L. E. Sjöberg, A. Gidudu, and R. Ssengendo*

The Uganda Gravimetric Geoid Model 2014 Computed by The KTH Method

Abstract: For many developing countries such as Uganda, precise gravimetric geoid determination is hindered by the low quantity and quality of the terrestrial gravity data. With only one gravity data point per 65 km^2, gravimetric geoid determination in Uganda appears an impossible task. However, recent advances in geoid modelling techniques coupled with the gravity-field anomalies from the Gravity Field and Steady-State Ocean Circulation Explorer (GOCE) satellite mission have opened new avenues for geoid determination especially for areas with sparse terrestrial gravity. The present study therefore investigates the computation of a gravimetric geoid model over Uganda (UGG2014) using the Least Squares Modification of Stokes formula with additive corrections. UGG2014 was derived from sparse terrestrial gravity data from the International Gravimetric Bureau, the 3 arc second SRTM ver4.1 Digital Elevation Model from CGIAR-CSI and the GOCE-only global geopotential model GO_CONS_GCF_2_TIM_R5. To compensate for the missing gravity data in the target area, we used the surface gravity anomalies extracted from the World Gravity Map 2012. Using 10 Global Navigation Satellite System (GNSS)/levelling data points distributed over Uganda, the RMS fit of the gravimetric geoid model before and after a 4-parameter fit is 11 cm and 7 cm respectively. These results show that UGG2014 agrees considerably better with GNSS/levelling than any other recent regional/global gravimetric geoid model. The results also emphasize the significant contribution of the GOCE satellite mission to the gravity field recovery, especially for areas with very limited terrestrial gravity data. With an RMS of 7 cm, UGG2014 is a significant step forward in the modelling of a "1-cm geoid" over Uganda despite the poor quality and quantity of the terrestrial gravity data used for its computation.

Keywords: Geoid; KTH method; least squares modification; Stokes' formula; Uganda

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1 Introduction

Over the last 30 years, the importance of the geoid has increased substantially due to the widespread use of Global Navigation Satellite Systems (GNSS) for positioning and navigation. GNSS, unlike traditional surveying instruments, has the ability to provide three-dimensional coordinates (latitude, longitude and height) anywhere in the world, any time irrespective of the weather. However the GNSS-determined heights, i.e. ellipsoidal heights, are geometrical heights. These cannot be used in surveying and engineering projects where orthometric heights are required. Hence the need for the determination of the geoid, since it is the reference surface for orthometric heights. Thus by combining the geoid and GNSS observations, the ellipsoidal heights can be converted to the physically meaningful orthometric heights.

The remove-compute-restore (RCR) is perhaps the most well-known approach to gravimetric geoid determination and has been applied in most parts of the world Zhang et al. (1998); Fotopoulos et al. (1999); Forsberg (2001). As an alternative, the Least Squares Modification of Stokes formula (LSMS) with additive corrections (AC), commonly called the KTH method, has been gaining prominence since winning the geoid modeling competition at the International Hotine-Marussi Symposium in 2009 Ågren et al. (2009a). The method was developed at the Royal Institute of Technology (KTH) Division of Geodesy by Sjöberg (1991, 2003a,b, 2005). Compared to other methods, this method is superior because it is the only method that minimizes the expected global mean square error of the estimated geoid height. Hence, in contrast to most other methods of modifying Stokes' formula, which only strive at reducing the truncation error, the KTH method matches the errors of truncation, gravity anomaly and the Global Geopotential Model (GGM) in a least squares sense.

In this study, we compute the gravimetric geoid model over Uganda using the KTH method. Although the method has been successfully applied in a number of countries such as Iran Kiamehr (2006), Tanzania Ulotu (2009), Central Turkey Abbak et al. (2012) and was used for the official gravimetric quasigeoid of Sweden Ågren et al. (2009a,b), the present study considers an area with unprecedented very limited terrestrial gravity data. Compared to Central Turkey, where there was one gravity data point per 22 km^2, the situation in Uganda is worse with only one gravity data point per 65 km^2. In addition to the limited nature of gravity data, its distribution is not uniform over the entire country, and its accuracy can only be estimated to approximately 9 mGal. Thus to determine a gravimetric geoid precise enough at least for engineering applications, the sparse terrestrial gravity data is optimally combined with the Gravity Field and Steady-State Ocean Circulation Explorer (GOCE) satellite mission gravity anomalies in a least squares solution. Therefore, the study also highlights the effect of newly published GGMs on regional geoid determination thus emphasizing the contribution of the GOCE satellite mission to the gravity field recovery, especially for countries with uneven and sparse terrestrial gravity data.

The applied version of the KTH method used in this study is presented in Section 2. In Section 3, the GNSS/levelling data and the required data for computation of the geoid are validated and evaluated using GNSS/levelling. In Section 4, some important computational options, namely gravity anomaly signal and error degree variances, choice of cap size, optimum least squares modification parameters and additive corrections, are discussed. In Section 5, the computed gravimetric geoid model (UGG2014) is evaluated both internally using error propagation and externally using GNSS/levelling. After that it is compared with some global/regional gravimetric models. Finally, conclusions are presented in Section 6.

2 The KTH Method

2.1 The Least Squares Estimator of the KTH method

The *Least Squares Estimator of the KTH method* is given by Sjöberg (2003b) as

$$\widetilde{N}^{L,M} = \frac{R}{4\pi\gamma} \iint_{\sigma_0} S^L(\psi)\Delta g d\sigma + c\sum_{n=0}^{M}\left(Q_n^L + s_n\right)\Delta g_n^{GGM} + \delta N_{comb}^T + \delta N_{dwc} + \delta N_{tot}^a + \delta N_{tot}^e \tag{1}$$

where σ_0 is the spherical cap, R is the mean Earth radius, γ is mean normal gravity on the reference ellipsoid, $S^L(\psi)$ is the modified Stokes' function, $c = R/2\gamma$, s_n are the modification parameters, M is the maximum degree of the GGM, L is the maximum degree of modification, Q_n^L are the Molodensky truncation coefficients, Δg is the unreduced surface gravity anomaly, Δg_n^{GGM} is the Laplace surface harmonic of the gravity anomaly determined by the GGM of degree n. The estimator in Eq. 1 is the so-called combined estimator Sjöberg (2003b), which means that the truncated Stokes' formula is applied to the unreduced surface gravity anomaly after which the final geoid height is determined by adding a number of additive corrections, i.e. δN_{comb}^T- the combined topographic correction, δN_{dwc}- the downward continuation correction, δN_{tot}^a- the total atmospheric correction and δN_{tot}^e- the total ellipsoidal correction. Below we highlight the additive corrections one by one.

The *combined topographic correction* is computed as Sjöberg (2000, 2001)

$$\delta N_{comb}^T(P) = -\frac{2\pi\mu}{\gamma}\left(H^2(P) + \frac{2}{3}\frac{H^3(P)}{R}\right) \tag{2}$$

where P is the computational point, H is the topographic height, μ is the product of the gravitational constant (G) and the standard topographic density (ρ), i.e. $\mu = G\rho$. Vermeer (2008) has questioned the exactness of the above formula for realistic terrains. However, as discussed in Sjöberg (2008, 2009), Eq. 2 corresponds to the negative of the so-called *topographic potential bias*, which in this case is the strict combined effect on the geoid height.

The *downward continuation (DWC) correction* can be written as Sjöberg (2003b,c)

$$\delta N_{dwc}^L = \delta N_{dwc}^{B,L} + \delta N_{dwc}^{te,L} \tag{3}$$

where $\delta N_{dwc}^{B,L}$ and $\delta N_{dwc}^{te,L}$ are the Bouguer shell effect and terrain effect, respectively , given by

$$\delta N_{dwc}^{B,L} = \delta N_{dwc}^{B} + c\sum_{n=2}^{\infty}\left[\left(\frac{R}{r_P}\right)^{n+1} - 1\right]\left(s_n^{\star} + Q_n^L\right)\Delta g_n \tag{3a}$$

with

$$\delta N_{dwc}^{B} \approx \frac{H(P)\Delta g(P)}{\gamma_0} + 3\frac{H(P)}{r_P}\zeta_P - \frac{H^2(P)}{2\gamma_0}\left(\frac{\partial\Delta g}{\partial H(P)}\right) \tag{3b}$$

and

$$\delta N_{dwc}^{te,L} \approx \frac{R}{4\pi\gamma_0} \iint_{\sigma_0} S^L(\psi)(H_P - H_Q)\left(\frac{\partial \Delta g}{\partial H}\right)_Q d\sigma_Q \quad (3c)$$

In the equations above, P and Q are the points on the earth surface and the running point on the sphere, respectively, $r_P = R + H(P)$, ζ_P is defined by Bruns' formula, i.e. $\zeta_P = T_P/\gamma$ where T_P is the disturbing potential for point P and γ is the normal gravity at the normal height of point P and Δg_n the Laplace harmonics in the sum in Eq. 3a is taken from a GGM, which requires the upper limit of the sum to be set equal to or below its maximum order.

Following Sjöberg (2001); Sjöberg and Nahavandchi (2000), the *combined atmospheric correction* can be computed as

$$\delta N_{comb}^{a} = \frac{\delta V_0^a}{\gamma} - \frac{2\pi R\rho_0}{\gamma}\sum_{n=2}^{M}\left(\frac{2}{n-1} - s_n - Q_n^L\right)H_n(P) - \frac{2\pi R\rho_0}{\gamma}\sum_{n=M+1}^{\infty}\left(\frac{2}{n-1} - \frac{n+2}{2n+1}Q_n^L\right)H_n(P) \quad (4)$$

where δV_0^a is the zero degree term of the atmospheric potential, ρ_0 is the atmospheric density at sea level, H_n is the Laplace surface harmonic of degree n for the topographic height and either $s_n^\star = s_n$ if $2 \le n \le M$or $s_n^\star = 0$ otherwise.

The *ellipsoidal correction to order* $e^2$of the modified Stokes' formula is given by Sjöberg (2004) as

$$\delta N_{total}^{e,L} = \frac{R}{2\gamma}\sum_{n=2}^{\infty}\left(\frac{2}{n-1} - s_n^\star - Q_n^L\right)\left(k\Delta g_n + \frac{a}{R}\delta g_n^e\right) \quad (5)$$

where δg_n^e is the Laplace harmonics of the ellipsoidal correction to the gravity anomaly, which can be decomposed into a series as shown by Sjöberg (2003d, 2004), $k = a/R - 1$ is a scale factor and a is the semi-major axis of the reference ellipsoid.

2.2 The expected global mean square error

The expected global mean square error (MSE) of the geoidal undulation estimator is developed by Sjöberg (1986, 1991, 2005) as:

$$\left(\delta \bar{N}^{L,M}\right)^2 = c^2\sum_{n=2}^{\infty}\left(\frac{2}{n-1} - s_n^\star - Q_n^L\right)^2\sigma_n^2 + c^2\sum_{n=2}^{M}\left(s_n^\star + Q_n^L\right)^2 dc_n^2 - c^2\sum_{n=M+1}^{\infty}\left(s_n^\star + Q_n^L\right)^2 c_n^2 \quad (6)$$

where either $s_n^\star = s_n$ if $2 \le n \le L$ or $s_n^\star = 0$ otherwise, c_n^2 is the gravity anomaly degree variance given by $c_n^2 = \frac{1}{4\pi}\iint_{\sigma}\Delta g_n^2 d\sigma$ and the global error degree variances for Δg^T and Δg^{GGM} are given by $\sigma_n^2 = E\left\{\frac{1}{4\pi}\iint_{\sigma_0}\left(\varepsilon_n^T\right)^2 d\sigma\right\}$ and $dc_n^2 = E\left\{\frac{1}{4\pi}\iint_{\sigma}\left(\varepsilon_n^{GGM}\right)^2 d\sigma\right\}$ respectively. Q_n^L are Molodenskii's truncation coefficients given by $Q_n^L = Q_n^L(\psi_0) = Q_n(\psi_0) - \sum_{k=0}^{L} E_{nk}s_k,$ with $Q_n(\psi_0) = \int_{\psi_0}^{\pi} S(\psi)P_n(\psi)\sin\psi d\psi$ and $E_{nk} = E_{nk}(\psi_0) = \frac{2k+1}{2}\int_{\psi_0}^{\pi} P_n(\cos\psi)\sin\psi d\psi . c = R/(2\gamma)$ is a scale factor and s_n are the least squares modification parameters. Therefore, the MSE is thus the sum of the variances from the terrestrial gravity data and the GGM plus the truncation bias squared Sjöberg (2005). In practice, the KTH method aims at selecting s_n in a least squares sense so as to minimise the MSE.

3 Validation and Evaluation of the Data sets Required for Geoid Determination

3.1 GNSS/levelling data

Over the years, GNSS/levelling data has become one of the standard tools for validating and evaluating global and local gravimetric geoid models. With improved precision, i.e. 1 to 2 cm, for the ellipsoidal heights, GNSS/levelling is nowadays used as an external measure of the accu-

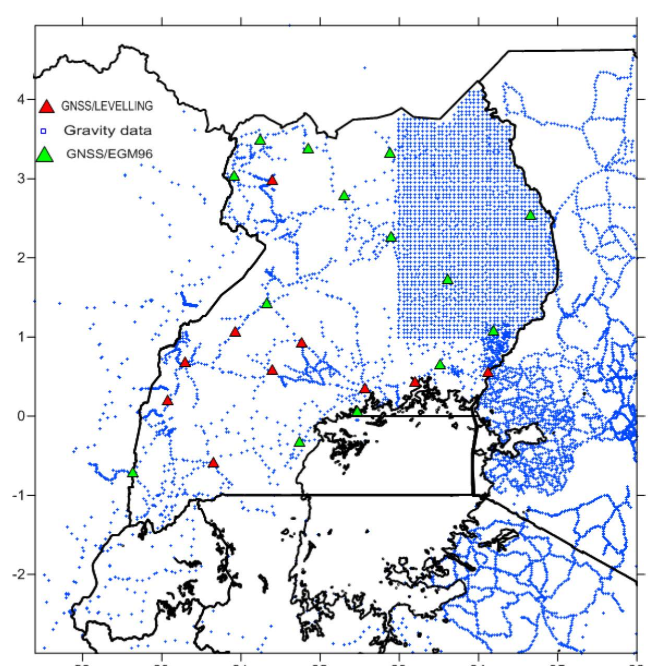

Figure 1: Location of the gravity data and GNSS/levelling/EGM96 benchmarks

Table 1: Statistics of the model comparison with 24 GNSS/EGM96 points (Unit: m)

Comparison	Resolution (")	Min	Max	Mean	Std.	RMS
H(EGM96)-H(ASTER)	1	−23.62	12.74	−2.34	8.36	8.51
H(EGM96)-H(SRTM3)	3	−8.48	8.95	−1.95	4.13	4.48
H(EGM96)-H(SRTM30)	30	−11.16	58.85	9.30	17.82	19.77

Table 2: Overview of the selected GGMs

Model	Year	Degree	Data	Reference
GO_CONS_GCF_2_TIM_R5	2014	280	GOCE	Brockmann et al. (2014)
GO_CONS_GCF_2_DIR_R5	2014	280	GOCE, GRACE, LAGEOS	Bruinsma et al. (2013)
JYY_GOCE04S	2014	230	GOCE	Yi et al. (2013)
GOGRA04S	2014	230	GOCE, GRACE	Yi et al. (2013)
DGM-1S	2012	250	GOCE, GRACE	Farahani et al. (2013)
GOCO03S	2012	250	GOCE, GRACE, CHAMP,LAGEOS	Mayer-Gürr et al. (2012)
EGM2008	2008	2190	GRACE, T, A*	Pavlis et al. (2008)

* T= Terrestrial Gravity data, A= Altimetry data

racy of the global and local gravimetric geoid models. Therefore, before the computation of UGG2014, the first task was to assess the availability of the GNSS/levelling data in the country. Unfortunately, there is either no information or little information about the location and general status of the levelling benchmarks in Uganda to the extent that as of 2014, there is no information pertaining to how many of the 3033 benchmarks that were established by the British Directorate of Overseas Surveys still exists. In addition, no GNSS observations have been made on levelled benchmarks. Therefore, as part of this study GNSS observations using Trimble R7 GNSS receivers were carried out on 24 points shown in Fig. 1 consisting of 10 Fundamental Benchmarks (FBM) of the Uganda vertical network, whose normal-orthometric heights were readily available from the National Mapping Agency, and 14 zero-order points of the Uganda Triangulation network without normal-orthometric heights. For the zero-order points, the GNSS ellipsoidal heights were transformed to physical heights using the National Geospatial-intelligence Agency NGA EGM96 (Lemoine et al., 1998) geoid calculator (http://earth-info.nga.mil/GandG/wgs84/gravitymod/egm96/intpt.html) and are therefore denoted as GNSS/EGM96.

3.2 Digital Elevation Models (DEM)

Height information derived from digital elevation models is very important in geoid computation, because heights are required in the computation of the Bouguer correction, which is used in the conversion of the surface free-air anomalies to Bouguer anomalies which are then used in the gridding procedure. In addition, heights are required in the computation of the combined topographic correction and the downward continuation(DWC) effect, which are additive corrections to the approximate geoid height. Thus any errors in the DEM will introduce errors in the gravity anomalies, the topographic correction and the DWC effect, thereby directly affecting the accuracy of the geoid estimate. In this study, 3 global DEMs, i.e.Consortium for Spatial Information (CSI) of the Consultative Group of International Agricultural Research (CGIAR), Italy (CGIAR-CSI SRTM ver4.1- http://www.cgiar-csi.org/data/srtm-90m-digital-elevation-database-v4-1), the SRTM30 data Version 2.0 (http://dds.cr.usgs.gov/srtm/version2_1/) and ASTER GDEM ver2 (http://www.jspacesystems.or.jp/ersdac/GDEM/E/index.html), were evaluated using the 24 GNSS/EGM96 data points referred to in Section 3.1. The evaluation was carried out by comparing the heights derived from the DEM with those of the same points derived from GNSS/EGM96. We used the 24 GNSS points but with the GNSS ellipsoidal heights transformed to physical heights using EGM96 Lemoine et al. (1998). The advantage with this dataset is that the heights derived from GNSS are consistent with the vertical georeferencing of both SRTM and ASTER Hirt et al. (2010). This minimises biases due to differences in vertical datum definitions. The results reported in Table 1 show that of the 3 DEMs, SRTM3 gives the best results in terms of the mean, standard deviation and RMS of the differences versus the GNSS/EGM96 heights. Its standard deviation and RMS are almost half those of ASTER, which indicates that in Uganda the quality of SRTM3 is clearly much better than ASTER. Therefore SRTM3 is selected for the final computation of UGG2014.

3.3 Global Geopotential Models

The current GGMs, representing the Earth's gravitational field, can be classified into three groups: satellite-only, combined and tailored gravity field models. The satellite-only GGMs are derived from the tracking and analysis of the orbits of artificial Earth satellites only. The combined GGMs include satellite gravity data, terrestrial gravity data and satellite altimetry data. The tailored GGMs are either satellite-only or combined models, which are adjusted and extended to higher degrees by using previously used or unused higher resolution gravity data. Currently a number of GGMs have been derived and made available freely to the scientific community and can be downloaded from the website of the International Centre for Global Earth Models (http://icgem.gfz-potsdam.de/ICGEM/). From this website we selected 6 satellite-only models and one combined model for evaluation. We choose these models because they were the most recently published GGMs at the time. In addition we also wanted to test how the GOCE-only models compare with the GOCE/GRACE/CHAMP models when fitted to GNSS/levelling data. Table 2 gives an overview of the models selected.

The GGMs were evaluated by comparing the geoid heights from GNSS/levelling with the GGM geoid heights computed using the MATLAB based software EGMlab Kiamehr and Eshagh (2008), which transforms the coefficients so that they refer to GRS80. The comparison was carried out both before and after the 4-parameter fitting with the parametric model used to minimize the effect of systematic biases emanating from commission and omission errors of the GGMs and biases from GNSS and levelling Fotopoulos (2013). The descriptive statistics of the differences between the GGM derived geoid heights and the GNSS/levelling geoid heights are reported in Table 3.

As expected, EGM2008 with standard deviations of 22 cm and 9 cm before and after the 4-parameter fitting is the GGM that best fits GNSS/levelling in Uganda. This is because EGM2008 is a combined model which includes terrestrial gravity anomalies and is also given as a series of spherical harmonic coefficients complete to d/o 2159 unlike all the other GGMs tested which are satellite-only GGMs with maximum degree and order up to 280. However, for the computation of UGG2014, we used the GOCE-only model GO_CONS_GCF_2_TIM_R5 up to degree 280, whose standard deviations of 37 cm and 29 cm before and after the 4-parameter fitting respectively are the lowest for the satellite-only GGMs. This was preferred in order to guard against correlations that may arise between the errors in the GGM and the terrestrial gravity anomalies in the case of the combined model Ågren (2004); Ågren et al. (2009b). However, these results also highlight the contribution of the GOCE satellite mission to the gravity field recovery as the difference in standard deviations between the GOCE-only model to d/o 280 and the combined model –EGM2008 complete to d/o 2159 is approximately only 17 cm.

3.4 Gravity anomalies

The terrestrial gravity data used in this study was downloaded from the International Gravimetric Bureau (BGI) gravity database (http://bgi.omp.obs-mip.fr/data-products/Gravity-Databases/Land-Gravity-data). The data covers the area which lies between $3°S \le \varphi \le 5°N$ in latitude and $28° \le \lambda \le 36°$ in longitude. The distribution of the data is presented in Fig. 1. 7839 gravity data points with an accuracy of approximately 20 mGal were provided. Of these, 3624 points lie within the boundaries of Uganda between $1.5°S \le \varphi \le 4.5°N$ in latitude and $29.5° \le \lambda \le 35°$ in longitude, producing a density of one point per 65 km^2. We can see that the gravity data is not uniformly distributed all over the entire country with a uniform distribution observed only in the North-Eastern part of the country where mineral deposits were suspected leading to a lot of gravity measurements undertaken between 1936 and 1975.

The geodetic datum of the gravity points is the GRS67 geodetic system with the normal gravity on the GRS67 ellipsoid determined to an accuracy of ±0.004 mGal. However, in this study our intention is to determine a gravimetric geoid model for Uganda relative to the GRS80 geodetic system using surface gravity anomalies following Molodensky's theory in which the gravity anomalies refer to the Earth's surface (the ground) instead of the geoid (Heiskanen and Moritz, 1967, pp. 287-328)(Hofmann-Wellenhof and Moritz, 2006, p. 289). We used the Somigliana-Pizzetti formula (Moritz, 1980, p. 131) to compute the normal gravity on the GRS80 reference ellipsoid with the normal height of the computational point computed as the sum of the normal-orthometric height of the gravity point and the geoid-to-quasi-geoid separation approximated by Eq. (8-103) in (Heiskanen and Moritz, 1967, pp. 327-328).

In addition to the uneven distribution of the gravity points, the KTH method requires gravity data outside the

Table 3: Statistics of the differences between GGM geoid heights and GNSS/levelling geoid heights for 10 GNSS/levelling points over Uganda (Unit: cm)

Model		Min	Max	Mean	Standard deviation
EGM2008	Before	23	87	49	22
	After	−18	13	0	9
GO_CONS_GCF_2_TIM_R5	Before	−22	115	34	37
	After	−63	26	0	29
GO_CONS_GCF_2_DIR_R5	Before	17	118	32	40
	After	−69	41	0	34
JYY_GOCE04S	Before	−35	128	36	46
	After	−69	43	0	36
GOGRA04S	Before	−33	121	38	43
	After	−69	43	0	35
DGM-1S	Before	−11	143	53	46
	After	−64	47	0	34
GOCO03S	Before	−48	98	21	44
	After	−54	43	0	32

extents of the study area (up to the cap size), this data is either sparsely available e.g. beyond the border with the Democratic Republic of the Congo or is completely missing e.g. beyond the border with South Sudan. To fill these gaps, surface gravity anomalies were extracted from the World Gravity Map 2012 Bonvalot et al. (2012).

The final grid of the surface gravity anomalies at a resolution of $1'x1'$was made as follows:

- Detection of outliers in the terrestrial gravity data using visual inspection, direct comparison with the WGM2012 surface gravity anomalies and the use of the cross validation approach Kiamehr (2007); Ulotu (2009). As a result a total of 812 gravity points representing 10.3 % of the terrestrial gravity data were identified as outliers and then removed from the gravity data.
- Using the Bouguer surface (removal of topographic masses) to convert the surface gravity anomalies into reduced gravity anomalies, which are assumed to be smoother than the original surface gravity anomalies. This technique was used to overcome the challenge of interpolating unreduced gravity anomalies since the KTH method works on the full gravity anomaly without any reduction Sjöberg (2003b). Then the reduced gravity anomalies were interpolated to a denser grid and finally the effect of the topographic masses were restored to the Bouguer anomaly grid resulting in to free-air anomalies.
- We used the method of Kriging with linear variograms Kiamehr (2007); Ulotu (2009) to construct the final grid. By cross-validation of all gravity data the accuracy of the surface gravity anomaly was estimated to be approximately 9 mGal. This certainly is a great improvement from the initial 20 mGal of the original terrestrial gravity data from BGI. This shows the effect of outlier detection on gravity surveys and subsequently on regional gravimetric geoid determination.

4 Determination of UGG2014

4.1 Signal and error degree variances

The error degree variances of the GGM are provided together with the GGM up to the maximum degree of the particular GGM. The problem is in determining the signal degree variances for higher degrees. In this study we numerically test three degree variance models, i.e. Kaula's rule Kaula (1963, 1966), the Tscherning and Rapp model Tscherning and Rapp (1974) and the Jekeli and Moritz model Jekeli (1978); Moritz (1976, 1977) with the selected GGM GO-CONS-GCF-TIM-R5 ($n_{\max} = 280$). The numerical results show that beyond degree 2000, the Jekeli/Moritz model yields degree variances approximately equal to zero, showing that there is no gravity anomaly power above this degree, which is unrealistic since we now (2014) know that GGMs can have gravity anomaly power complete to degree and order 2159 (e.g. EGM2008; Pavlis et al. (2008)). Beyond degree 2500, the Kaula model yields too much power (almost twice as much

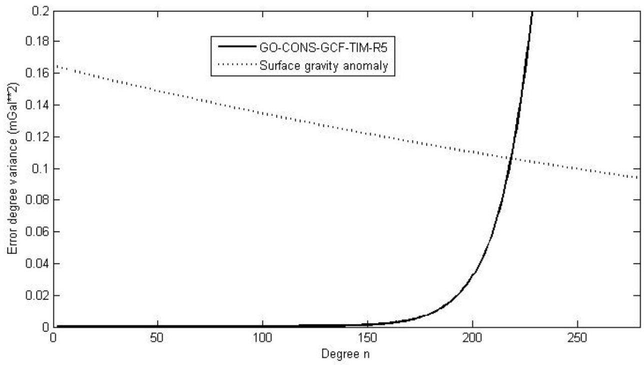

Figure 2: Error degree variances of GO-CONS-GCF-TIM-R5 and surface gravity anomaly

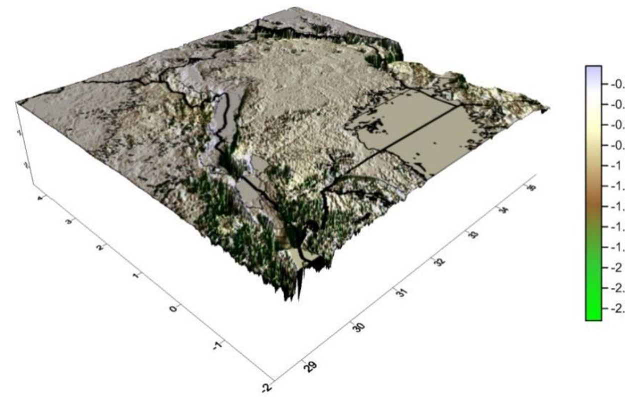

Figure 3: 3D surface view of the combined topographic corrections overlaid with relief and contour layers (Unit: m and contour interval = 0.1 m)

when compared with the Tscherning/Rapp model). This makes it unsuitable for modelling degree variances for higher frequencies. Compared with EGM2008, the Tscherning/Rapp model fits the degree variances of EGM2008 reasonably well for the frequencies between degrees 280 and 1000. Based on the above numerical results and the findings of Ågren (2004), we use the Tscherning and Rapp model to determine the degree variances for the degrees higher than the maximum degree of our selected GGM. To account for the error degree variances of the surface gravity anomaly, we use the correlated model as recommended by Ågren (2004). This model is based on a combination of the white noise model (cf. Rummel (1997); Jekeli and Rapp (1980)) and the reciprocal distance model of Sjöberg (1986). The reciprocal distance model provides the power for the low degrees while the white noise model provides the power for the higher degrees of the spectrum up to the Nyquist degree, M_N, which is defined by Ågren (2004) as$\pi/\Delta\phi$, where $\Delta\phi$ is the block size of the gravity anomaly grid in question (in our caseM_N = 10800 since the block size of the gravity anomaly grid is$60''$), beyond which degree, the power drops down to almost zero as would be expected.

To determine the weighting scheme of the gravity anomaly data, we plot the error degree variances of the satellite-only GGM GO-CONS-GCF-TIM-R5, and the surface gravity anomaly error degree variances determined by the correlated model (Fig. 2). We can see that up to about degree 200, the GOCE-only model is better than the surface gravity anomalies. This represents the upper degree for which the GOCE model is believed to be better than the terrestrial gravity anomalies (cf. Ågren et al. (2009b)).

4.2 Choice of the cap size

Theoretically there is no optimum cap size for the numerical integration by the KTH method provided homogeneous gravity data is available Sjöberg (2003b). However, to reduce the effect of undetected systematic/gross errors in the gravity data, a limited cap size of a few degrees is usually used. On the basis of gravity data availability, a cap size of 1° was used for the determination of UGG2014 so as to optimise the available gravity data while at the same time reducing the influence of the WGM2012 gravity data on the surface gravity anomalies.

4.3 Determination of the Optimum Least Squares Modification Parameters

The determination of the optimum least squares modification parameters has been studied in detail by Ågren (2004); Sjöberg (1991). Based on these studies the badly conditioned system of equations for the unbiased least squares estimator in the KTH method is solved using Singular Value Decomposition. In this study we use the correlated model to compute the least squares modification parameters. We numerically study the behaviour of the parameters using different standard deviations for the terrestrial gravity data (σ = 1 mGal, 5 mGal, 9 mGal and 20 mGal), a correlation length of 0.2° for the reciprocal distance model and M_N = 10800. Our results show that (although the parameters s_n depend on the gravity anomaly degree variances which are not accurately known), the least squares method is rather insensitive to the choice of the weights such that even with very low quality terrestrial gravity data (in our case σ = 9 mGal), the resulting least squares modification parameters are insensitive to the long-wavelength gravity anomaly errors and at the same time yield a low truncation error (cf. Ågren et al. (2004)).

4.4 Determination of Approximate Geoid Height

The approximate geoid height model for Uganda was computed from GGM GO_CONS_GCF_TIM_R5 and a $1'x1'$grid of the surface gravity anomalies. It has the following statistics: minimum = −17.62 m, maximum = −4.67 m, mean = −12.56 m, standard deviation = 2.47 m and RMS = 12.81 m. This implies that the geoid is located below the reference ellipsoid.

4.5 Additive Corrections

The *combined topographic correction* is computed according to Eq. 2 where H is extracted from the 3”x3” SRTM3 DEM. The results in metres are presented in Fig. 3, where we can see that generally the combined topographic corrections are between −0.2 m and −1.0 m over the study area with the lowest correction observed along the Great Rift Valley and the highest correction observed on the Rwenzori Mountains along the western border of the country.

The downward continuation correction, illustrated in Fig. 4, is computed according to Eq. 3 using the SRTM3 DEM and the chosen GGM GO_CONS_GCF_TIM_R5 with M = 280. It is clear that this correction is large especially around the Rwenzori Mountains along the western border and around Mountain Elgon along the eastern border with Kenya where it ranges from 1 m to 2.4 m. For the rest of the country, this correction is still large as it ranges between 0.4 m and −0.2 m. Overall, the statistics of the DWC correction are: minimum = −0.23 m, maximum = 2.45 m, mean = −0.00 m, standard deviation = 0.08 m and RMS = 0.08 m.

By combining the *combined topographic correction* and *DWC correction* we obtain the total topographic effect

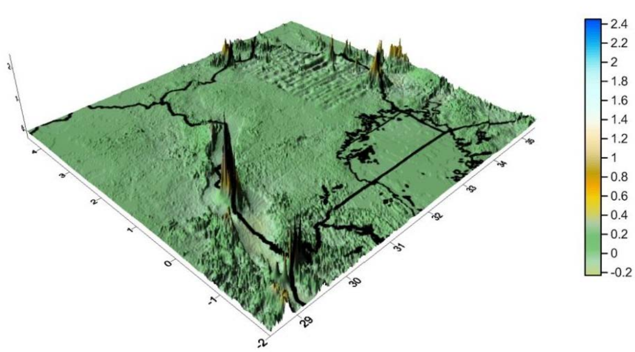

Figure 4: The Downward Continuation Correction (Unit: m)

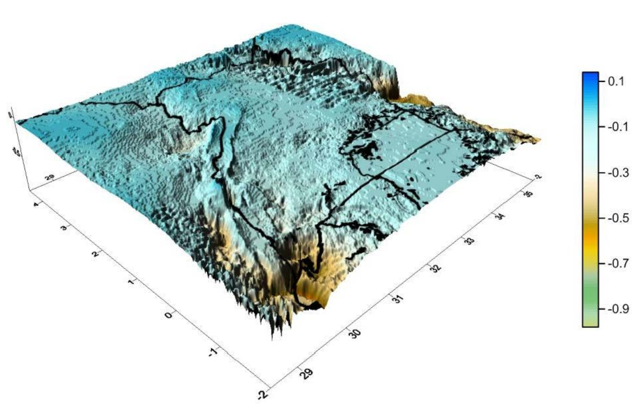

Figure 5: The total topographic effect (Units: metres)

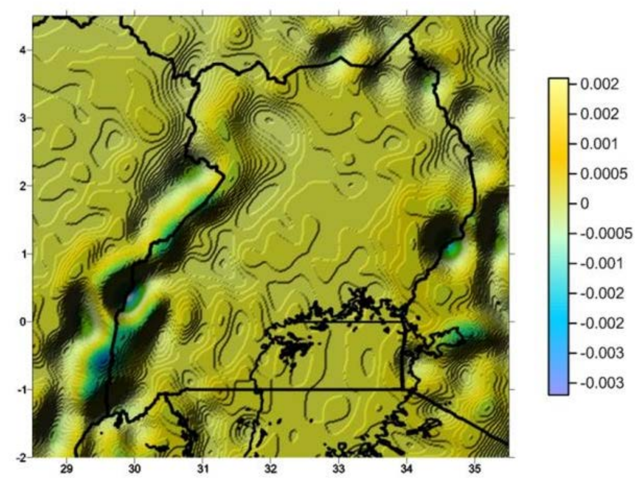

Figure 6: The total atmospheric correction (units: metres)

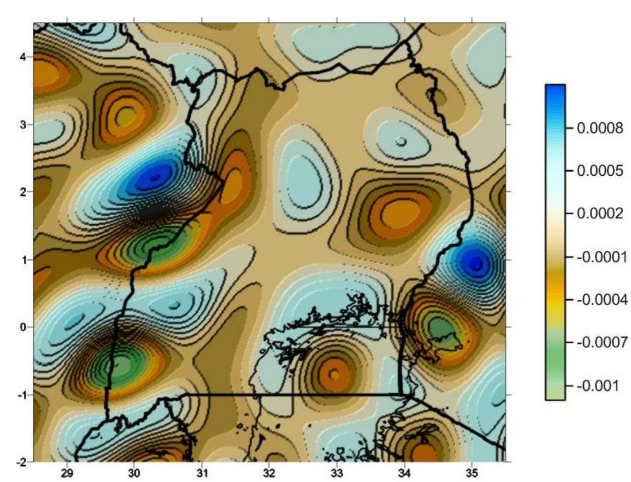

Figure 7: The ellipsoidal correction (units: metres)

on the geoid height as

$$\delta N^T_{tot} = \delta N^T_{comb} + \delta N_{DWC} \quad (7)$$

The magnitude of the total topographic effect in metres is illustrated in Fig. 5. The counteracting effect of the DWC on the combined topographic correction is depicted by the reduction in the magnitude of the total topographic effect whose statistics are: minimum = −0.98 m, maximum = 0.14 m, mean = −0.18 m, standard deviation = 0.11 m and RMS = 0.11 m.

The *total atmospheric correction* in the KTH method is dependent on the type of GGM used in the modification Sjöberg (2001). In our case we use the GGM GO_CONS_GCF_TIM_R5, which is a GOCE-only GGM to degree 280. Thus the total atmospheric effect on the geoid height is computed according to Eq. 4 and illustrated in Fig. 6. This correction is very small (within 3 mm) with the maximum absolute values observed along the Great Rift Valley.

The *ellipsoidal correction* to the modified Stokes formula is computed according to Eq. 5 using the selected GGM GO_CONS_GCF_TIM_R5 with maximum degree = 280. The result is illustrated in metres in Fig. 7. This correction is very small (within 1 mm) mainly because in the KTH method it depends on the cap size which is only 1°. In addition, the GGM gravity anomaly is computed based on the Mean Earth Ellipsoid instead of the Mean Earth Sphere following studies of Sjöberg (2003d,e, 2004).

5 Internal and External Accuracy Assessment of UGG2014

From Eq. 1, we can identify three major sources of error namely errors due to truncation of the integration cap, errors emanating from the observed terrestrial gravity data and errors stemming from the GGM. In addition, errors emanating from the additive corrections especially the total topographic effect are also noted. The internal accuracy of UGG2014 represented by the RMS of the geoidal undulation estimator is computed by Eq. 6 and reported in Table 4 for the 3 GGMs tested namely a GOCE-only model, a GOCE/GRACE/CHAMP/LAGEOS model and a combined model.

From the Table, we can see that the GOCE-only model has the lowest estimated geoid height RMS value of the 3 GGMs compared. This emphasizes the important contribution of the GOCE satellite data to gravity field recovery as model GO_CONS_GCF_2_TIM_R5 includes all the GOCE observations for the entire mission Brockmann et al. (2014), whereas model GOCO03S includes only 18 months of the GOCE observations Mayer-Gürr et al. (2012). In addition, the contribution of the GGM to the RMSE is only 4.6 cm which is less than half the contribution from the terrestrial gravity data. This shows the significant progress brought about by the improved knowledge of the long and medium wavelengths of the Earth's gravity field as a result of the GOCE gravity satellite mission. At the same time it highlights the need for accurate terrestrial gravity data, since the biggest contribution stems from the terrestrial gravity data, which as we have earlier noted, is of a very low quality.

The internal accuracy assessment above is based on error propagation and therefore cannot be used alone to tell how good a geoid model is. The external accuracy assessment on the other hand is based on independent data sets (e.g. GNSS/levelling data) and is therefore a good indicator of the accuracy of the gravimetric geoid model. The assessment is usually carried out by comparing the estimates of the geoid heights ($N^{UGG2014/GGM}$) with that obtained from GNSS/levelling ($N^{GNSS/LEV.}$) based on the difference

$$\Delta N_i = N_i^{GNSS/LEV} - N_i^{UGG2014/GGM} = h_i - H_i^{NO} - N_i^{UGG2014/GGM} \quad (8)$$

where h and H^{NO} are the ellipsoidal and normal-orthometric heights respectively. From the above model, the statistics of the residuals for the 10 FBMs are presented in Table 5 together with a comparison made with the regional model for Africa i.e. African geoid Merry et al. (2005) and two combined GGMs, i.e. EGM2008 Pavlis et al. (2008) and EIGEN-6C3stat Förste et al. (2012). The residuals are also fitted using a 4-parameter model in or-

Table 4: Global RMS values for the Unbiased LSM Estimator for 3 GGMs

GGM	Expected Error	Estimated RMS
GO_CONS_GCF_2_TIM_R5 ($n_{\max}$ =280)	Total	11.5
	Δg only	10.3
	GGM only	4.6
	Truncation	2.2
GOCO03S ($n_{\max}$ = 250)	Total	14.9
	Δg only	12.2
	GGM only	8.0
	Truncation	3.0
EGM2008 ($n_{\max}$ = 360)	Total	11.8
	Δg only	8.0
	GGM only	8.2
	Truncation	2.8

Table 5: The GNSS/levelling residuals over the 10 FBMs before and after the 4-parameter fit (Unit: cm)

GGM/Regional Geoid		Min	Max	Mean	RMS
UGG2014	Before	−14.6	23.6	5.8	**11.6**
	After	−11.9	12.2	0.0	**7.4**
EGM2008	Before	22.5	86.7	48.8	**53.0**
($n_{\max}$ = 2190)	After	−18.0	13.0	0.00	**8.7**
EIGEN_6C3stat	Before	73.4	116.1	96.3	**97.2**
($n_{\max}$ = 1949)	After	−14.7	11.1	0.0	**8.2**
African Geoid (2007)	Before	−184.6	−91.4	−134.0	**138.7**
	After	−17.5	16.3	0.0	**10.4**

der to minimise the effect of a number of systematic biases stemming from GNSS/levelling data and the gravimetric geoid (cf. Fotopoulos (2013); Kiamehr and Sjöberg (2005). We can see that UGG2014 has the best agreement (11.6 cm before and 7.4 cm after) among all the gravimetric models. Most significantly its agreement before the parameter fitting (11.6 cm) is much better than any of the other gravimetric models. This highlights the advantages of combining terrestrial and satellite gravity data using the KTH method to determine a regional geoid even for areas like Uganda with very limited terrestrial gravity data. After applying the 4-parameter fit, the absolute agreement of all models recovers considerably to within +3 cm the fit of UGG2014 (7.4 cm).

Based on the RMS values of 11.6 cm and 7.4 cm before and after the parameter fitting, respectively, and assuming that the standard errors of the ellipsoidal heights and the normal-orthometric heights are 2.2 cm and 1.0 cm respectively, by simple error propagation the standard error of UGG2014 before and after can be estimated as $\sqrt{(11.6)^2-(2.2)^2-(1.0)^2} = 11.3$ cm and $\sqrt{(7.4)^2-(2.2)^2-(1.0)^2} = 7.0$ cm. We can see that the 4-parameter model has reduced the standard error of UGG2014 by 4.3 cm or 38% by absorbing the systematic biases.

Let us finally compare the internal and external estimates of the accuracy of UGG2014. Considering that the internal estimate is a pure error propagation of random/stochastic errors, while the external error includes systematic errors as well, the latter should exceed the former. After the 4-parameter fit, UGG2014 experiences the opposite: internal and external error estimates are 11.5 cm and 7.4 cm. A probable reason for this odd result could be that the applied error degree variances for the GGM and gravity anomaly observations are too large, which will affect the internal error estimate directly, while the external estimate is only indirectly affected through the geoid height estimates. As a result, the internal accuracy estimate is likely rather poor, while the estimated external accuracy, which is much more important, is more realistic.

6 Conclusions

The main purpose of this paper has been to present the computation of the gravimetric geoid model UGG2014 over Uganda by combining very limited and sparse terrestrial gravity data with a recently published GGM. The 11.6 cm and 7.4 cm RMSE obtained by UGG2014 before and after the 4-parameter fit, respectively, are very satisfactory given the poor quality and quantity of the terrestrial data used. If the standard errors for GNSS and levelling are taken as 2.2 cm and 1.0 cm, respectively, (which is reasonably realistic given the current state of the vertical network in Uganda), then the propagated RMSE for the fitted gravimetric geoid becomes 7 cm. Although this standard error is much larger than the 1 cm standard error anticipated for local/regional gravimetric geoid models in many countries, it represents significant progress since UGG2014 is the first regional/local gravimetric geoid model over Uganda. Our results also point out the significant contribution of the GOCE satellite mission to gravimetric geoid determination, especially for areas with sparse terrestrial gravity data. As part of future work, we plan on densifying the GNSS/levelling network over the entire country so as to provide a much better homogeneous data set that can be used for validating and evaluating global and regional gravimetric geoid models. We also anticipate that improvements in terrestrial gravity coverage as part of increased mineral exploration in the country will provide more gravity data that can be used to improve the accuracy of the gravimetric geoid model.

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