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Propagation of SH waves in a regular non-homogeneous monoclinic crustal layer lying over a non-homogeneous semi-infinite medium

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Abstract: This study discusses the dispersion equation for SH waves in a non-homogeneous monoclinic layer over a semi-infinite isotropic medium. The wave velocity equation has been obtained. In the isotropic case, when the non-homogeneity is absent, the dispersion equation reduces to a standard SH wave equation. The dispersion curves are depicted by means of graphs for different values of non-homogeneity parameters for the layer and semi-infinite medium.

Keywords: differential equations; monoclinic; non-homogeneity; SH waves.

1 Introduction

The formulations and solutions of many problems of linear wave propagation for homogeneous media are available in the literature of continuum mechanics of solids. In recent years, however, sufficient interest has risen in the problem connected with bodies whose mechanical properties are functions of space, i.e. non-homogeneous bodies. This interest is mainly due to the advent of solid rocket propellants, polymeric materials and growing demand for engineering and industrial applications.

The propagation of surface waves in elastic media is of considerable importance in earthquake engineering and seismology on account of the occurrence of stratification in the earth's crust, as the earth is made up of different layers. As a result, the theory of surface waves has been developed by Stoneley [1], Bullen [2], Ewing et al. [3], Hunter [4] and Jeffreys [5].

Many results of theoretical and experimental studies revealed that the real earth is considerably more complicated than the models presented earlier. This has led to a need for more realistic representation as a medium through which the seismic waves propagate. The wave propagation in crystalline media plays a very interesting role in geophysics and also in ultrasonic and signal processing. Monoclinic medium is an example of a body where the non-homogeneity characteristic is one of the most important features. Many authors have studied the propagation of different waves in different media with non-homogeneity.

Sezawa [6] studied the dispersion of elastic waves propagating on curved surfaces. The transmission of elastic waves through a stratified solid medium was first studied by Thomson [7]. Haskell [8] examined the dispersion of surface waves in multilayered media. Biot [9] studied the influence of gravity on Rayleigh waves, assuming that the force of gravity creates a type of initial stress of a hydrostatic nature and the medium is incompressible.

Propagation of Love waves in a non-homogeneous stratum of finite depth sandwiched between two semi-infinite isotropic media had been studied earlier by Sinha [10]. Roy [11] studied wave propagation in a thin two-layered laminated medium with couple stress under initial stress, while Datta [12] studied the effect of gravity on Rayleigh wave propagation in a homogeneous, isotropic elastic solid medium. The effects of irregularities on the propagation of guided SH waves was studied by Chattopadhyay et al. [13]. Goda [14] examined the effect of non-homogeneity and anisotropy on Stoneley waves, while Gupta et al. [15] investigated the influence of linearly varying density and rigidity on torsional surface waves in an inhomogeneous crustal layer.

Some of the recent notable works on the propagation of seismic waves in various media with different geometries are due to Chattopadhyay et al. [16–18].

Recently, Sethi et al. [19] investigated the surface waves in homogeneous viscoelastic media of a higher order under the influence of surface stresses.

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In this study, we consider the propagation of SH waves in a regular monoclinic crustal layer over an isotropic semi-infinite medium. The dispersion relation is found in closed form and matched with the classical Love wave equation as a particular case. The dispersion curves are depicted by means of graphs for different values of the non-homogeneity parameters. The influence of non-homogeneity parameters, wave number and layer thickness on the dimensionless phase velocity has been studied.

2 Formulation of the problem

Let us denote by ρ_i, u_i (i=1, 2), the densities and displacements in a monoclinic layer (of thickness H) and semi-infinite isotropic medium, respectively. The z-axis is taken along the interface between the layer and the semi-infinite medium, while the y-axis is taken vertically downwards as shown in Figure 1.

First, we will deduce the equation of motion for the propagation of SH waves in the monoclinic layer. We have the following strain-displacement relations:

$$S_1=\frac{\partial u}{\partial x},\ S_2=\frac{\partial v}{\partial y},\ S_3=\frac{\partial w}{\partial z},\ S_4=\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z},\ S_5=\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x},$$
$$S_6=\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}, \tag{1}$$

where u, v, w are displacements along the x, y, z axes, respectively and S_i (i=1, 2,…, 6) denote the strain components.

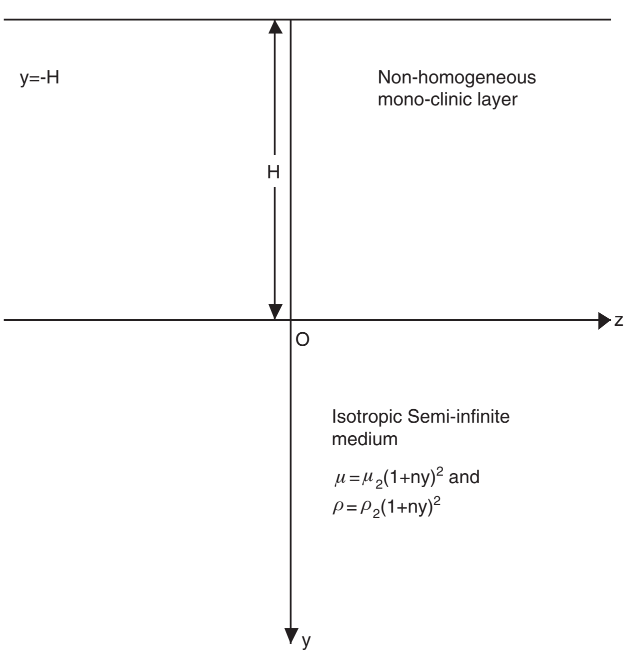

Figure 1: The geometry of the problem.

The stress-strain relations for a rotated y-cut quartz plate, which exhibits monoclinic symmetry with x being the diagonal axis are as follows:

$$\begin{aligned}
T_1&=C_{11}S_1+C_{12}S_2+C_{13}S_3+C_{14}S_4,\\
T_2&=C_{12}S_1+C_{22}S_2+C_{13}S_3+C_{14}S_4,\\
T_3&=C_{13}S_1+C_{23}S_2+C_{33}S_3+C_{34}S_4,\\
T_4&=C_{14}S_1+C_{24}S_2+C_{34}S_3+C_{44}S_4,\\
T_5&=C_{55}S_5+C_{56}S_6,\\
T_6&=C_{56}S_5+C_{66}S_6,
\end{aligned} \tag{2}$$

where T_i (i=1, 2,…, 6) are stress components and $C_{ij}=C_{ji}$ (i, j=1, 2,…, 6) are the elastic constants.

The equations of motion in the absence of body forces are as follows:

$$\frac{\partial T_1}{\partial x}+\frac{\partial T_6}{\partial y}+\frac{\partial T_5}{\partial z}=\rho\frac{\partial^2 u}{\partial t^2},$$
$$\frac{\partial T_6}{\partial x}+\frac{\partial T_2}{\partial y}+\frac{\partial T_4}{\partial z}=\rho\frac{\partial^2 v}{\partial t^2},$$

and

$$\frac{\partial T_5}{\partial x}+\frac{\partial T_4}{\partial y}+\frac{\partial T_3}{\partial z}=\rho\frac{\partial^2 w}{\partial t^2}, \tag{3}$$

where ρ is the density of the upper monoclinic layer.

For SH waves propagating in the z-direction with the displacement only in the x-direction, we have

$$u=u(y,\ z,\ t),\ v=0,\ w=0. \tag{4}$$

Introducing Eq. (4) to Eq. (1), we obtain

$$S_1=0,\ S_2=0,\ S_3=0,\ S_4=0,\ S_5=\frac{\partial u}{\partial z},\ S_6=\frac{\partial u}{\partial y}, \tag{5}$$

Introducing Eq. (5) to Eq. (2), we obtain

$$T_1=T_2=T_3=T_4=0,\ T_5=C_{55}\frac{\partial u}{\partial z}+C_{56}\frac{\partial u}{\partial y}$$
$$\text{and } T_6=C_{56}\frac{\partial u}{\partial z}+C_{66}\frac{\partial u}{\partial y} \tag{6}$$

3 Solution for monoclinic layer

Let the non-homogeneities for the monoclinic layer be considered as

$$C_{66}=C'_{66}e^{my},\ C_{56}=C'_{56}e^{my},\ C_{55}=C'_{55}e^{my},\ \rho=\rho_1 e^{my}. \tag{7}$$

Introducing Eqs. (4), (6) and (7) into Eq. (3), we obtain the non-vanishing equation of motion as follows:

$$C'_{66}\frac{\partial^2 u_1}{\partial y^2}+2C'_{56}\frac{\partial^2 u_1}{\partial y\partial z}+C'_{55}\frac{\partial^2 u_1}{\partial z^2}+mC'_{56}\frac{\partial u_1}{\partial z}+mC'_{66}\frac{\partial u_1}{\partial y}=\rho_1\frac{\partial^2 u_1}{\partial t^2}. \tag{8}$$

We seek a solution of Eq. (8) is of the following form:

$$u_1(y,z,t)=U_1(y)e^{iK(z-ct)}, \tag{9}$$

where K is the wave number and c is the velocity of SH waves.

By inserting Eq. (9) into Eq. (8), we obtain

$$\frac{d^2U_1}{dy^2}+\left(2iK\frac{C'_{56}}{C'_{66}}+m\right)\frac{dU_1}{dy}+\left[\frac{C'_{55}(-k^2)+ikmC'_{56}+\rho_1\omega^2}{C'_{66}}\right]U_1=0 \tag{10}$$

Using $U_1=V(y)e^{-a_1y/2}$, where $a_1=\left(2iK\frac{C'_{56}}{C'_{56}}+m\right)$ in Eq. (10), we obtain

$$\frac{d^2V}{dy^2}+\left[\frac{-a_1^{\,2}}{4}-\frac{C'_{55}}{C'_{66}}K^2+iKm\frac{C'_{56}}{C'_{66}}+\frac{\rho_1\omega^2}{C'_{66}}\right]V=0. \tag{11}$$

The solution of Eq. (11) is given by following expression:

$$V(y)=(A\cos Ty+B\sin Ty),$$

where

$$T^2=K^2\left[-\frac{m^2}{4K^2}+\left(\frac{C'_{56}}{C'_{66}}\right)^2-\frac{C'_{55}}{C'_{66}}+\frac{c^2}{\beta_1^{\,2}}\right]$$

with

$$\beta_1^{\,2}=\frac{C'_{66}}{\rho_1}.$$

Hence, for the upper monoclinic layer, the desired solution is given by the following expression:

$$u_1(y,z,t)=[A\cos Ty+B\sin Ty]e^{-a_1y/2}\,e^{i(Kz-\omega t)} \tag{12}$$

4 Solution for semi-infinite half space

For propagation of Love waves, we have

$$u=w=0 \text{ and } v=v(y,z,t). \tag{13}$$

The equation governing the propagation of Love waves in homogeneous isotropic elastic medium in the absence of body forces are as follows:

$$\frac{\partial}{\partial x}\tau_{xx}+\frac{\partial}{\partial y}\tau_{yx}+\frac{\partial}{\partial z}\tau_{zx}=\rho\frac{\partial^2 u}{\partial t^2},$$
$$\frac{\partial}{\partial x}\tau_{xy}+\frac{\partial}{\partial y}\tau_{yy}+\frac{\partial}{\partial z}\tau_{zy}=\rho\frac{\partial^2 v}{\partial t^2},$$
$$\frac{\partial}{\partial x}\tau_{xz}+\frac{\partial}{\partial y}\tau_{yz}+\frac{\partial}{\partial z}\tau_{zz}=\rho\frac{\partial^2 w}{\partial t^2}, \tag{14}$$

The stress-strain relations for general isotropic, elastic medium are

$$\tau_{ij}=\lambda\Delta\delta_{ij}+2\mu\varepsilon_{ij}, \tag{15}$$

where (λ, μ) are the Lame's constants and Δ is the dilatation.

$$\varepsilon_{ij}=\frac{1}{2}\left[\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right]. \tag{16}$$

Introducing Eqs. (4), (15) and (16) into Eq. (14), we obtain

$$\frac{\partial}{\partial y}\left(\mu\frac{\partial u_2}{\partial y}\right)+\frac{\partial}{\partial z}\left(\mu\frac{\partial u_2}{\partial z}\right)=\rho\frac{\partial^2 u_2}{\partial t} \tag{17}$$

For a wave propagating in the z-direction, we seek a solution of Eq. (17) in the form

$$u_2=W(y)\,e^{i(kz-wt)} \tag{18}$$

By inserting Eq. (18) into Eq. (17), we obtain

$$\frac{d^2W}{dy^2}+\frac{1}{\mu}\frac{d\mu}{dy}\frac{dW}{dy}+K^2\left(\frac{\rho c^2}{\mu}-1\right)W=0 \tag{19}$$

To eliminate $\frac{dW}{dy}$, we introduce $W=\frac{W_1}{\sqrt{\mu}}$ in Eq. (19), to obtain:

$$\frac{d^2W_1}{dy^2}-\frac{1}{2\mu}\frac{d^2\mu}{dy^2}W_1+\frac{1}{4\mu^2}\left(\frac{d\mu}{dy}\right)^2W_1+k^2\left(\frac{\rho c^2}{\mu}-1\right)W_1=0 \tag{20}$$

We assume variations in rigidity and density as follows:

$$\mu=\mu_2(1+ny)^2;\ \rho=\rho_2(1+ny)^2 \tag{21}$$

Introducing Eq. (21) in Eq. (20), we obtain

$$\frac{d^2W_1}{dy^2}-T_1^{\,2}W_1=0;$$
$$\text{where } T_1^{\,2}=K^2\left(1-\frac{c^2}{\beta_2^{\,2}}\right),\ \beta_2=\sqrt{\frac{\mu_2}{\rho_2}} \tag{22}$$

Thus, the solution of Eq. (22) is given by the following expression:

$$W_1=e^{T_1 y}+e^{-T_1 y}.$$

Hence, the desired displacement component for the non-homogenous half space is given by the following expression:

$$u_2(y,\ z,\ t)=\frac{Ce^{-T_1 y}}{1+ny}e^{i(Kz-wt)} \tag{23}$$

5 Boundary conditions

The boundary conditions are as follows:

(i) The upper monoclinic layer is stress-free, i.e. $T_6=0$, at $y=-H$;

$$C_{56}\frac{\partial u_1}{\partial z}+C_{66}\frac{\partial u_1}{\partial y}=0 \quad \text{at } y=-H, \tag{24}$$

(ii) The stresses are continuous at the common interface;

$$C_{56}\frac{\partial u_1}{\partial z}+C_{66}\frac{\partial u_1}{\partial y}=\mu_2\frac{\partial u_2}{\partial y} \quad \text{at } y=0, \tag{25}$$

(iii) The displacements are continuous at the common interface;

$$u_1=u_2 \quad \text{at } y=0, \tag{26}$$

Applying boundary conditions (24), (25) and (26) to Eqs. (12) and (23), the following system of equations is obtained:

$$A\left[C'_{66}T\sin TH+\left(C'_{56}ik-\frac{a_1}{2}C'_{66}\right)\cos TH\right]$$
$$+B\left[\left(-ikC'_{56}+\frac{a_1}{2}C'_{66}\right)\sin TH+C'_{66}T\cos TH\right]=0 \tag{27}$$

$$A\left[C'_{56}ik-\frac{a_1}{2}C'_{66}\right]+BC'_{66}T=-\mu_2(T_1+n)C, \tag{28}$$

$$A=C, \tag{29}$$

Finally, eliminating the constants A, B, C from Eqs. (27), (28) and (29), we obtain

$$\mathrm{Det}(D_{ij})=0,\ \text{where } i,\ j=1,\ 2,\ 3, \tag{30}$$

where

$$D_{11}=C'_{66}\ T\sin TH-\frac{m}{2}C'_{56}\cos TH;$$

$$D_{12}=\frac{m}{2}C'_{56}\sin TH+C'_{66}T\cos TH;\ D_{13}=0;$$

$$D_{21}=\frac{-m}{2}C'_{56};\ D_{22}=C'_{66}T;\ D_{23}=(T_1+n)\mu_2;$$

$$D_{31}=1;\ D_{32}=0;\ D_{33}=-1.$$

After simplification, Eq. (30) takes the form

$$\tan(TH)=\frac{A_1}{A_2} \tag{31}$$

where

$$A_1=T(T_1+n)\frac{\mu_2}{C'_{66}}$$

$$A_2=T^2+\frac{m^2}{4}-(T_1+n)\frac{\mu_2}{C'_{66}}\frac{m}{2}$$

Finally, introducing the values of T and T_1 in Eq. (31), we obtain

$$\tan\left(KH\sqrt{\left[-\frac{m^2}{4K^2}+\left(\frac{C'_{56}}{C'_{66}}\right)^2-\frac{C'_{55}}{C'_{66}}+\frac{c^2}{\beta_1^{\,2}}\right]}\right)=\frac{A_1}{A_2} \tag{32}$$

where

$$A_1=\frac{\mu_2}{C'_{66}}\left[\sqrt{1-\frac{c^2}{\beta_2^{\,2}}}+\frac{n}{k}\right]\sqrt{\left[-\frac{m^2}{4K^2}+\left(\frac{C'_{56}}{C'_{66}}\right)^2-\frac{C'_{55}}{C'_{66}}+\frac{c^2}{\beta_1^{\,2}}\right]}$$

$$A_2=\left(\frac{C'_{56}}{C'_{66}}\right)^2-\frac{C'_{55}}{C'_{66}}+\frac{c^2}{\beta_1^{\,2}}-\frac{\mu_2}{C'_{66}}\frac{m}{2K}\left[\sqrt{1-\frac{c^2}{\beta_2^{\,2}}}+\frac{n}{K}\right]$$

Here, Eq. (32) represents the dispersion equation for the propagation of SH waves in a non-homogeneous monoclinic layer lying over an isotropic non-homogeneous semi-infinite medium.

6 Particular cases

Case (I): When $C'_{66}=C'_{55}=\mu_1$, $C'_{56}=0$, Eq. (32) reduces to the form

$$\tan\left(KH\sqrt{-1+\frac{c^2}{\beta_1^{\,2}}}\right)=\frac{A_3}{A_4}, \tag{33}$$

where

$$A_3 = \frac{\mu_2}{\mu_1}\left[\sqrt{1-\frac{c^2}{\beta_2^{\,2}}+\frac{n}{K}}\right],$$

$$A_4 = \sqrt{-1+\frac{c^2}{\beta_1^{\,2}}}.$$

Here, Eq. (33) represents the wave velocity equation for propagation of SH waves in a non-homogeneous isotropic layer lying over an isotropic non-homogeneous semi-infinite medium.

Case (II): When $m=0,\ C'_{66}=C'_{55}=\mu_1,$ $C'_{56}=0,$ Eq. (32) takes the following form:

$$\tan\left(KH\sqrt{\frac{c^2}{\beta_1^{\,2}}-1}\right)=\frac{A_5}{A_6}, \tag{34}$$

where

$$A_5 = \frac{\mu_2}{\mu_1}\left[\sqrt{1-\frac{c^2}{\beta_2^{\,2}}+\frac{n}{K}}\right],$$

$$A_6 = \sqrt{\frac{c^2}{\beta_1^{\,2}}-1}.$$

Here, Eq. (34) represents the dispersion relation for the propagation of SH waves in an isotropic homogeneous layer lying over an isotropic non-homogeneous semi-infinite medium.

Case (III): When m=0, n=0,

$$C'_{66}=C'_{55}=\mu_1,\ C'_{56}=0,$$

Eq. (32) takes the following form:

$$\tan\left(KH\sqrt{\frac{c^2}{\beta_1^{\,2}}-1}\right)=\frac{A_7}{A_8}, \tag{35}$$

where

$$A_7 = \frac{\mu_2}{\mu_1}\left[\sqrt{1-\frac{c^2}{\beta_2^{\,2}}}\right],$$

$$A_8 = \sqrt{\frac{c^2}{\beta_1^{\,2}}-1}$$

Here, Eq. (35) represents the dispersion equation for propagation of SH waves in an isotropic homogeneous layer lying over an isotropic homogeneous semi-infinite medium, which is in complete agreement with the corresponding classical result for Love waves.

Case (IV): When m=0, Eq. (32) takes the following form:

$$\tan\left(KH\sqrt{\frac{c^2}{\beta_1^{\,2}}+\left(\frac{C'_{56}}{C'_{66}}\right)^2-\frac{C'_{55}}{C'_{66}}}\right)=\frac{A_9}{A_{10}} \tag{36}$$

where

$$A_9 = \frac{\mu_2}{C'_{66}}\left[\sqrt{1-\frac{c^2}{\beta_2^{\,2}}+\frac{n}{K}}\right],$$

$$A_{10} = \sqrt{\left[\left(\frac{C'_{56}}{C'_{66}}\right)^2-\frac{C'_{55}}{C'_{66}}\right]+\frac{c^2}{\beta_1^{\,2}}}.$$

Here, Eq. (36) represents the dispersion relation for the propagation of SH waves in a homogeneous monoclinic layer lying over an isotropic non-homogeneous semi-infinite medium.

Case (V): When m=0, n=0, Eq. (32) takes the following form:

$$\tan\left(KH\sqrt{\frac{c^2}{\beta_1^{\,2}}+\left(\frac{C'_{56}}{C'_{66}}\right)^2-\frac{C'_{55}}{C'_{66}}}\right)=\frac{A_{11}}{A_{12}}, \tag{37}$$

where

$$A_{11} = \frac{\mu_2}{C'_{66}}\left[\sqrt{1-\frac{c^2}{\beta_2^{\,2}}}\right],$$

$$A_{12} = \sqrt{\left[\left(\frac{C'_{56}}{C'_{66}}\right)^2-\frac{C'_{55}}{C'_{66}}\right]+\frac{c^2}{\beta_1^{\,2}}}.$$

Here, Eq. (37) represents the wave velocity equation for propagation of SH waves in a homogeneous monoclinic layer lying over an isotropic homogeneous semi-infinite medium, which is in complete agreement with the corresponding classical result given by Chattopadhyay et al. [13].

7 Numerical computations and discussion

To study the effects of various dispersion non-homogeneities on the propagation of SH waves propagating in a non-homogeneous monoclinic layer lying over a non-homogeneous semi-infinite media, the phase velocity is calculated numerically with the help of MATLAB for Eq. (32). We assume the following values for the constants:

For the monoclinic layer (Tierstein [20])

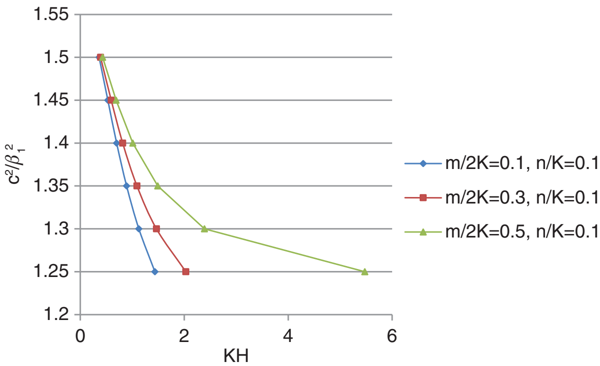

Figure 2: Variation of the dimensionless phase velocity $(c/\beta_1)^2$ against the dimensionless wave number KH, demonstrating the influence of non-homogeneity associated with the monoclinic crustal layer.

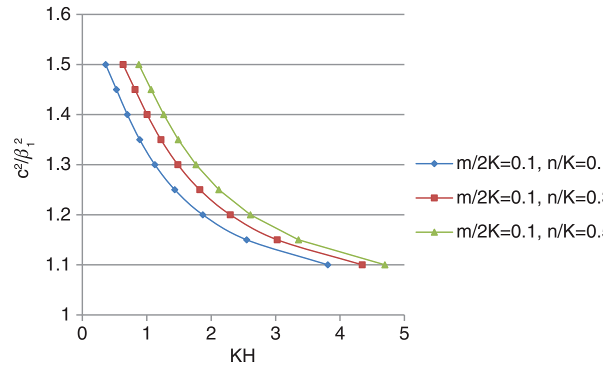

Figure 3: Variation of the dimensionless phase velocity $(c/\beta_1)^2$ against the dimensionless wave number KH, demonstrating the influence of non-homogeneity associated with the half-space.

$$C'_{55}=94\times10^9\ \mathrm{N/m^2},\ C'_{56}=-11\times10^9\ \mathrm{N/m^2},$$
$$C'_{66}=93\times10^9\ \mathrm{N/m^2},\ \rho_1=7450\ \mathrm{Kg/m^3}.$$

For the semi-infinite medium (Gubbins [21])

$$\mu_2=6.54\times10^{10}\ \mathrm{N/m^2},\ \rho_2=3409\ \mathrm{Kg/m^3}.$$

The effect of exponentially varying elastic parameters and density on SH waves in a non-homogeneous monoclinic crustal layer over a non-homogeneous half space is discussed in the following way by means of the respective graphs.

Figure 2 shows the effect of the non-homogeneity parameter m/2K measuring the rigidity of the monoclinic crustal layer when the non-homogeneity of the half space (i.e. rigidity and density varying quadratically with depth) is taken into consideration. The following observations and effects are notable and discussed below.

(a) For a particular dimensionless wave number KH and a fixed value of the non-homogeneity of the half space, i.e. n/K=0.1, the dimensionless phase velocity $(c/\beta_1)^2$ of SH waves increases, as the value of m/2K increases from 0.1 to 0.5.
(b) For various values of m/2K and a fixed value of n/K, the phase velocity $(c/\beta_1)^2$ increases as the wave number decreases in all curves 1–3.
(c) Curve 1 (for m/2K=0.1) is steeper than curve 2 (for m/2K=0.3) which, in turn, is steeper than curve 3 (for m/2K=0.5). This reveals that the dimensionless non-homogeneity factor m/2K has a prominent effect on SH wave propagation.
(d) Curve 1 (m/2K=1.0, n/K=0.1), Curve 2 (m/2K=0.3, n/K=0.1) and Curve 3 (m/2K=0.5, n/K=0.1) coincide as the wave number approaches 0.4.

Figure 3 shows the effect of the non-homogeneity parameter n/K accounting for the rigidity and density of the non-homogeneous half-space when the elastic parameters and density vary exponentially with depth. The following observations and effects are notable and discussed below.

(a) For a particular dimensionless wave number KH and a fixed value of the non-homogeneity of the layer, i.e. m/2K=0.5, the dimensionless phase velocity $(c/\beta_1)^2$ of SH waves increases, as the value of n/K increases from 0.1 to 0.5.
(b) For various values of n/K and a fixed value of m/2K, the phase velocity increases as the wave number decreases in all curves 1–3.
(c) Curve 1 (for n/K=0.1) is steeper than the curve 2 (for n/K=0.3) which, in turn, is steeper than curve 3 (for n/K=0.5). This reveals that the dimensionless non-homogeneity factor n/K has a prominent effect on SH wave propagation.

8 Conclusions

Here, we have studied the propagation of SH waves in a non-homogeneous monoclinic crustal layer lying over a non-homogeneous semi-infinite medium. Closed form solutions have been derived separately for the displacements in the monoclinic layer and the half-space. By using the asymptotic expansion of Whittaker's function, we have derived the wave velocity equation for SH waves in a compact form. The dimensionless phase velocity is calculated numerically with the help of MATLAB. The effect of various dimensionless elastic parameters and non-homogeneity factors on the dimensionless phase velocity $(c/\beta_1)^2$ have been shown graphically. Our main observations are listed below:

1. For various values of m/2K and fixed value of n/K, the phase velocity $(c/\beta_1)^2$ increases as the wave number decreases.
2. For a particular dimensionless wave number KH and a fixed value of the non-homogeneity parameter of half space, i.e. n/K, the dimensionless phase velocity $(c/\beta_1)^2$ of SH waves increases, as the value of m/2K increases.
3. For a particular dimensionless wave number KH and a fixed value of the non-homogeneity parameter of the layer, i.e. m/2K, the dimensionless phase velocity $(c/\beta_1)^2$ of SH waves increases, as the value of n/K increases.
4. In the absence of all non-homogeneities (in the density and rigidity of the monoclinic layer, as well as in the semi-infinite/half-space), the dispersion equation for the propagation of SH waves in a homogeneous monoclinic layer lying over an isotropic homogeneous semi-infinite medium is in complete agreement with the classical dispersion equation.
5. In the absence of all non-homogeneities (in density and rigidity and $C'_{66}=C'_{55}=\mu_1$, $C'_{56}=0$), the dispersion equation for the propagation of SH waves in an isotropic homogeneous layer lying over an isotropic homogeneous semi-infinite medium is in complete agreement with the classical dispersion equation of Love waves.

The wave propagation in crystalline media (monoclinic media) plays a very important role in geophysics and also in ultrasonic and signal processing. This study may be helpful in understanding the cause and estimate the damage due to earthquakes. This study may help in predicting the behavior of SH waves in non-homogeneous crystalline geological media.

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