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Filtration model of plastic flow

Abstract: A filtration model for plastic flow based on the idea of a deformed material considered as a two-phase heterogeneous medium has been suggested. In this approach, the wave displacement is regarded as a shock transition in the medium. One of the phases (the excited one) is responsible for system restructuring, and the other phase (the normal one) is unrelated to structural transformations. The plastic wave is the result of the interaction of these two phases. The governing equations for the filtration model are obtained. They include the laws of momentum and mass conservation, as well as the filtration ratio of the phases.

Keywords: filtration model; heterogeneous medium; plastic deformation; plastic wave.

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1 Introduction

One of the major issues in the physics of strength and plasticity is the explanation of the observed inhomogeneities of plastic flow in materials, as well as its evolution and corresponding stages observed during experiments [1, 2]. To date, methods used in modern physical materials science such as scanning and transmission electron microscopy, as well as double-exposure speckle-interferometry, have shown that the process of plastic deformation is of wave nature [3–5]. This is supported by observed strain-stress distributions at the boundary “surface layer-substrate” in a “staggered” order (the “checkerboard” effect) [3, 4], along with the observed nonuniform distribution of displacement fields and deformation [5]. These facts indicate that there are regimes or zones in the material that are not involved in the evolving plastic deformation. The characteristic micro- and macro-scales of these inhomogeneities according to Ref. [5] can range from ~1 μm to ~1 mm. The observed corresponding stages of plastic deformation are due to the changing nature of deformation localization and increase in the number of equidistant localization sites at the stages of linear and parabolic hardening, whereas at the stage of prefracture the collapse of plasticity wave occurs.

Studies of the dislocation substructures [6, 7] at various stages of plastic deformation indicated that the transition from one stage to another is accompanied by the transformation of one type of substructure to another and during the transition process two types of substructures can exist simultaneously. The combination of these experimental facts leads to the conclusion that the cause of the observed regularities of plastic flow is the collective nature of the changes of the internal structure [8–12]. To describe this type of plastic deformation, ideas from the mechanics of heterogeneous media can be applied [13]. Such an approach was also independently adopted in Ref. [14] for the study of phase transitions, plastic deformation, and other structural transformations in solids. The peculiarity of this approach is the split of the entire ensemble of the structural elements of the medium (atoms, defects, etc.) into two subsystems: the excited one, responsible for the system restructuring and the normal one which remains unexcited and not related to structural transformations. After such splitting, the resulting heterogeneous mixture is represented by a set of several continua (phases), each of which is described by the respective conservation laws and constitutive equations. The proposed model in this article provides an explanation of nonuniform distribution of displacements under uniaxial deformation [5] using the laws of momentum and mass conservation. As discussed in Ref. [15], plastic deformation of polycrystals occurs due to “microshifts” and “macroshifts” that emerge as a current of a fast-moving phase in between weakly deformable and inactive blocks. This can be viewed as a current in a two-phase heterogeneous mixture. The first component is identified with the microshifts, and the second one with the macroshifts. Balance laws and constitutive equations can then be mathematically expressed by the following set of conservation laws for the mass and momentum of the two phases.

$$\frac{\partial \rho_1}{\partial t} + \operatorname{div} \rho_1 \mathbf{w} = I_{21}; \quad (1)$$

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$$\rho_1\frac{d_1\mathbf{w}}{dt}=\mathrm{div}\boldsymbol{\sigma}_1+\mathbf{p}_{21}-I_{21}\mathbf{w};\ \frac{d_1}{dt}=\frac{\partial}{\partial t}+\mathbf{w}\cdot\nabla \tag{2}$$

$$\frac{\partial\rho_2}{\partial t}+\mathrm{div}\rho_2\mathbf{u}=I_{12}; \tag{3}$$

$$\rho_2\frac{d_2\mathbf{u}}{dt}=\mathrm{div}\boldsymbol{\sigma}_2+\mathbf{p}_{12}-I_{12}\mathbf{u};\ \frac{d_2}{dt}=\frac{\partial}{\partial t}+\mathbf{u}\cdot\nabla \tag{4}$$

where $\rho_1=\alpha\rho_e$, $\rho_2=(1\text{-}\alpha)\rho_s$ and $\boldsymbol{\sigma}_1=\alpha\boldsymbol{\sigma}$, $\boldsymbol{\sigma}_2=(1\text{-}\alpha)\boldsymbol{\sigma}$. The quantities ρ_e and ρ_s are the true densities of phases; α is the volume fraction of the first phase; $\boldsymbol{\sigma}$ is the total tensor stress of the whole mixture; $\mathbf{p}_{21}=\text{-}\mathbf{p}_{12}$ and $I_{21}=\text{-}I_{12}$ denote the exchange intensities of momentum and mass; whereas $\mathbf{w}$ and $\mathbf{u}$ are the velocities of the first and the second phases, respectively. The intensity of the pulse exchange between the phases can be represented as $\mathbf{p}_{21}=\mathbf{R}_{21}+I_{21}\mathbf{u}_{21}$, where $\boldsymbol{R}_{21}$ is the interphase force associated with friction and other interaction forces, and $I_{21}\mathbf{u}_{21}$ is another force associated with flow and phase transformations. We assume that the intensity of mass transfer is small compared to the intensity of the momentum exchange and the mixture components interact according to Rakhmatulin's scheme [16].

Consequently, $\mathbf{p}_{21}=\mathbf{R}_{21}$ and $\mathbf{R}_{21}=\text{-}p\nabla\alpha+\mathbf{F}_{21}$, where the force $\mathbf{F}_{21}=\frac{1}{K}\alpha\rho_e(1\text{-}\alpha)(\mathbf{u}\text{-}\mathbf{w})$ is associated with the high-speed nonequilibrium phases, with K being a constant. In view of all the above facts, the system of Eqs. (1)–(4) takes the form:

$$\rho_1\frac{d_1\mathbf{w}}{dt}=\alpha\mathrm{div}\boldsymbol{\sigma}+\alpha\rho_e(1\text{-}\alpha)(\mathbf{u}\text{-}\mathbf{w})/K \tag{5}$$

$$\frac{d_1\rho_1}{dt}+\rho_1\mathrm{div}\mathbf{w}=0 \tag{6}$$

$$\rho_2\frac{d_2\mathbf{u}}{dt}=(1\text{-}\alpha)\mathrm{div}\boldsymbol{\sigma}\text{-}\alpha\rho_e(1\text{-}\alpha)(\mathbf{u}\text{-}\mathbf{w})/K \tag{7}$$

$$\frac{d_2\rho_2}{dt}+\rho_2\mathrm{div}\mathbf{u}=0 \tag{8}$$

In Eq. (5) we assume that the inertia term $\rho_1\frac{d_1\mathbf{w}}{dt}\approx 0$; then adding Eqs. (5) and (7) leads to the following relation:

$$\alpha div\boldsymbol{\sigma}=\text{-}\alpha(1\text{-}\alpha)\rho_e(\mathbf{u}\text{-}\mathbf{w})/K. \tag{9}$$

Equation (9) is a consequence of the law of momentum conservation for the first phase and may be viewed as being analogous to Darcy's law in the filtration theory [13]. The meaning of the constant $1/K$ is that it is a factor of resistance to movement of the first phase within the second.

The system of Eqs. (5)–(8) must be closed by the equation of state. As the second phase consists of weakly deformable blocks, we may take ρ_s=constant. For the first phase, we assume that $\rho_e=F(P)$ with P denoting pressure. Now let us consider the problem within a one-dimensional setting, by also assuming that the overall stress in a heterogeneous mixture depends on the pressure $\sigma=-P$. Then Eqs. (5)–(8) along with Eq. (9) and the respective equations of state will give:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=\text{-}\frac{1}{(1\text{-}\alpha)\rho_s}\frac{\partial P}{\partial x}; \tag{10}$$

$$\frac{\partial\alpha}{\partial t}+u\frac{\partial\alpha}{\partial x}=(1\text{-}\alpha)\frac{\partial u}{\partial x}; \tag{11}$$

$$\frac{\partial\rho_1}{\partial t}+\frac{\partial\rho_1 w}{\partial x}=0 \tag{12}$$

2 Results and discussion

We seek a solution in the form of a traveling wave $\alpha(x\text{-}u_0t)$, $u(x\text{-}u_0t)$, $w(x\text{-}u_0t)$, $\rho_1(x\text{-}u_0t)$, $P(x\text{-}u_0t)$. Then

$$u'(u\text{-}u_0)=\text{-}\frac{1}{(1\text{-}\alpha)\rho_s}P' \tag{13}$$

$$\alpha'(u\text{-}u_0)=(1\text{-}\alpha)u' \tag{14}$$

$$\text{-}u_0\rho_1'+(\rho_1 w)'=0 \tag{15}$$

where the prime denotes derivative with respect to corresponding traveling wave variables.

The first integrals of Eqs. (13)–(15) are:

$$\alpha=1\text{-}\frac{C_1}{u\text{-}u_0} \tag{16}$$

$$P=(C_2\text{-}C_1\rho_s u) \tag{17}$$

$$\alpha\rho_e\left(u\text{-}u_0\text{-}\frac{KP'}{B(1\text{-}\alpha)\rho_e}\right)=C_3 \tag{18}$$

Transforming Eq. (18) with the help of Eqs. (16) and (17) and using the variable $\bar{u}=u\text{-}u_0$, we obtain the following equation containing the rate of the second phase:

$$\frac{d\bar{u}}{d\eta}=\frac{C_3\text{-}(\bar{u}\text{-}C_1)\rho_e}{\bar{u}\text{-}C_1} \tag{19}$$

where $d\eta=\frac{d\xi}{K\rho_s}$. Next, we consider the case $\rho_e=AP$ [17], for which

$$\frac{d\bar{u}}{d\eta}=\frac{(\bar{u}-\bar{u}_1)(\bar{u}-\bar{u}_2)}{\bar{u}-C_1}, \tag{20}$$

where $\bar{u}_1$, and $\bar{u}_2$ denote the velocities of the second phase on the boundary of the localized zone. Integration of this equation leads to:

$$\left(\frac{C_1-\bar{u}_2}{\bar{u}_1-\bar{u}_2}\right)\ln(\bar{u}-\bar{u}_2)-\left(\frac{C_1-\bar{u}_1}{\bar{u}_2-\bar{u}_1}\right)\ln(\bar{u}-\bar{u}_1)=d\eta+C \tag{21}$$

To determine the constants involved in Eqs. (20) and (21), the following boundary conditions are used:

$$\bar{u}(0)=\bar{u}_1,\ \bar{u}(L)=\bar{u}_2,\ \bar{u}'(0)=0,\ \bar{u}'(L)=0,\ \alpha(0)=\alpha_1,\ \alpha(L)=\alpha_2 \tag{22}$$

Then, by Eq. (22), the first integrals will take the form:

$$\begin{aligned}&(1-\alpha_1)\bar{u}_1=C_1\\&P_1=C_2-C_1\rho_s\bar{u}_1\\&-\bar{u}_1^2C_1\rho+(C_1^2\rho+C_2)\bar{u}_1-C_3-C_1C_2=0\\&(1-\alpha_2)\bar{u}_2=C_1\\&P_2=C_2-C_1\rho_s\bar{u}_2\\&-\bar{u}_2^2C_1\rho+(C_1^2\rho+C_2)\bar{u}_2-C_3-C_1C_2=0\end{aligned} \tag{23}$$

Returning to Eqs. (21) and (23) by using the variable u, we construct the speed plotted for the second phase for the case $u_1>u_2$ and $\alpha_1<\alpha_2$ with respect to coordinate x at various times (Figure 1). This clearly shows that a kind of "shock transition" occurs. Consequently, there are areas of the deformable material, which are not involved in the plastic deformation. This is confirmed by experimental observation [5]. The speed of the containment chamber may be defined as $u_0=\frac{(\alpha_1-1)u_1+(1-\alpha_2)u_2}{\alpha_1-\alpha_2}$. If $u_1=0$ and $u_2=u_*$, where u_* is the velocity of the traverse beam, the values of the marginal rate of localization exceeds the rate of the traverse beam in the testing machine, which is also consistent with the experiment. The case $u_1<u_2$ and $\alpha_1>\alpha_2$ also allows the existence of shock transition. Note that similar relationships were obtained in Ref. [18] for fixed dynamical structures, and in Ref. [19] for a shock wave in an ideal gas.

We define the width of the shock transition with a relation having the following form: $l=\frac{u_1-u_2}{\max\left(\frac{du}{dx}\right)}$. The evaluation of this magnitude shows that it has the value of ~10 μm, which coincides with the characteristic length scales of heterogeneity observed in the experiment. Also note that the free path of dislocation motion in materials is of the same order of magnitude [20].

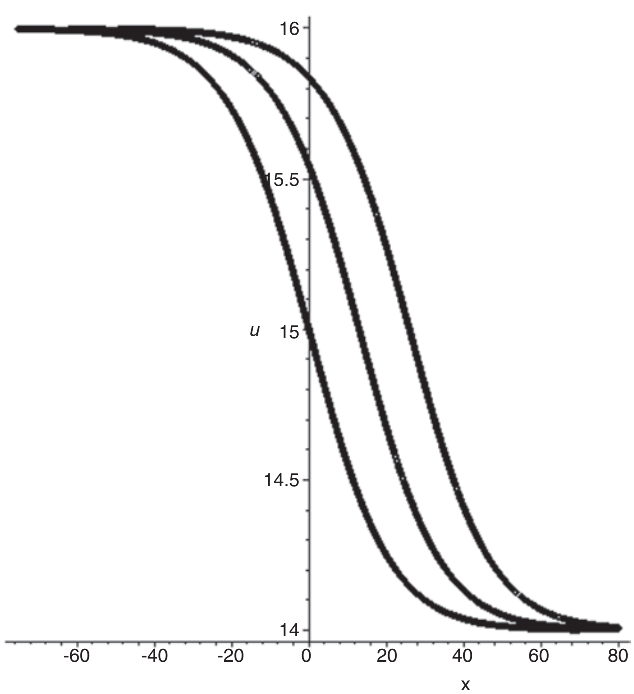

Figure 1 Velocity of the second phase with respect to the spatial coordinates at various points of time (1–t=0, 2–t=1, 3–t=2).

3 Conclusions

1. The system of governing equations of the filtration plasticity model is provided. A solution is obtained in the form of shock transition. Its width coincides with the characteristic values of the scale of the inhomogeneity of deformation.
2. It is shown that the maximum speed localization front exceeds the rate of crosshead of the testing machine, which corresponds to the experimental observations.
3. In this article, emphasis has been placed mainly on Russian literature on the topic as this is not well known in the West. It is noted, in thin connection, that excessive literature on this topic of considering a generalized continuum medium as a superposition of "normal" and "excited" states was advanced by the last author and his coworkers in a series of publication [21–32].

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