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On the constitutive equations of fresh mortar

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Abstract: A preliminary constitutive model is suggested for fresh mortar by taking into account the rheological properties of the base matrix (concrete matrix), as well as the interaction of the aggregates (concrete suspensions). The model may be applicable for interventions in monuments and restoration/protection of objects of cultural heritage.

Keywords: constitutive equations; mortar; rheology; viscosity; yield stress.

1 Introduction

Rheology, defined as "the study of deformation and flow", provides a measure between shear stress and rate of deformation. The corresponding constitutive equation can be employed to describe mathematically the flow of fresh concrete. Concrete composed of cement particles, aggregates, water and air, can be characterized as suspended solid particles (aggregate) in viscous media (cement paste) [1–5]. Numerous constitutive equations have been proposed to characterize the rheology of fresh concrete as suspensions, but only Bingham model and Herschel and Bulkley (HB) model have received wide acceptance. For normal concrete, experimental data have confirmed that the flow of fresh concrete follows Bingham's material model, i.e.:

$$\tau=\tau_0+n\dot{\gamma} \tag{1}$$

In which τ is the shear stress (Pa), τ_0 the yield stress (Pa), n the plastic viscosity (Pa s), and $\dot{\gamma}$ the shear strain rate (1/s). τ_0 and $\dot{\gamma}$ are referred to as Bingham material properties with the first property providing a measure of the shear stress required to initiate flow and the second one a measure of the material resistance to flow after the material begins to flow. These two rheological properties are therefore needed to quantitatively characterize the flow of fresh concrete. Murata and Kikukawa [6] implemented Roscoe's [7] equation to quantify the plastic viscosity of concrete, and proposed the following methodology: Calculate the plastic viscosity of cement paste by postulating that cement particles are suspended in water, i.e. there are no physical or chemical interactions between the cement particles and water. Then by recognizing that Roscoe's equation was developed with the premise that the particles are solid, spherical, and identical in shape and size, they proposed an extension to account for the irregularly shaped and non-uniform size of the particles. They proposed the following relation

$${}^{i}n_r=\frac{{}^{i}n}{{}^{i}n_0}=\left(1-\frac{{}^{i}\varphi}{{}^{i}C}\right)^{-{}^{i}k} \tag{2}$$

where ${}^{i}k={}^{i}k_1\,\varphi+{}^{i}k_2$, superscript "$i$" is set equal to 1 for cement paste, 1gr becomes the relative plastic viscosity of cement paste, 1g the plastic viscosity of cement paste, 1g0 the plastic viscosity of water, 1u the volumetric concentration of cement, 1C the percentage of absolute volume of cement, and 1k the coefficient of agglomerated cement particles and coefficients ${}^{i}k_1$, ${}^{i}k_2$ are constant and are found through regression.

2 Mortar

Models proposed in the literature characterizing the flow of fresh mortar are cited in Lu et al. [1], Epsing [8], and Hu [9]. However, the majority of these models are phenomenological. In this research it is postulated that the flow can be represented by three interactions: static interaction between particles, dynamic interaction between particles, and collision between particles and that these three interactions are independent. Accordingly, the model can be represented by

$$\tau=\tau_0+\tau_{DI}+\tau_{\text{collisions}} \tag{3}$$

where τ_0 is the shear stress due to static interaction between the particles, τ_{DI} is due to dynamic interaction between the particles, and $\tau_{\text{collisions}}$ is due to collisions of the particles. Toward the development of a fundamental constitutive rheological model for mortar, Lu et al. [1]

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assumed that the particles were rigid, non-cohesive, and well distributed and that the amount of air was negligible. They have, also, accounted for the high concentration of suspended particles, the different size and shape of the particles, and the interaction and collision of the particles during flow which are necessary requirements to afford a representative description of the flow of fresh mortar [1].

3 Existing constitutive equations for fresh mortar

The proposed model for characterizing the flow of fresh mortar postulates that the shear stress arises due to different causes and the components are additive [1]. Accordingly, shear stress is the sum of stresses due to static interaction, dynamic interaction, and particle collisions.

3.1 Yield stress

Yield stress, which is one of Bingham's rheological properties, is the term that accounts for the static interaction between the particles. Yield stress proposed by Chidiac et al. [10] is adopted for this study and is given by

$$\frac{\tau(y)}{\tau_i}=\frac{4(y^3-y^7)}{4-25y^3+42y^5-25y^7+4y^{10}} \tag{4}$$

where the function y is dependent on the volume fraction of the solid material, i.e.

$$y(\varphi)=\left(\frac{\varphi}{\varphi_{\max}}\right)^{\frac{1}{3}}\left(1-\alpha\left(\frac{m_c}{m_w}+\rho_w V_{\text{air}}\right)\right) \tag{5}$$

where τ_i is the "intrinsic" yield stress and is a function of the shape of the particles; $y(\varphi)$ is the ratio of particle size to cell size; φ is the volumetric fraction of solid material refers to packing density; $\varphi_{\max}$ is the maximum packing density of the whole mixture; m_c and m_w are the mass of gravel and water of the mixture, respectively; ρ_w is the density of water; V_{air} is the volume of air; and α is a fitting parameter.

3.2 Particle interactions

Lu et al. [1] postulated that the interaction between two adjacent particles can be mathematically represented by the model shown in Figure 1. Accordingly, the effect of a two particle interaction takes the following form:

$$\tau_{DI}=n_p\left(1+\frac{y}{1+y}\right)\dot{\gamma} \tag{6}$$

where

$$y=\frac{3\varphi_A}{1+1.65\varphi_A} \tag{7}$$

where n_p is the plastic viscosity of cement paste and φ_A is the packing density of aggregate. However, for mortar there are more than two particles that are interacting at one time. To account for multi-particle interaction, the cell method developed by Chidiac et al. [10] is employed.

The concept consists of a rigid particle surrounded by fluid. By accepting the cell as a representative volume, it implies that the particles, which are located at the center, do not come in contact with each other and that the particle interaction is limited to the interaction of the cells. Accordingly, Chidiac et al. [10] have developed an equivalent model that accounts for cell interaction, and is given by

$$\frac{\tau_{DI}(y)}{4n_w n_i}=\frac{y^3-y^7}{4-25y^3+42y^5-25y^7+4y^{10}} \tag{8}$$

where

$$y(\varphi)=\left(\frac{\varphi}{\varphi_{\max}}\right)^{\frac{1}{3}}\left(1-\beta\frac{m_c}{m_w}\right) \tag{9}$$

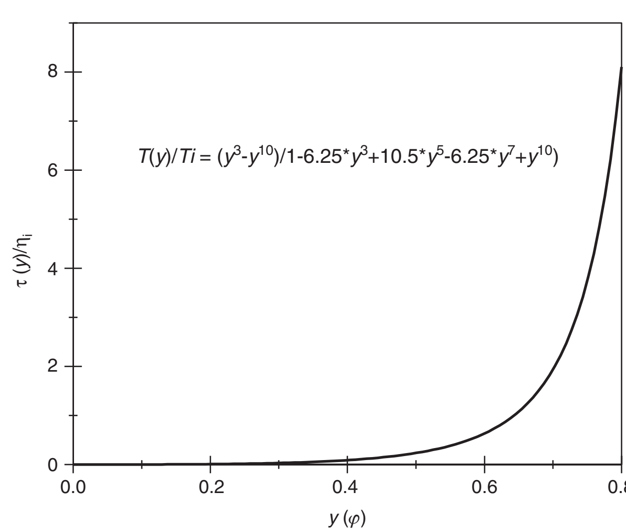


Figure 1: Plot of $\tau(y)/\tau_i$ vs. $y(\varphi)$ relation.

where n_w is the viscosity of water, n_i is the intrinsic viscosity and is a function of the particle shape, m_c is the mass of cement in the mixture, m_w is the mass of water and β is a fitting parameter. Here we did not count for the air volume in the mixture. In this study no collisions are taken in the account.

The corresponding relation between yield stress and their parameter *w/c* is given in Figure 2.

4 Evaluation of the mortar constitutive equations

Evaluation of the model consists of two parts: the first part is to validate the rheological properties including viscosity and yield stress based on experimental measurements carried out by Ferraris and deLarrard [11], and the second part is to evaluate the ability of the constitutive equation to characterize the mortar flow using experimental work reported by Hu [9].

4.1 Rheological properties

Ferraris and deLarrard [11] carried out an extensive testing program to measure the rheological properties of mortar and concrete. The data corresponding to normal slump concrete were used to demonstrate the adequacy of the proposed model to predict the rheological properties of concrete [3]. In this study, the data corresponding to normal mortar are used to evaluate rheological properties of mortar. Details of the experimental program are reported in Ferraris and deLarrard [11] and the experimental results are shown in Figure 3.

4.2 Constitutive flow of mortar

An experimental study was conducted by Hu [9] to investigate the flow characteristics of fresh mortar. The mortar mixture was composed of type I Portland cement, river-sand fine aggregate, and water. The mixture proportions were 0.4 water to cement ratio, and 2.0 sand to cement ratio. The particle size ranged between 0.6 mm and 1.18 mm. A Brookfield rheometer was used to measure shear stress versus shear strain rate. The loading and unloading sequences, shown in Figure 3, indicate an initial pre-shear cycle prior to the commencement of the test. The test includes a loading cycle, referred to as up-curve, and an unloading cycle, referred to as down-curve. The rate is constant for both the up and down curve. It should be noted that the loading and unloading sequences shown in Figure 3 do not permit the determination of shear stress growth which is the maximum stress from rest and is equal to the static yield stress. The equivalent term in the proposed model is t_o, the shear stress due tostatic interaction between the particles. Experimental results from Hu [9] are shown in Figure 3. An understanding of these results is merited prior to the application of the proposed constitutive equations. The results show a jump in the shear stress at the onset of the up-curve before quickly decreasing to shear stress values in the ramp of 500 Pa. Subsequently, the experimental results indicate a small increase in shear stress as the strain rate went from 20 s^{-1} to 100 s^{-1}. Although the pre-shear cycle was intended to break-down the structure, the recorded response indicates a significant resistance to aggregate movement as they move through the

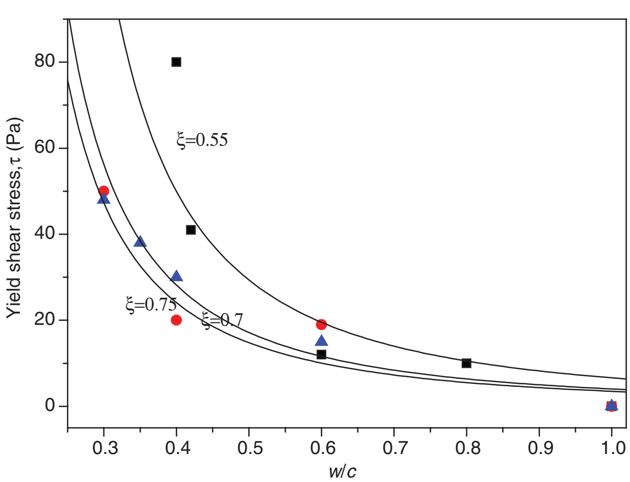


Figure 2: Plot of yield stress as a function of the water-to-cement ratio *w/c*.

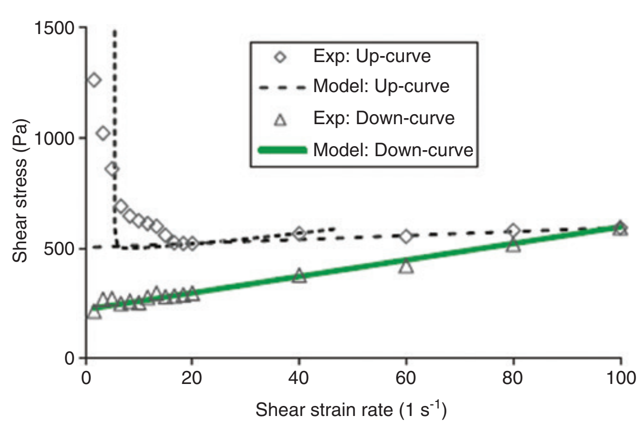


Figure 3: Shear stress vs. shear strain rate fitting with experimental data.

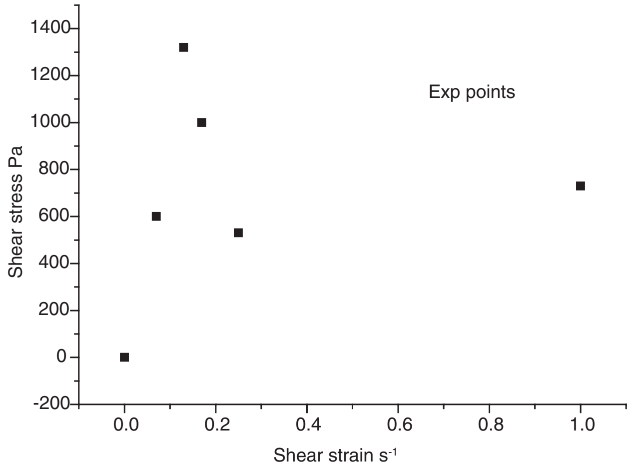


Figure 4: Experimental data of shear stress vs. shear strain (Hu [9]).

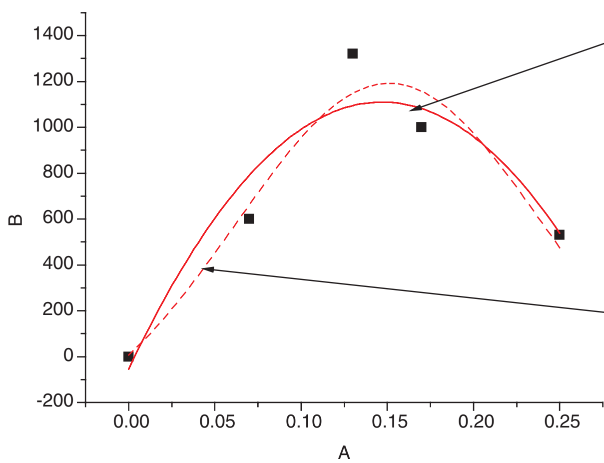
cement paste at low speed. The scientific interpretation of these results suggests that the Reynolds number is low at the onset of the test, and thus the drag force is proportional to the velocity of the aggregates and plastic viscosity of the cement paste.

5 Proposed model for mortar

The proposed model for the shear stress as a function of the shear strain is based on the distribution

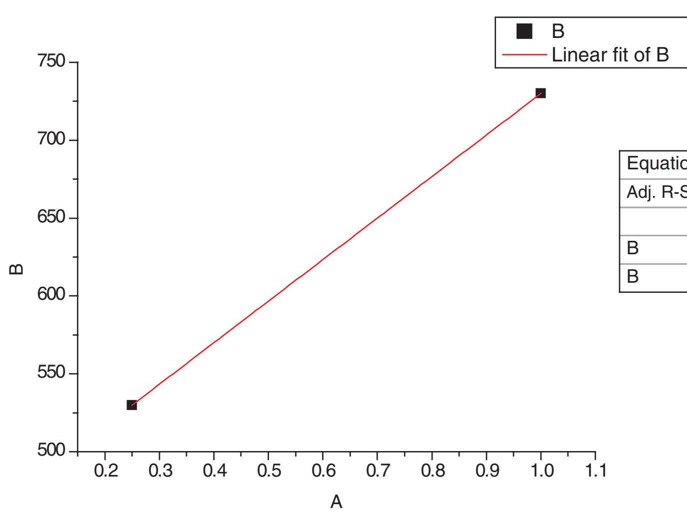
$$\tau(\gamma)=B+A\exp(-\alpha(\gamma-\gamma_1))^2 \tag{10}$$

where the constants A, B are computed from the following equations



Model	Polynomial		
Adj. R-Square	0.80684		
		Value	Standard error
B	Intercept	-55,25915	209,71855
B	B1	15858,58241	3736,71334
B	B2	-53940,15321	14276,35263

Equation	y=y0 + (A/(w*sqrt(PI/2)))*exp(-2*((x-xc)/w)^2)		
Adj. R-Square	0.75649		
		Value	Standard error
B	y0	-353,57209	1088,97209
B	xc	0,15116	0,01521
B	w	0,17717	0,11403
B	A	343,1401	442,04923
B	sigma	0,08859	
B	FWHM	0,20861	
B	Height	1545,30455	



Equation	y=a+b*x		
Adj. R-Square	--		
		Value	Standard error
B	Intercept	463,3333	--
B	Slope	266,6666	--

Figure 5: Fitting of the experimental data with the proposed model.

$$A = \frac{\tau_{\lim}}{1\text{-}\exp(-\alpha\gamma_1^2)} \qquad B = -A\exp(-\alpha\gamma_1^2) \tag{11}$$

In the above equations the parameters $\tau_{\lim}$ and γ_1 indicate the limited value of the shear stress and shear strain at the point where the experimental points change to linear part, i.e. in the experimental data (Figure 4) $\tau_{\lim}$=550 Pa and γ_1=0.25 s^{-1}. The experimental data proposed by Hu [9] are shown in Figure 4.

The fitting of the experimental points with the proposed model is shown in Figure 5.

6 Conclusions

In this study we proposed a new model for predicted the experimental data published by Hu [9] for the shear stress vs. shear rate for mortar. The model predicts well the experimental points up to point where the curve starts to become linear. The parameters are computed using nonlinear fitting procedure.

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