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Surface waves in homogenous visco-elastic media of higher order under the influence of surface stresses

Abstract: The aim of the present paper is to investigate the surface waves in a homogeneous, isotropic, visco-elastic solid medium of n^{th} order, including time rate of strain under the influence of surface stresses. The theory of generalized surface waves is developed to investigate particular cases of waves such as the Stoneley, Rayleigh, and Love waves. Corresponding equations have been obtained for different cases. These are reduced to classical results, when the effects of surface stresses and viscosity are ignored.

Keywords: surface stresses; surface waves; visco-elasticity.

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1 Introduction

The propagation of surface waves in elastic media is of considerable importance in earthquake engineering and seismology due to the stratification in the earth's crust. As a result, the theory of surface waves has been developed by Stoneley [1], Bullen [2], Ewing et al. [3], Hunters [4], and Jeffreys [5].

The effect of gravity on wave propagation in an elastic solid medium was first considered by Bromwich [6] who treated gravity as a type of body force. Love [7] extended the work of Bromwich [6] in investigating the influence of gravity on surface waves and showed that the Rayleigh wave velocity may be affected significantly by the gravity field. Sezawa [8] studied the dispersion of elastic waves propagating on curved surfaces.

The transmission of elastic waves through a stratified solid medium was first studied by Thomson [9]. Haskell [10] examined the dispersion of surface waves in multilayered media. Biot [11] studied the influence of gravity on Rayleigh waves, assuming the force of gravity to create a type of initial stress of hydrostatic nature and the medium to be incompressible. De and Sengupta [12] examined several problems of elastic waves and vibrations under the influence of gravity field. In another work, Sengupta and Acharya [13] studied the influence of gravity on the propagation of waves in a thermoelastic layer. Brunelle [14], meanwhile, analyzed surface wave propagation under initial tension or compression. Roy [15] studied wave propagation in a thin two-layered laminated medium (with stress couples) under initial stress, while Datta [16] studied the effect of gravity on Rayleigh wave propagation in a homogeneous, isotropic elastic solid medium. Goda [17] examined the effect of inhomogeneity and anisotropy on Stoneley waves, while Abd-Alla and Ahmed [18] studied the Rayleigh waves in an orthotropic thermoelastic medium under gravity field and initial stress. Bland [19], Flugge [20], and Voigt [21] all analyzed wave propagation in visco-elastic media. Recently, Sethi and Gupta [22] studied surface waves in non-homogeneous, general visco-elastic media of higher order.

Gurtin and Murdoch [23], Chandrasekharaiah [24], and other authors [25–28] all reported that surface stress plays a vital role in the propagation of waves due to the fact that the surface of a body exhibits properties that are quite different than those associated with the interior of the medium. In fact, surface tension, which is generally accounted for in the theory of liquids may be considered as a particular case of surface stress. The presence of surface stress on the boundary of bodies has been detected in some particular type of crystals, whose orders of magnitude agree with the predictions made by molecular theory [23]. Compressive surface stress is involved in the case of shot peening of ductile metals [23], and its knowledge is quite useful for the shaping of aircraft wing panels.

A few problems on the propagation of plane waves in homogeneous and isotropic materials have been considered [23]. Though the concept of surface stress is comparatively new, a few authors [24, 25] investigated problems based on the effect of surface stress. Pal et al. [26], in particular, investigated the effect of surface stress on the propagation of surface waves.

In the present paper, the problem of n^{th} order visco-elastic surface waves involving time rates of strain in a homogeneous and isotropic medium, under the influence of surface stresses, is studied. Biot's theory of incremental deformations is used to obtain the wave velocity equation for Stoneley, Rayleigh and Love waves. These equations are in complete agreement with the corresponding classical results in the absence of surface stresses and viscosity.

2 Formulation of the problem

Consider M_1 and M_2 to be two homogeneous, visco-elastic, isotropic, semi-infinite media welded in contact to prevent any relative motion or sliding before or after the occurrence of any disturbance. Suppose that the media are separated by a plane horizontal boundary, which extends to an infinite large distance from the origin, M_2 is taken to be above M_1, and the mechanical properties of M_1 are different from those of M_2.

As a reference coordinate system, we consider a set of orthogonal cartesian axes Oxyz, with the origin O being at an arbitrary point on the boundary, and Oz pointing outward normal to M_1 (Figure 1). Consider the possibility of a type of wave traveling in the positive x-direction, in such a way that the disturbance is largely confined to the neighborhood of the boundary and, at any instant, all particles in any line parallel to Oy have equal displacement and all partial derivatives with respect to y are zero. These two assumptions suggest that the wave is a surface wave.

Further let us assume that "u, v, w" are the components of displacement at any point (x, y, z) at any time t. The dynamical equations of motion for a three-dimensional isotropic, visco-elastic solid medium (e.g., Biot' [11]) are as follows:

$$\frac{\partial\tau_{11}}{\partial x}+\frac{\partial\tau_{12}}{\partial y}+\frac{\partial\tau_{13}}{\partial z}=\rho\frac{\partial^2 u}{\partial t^2}, \tag{1a}$$

$$\frac{\partial\tau_{12}}{\partial x}+\frac{\partial\tau_{22}}{\partial y}+\frac{\partial\tau_{23}}{\partial z}=\rho\frac{\partial^2 v}{\partial t^2}, \tag{1b}$$

$$\frac{\partial\tau_{13}}{\partial x}+\frac{\partial\tau_{23}}{\partial y}+\frac{\partial\tau_{33}}{\partial z}=\rho\frac{\partial^2 w}{\partial t^2}, \tag{1c}$$

where ρ is the density of the material medium, and $\tau_{ij}=\tau_{ji}$ are the stress components.

The stress-strain relations for a general isotropic, visco-elastic medium are assumed to be given by the following:

$$\tau_{ij}=D_\lambda\,\Delta\,\delta_{ij}+2\,D_\mu\,e_{ij}\,, \tag{2}$$

where

$$D_\lambda=\sum_{K=0}^{n}\lambda_K\frac{\partial^K}{\partial t^K},\quad D_u=\sum_{K=0}^{n}\mu_K\frac{\partial^K}{\partial t^K},$$
$$D'_\lambda=\sum_{K=0}^{n}\lambda'_K\frac{\partial^K}{\partial t^K},\quad D'_\mu=\sum_{K=0}^{n}\mu'_K\frac{\partial^K}{\partial t^K}, \tag{3}$$
$$\Delta=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z},\ \nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}.$$

The coefficients λ_0, μ_0, λ'_0, μ'_0, are constants and λ_K, μ_K (K=1, 2, ... n) are the parameters associated with K^{th} order visco-elasticity.

Introducing Eqs. (2) and (3) to Eqs. (1a), (1b) and (1c), we obtain:

$$(D_\lambda+D_\mu)\frac{\partial\Delta}{\partial x}+D_\mu\nabla^2 u=\rho\frac{\partial^2 u}{\partial t^2}, \tag{4a}$$

$$D_\mu\nabla^2 v=\rho\frac{\partial^2 v}{\partial t^2}, \tag{4b}$$

$$(D_\lambda+D_\mu)\frac{\partial\Delta}{\partial z}+D_\mu\nabla^2 w=\rho\frac{\partial^2 w}{\partial t^2}. \tag{4c}$$

To investigate the propagation of a surface wave along the direction of Ox, we introduce the displacement potentials ϕ (x, z, t) and ψ (x, z, t), which are related to the displacement components as follows:

$$u=\frac{\partial\phi}{\partial x}-\frac{\partial\psi}{\partial z},\ w=\frac{\partial\phi}{\partial z}+\frac{\partial\psi}{\partial x}. \tag{5}$$

The displacement potentials ϕ and ψ in the above equations are two distinct "potentials", whose Laplacians specify the dilatation and rotation given by:

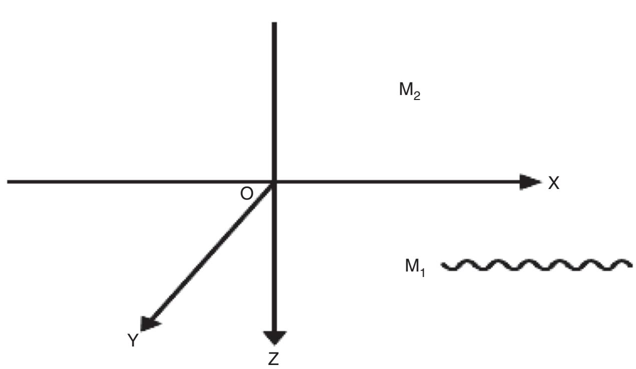

Figure 1 Two media, M_1 and M_2, in contact.

$$\nabla^2\phi=\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=\Delta,\quad \nabla^2\psi=\frac{\partial w}{\partial x}-\frac{\partial u}{\partial z}=2\Omega.$$

These are associated with P-waves and SV-waves.

Substituting Eq. (5) into Eqs. (4a), (4b) and (4c), we obtain:

$$\mathrm{H_T}\,\nabla^2\phi=\frac{\partial^2\phi}{\partial t^2}, \tag{6a}$$

$$\mathrm{H_\mu}\nabla^2 \mathrm{v}=\frac{\partial^2 v}{\partial t^2}, \tag{6b}$$

$$\mathrm{H_\mu}\nabla^2\psi=\frac{\partial^2\psi}{\partial t^2}, \tag{6c}$$

where

$$\mathrm{H_T}=\sum_{K=0}^{n}U_{KT}^2\frac{\partial^K}{\partial t^K},\ \mathrm{H_\mu}=\sum_{K=0}^{n}U_{KS}^2\frac{\partial^K}{\partial t^K}, \tag{7}$$

with $\mathrm{U_{KT}^2}=\frac{\lambda_K+2\mu_K}{\rho_0}$, $\mathrm{U_{KS}^2}=\frac{\mu_K}{\rho_0}$.

Similar relations for medium M_2 can be obtained by replacing λ_K, μ_K, ρ by λ'_K, μ'_K, and ρ'.

3 Solution of the problem

Now our main objective is to solve Eqs. (6a), (6b), and (6c).

We seek a solution of the following form:

$$(\phi,\psi,v)=[f(z), V(z), h(z)]\,e^{i\eta(x-ct)}. \tag{8}$$

By inserting Eqs. (8) and (7) into Eqs. (6a), (6b) and (6c), we obtain a set of differential equations for the medium M_1 as follows:

$$\begin{aligned}&\frac{d^2 f}{dz^2}+\mathrm{H}_1^{\,2}f=0,\\ &\frac{d^2 h}{dz^2}+K_1^{\,2}h=0,\\ &\frac{d^2 V}{dz^2}+K_1^{\,2}V=0,\end{aligned} \tag{9}$$

where

$$H_1^{\,2}=\frac{\eta^2c^2}{\sum_{K=0}^{n}U_{KT}^2(-i\eta c)^K}-\eta^2,\ K_1^{\,2}=\frac{\eta^2c^2}{\sum_{K=0}^{n}U_{KS}^2(-i\eta c)^K}-\eta^2. \tag{10}$$

The corresponding equations for medium M_2 are given by:

$$\begin{aligned}&\frac{d^2 f'}{dz^2}+\mathrm{H}_1'^2 f'=0,\\ &\frac{d^2 h'}{dz^2}+\mathrm{K}_1'^2\,\mathrm{h}=0,\\ &\frac{d^2 V'}{dz^2}+K_1'^2V'=0,\end{aligned} \tag{11}$$

where

$$H_1'^2=\frac{\eta^2c^2}{\sum_{K=0}^{n}U_{KT}'^2(-i\eta c)^K}-\eta^2,\ K_1'^2=\frac{\eta^2c^2}{\sum_{K=0}^{n}U_{KS}'^2(-i\eta c)^K}-\eta^2. \tag{12}$$

The governing equations for media M_1 and M_2 must have exponential solutions such that f, V, and h describe the surface waves. They must become vanishingly small as $z\to\infty$.

Hence, for the medium M_1, the desired solutions are given by the following expressions:

$$\begin{aligned}\phi(x,z,t)&=Ae^{-H_1z+i\eta(x-ct)},\\ \psi(x,z,t)&=Be^{-K_1z+i\eta(x-ct)},\\ v(x,z,t)&=Ee^{-K_1z+i\eta(x-ct)}.\end{aligned} \tag{13}$$

Similarly for medium M_2 (for the region $0\le z<-\infty$) they are given by the following expressions:

$$\begin{aligned}\phi'(x,z,t)&=A'e^{H_1'z+i\eta(x-ct)},\\ \psi'(x,z,t)&=B'e^{K_1'z+i\eta(x-ct)},\\ v'(x,z,t)&=E'e^{K_1'z+i\eta(x-ct)}.\end{aligned} \tag{14}$$

4 Boundary conditions

We assume that the plane z=0 is a material layer that adheres to its neighboring layer without slipping. The layer is capable of supporting its own stress as represented by a surface stress tensor $\sum_{i\alpha}$, which obeys the equation given by Chandrashekraiah [24], i.e.,

$$\begin{aligned}\sum_{i\alpha}&=[\delta_{i\alpha}\{\sigma+(\lambda_d+\sigma)u_{\gamma,\gamma}\}+\mu_d u_{i,\alpha}+(\mu_d-\sigma)u_{\alpha,i}]\ \text{for i, }\alpha,\gamma=1,\ 2,\\ &=\sigma u_{3,\alpha}\ \text{for i}=3.\end{aligned} \tag{15}$$

Here, λ_d and μ_d are the Lame's moduli of the material boundary, and σ is the residual surface tension on the layer z=0. The forces on the bounding surface are governed by the surface stress tensor $\sum_{i\alpha}$. The dimensions of λ_d, μ_d and σ are N/m.

(i) The displacement components at the boundary surface between the media M_1 and M_2 must be continuous at all times and positions.
i.e., [u, v, w] M_1=[u, v, w] M_2 at z=0, respectively. We also have

(ii) $\tau_{i3}+\sum_{i\alpha,\alpha}-\rho_1\frac{\partial^2 u_i}{\partial t^2}=\tau'_{i3}$, at z=0, ($u_1$=u, u_2=v, u_3=w).

Here, ρ_1 is the mass per unit area of the layer, and τ_{ij} and τ'_{ij} are the stress tensors in the interior of media M_1 and M_2, respectively. The dimension of conventional stress tensor τ_{i3} is force per unit area and that of stress tensor $\sum_{i\alpha,\alpha}$ is force per unit length, and these further obey the law given by Gurtin and Murdoch [23]:

$$\tau_{ij}=D_\lambda\delta_{ij}\, u_{k,k}+D_\mu(u_{i,j}+u_{j,i}).$$

Hence, the boundary conditions become:

$$D_\mu\left(2\frac{\partial^2\varphi}{\partial x\partial z}+\frac{\partial^2\psi}{\partial x^2}-\frac{\partial^2\psi}{\partial z^2}\right)+\left((\lambda_d+2\mu_d)\frac{\partial^2}{\partial x^2}-\rho_1\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial\varphi}{\partial x}-\frac{\partial\psi}{\partial z}\right)$$
$$=D'_\mu\left(2\frac{\partial^2\varphi'}{\partial x\partial z}+\frac{\partial^2\psi'}{\partial x^2}-\frac{\partial^2\psi'}{\partial z^2}\right)$$

$$\left(\mu_d\frac{\partial^2}{\partial x^2}-\rho_1\frac{\partial^2}{\partial t^2}+D_\mu\frac{\partial}{\partial z}\right)v=D'_\mu\frac{\partial v'}{\partial z},$$

$$2D_\mu\left(\frac{\partial^2\psi}{\partial x\partial z}-\frac{\partial^2\varphi}{\partial x^2}\right)+\frac{D_\mu}{H_\mu}\frac{\partial^2\varphi}{\partial t^2}+\left(\sigma\frac{\partial^2}{\partial x^2}-\rho_1\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial\varphi}{\partial z}+\frac{\partial\psi}{\partial x}\right)$$
$$=2D'_\mu\left(\frac{\partial^2\psi'}{\partial x\partial z}-\frac{\partial^2\varphi'}{\partial x^2}\right)+\frac{D'_\mu}{H'_\mu}\frac{\partial^2\varphi'}{\partial t^2}. \tag{16}$$

Applying boundary conditions (16) to (13) and (14), the following system of equations is obtained:

$$A-i\,T_1B-A'-i\,T'_1B'=0, \tag{17a}$$

$$E=E', \tag{17b}$$

$$T_2A-i\,B+T'_2A'+i\,B'=0, \tag{17c}$$

$$\mu^*_K[A\,(2i\,T_2+i\,F\eta)+B\{(1+T_1^2)+\eta FT_1\}]+\mu'^*_K[(2i\,T'_2)A'$$
$$-(1+T_1'^2)B']=0, \tag{17d}$$

$$E\left[\mu_d-c^2\rho_1+\frac{\mu^*_K}{\eta}T_1\right]+\left[\frac{\mu'^*_K}{\eta}T'_1E'\right]=0, \tag{17e}$$

$$\mu^*_K[A\,(2-S^2+\eta HT_2)+B(-2i\,T_1-iH\eta)]-\mu'^*_K[A'(2-S'^2)$$
$$+B'(2iT'_1)]=0, \tag{17f}$$

where we have taken

$$F=\frac{\lambda_d+2\mu_d-\rho_1c^2}{\mu^*_K},\ H=\frac{\sigma-\rho_1c^2}{\mu^*_K} \tag{18}$$

and

$$\mu^*_K=\sum_{K=0}^{n}\mu_K(-i\eta c)^K,\ \mu'^*_K=\sum_{K=0}^{n}\mu'_K(-i\eta c)^K,$$
$$T_2^2=\frac{c^2}{\sum_{K=0}^{n}U^2_{KT}(-i\eta c)^K}-1,$$
$$T_1^2=\frac{c^2}{\sum_{K=0}^{n}U^2_{KS}(-i\eta c)^K}-1,\ T_2'^2=\frac{c^2}{\sum_{K=0}^{n}U'^2_{KT}(-i\eta c)^K}-1,$$
$$T_1'^2=\frac{c^2}{\sum_{K=0}^{n}U'^2_{KS}(-i\eta c)^K}-1,$$
$$S^2=\frac{c^2}{\sum_{K=0}^{n}U^2_{KS}(-i\eta c)^K},\ S'^2=\frac{c^2}{\sum_{K=0}^{n}U'^2_{KS}(-i\eta c)^K}. \tag{19}$$

From Eqs. (17b) and (17e), we obtain E=E′=0. Thus, there is no propagation of displacement v, and hence, SH-waves are decoupled in this case.

Finally, eliminating the constants A, B, A′, B′ from Eqs. (17a), (17c), (17d) and (17f), we obtain:

$$\det(a_{ij})=0,\ i,j=1,2,3,4, \tag{20}$$

where

$$a_{11}=1,\ a_{12}=-i\,T_1,\ a_{13}=-1,\ a_{14}=-iT'_1,$$
$$a_{21}=T_2,\ a_{22}=-i,\ a_{23}=T'_2,\ a_{24}=i,$$
$$a_{31}=\mu^*_K(2i\,T_2+i\,F\eta),\ a_{32}=\mu^*_K\{(1+T_1^2)+\eta FT_1\},$$
$$a_{33}=\mu'^*_K(2i\,T'_2),\ a_{34}=-\mu'^*_K(1+T_1'^2),$$
$$a_{41}=\mu^*_K(2-S^2+\eta H\,T_2),\ a_{42}=\mu^*_K(-2i\,T_1-i\,H\eta),$$
$$a_{43}=\mu'^*_K-(2-S'^2),\ a_{44}=-\mu'^*_K(2\,i\,T'_1).$$

Here, Eq. (20) represents the wave velocity dispersion equation for interface waves in homogeneous, visco-elastic solid media under the influence of surface stresses, where the viscosity is of general n^{th} order involving time rates of change of strain.

5 Particular cases

5.1 Stoneley waves

Stoneley waves are a generalized form of Rayleigh waves propagating along the common boundary of two semi-infinite media, M_1 and M_2. Therefore, Eq. (20) determines the wave velocity equation for Stoneley waves in homogeneous, visco-elastic, solid media of n^{th} order involving time rates of strain under the influence of surface stresses.

Clearly from Eq. (20), it follows that the wave velocity of the Stoneley waves depends upon the surface stresses and viscosity.

Thus, after simplification, Eq. (20) is reduced to:

$$\eta^2 FH(1-T_1T_2)(1-T_1'T_2')-\eta\frac{\mu_K'^*}{\mu_K^*}[F\{T_1'(T_1T_2-1)(2-M') + \frac{\mu_K'^*}{\mu_K^*}T_1(T_1'T_2'-1)(2-M)\}+H\{T_2'(T_1T_2-1)(2-L') + \frac{\mu_K^*}{\mu_K'^*}T_2(T_1'T_2'-1)(2-L)\}]-\Delta=0, \tag{21}$$

where

$$\Delta=\left(\frac{\mu_K'^*}{\mu_K^*}\right)^2(1-T_1T_2)(M'L'-4T_1'T_2')+\frac{\mu_K'^*}{\mu_K^*}[2(T_1+T_1')(MT_2'+M'T_2) +2(T_2+T_2')(LT_2'+L'T_2)-(1+T_1T_2)(ML'+4T_1'T_2)-(LM'+4T_1T_2') -T_1'(LM'T_2+4T_1T_2')]+(1-T_1'T_2')(ML-4T_1T_2) \tag{22}$$

and $L=(1+T_1^2)$, $L'=(1+T_1'^2)$, $M=(2-S^2)$, $M=(2-S'^2)$.

Due to the presence of wave number k in Eq. (21), it follows that Stoneley waves are dispersive in nature.

In the absence of surface stresses, we take λ_d, μ_d, σ and ρ_1 to be equal to 0 so that F=H=0. Then Eq. (21) is reduced to:

$$\left(\frac{\mu_K'^*}{\mu_K^*}\right)^2(1-T_1T_2)(M'L'-4T_1'T_2')+\frac{\mu_K'^*}{\mu_K^*}[2(T_1+T_1')(MT_2'+M'T_2) +2(T_2+T_2')(LT_2'+L'T_2)-(1+T_1T_2)(ML'+4T_1'T_2) -(LM'+4T_1T_2')-T_1'(LM'T_2+4T_1T_2')]+(1-T_1'T_2')(ML-4T_1T_2)=0 \tag{23}$$

Here, Eq. (23) represents the wave velocity equation of Stoneley waves in homogeneous visco-elastic media, which is in complete agreement with the corresponding classical result. In the absence of viscous effects, Eq. (23) is reduced to:

$$(1-R'S')\{(2-s^2)^2-4RS-4\frac{\mu_0'}{\mu_0}(1-s^2-RS)\}+(1-RS)\{(2-q^2s^2)^2 -4R'S'-4\frac{\mu_0'}{\mu_0}(1-q^2s^2-R'S')\}-q^2s^4\frac{\mu_0'}{\mu_0}(2+SR'+S'R)=0, \tag{24}$$

where

$s=c/b$, $S=(1-s^2)^{1/2}$, $R=(1-e^2s^2)^{1/2}$, $S'=(1-q^2s^2)^{1/2}$, $R'=(-t^2s^2)^{1/2}$,

$$e=b/a,\ q=b/b',\ t=b/a',\ a^2=\frac{\lambda_0+2\mu_0}{\rho_0},\ b^2=\frac{\mu_0}{\rho_0},$$

$$a'^2=\frac{\lambda_0'+2\mu_0'}{\rho_0'},\ b'^2=\frac{\mu_0'}{\rho_0'}.$$

Thus, Eq. (24) represents the wave velocity equation of Stoneley waves in elastic media, which is in complete agreement with the corresponding classical result.

5.2 Rayleigh waves

To investigate the possibility of Rayleigh waves in a homogeneous semi-infinite visco-elastic media, we replace medium M_2 by vacuum, in the proceeding problem. We also note that the SH-wave is decoupled in this case. By applying the boundary conditions:

$$\tau_{13}+\sum_{1\alpha,\alpha}-\rho_1\frac{\partial^2 u}{\partial t^2}=0,\qquad \tau_{33}+\sum_{3\alpha,\alpha}-\rho_1\frac{\partial^2 w}{\partial t^2}=0. \tag{25}$$

Eqs. (17d) and (17f) are reduced to:

$$A(2iT_2+iF\eta)+B\{(1+T_1^2)+\eta FT_1\}=0, \tag{26a}$$

$$A(2-S^2+\eta HT_2)+B(-2iT_1-iH\eta)=0. \tag{26b}$$

Eliminating A and B from Eqs. (26a) and (26b), we obtain:

$$(2iT_2+iF\eta)(-2iT_1-iH\eta)-\{(1+T_1^2)+\eta FT_1\}(2-S^2+\eta HT_2)=0. \tag{27}$$

Upon simplification, Eq. (27) gives the following:

$$\eta^2 FH(1-T_1T_2)+\eta[2FT_1+2HT-(2-S^2)FT_1-(1+T_1^2)HT_2] +4T_1T_2-(1+T_1^2)(2-S^2)=0. \tag{28}$$

Here, Eq. (28) represents the wave velocity equation for Rayleigh waves in a homogeneous, visco-elastic solid medium of n^{th} order involving time rates of strain under the influence of surface stresses. Thus, from Eq. (28), we conclude that Rayleigh waves depend on the residual surface tension, surface stresses and viscosity.

Meanwhile, in the case of conventional stress free boundary, Eq. (28) becomes:

$$4T_1T_2-(1+T_1^2)(2-S^2)=0. \tag{29}$$

Thus, Eq. (29) represents the wave velocity equation for Rayleigh waves in a homogeneous, visco-elastic solid medium of n^{th} order involving time rates of strain. This equation, of course, is in complete agreement with the corresponding classical result, when the effect of viscosity is neglected.

5.3 Love waves

To investigate the possibility of Love waves in a homogeneous visco-elastic solid, we restrict medium M_2 by two

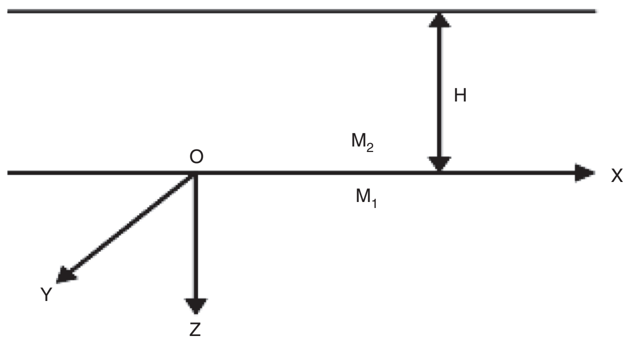

Figure 2 Configuration for Love waves.

horizontal plane surfaces at a distance H-apart, while M_1 remains infinite (see Figure 2).

For medium M_1, the displacement component "v" remains same as in general case given by Eq. (13). For the medium M_2, we preserve the full solution, since the displacement component along the y-axis (i.e., v) no longer diminishes with increasing distance from the boundary surface of two media.

Thus:

$$v' = E_1 e^{-K_1' z + i\eta(x-ct)} + E_2 e^{K_1' z + i\eta(x-ct)}. \tag{30}$$

(i) In this case, the boundary conditions are given by: v and τ_{32} are continuous at z=0,
(ii) $\tau'_{32}=0$ at z=-H,

where we take

$$\tau_{23} + \sum_{2\alpha,\alpha} -\rho_1 \frac{\partial^2 v}{\partial t^2} = 0.$$

Applying boundary conditions (i) and (ii) to Eqs. (13), (16) and (30), we obtain:

$$E = E_1 + E_2, \tag{31}$$

$$-K_1 \mu_K^* E = \mu_K'^* [-K_1' E_1 + K_1' E_2], \tag{32}$$

$$[\mu_K'^* K_1' \mu_d - \rho_1 c^2] e^{K_1' H} E_1 - [\mu_K'^* K_1' - (\mu_d - \rho_1 c^2)] e^{-K_1' H} E_2 = 0. \tag{33}$$

Upon eliminating the constants E, E_1 and E_2 from Eqs. (31), (32) and (33), we obtain:

$$\tan(i K_1' H) = -i \frac{[\mu_K^* K_1 \mu_K'^* K_1' + \mu_K'^* K_1' (\mu_d - \rho_1 c^2)]}{[\mu_K'^{*2} K_1'^2 + \mu_K^* K_1 (\mu_d - \rho_1 c^2)]}. \tag{34}$$

In this case, Eq. (34) represents the wave velocity equation for Love waves in a homogeneous, visco-elastic solid medium of n^{th} order involving time rates of strain under the influence of surface stresses. Clearly, it depends upon the viscous parameter μ_d and is independent of the residual surface tension σ. Moreover, when surface stresses and viscous effects are ignored, this equation reduces to the corresponding classical result of Love waves.

6 Discussion and conclusions

The present study reveals the effects of surface stresses, residual surface tension, and viscosity on the wave velocity equations corresponding to the Stoneley, Rayleigh, and Love waves. is the process by which visco-elastic surface waves is affected is identified through the time rates of strain parameters. These parameters influence the wave velocity depending on the corresponding constants characterizing the visco-elastic behavior of the material. Special cases of this study in homogeneous elastic medium are discussed by several authors, including Chandrasekharaiah [24] and Gurtin and Murdoch [23, 27, 28].

The wave velocity equation for Rayleigh waves under the influence of surface stresses is dispersive due to the presence of the wave number. It also depends on the viscosity, residual surface tension and surface stresses. Our results are in complete agreement with the corresponding classical results when surface stresses and viscous effects are neglected.

By contrast, Love waves do not depend on the residual surface tension σ. These are only affected by such factors as viscosity, Lame moduli of material boundary, and surface stresses. In the absence of surface stresses and other effects, the dispersion equation is in complete agreement with the corresponding classical result.

It is noted that the wave velocity equation of Stoneley waves is very similar to the corresponding problem in the classical theory of elasticity. Here, we also observed the dispersion of waves due to the presence of wave number, surface stresses, and visco-elastic nature of the solid. Moreover, the wave velocity equation of this generalized type of surface waves in homogeneous visco-elastic solid media of higher order under the influence of surface stresses is in complete agreement with the corresponding classical results when surface stresses and viscous effects are neglected. Finally, the solution of wave velocity equation for Stoneley waves cannot be determined by easy analytical methods. One needs to apply numerical techniques to solve the relevant detrimental equation by choosing suitable values of physical constants for both media M_1 and M_2.

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