$$
$$\Rightarrow \eta_P = H_0^{NOrt} - H_P^{NOrt} - h_0 + h_P + \eta_0 \tag{3}$$

As reinforced by reference [15], this is the most advisable alternative these days, specially under the aspect of vertical reference systems consecution, in the form of Eq. 4. Accordingly to this reference, modern geodetic techniques, mainly those based on satellite positioning and navigation, present higher accuracy requirements than those provided by traditional absolute approach described in Eq. 2. Therefore, the use of the relative method to correlate ellipsoidal and normal-orthometric heights with normal-geoid height was adopted in the present paper in the form:

$$\eta_P = H_0^{NOrt} - H_P^{NOrt} - h_0 + h_P + \eta_0 \Rightarrow$$
$$\Rightarrow \Delta\eta = \Delta h - \Delta H^{NOrt} \quad (4)$$

Still, since the employed data are acquired according to different permanent tide systems, it was necessary to perform the their compatibilization in such aspect for the purpose of manipulating and integrating them [16-18]. In this way, all variables were compatibilized to the zero-tide system for calculations and to mean-tide system for results presentation using mathematical formulation constant of the last reference and rewritten in Eq. 5. In such equation, $k = 0,3$ e $h = 0,6$ are the conventional Love numbers related to tide, and ϕ is the geocentric latitude of the calculation point.

$$h_{mean-tide} = h_{tide-free} - \left\{(1+k-h)\left[-0.198\times \left(\frac{3}{2}sen^2\phi - \frac{1}{2}\right)\right]\right\} \quad (5)$$

2.3.2 Evaluation criteria

The criteria employed to choose which model best suits for modelling the normal-geoid height, in a given calculation point P, took into consideration the methodology presented in Eq. 6. In such equation, it must occur the minimization of a factor named θ_P, which is equal to the module of the difference between the reference normal-geoid height module η_P^{ref}, obtained by means of Eq. 3, and the calculated normal-geoid height module η_P^{calc}, calculated directly from the GGMs.

$$\left\{\begin{array}{l} [\eta_P]_{EGM2008} \\ [\eta_P]_{EIGEN-6C4} \\ [\eta_P]_{GOCO05C} \\ [\eta_P]_{XGM2016} \end{array}\right. \Rightarrow \quad min||\eta_P{}^{ref}| \; - \; |\eta_P{}^{calc}|| \quad (6)$$

With respect to the standard adopted for point P_0, its choice was determined according to the one that for the whole set of points was able to minimize the factor θ_m, as regulated by Eq. 7.

$$P_0 \Rightarrow min(\theta_m) \quad (7)$$

$$\text{where } \theta_m = \frac{1}{n}\sum_{i=1}^{n}\theta_i$$

The modeling and analysis in different degrees of development and according to different geopotential funcionals to represent normal-geoid heights was carried out similarly to Eq. 6. This is indirectly represented in the applicable results, and its details are omitted from the present study.

3 Results and discussions

3.1 National wide results

In a National sense, Table 2 and Fig. 5 present the differences statistics between GNSS/levelling and GGMs solutions. In Fig. 5, inserted here to enable better visual comprehension of Table 2, continuous lines represent normal-geoid heights modelled by height anomalies, while the broken lines represent their modeling by geoid heights.

Still in Fig. 5, yellow colored lines are related to EGM2008 results, blue colored ones are related to EIGEN-6C4, green colored ones are related to GOCO05C results, and orange ones are related to XGM2016. The red dotted line at the bottom of the figure represent the GGM configuration which provided minimum mean difference for the whole set of points, named the optimal configuration - GGM XGM2016, degree 719, functional height anomaly. For this configuration, mean discrepancy value is $\theta_m = 0.1461\,m$

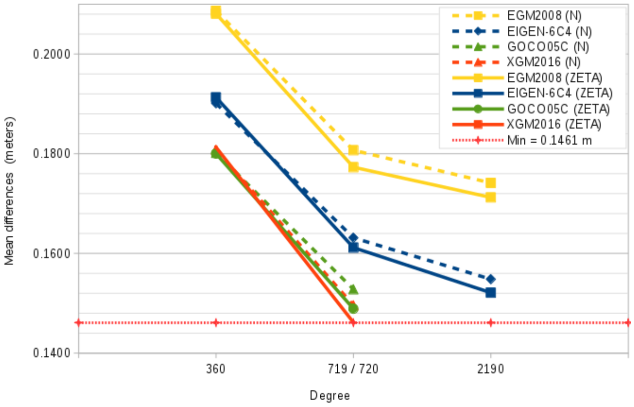


Figure 5: National mean differences between GNSS/levelling and GGMs solutions.

Particularly in Table 2, there are some remarkable values which must be deepened. For example, one should notice that all the minimum differences are equal to $0.0000\,m$; they mean that, except from the reference point P_0, the next point which normal-geoid height was deter-

Table 2: Mean differences statistics according to different degrees and geopotential functionals.

Model	Degree	Functional	Max (m)	Mean (m)	Min (m)	RMS
EGM2008	2190	N	2.9766	0.1741	0.0000	0.2399
		ζ	2.9651	0.1712	0.0000	0.2354
	720	N	2.9292	0.1807	0.0000	0.2385
		ζ	2.9107	0.1773	0.0000	0.2337
	360	N	2.5203	0.2087	0.0000	0.2362
		ζ	2.5009	0.2081	0.0000	0.2328
EIGEN6C4	2190	N	1.1824	0.1548	0.0000	0.1637
		ζ	1.1463	0.1521	0.0000	0.1613
	720	N	1.1595	0.1631	0.0000	0.1665
		ζ	1.1208	0.1612	0.0000	0.1632
	360	N	1.3147	0.1901	0.0000	0.1723
		ζ	1.3088	0.1913	0.0000	0.1723
GOCO05C	720	N	1.1345	0.1528	0.0000	0.1554
		ζ	1.1166	0.1489	0.0000	0.1504
	360	N	1.3084	0.1802	0.0000	0.1681
		ζ	1.3091	0.1800	0.0000	0.1662
XGM2016	719	N	0.9738	0.1496	0.0000	0.1518
		ζ	0.9596	0.1461	0.0000	0.1480
	360	N	1.3302	0.1808	0.0000	0.1669
		ζ	1.3221	0.1813	0.0000	0.1662

mined taking the first one as a reference has a θ factor minor than $10^{-4}\,m$ - for example, for GGM EGM2008, degree 2190 and geoid functional, the minimum value for θ is $2.2161 \times 10^{-5}\,m \cong 0.0000\,m$.

Furthermore, it is possible to notice from both Fig. 5 and Table 2 that discrepancies are considerably higher for degree 360, independently of model or functional; such result is plenty acceptable and related to the omission error associated to the model truncation. Since they produce an increase of up to $5\,cm$ in the mean difference through the optimal configuration (see Table 2, GGM EGM2008, degree 360), they were considered to be unsuitable for normal-geoid heights modeling.

In addition, considering the main GGMs evaluation and comparison as proposed in the Introduction and as regulated by Eq. 7, as well as the case when all the models are developed up to degree 719/720, maximum possible degree for the recent GGMs, it is possible to verify that the mean discrepancy between values compared for XGM2016, GOCO05C, EIGEN-6C4 and EGM2008 is $0.1496\,cm$, $0.1528\,cm$, $0.1631\,cm$ and $0.1807\,cm$, respectively for geoid functional, and $0.1461\,cm$, $0.1489\,cm$, $0.1612\,cm$ and $0.1773\,cm$, respectively for height anomaly functional. This states a better performance of both the modern GGMs, XGM2016 and GOCO05C, when compared to the older ones - EIGEN-6C4 and EGM2008.

Considering, in a second moment, all GGMs developed up to their individual maximum possible degree - EGM2008 and EIGEN-6C4 up to 2190, GOCO05C up to 720 and XGM2016 up to 719, it is possible to verify that the advantage of both modern GGMs is kept: mean discrepancy between values compared for XGM2016, GOCO05C, EIGEN-6C4 and EGM2008 is $0.1496\,cm$, $0.1528\,cm$, $0.1548\,cm$ and $0.1741\,cm$, respectively for geoid functional, and $0.1461\,cm$, $0.1489\,cm$, $0.1521\,cm$ and $0.1712\,cm$, respectively for height anomaly functional. Also under this heading, it is possible to notice a centimetric better performance of XGM2016 and GOCO05C, specially when compared to EGM2008, and a milimetric advantage when compared to EIGEN-6C4. This last one may be considered not significant, since the input data have centimetric precisions and the general result may indicate an equivalent result for these three GGMs. However, it is remarkable to notice that such equivalence and theoretical equality is achieved even with big disparity of possible maximum degrees of development for these GGMs - i.e. 2190 for EIGEN-6C4 and 719/720 for XGM2016 and GOCO05C.

With respect to the optimal configuration, Fig. 6 presents a map with the individual results by GNSS/BM station location. Once again, attempting to Fig. 6 in comparison with Fig. 3, it is possible to verify that there is no strict national relation between low adjustment of GNSS/BM stations to the optimal configuration and the fact of belonging to BVRN branches. Most of these stations located in southeast and south portions of Brazil are well-behaved when modelled with the presented approach. Though, when handling the north part of these stations, specially the north-south section between 60° W and 65° W, the discrepancies are evidently connected to unknown quality input data. Regional behavior of normal-geoid heights modeling is considered in the next item of this paper.

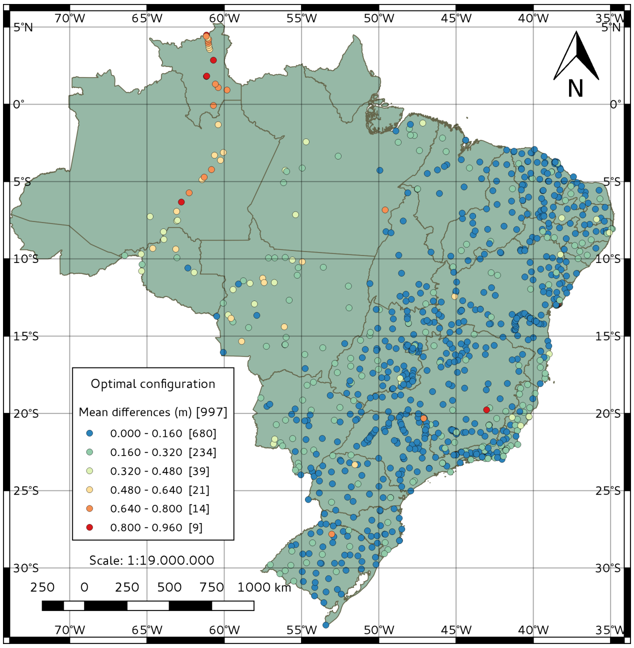


Figure 6: National mean differences map between GNSS/levelling and GGMs solutions.

3.2 Regional wide results

As an attempt to understand regional behavior of each GGM, it was considered an approach according to each one of the five different geographic regions in Brazil, as presented in Fig. 7. These regions are: North, Northeast, Central-West, Southeast and South. The 997 GNSS/BM stations were then divided by belonged region as follows, and minor analysis similar to the national one were carried out.

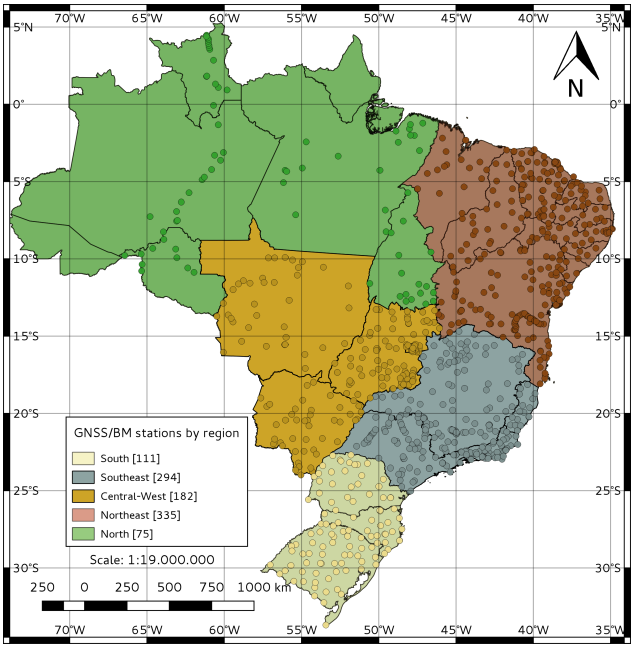


Figure 7: GNSS/BM stations divided by Brazilian geographic regions.

- 75 stations in North region;
- 335 stations in Northeast region;
- 182 stations in Central-West region;
- 294 stations in Southeast region; and
- 111 stations in South region.

The results achieved in these regional analysis, expressed by mean differences values for each set of stations according to GGM configuration (functional and degree), is presented in Fig. 8. In addition, individual differences disposal by station location is presented in Fig. 9. This last figure is particularly important to the present study, since it enables a visual inspection of the GNSS/BM stations accordingly to their individual performance, as well as the identification of possible GNSS/BM stations with observations acquirement issues, generating though incorrect altimetric data.

The first observation on the regional results is that, for brevity reasons, it is only provided the mean discrepancy for each set of points θ_m for each configuration in Fig. 8. Through their analysis, it is important to notice, once again, the outstanding performance of the recent combined GGMs when compared to the older ones: for North, Central-West, and Southeast regions, the results for GOCO05C and XGM2016 are better fitted than the results for EGM2008 and EIGEN-6C4, even considering the differences in maximum possible development degrees already mentioned. Particularly in North and Central-West region, it is noticeable the advantages of the XGM2016 when compared to EGM2008; the difference between EGM2008 best results and XGM2016 best results, for example, differ in up to 7 *cm* for the first area and in up to 10 *cm* for the second one. This can be explained by the new model processing method and, mainly, the integration of gravimetric data from different and better quality sources for modern results.

An additional analysis can be performed considering mean optimal differences for each area according to different GGMs and the location for each GNSS/BM station. This analysis may be conducted observing both Fig. 8 and 9 and, still, Fig. 3, which highlights the normal-orthometric heights accuracy and the existence of branch stations. It is possible to verify that, as already indicated, there is a strong relation between the quality of the input data and the calculated mean adequacy of the GGMs for modeling normal-geoid heights. Notice that, as shown in Fig. 8, the two worst results for such modeling occur exactly for North and Central-West regions, which contain a larger number of branch stations or even those ones which normal-orthometric height accuracy are under aimed values. Furthermore, comparing Fig. 9 and 3, one may notice that most part of these branch stations provide the worst discrepancy results even for optimal configuration - once again, remark the north-south section between 60° W and 65° W in the North region, where the discrepancies are evidently connected to unknown quality input data.

However, searching for this standard in other regions does not result positively. For example, consider the continuous east-west section in Northeast region between 14° S and 15° S; these stations are highlighted as branch stations of BVRN but do not provide the worst results in this set. Individually, their performance indicates since milimetric discrepancies up to even $10\,cm$ discrepancies, when considered the optimal configuration. Thus, this indicates that there is no direct dependency between the GNSS/BM condition as a branch station and bad adequacy for the intended modeling. Meanwhile, there is a persistence in considering low quality input data as a limiting factor for the applied method, even more the reliability of the used data.

As a result of regional wide analysis, it is possible to perform a recommendation for normal-geoid heights modeling in each region according to different GGMs, as presented in Fig. 10. However, this recommendation may be replaced by national wide optimal configuration - GGM XGM2016, degree 719, functional height anomaly, as shown in item 3.1 - since maximum mean discrepancies differ from this configuration and regional wide optimal configuration in $1\,cm$ in Northeast region and $0.5\,cm$ in South region.

One last observation lies in the fact that mean value of mean regional wide discrepancies are different of the mean national wide discrepancy, considering all of them according to appropriate optimal configurations. This is because in each case different choices for point P_0 were made, as stated in Eq. 7.

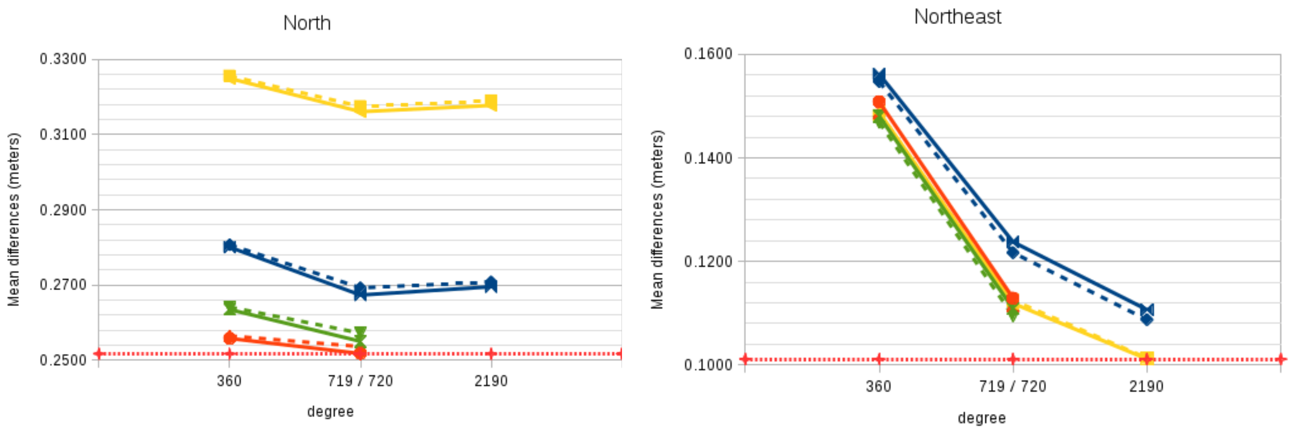
4 Conclusions and outlook

It was carried out an evaluation and a comparison between two different generations of combined Global Geopotential Models: the first one, integrated by established and already deeply studied models EGM2008 and EIGEN-6C4; the second one, integrated by new and innovative models GOCO05C and XGM2016. This comparative evaluation was based on GNSS/levelling solutions taken over 997 GNSS/BM stations all along Brazil, according to a relative approach and using the cited GGMs in different degrees of development and geopotential functionals.

Throughout the study developed and the main results achieved, it was clearly verified superior results for both modern GGMs, specially for XGM2016 - which provided the national optimal configuration [GGM XGM2016, functional height anomaly, degree 719]. Both modern GGMs achieved better results, or at least equivalent results, for normal-geoid heights modeling when compared do the established ones. This indicates an outstanding performance despite the disparity of maximum degrees of development for the GGMs - 2190 for EGM2008 and EIGEN-6C4, and 719/720 for XGM2016 and GOCO05C.

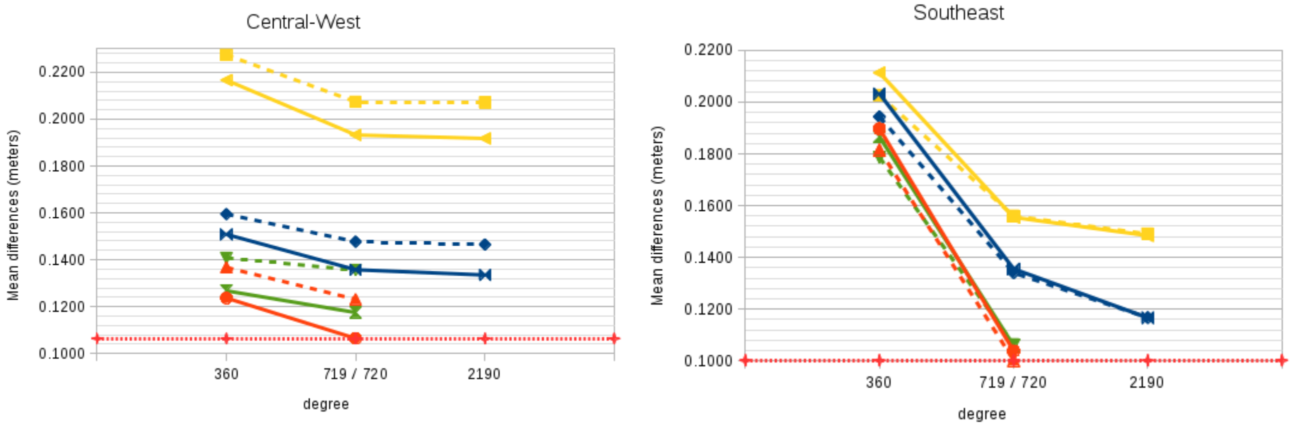
Furthermore, it was held a regional wide analysis, according to five Brazilian geographic regions. As a result for this analysis, it was possible to perform a recommendation for normal-geoid heights modeling in each region according to different GGMs. Though, this recommendation may be replaced by national wide optimal configuration, since maximum mean discrepancies do not differ significantly from this configuration and regional wide optimal configurations.

This result reaffirms the positioning of modern combined GGMs as an exponent in the history of development of combined models as initially suggested by reference [6], with a view to the wide dissemination of their results and the potentiality of their use. Furthermore, it even brings a positive expectation about the upcoming development of combined GGMs expandable up to higher degrees, as expected for EGM2020 [8] and on getting even better adequacy for the studied modeling.



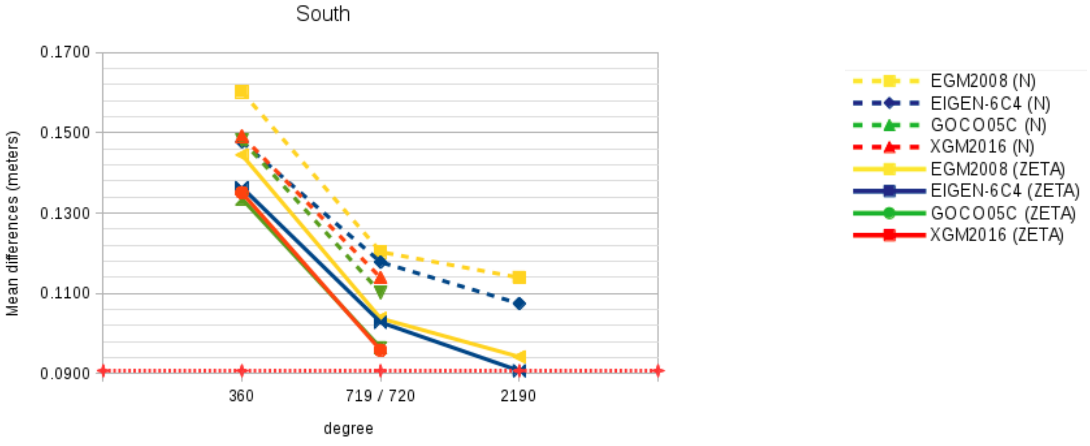
(a) North region (min = 0.2517 m)

(b) Northeast region (min = 0.1011 m)



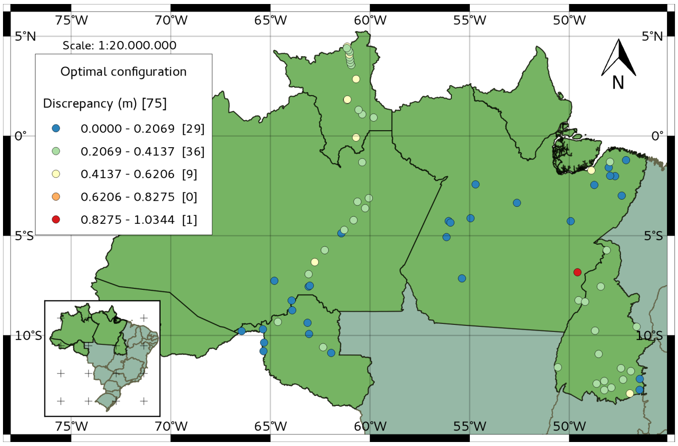
(c) Central-West region (min = 0.1064 m)

(d) Southeast region (min = 0.1002 m)

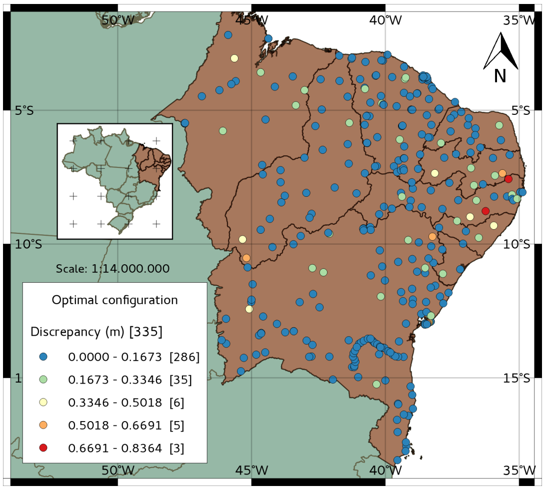


(e) South region (min = 0.0907 m)

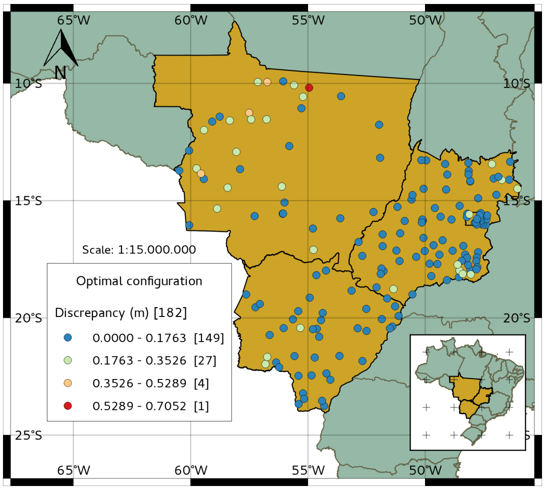
Figure 8: Regional mean differences between GNSS/levelling and GGMs solutions, respectively in: a) North region; b) Northeast region; c) Central-West region; d) Southeast region; and e) South region.



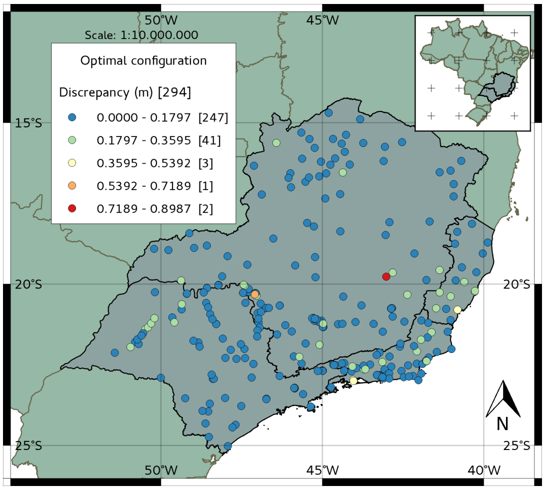
(a) North region



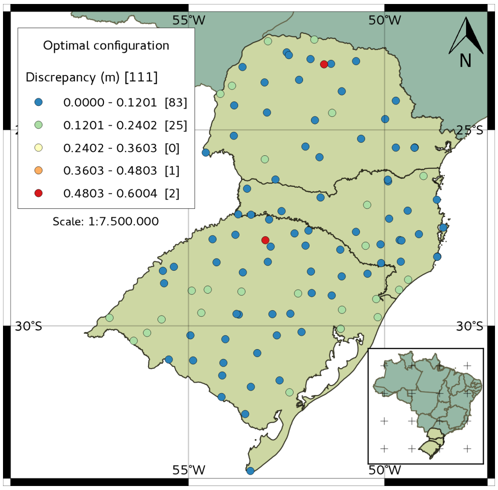
(b) Northeast region



(c) Central-West region



(d) Southeast region



(e) South region

Figure 9: Regional mean differences maps between GNSS/levelling and GGMs solutions, respectively in: a) North region; b) Northeast region; c) Central-West region; d) Southeast region; and e) South region.

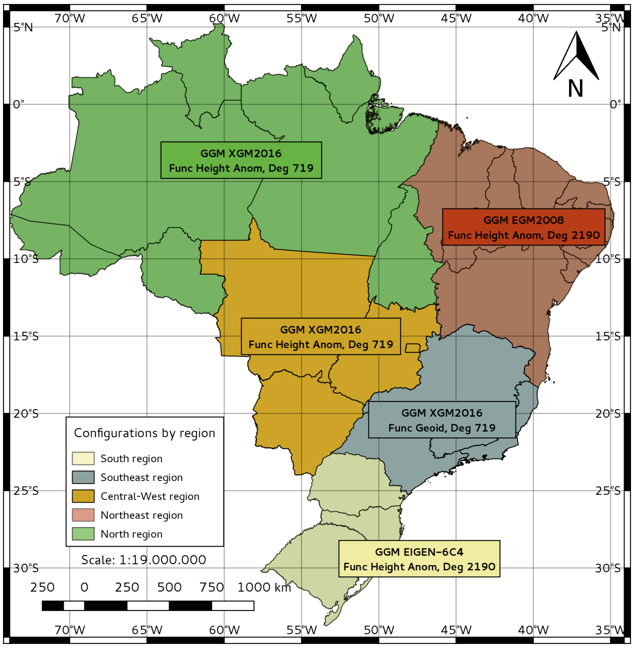


Figure 10: GGMs configuration recommendation for regional wide normal-geoid heights modeling.

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