**Research Article**

Lu Chen*

# Interval-valued T-spherical fuzzy extended power aggregation operators and their application in multi-criteria decision-making

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**Abstract:** As an effective tool to show the fuzziness of qualitative information, the interval-valued T-spherical fuzzy set can utilize three kinds of information, namely, membership, abstinence, and non-membership, to show the opinions of decision-maker. Given this advantage, many interval-valued T-spherical fuzzy multi-criteria decision-making (IVTSF-MCDM) methods have been designed. However, most of the existing IVTSF-MCDM methods have a common limitation that the inability to effectively show the impacts of extreme data. To address this limitation, this study develops a novel MCDM method based on interval-valued T-spherical fuzzy extended power aggregation operator. First, interval-valued T-spherical fuzzy cross-entropy (CE) and interval-valued T-spherical fuzzy symmetrical CE are defined to measure the difference between two interval-valued T-spherical fuzzy numbers, which are used to determine criteria weights in MCDM. Second, interval-valued T-spherical fuzzy extended power average operator and interval-valued T-spherical fuzzy extended power geometric operator are proposed, and their properties are investigated. Moreover, in view of that criteria may be assigned to different weights, this study defines interval-valued T-spherical fuzzy extended power weighted average operator and interval-valued T-spherical fuzzy extended power weighted geometric operator to derive the order of alternatives. Finally, the applicability of the proposed method is validated by the case about investment country selection, while the sensitivity and comparison analyses are also conducted to further prove its advantages and effectiveness.

**Keywords:** interval-value T-spherical fuzzy set, extended power aggregation operator, multi-criteria decision making, cross-entropy

## 1 Introduction

Multi-criteria decision-making (MCDM) is an important research field in decision science. Since the evaluation information provided by decision-makers (DMs) is usually ambiguous and uncertain in MCDM, some fuzzy sets have been adopted to express uncertain evaluation information, including fuzzy set intuitionistic fuzzy set, Pythagorean fuzzy set, and q-rung orthopair fuzzy set [1–4]. In actual situations, DMs' opinions involve more answers of types: yes, no, abstain, and refusal. Voting is a suitable example of such situation as human voters may be classified into four types: vote for, abstain, vote against, and refusal of the voting. To deal with this situation, Cuong presented the picture fuzzy set [5]. The picture fuzzy set includes three kinds of degrees, namely, the membership degree $\mu(0 \le \mu \le 1)$, abstinence degree $\eta(0 \le \eta \le 1)$, and non-membership degree $\nu(0 \le \nu \le 1)$, and the restriction is that the sum of the three degrees must not exceed one, i.e., $\mu + \eta + \nu \le 1$. Subsequently, inspired by ideas of q-rung orthopair fuzzy set, Mahmood et al. [6] extended picture fuzzy set to

*** Corresponding author: Lu Chen**, College of Criminal Justice, Henan University of Economics and Law, Zhengzhou, 450016, China, e-mail: chenlu_chen@hotmail.com

define T-spherical fuzzy set (TSFS), which satisfies the condition $\mu^q + \eta^q + \nu^q \leq 1 (q \geq 1)$. However, DMs may choose interval numbers to express their opinions since their cognitions of the evaluation object are uncertain. Thus, Ullah et al. [7] proposed interval-valued TSFS (IVTSFS) on the basis of TSFS, in which the membership degree, abstinence degree, and non-membership degree are expressed in form of interval numbers. Obviously, IVTSFS can more intuitively show the perceptions of DMs by comparing with the aforementioned fuzzy sets.

Taking the advantages of IVTSFS, some interval-valued T-spherical fuzzy MCDM (IVTSF-MCDM) methods have been proposed [6–13]. For example, Ullah et al. proposed interval-valued T-spherical fuzzy weighted average (IVTSFWA) operator and interval-valued T-spherical fuzzy weighted geometric (IVTSFWG) operator, and used these aggregation operators to handle MCDM problems [7]. Subsequently, Jin et al. [8] developed some Hamacher aggregation operators in the environment of IVTSFSs, including interval-valued T-spherical fuzzy Hamacher weighted average (IVTSFHWA) and interval-valued T-spherical fuzzy Hamacher weighted geometric (IVTSFHWG) operators, and utilized these operators to design MCDM method. Considering that Dombi t-norm and Dombi t-conorm have more flexibility, Ullah et al. [9] presented the concepts of interval-valued T-spherical fuzzy Dombi weighted average (IVTSFDWA) and interval-valued T-spherical fuzzy Dombi weighted geometric (IVTSFDWG) operators, and applied these operators into MCDM. Inspired by Frank t-norm and Frank t-conorm, Hussain et al. [10] defined interval-valued T-spherical fuzzy Frank aggregation operators, which are used to assess the business proposals. In view of the aforementioned operators that cannot reflect the interrelationships between two criteria in MCDM, Akram et al. [11] extended Bonferroni mean operators to interval-valued T-spherical fuzzy context. In fact, there are some relationships among more than two criteria. Thus, Yang, et al. [12] proposed interval-valued T-spherical fuzzy weighted Muirhead mean (IVTSFWMM) operator and established the decision-making method of digitalization solutions of medical system based on this operator. Although these interval-valued T-spherical fuzzy aggregation operators are from different mathematical perspective, they ignore the impacts of extreme data, such as too high and too low evaluation results, on the final result in MCDM. To address this issue, Xu and Yager [13] proposed the power average (PA) operator and power geometric (PG) operator, which regard extreme data as erroneous or biased information. Thus, PA and PG operators assign smaller weights to extreme data. Some scholars [14–18] have applied PA and PG operators to solve MCDM problems with different fuzzy sets. However, extreme data may be important in some cases. For example, five professors' assessments about the innovativeness of an article are expressed in form of interval value T-spherical fuzzy numbers (IVTSFNs), where the full professor gives a IVTSFN ([0.1, 0.2], [0.6, 0.6], [0.3, 0.4]), and four assistant professors' evaluations are ([0.3, 0.5], [0.4, 0.6], [0.2, 0.6]). Although the full professor' opinion differs from that of assistant professors, his opinion is more important than that of others. It is worth mentioning that full professor' opinions are regarded as biased information and assigned to a smaller weight when using PA or PG operator. Obviously, PA or PG operator may lead to unreasonable results in some cases. To overcome this shortcoming, Xiong et al. [19] and Xiong et al. [20] introduced the extended power average (EPA) operator and extended power geometric (EPG) operator, respectively, which set an adjustable parameter to reflect the preferences of DMs on extreme data on the basic of PA and PG operators. Obviously, extended power aggregation operators can describe more important information than PA and PG. However, there are few studies on extended power aggregation operators in IVTSF-MCDM problems. Hence, it is valuable to employ EPA and EPG operators to solve IVTSF-MCDM problems.

In IVTSF-MCDM, different criteria usually have different impacts on the decision results. In other words, different criteria should be assigned with different weights. The maximizing deviation approach is very effective in determining criteria weights, which relies on the differences between DMs' opinions under each criterion [21]. Therefore, how to choose a suitable tool to measure the differences between data is material for the maximizing deviation approach. As an important concept of information theory, cross-entropy (CE) can measure the difference between two probability distributions. In other words, CE can reflect the gap of the data to some extent. Therefore, it is very effective to combine CE with the maximizing deviation approach to determine criteria weights. The CE has been extended to the different fuzzy environment, such as intuitionistic fuzzy CE [22], Pythagorean fuzzy CE [23], q-rung orthopair fuzzy CE [24], picture fuzzy CE [25], and T-spherical fuzzy CE [26]. These existing CEs have been successfully applied into MCDM problems. However, few scholars explore CE under interval-valued T-spherical fuzzy context. Therefore, this article designs interval-valued T-spherical fuzzy CE (IVTSFCE) and interval-valued T-spherical fuzzy symmetric CE (IVTSFSCE) to calculate the differences between two IVTSFNs. Given this characteristic of IVTSFSCE, it can be combined with the maximizing deviation approach to derive criteria weights in IVTSF-MCDM problems, and it provides the necessary support for the proposed MCDM method.

According to the aforementioned analysis, this article intends to propose a novel IVTSF-MCDM method that can reflect the impact of extreme data on the ranking results of alternatives. The motivation for the proposed method can be as follows:

(1) The previous researches of CE [23–26] cannot deal with interval-valued T-spherical fuzzy information.
(2) Some existing IVTSF-MCDM methods [7–11] cannot consider the influence of extreme data on the order of alternatives, which leads to an unreasonable result.

To address these key issues, a novel MCDM method based on interval-valued T-spherical fuzzy extended power aggregation operators is designed. First, the IVTSFCE and IVTSFSCE are developed and combined with the maximizing deviation approach to derive the weights of criteria. Second, some interval-valued T-spherical fuzzy extended power aggregation operators, such as interval-valued T-spherical fuzzy extended power average (IVTSFEPA), interval-valued T-spherical fuzzy extended power geometric (IVTSFEPG), interval-valued T-spherical fuzzy extended power weighted average (IVTSFEPWA), and interval-valued T-spherical fuzzy extended power weighted geometric (IVTSFEPWG) operators, are proposed to reflect the effect of extreme data. In the end, an example of investment country selection is presented to prove the availability of the proposed method, while the sensitivity and comparative analysis are used to prove the advantages of the proposed method. In a summary, the contributions of this article contain:

(1) The IVTSFCE and IVTSFSCE are developed to measure the differences between two IVTSFNs.
(2) Some interval-valued T-spherical fuzzy extended power aggregation operators, such as IVTSFEPA, IVTSF-EPG, IVTSFEPWA and IVTSFEPWG operators, are presented to show the effect of extreme data.
(3) A novel MCDM method is designed using IVTSFCE and interval-valued T-spherical fuzzy extended power aggregation operators.

The framework of this article includes: in Section 2, some basic knowledge about IVTSFS, PA, and PG operators is recalled, while IVTSFCE and IVTSFSCE are defined. In Section 3, some interval-valued T-spherical fuzzy extended power aggregation operators are presented and their properties are investigated. In Section 4, a novel MCDM method is proposed on the basis of IVTSFCE and interval-valued T-spherical fuzzy extended power aggregation operators. Section 5 presents an application of the proposed method into an illustrative case. Additionally, the sensitivity and comparison analyses prove the merit of the proposed method. In Section 6, some conclusions are given.

# 2 Preliminaries

In this section, the IVTSFS, PA, and PG operators are recalled, and the concepts of IVTSFCE and IVTSFSCE are defined.

## 2.1 IVTSFS

**Definition 1.** (Ullah et al. [7]) Let $X$ be a finite set, and the IVTSFS on $X$ is expressed as:

$$T = \{([\mu_x^L, \mu_x^U],\ [\eta_x^L, \eta_x^U],\ [\nu_x^L, \nu_x^U])|x \in X\}, \tag{1}$$

where $[\mu_x^L, \mu_x^U]$, $[\eta_x^L, \eta_x^U]$, and $[\nu_x^L, \nu_x^U]$ denote the elements $x \in X$ satisfying the degree of membership, abstinence, and non-membership of the set $T$, respectively, and meet $0 \le \mu_x^L \le \mu_x^U \le 1, 0 \le \eta_x^L \le \eta_x^U \le 1, 0 \le \nu_x^L \le \nu_x^U \le 1$ and $0 \le (\mu_x^U)^q + (\eta_x^U)^q + \nu_x^U)^q \le 1 (q \ge 1)$.

For the convenience of expression, we denote $([\mu_x^L, \mu_x^U], [\eta_x^L, \eta_x^U], [\nu_x^L, \nu_x^U])$ as $\alpha = ([\mu^L, \mu^U], [\eta^L, \eta^U], [\nu^L, \nu^U])$ and call $\alpha$ the interval T-spherical fuzzy number (IVTSFN).

**Definition 2.** (Ullah et al. [7]) Let two IVTSFNs be $\alpha_1 = ([\mu_1^L, \mu_1^U], [\eta_1^L, \eta_1^U], [\nu_1^L, \nu_1^U])$ and $\alpha_2 = ([\mu_2^L, \mu_2^U], [\eta_2^L, \eta_2^U], [\nu_2^L, \nu_2^U])$, $\lambda > 0$, then

(1) $\alpha_1 \oplus \alpha_2 = ([((\mu_1^L)^q + (\mu_2^L)^q - (\mu_1^L\mu_2^L)^q)^{1/q}, ((\mu_1^U)^q + (\mu_2^U)^q - (\mu_1^U\mu_2^U)^q)^{1/q}], [\eta_1^L\eta_2^L, \eta_1^U\eta_2^U], [\nu_1^L\nu_2^L, \nu_1^U\nu_2^U])$

(2) $\alpha_1 \otimes \alpha_2 = ([\mu_1^L\mu_2^L, \mu_1^U\mu_2^U], [((\eta_1^L)^q + (\eta_2^L)^q - (\eta_1^L\eta_2^L)^q)^{1/q}, ((\eta_1^U)^q + (\eta_2^U)^q - (\eta_1^U\eta_2^U)^q)^{1/q}],$
$[((\nu_1^L)^q + (\nu_2^L)^q - (\nu_1^L\nu_2^L)^q)^{1/q}, ((\nu_1^U)^q + (\nu_2^U)^q - (\nu_1^U\nu_2^U)^q)^{1/q}])$

(3) $\lambda\alpha_1 = ([(1 - (1 - (\mu_1^L)^q)^\lambda)^{1/q}, (1 - (1 - (\mu_1^U)^q)^\lambda)^{1/q}], [(\eta_1^L)^\lambda, (\eta_1^U)^\lambda], [(\nu_1^L)^\lambda, (\nu_1^U)^\lambda])$

(4) $(\alpha_1)^\lambda = ([(\mu_1^L)^\lambda, (\mu_1^U)^\lambda], [(1 - (1 - (\eta_1^L)^q)^\lambda)^{1/q}, (1 - (1 - (\eta_1^U)^q)^\lambda)^{1/q}], [(1 - (1 - (\nu_1^L)^q)^\lambda)^{1/q}, (1 - (1 - (\nu_1^U)^q)^\lambda)^{1/q}])$

**Definition 3.** (Ullah et al. [7]) Let an IVTSFN be $\alpha = ([\mu^L, \mu^U], [\eta^L, \eta^U], [\nu^L, \nu^U])$, then its score function $sc(\alpha)$ is as follows:

$$sc(\alpha) = \frac{(\mu^L)^q(1 - (\eta^L)^q - (\nu^L)^q) + (\mu^U)^q(1 - (\eta^U)^q - (\nu^U)^q)}{3}. \tag{2}$$

According to the score function of IVTSFN, Ullah et al. [7] presented the comparison laws for two IVTSFNs, which is as follows: (1) if $sc(\alpha_1) > sc(\alpha_2)$, then $\alpha_1 > \alpha_2$; (2) if $sc(\alpha_1) = sc(\alpha_2)$, then $\alpha_1 \approx \alpha_2$.

## 2.2 Extended power aggregation operators

To depict the effect of extreme data, Xiong et al. [19] and Xiong et al. [20] proposed EPA operator and EPG operator, respectively.

**Definition 4.** (Xiong et al. [19]; Xiong et al. [20]) Let a set of nonnegative numbers $a_1, a_2, \ldots, a_n$, then

(1) The EPA operator is defined as:

$$EPA^\xi(a_1, a_2, \ldots, a_n) = \sum_{k=1}^{n} \frac{(\xi + t(a_k))a_k}{\sum_{k=1}^{n}(\xi + t(a_k))}. \tag{3}$$

(2) The EPG operator is defined as:

$$EPG^\xi(a_1, a_2, \ldots, a_n) = \prod_{k=1}^{n} (a_k)^{\frac{(\xi + t(a_k))}{\sum_{j=1}^{n}(\xi + t(a_k))}}, \tag{4}$$

where the parameter $\xi \in (-\infty, 1 - n] \cup [0, +\infty)$, $t(a_k) = \sum_{l=1,\, l \ne k}^{n} Sup(a_k, a_l)$, $Sup(a_k, a_l)$ denotes the support of $a_l$ for $a_k$ and satisfies three conditions: (1) $Sup(a_k, a_l) \in [0, 1]$; (2) $Sup(a_k, a_l) = Sup(a_l, a_k)$; (3) if $|a_k - a_l| \le |a_p - a_q|$, then $Sup(a_k, a_l) \ge Sup(a_p, a_q)$.

It is worth noting that if DMs consider the extreme values to be error or bias information, the parameter takes $\xi \in [0, +\infty)$. If DMs regard the extreme values as important information, the parameter takes $\xi \in (-\infty, 1 - n]$.

## 2.3 IVTSFCE

To measure the difference between two IVISFNs, this subsection proposes the concept of IVTSFCE.

**Definition 5.** Let two IVISFNs be $\alpha_1 = ([\mu_1^L, \mu_1^U], [\eta_1^L, \eta_1^U], [v_1^L, v_1^U])$ and $\alpha_2 = ([\mu_2^L, \mu_2^U], [\eta_2^L, \eta_2^U], [v_2^L, v_2^U])$, then IVTSFCE is calculated as:

$$\begin{aligned} \mathrm{CE}(\alpha_1, \alpha_2) = \frac{1}{6\ln 2}\Bigg(&\mu_1^L \ln\left(\frac{2\mu_1^L}{\mu_1^L + \mu_2^L}\right) + (1-\mu_1^L)\ln\left(\frac{2(1-\mu_1^L)}{2-\mu_1^L-\mu_2^L}\right) + \mu_1^U \ln\left(\frac{2\mu_1^U}{\mu_1^U+\mu_2^U}\right) \\ &+ (1-\mu_1^U)\ln\left(\frac{2(1-\mu_1^U)}{2-\mu_1^U-\mu_2^U}\right) + \eta_1^L \ln\left(\frac{2\eta_1^L}{\eta_1^L+\eta_2^L}\right) + (1-\eta_1^L)\ln\left(\frac{2(1-\eta_1^L)}{2-\eta_1^L-\eta_2^L}\right) \\ &+ \eta_1^U \ln\left(\frac{2\eta_1^U}{\eta_1^U+\eta_2^U}\right) + (1-\eta_1^U)\ln\left(\frac{2(1-\eta_1^U)}{2-\eta_1^U-\eta_2^U}\right) + v_1^L \ln\left(\frac{2v_1^L}{v_1^L+v_2^L}\right) \\ &+ (1-v_1^L)\ln\left(\frac{2(1-v_1^L)}{2-v_1^L-v_2^L}\right) + v_1^U \ln\left(\frac{2v_1^U}{v_1^U+v_2^U}\right) + (1-v_1^U)\ln\left(\frac{2(1-v_1^U)}{2-v_1^U-v_2^U}\right)\Bigg). \end{aligned} \tag{5}$$

It is worth noting that $\mathrm{CE}(\alpha_1, \alpha_2)$ does not satisfy symmetry. Thus, this study proposes IVTSFSCE.

**Definition 6.** Let two IVISFNs be $\alpha_1 = ([\mu_1^L, \mu_1^U], [\eta_1^L, \eta_1^U], [v_1^L, v_1^U])$ and $\alpha_2 = ([\mu_2^L, \mu_2^U], [\eta_2^L, \eta_2^U], [v_2^L, v_2^U])$, then IVTSFSCE can be defined as:

$$\mathrm{DE}(\alpha_1, \alpha_2) = \frac{1}{2}(\mathrm{CE}(\alpha_1, \alpha_2) + \mathrm{CE}(\alpha_2, \alpha_1)). \tag{6}$$

**Property 1.** Let $\mathrm{DE}(\alpha_1, \alpha_2)$ be the symmetric CE between $\alpha_1$ and $\alpha_2$, then it meets the following properties:
(1) $\mathrm{DE}(\alpha_1, \alpha_2) = \mathrm{DE}(\alpha_2, \alpha_1)$;
(2) $0 \le \mathrm{DE}(\alpha_1, \alpha_2) \le 1$;
(3) $\mathrm{DE}(\alpha_1, \alpha_2) = 0$ if and only if $\alpha_1 = \alpha_2$.

**Proof. Property (1)** is obvious; thus, this article proves properties (2) and (3).

**Property (2):** It is easy to have

$$0 \le \mu_1^L \ln\left(\frac{2\mu_1^L}{\mu_1^L+\mu_2^L}\right) + (1-\mu_1^L)\ln\left(\frac{2(1-\mu_1^L)}{2-\mu_1^L-\mu_2^L}\right) \le \mu_1^L \ln 2 + (1-\mu_1^L)\ln 2.$$

Thus, we can derive

$$\begin{aligned} \mathrm{CE}(\alpha_1, \alpha_2) &\le \frac{1}{6\ln 2}(\mu_1^L \ln 2 + (1-\mu_1^L)\ln 2 + \mu_1^U \ln 2 + (1-\mu_1^U)\ln 2 + \eta_1^L \ln 2 + (1-\eta_1^L)\ln 2 \\ &\quad + \eta_1^U \ln 2 + (1-\eta_1^U)\ln 2 + v_1^L \ln 2 + (1-v_1^L)\ln 2 + v_1^U \ln 2 + (1-v_1^U)\ln 2) \\ &= \frac{1}{6\ln 2} \times 6\ln 2 = 1. \end{aligned}$$

Furthermore, we can have $0 \le \mathrm{DE}(\alpha_1, \alpha_2) \le 1$.

**Property (3):** Let the function be $f(t) = -\ln t$. Then, its first-order and second-order derivatives are $f'(t) = -1/t < 0$ and $f''(t) = 1/t^2 > 0$, respectively, which implies that $f(x)$ is a concave function. According to the properties of concave functions, there must exist $-\ln(\lambda_1 t_1 + \lambda_2 t_2) \le -\lambda_1 \ln t_1 - \lambda_2 \ln t_2$, with $\lambda_1 + \lambda_2 = 1$ and $\lambda_1, \lambda_2 \in (0, 1)$. Moreover, $-\ln(\lambda_1 t_1 + \lambda_2 t_2) = -\lambda_1 \ln t_1 - \lambda_2 \ln t_2$ if and only if $t_1 = t_2$.

Assume $t_1 = (\mu_1^L + \mu_2^L)/2\mu_1^L$, $t_2 = (2 - \mu_1^L - \mu_2^L)/(2 - 2\mu_1^L)$, $\lambda_1 = \mu_1^L$ and $\lambda_2 = 1 - \mu_1^L$. Then, we can deduce that $\mu_1^L \ln(2\mu_1^L/(\mu_1^L + \mu_2^L)) + (1 - \mu_1^L)\ln(2(1 - \mu_1^L)/(2 - \mu_1^L - \mu_2^L)) = 0$ if and only if $(\mu_1^L + \mu_2^L)/\mu_1^L = (2 - \mu_1^L - \mu_2^L)/(2 - 2\mu_1^L) \Rightarrow \mu_1^L = \mu_2^L$. Furthermore, we can have $\mathrm{DE}(\alpha_1, \alpha_2) = 0$ if and only if $\alpha_1 = \alpha_2$. □

# 3 Interval-valued T-spherical fuzzy extended power aggregation operators

Inspired by EPA and EPG operators, this section proposes four interval-valued T-spherical fuzzy extended power aggregation operators, such as IVTSFEPA, IVTSFEPG, IVTSFEPWA, and IVTSFEPWG operators.

## 3.1 IVTSFEPA and IVTSFEPG operators

**Definition 7.** Let a set of IVTSFNs be $\alpha_k = ([\mu_k^L, \mu_k^U], [\eta_k^L, \eta_k^U], [\nu_k^L, \nu_k^U])(k = 1, 2,..., n)$, then

(1) The IVTSFEPA operator is expressed as:

$$\text{IVTSFEPA}^{\xi}(\alpha_1, \alpha_2,..., \alpha_n) = \underset{k=1}{\overset{n}{\oplus}} \frac{\xi + t(\alpha_k)}{\sum_{k=1}^{n}(\xi + t(\alpha_k))}\alpha_k. \tag{7}$$

(2) The IVTSFEPG operator is defined as:

$$\text{IVTSFEPG}^{\xi}(\alpha_1, \alpha_2,..., \alpha_n) = \underset{k=1}{\overset{n}{\otimes}} (\alpha_k)^{\frac{\xi + t(\alpha_k)}{\sum_{k=1}^{n}(\xi + t(\alpha_k))}}, \tag{8}$$

where the parameter $\xi \in (-\infty, 1 - n] \cup [0, +\infty)$, , $\text{Sup}(\alpha_k, \alpha_l)$ denotes the support of $\alpha_l$ for $\alpha_k$ and satisfies three conditions: (1) $\text{Sup}(\alpha_k, \alpha_l) \in [0, 1]$; (2) $\text{Sup}(\alpha_k, \alpha_l) = \text{Sup}(\alpha_l, \alpha_k)$; (3) if $\text{DE}(\alpha_k, \alpha_l) \leq \text{DE}(\alpha_p, \alpha_q)$, then $\text{Sup}(\alpha_k, \alpha_l) \geq \text{Sup}(\alpha_p, \alpha_q)$.

According to Definition 7, the support $\text{Sup}(\alpha_k, \alpha_l)$ is similar to the similarity between two IVTSFNs. If two IVTSFNs are more similar, their support degree is larger. Thus, this study assumes $\text{Sup}(\alpha_k, \alpha_l) = 1 - \text{DE}(\alpha_k, \alpha_l)$.

**Theorem 1.** *Let a set of IVTSFNs be $\alpha_k = ([\mu_k^L, \mu_k^U], [\eta_k^L, \eta_k^U], [\nu_k^L, \nu_k^U])(k = 1, 2,..., n)$, then the results obtained by IVTSFEPA and IVTSFEPG operators are still an IVTSFN. The corresponding calculation results are*

$$\begin{aligned}
&\text{IVTSFEPA}^{\xi}(\alpha_1, \alpha_2,..., \alpha_n)\\
&= \left\{\left[\sqrt[q]{1 - \prod_{k=1}^{n}(1 - (\mu_k^L)^q)^{\frac{\xi + t(\alpha_k)}{\sum_{k=1}^{n}(\xi + t(\alpha_k))}}}, \sqrt[q]{1 - \prod_{k=1}^{n}(1 - (\mu_k^U)^q)^{\frac{\xi + t(\alpha_k)}{\sum_{k=1}^{n}(\xi + t(\alpha_k))}}}\right], \left[\prod_{k=1}^{n}(\eta_k^L)^{\frac{\xi + t(\alpha_k)}{\sum_{k=1}^{n}(\xi + t(\alpha_k))}}, \prod_{k=1}^{n}(\eta_k^U)^{\frac{\xi + t(\alpha_k)}{\sum_{k=1}^{n}(\xi + t(\alpha_k))}}\right],\right.\\
&\left.\left[\prod_{k=1}^{n}(\nu_k^L)^{\frac{\xi + t(\alpha_k)}{\sum_{k=1}^{n}(\xi + t(\alpha_k))}}, \prod_{k=1}^{n}(\nu_k^U)^{\frac{\xi + t(\alpha_k)}{\sum_{k=1}^{n}(\xi + t(\alpha_k))}}\right]\right\},
\end{aligned} \tag{9}$$

$$\begin{aligned}
&\text{IVTSFEPG}^{\xi}(\alpha_1, \alpha_2,..., \alpha_n)\\
&= \left\{\left[\prod_{k=1}^{n}(\mu_i^L)^{\frac{\xi + t(\alpha_k)}{\sum_{k=1}^{n}(\xi + t(\alpha_k))}}, \prod_{k=1}^{n}(\mu_k^U)^{\frac{\xi + t(\alpha_k)}{\sum_{k=1}^{n}(\xi + t(\alpha_k))}}\right], \left[\sqrt[q]{1 - \prod_{k=1}^{n}(1 - (\eta_k^L)^q)^{\frac{\xi + t(\alpha_k)}{\sum_{k=1}^{n}(\xi + t(\alpha_k))}}}, \sqrt[q]{1 - \prod_{k=1}^{n}(1 - (\eta_k^U)^q)^{\frac{\xi + t(\alpha_k)}{\sum_{k=1}^{n}(\xi + t(\alpha_k))}}}\right]\right.\\
&\left., \left[\sqrt[q]{1 - \prod_{k=1}^{n}(1 - (\nu_k^L)^q)^{\frac{\xi + t(\alpha_k)}{\sum_{k=1}^{n}(\xi + t(\alpha_k))}}}, \sqrt[q]{1 - \prod_{k=1}^{n}(1 - (\nu_k^U)^q)^{\frac{\xi + t(\alpha_k)}{\sum_{k=1}^{n}(\xi + t(\alpha_k))}}}\right]\right\}.
\end{aligned} \tag{10}$$

**Proof.** According to the operator law of IVTSFE in Definition 2, we have

$$\begin{aligned}
\underset{k=1}{\overset{n}{\oplus}}(\xi + t(\alpha_k))\alpha_k = &\left\{\left[\sqrt[q]{1 - \prod_{i=1}^{n}(1 - (\mu_i^L)^q)^{\xi + t(\alpha_k)}}, \sqrt[q]{1 - \prod_{k=1}^{n}(1 - (\mu_k^U)^q)^{\xi + t(\alpha_k)}}\right],\right.\\
&\left.\left[\prod_{k=1}^{n}(\eta_k^L)^{\xi + t(\alpha_k)}, \prod_{i=1}^{n}(\eta_i^U)^{\xi + t(\alpha_k)}\right], \left[\prod_{k=1}^{n}(\nu_k^L)^{\xi + t(\alpha_k)}, \prod_{i=1}^{n}(\nu_i^U)^{\xi + t(\alpha_k)}\right]\right\}.
\end{aligned} \tag{11}$$

Based on the first mathematical induction, we obtain

$$(\xi + t(\alpha_1))\alpha_1 = \{[\sqrt[q]{1-(1-(\mu_1^L)^q)^{\xi+t(\alpha_1)}}, \sqrt[q]{1-(1-(\mu_1^U)^q)^{\xi+t(\alpha_1)}}],[(\eta_1^L)^{\xi+t(\alpha_1)}, (\eta_1^U)^{\xi+t(\alpha_1)}], [(\nu_1^L)^{\xi+t(\alpha_1)}, (\nu_1^U)^{\xi+t(\alpha_1)}]\}$$

$$(\xi + t(\alpha_2))\alpha_2 = \{[\sqrt[q]{1-(1-(\mu_2^L)^q)^{\xi+t(\alpha_2)}}, \sqrt[q]{1-(1-(\mu_2^U)^q)^{\xi+t(\alpha_2)}}],[(\eta_2^L)^{\xi+t(\alpha_2)}, (\eta_2^U)^{\xi+t(\alpha_2)}], [(\nu_2^L)^{\xi+t(\alpha_2)}, (\nu_2^U)^{\xi+t(\alpha_2)}]\}$$

Then, we can obtain

$$\begin{aligned}\mathop{\oplus}_{k=1}^{2}(\xi + t(\alpha_k))\alpha_k = \ & \{[\sqrt[q]{1-(1-(\mu_1^L)^q)^{\xi+t(\alpha_1)}+1-(1-(\mu_2^L)^q)^{\xi+t(\alpha_2)}-(1-(1-(\mu_1^L)^q)^{\xi+t(\alpha_1)})\times(1-(1-(\mu_2^L)^q)^{\xi+t(\alpha_2)})},\\ & \sqrt[q]{1-(1-(\mu_1^U)^q)^{\xi+t(\alpha_1)}+1-(1-(\mu_2^U)^q)^{\xi+t(\alpha_2)}-(1-(1-(\mu_1^U)^q)^{\xi+t(\alpha_1)})\times(1-(1-(\mu_2^U)^q)^{\xi+t(\alpha_2)})}],\\ & [(\eta_1^L)^{\xi+t(\alpha_1)}\times(\eta_2^L)^{\xi+t(\alpha_1)}, (\eta_1^U)^{\xi+t(\alpha_1)}\times(\eta_1^U)^{\xi+t(\alpha_1)}], [(\nu_1^L)^{\xi+t(\alpha_1)}\times(\nu_2^L)^{\xi+t(\alpha_1)}, (\nu_1^U)^{\xi+t(\alpha_1)}\times(\nu_1^U)^{\xi+t(\alpha_1)}]\}\\ = \ & \left\{\left[\sqrt[q]{1-\prod_{k=1}^{2}(1-(\mu_k^L)^q)^{\xi+t(\alpha_k)}}, \sqrt[q]{1-\prod_{k=1}^{2}(1-(\mu_k^U)^q)^{\xi+t(\alpha_k)}}\right],\left[\prod_{k=1}^{2}(\eta_k^L)^{\xi+t(\alpha_k)}, \prod_{k=1}^{2}(\eta_k^U)^{\xi+t(\alpha_k)}\right],\right.\\ & \left.\left[\prod_{k=1}^{2}(\nu_k^L)^{\xi+t(\alpha_k)}, \prod_{k=1}^{2}(\nu_k^U)^{\xi+t(\alpha_k)}\right]\right\}.\end{aligned}$$

Furthermore, we assume equation (9) holds when $n = r$, and obtain

$$\begin{aligned}\mathop{\oplus}_{k=1}^{r}(\xi + t(\alpha_k))\alpha_k = & \left\{\left[\sqrt[q]{1-\prod_{i=1}^{r}(1-(\mu_i^L)^q)^{\xi+t(\alpha_k)}}, \sqrt[q]{1-\prod_{k=1}^{r}(1-(\mu_k^U)^q)^{\xi+t(\alpha_k)}}\right],\right.\\ & \left.\left[\prod_{k=1}^{r}(\eta_k^L)^{\xi+t(\alpha_k)}, \prod_{i=1}^{r}(\eta_i^U)^{\xi+t(\alpha_k)}\right],\left[\prod_{k=1}^{r}(\nu_k^L)^{\xi+t(\alpha_k)}, \prod_{i=1}^{r}(\nu_i^U)^{\xi+t(\alpha_k)}\right]\right\}.\end{aligned}$$

Final, when $n = r + 1$, we have

$$\begin{aligned}&\left(\mathop{\oplus}_{k=1}^{r}(\xi + t(\alpha_k))\alpha_k\right)\oplus(\xi + t(\alpha_{r+1}))\alpha_{r+1}\\ &= \left\{\left[\sqrt[q]{1-\prod_{k=1}^{r}(1-(\mu_k^L)^q)^{\xi+t(\alpha_k)}+1-(1-(\mu_{r+1}^L)^q)^{\xi+t(\alpha_{r+1})}-\left(1-\prod_{k=1}^{r}(1-(\mu_k^L)^q)^{\xi+t(\alpha_k)}\right)\times(1-(1-(\mu_{r+1}^L)^q)^{\xi+t(\alpha_{r+1})}\right)},\right.\\ &\left.\sqrt[q]{1-\prod_{k=1}^{r}(1-(\mu_k^U)^q)^{\xi+t(\alpha_k)}+1-(1-(\mu_{r+1}^U)^q)^{\xi+t(\alpha_{r+1})}-\left(1-\prod_{k=1}^{r}(1-(\mu_k^U)^q)^{\xi+t(\alpha_k)}\right)\times(1-(1-(\mu_{r+1}^U)^q)^{\xi+t(\alpha_{r+1})}\right)}\right],\\ &\left[\prod_{k=1}^{r}(\eta_k^L)^{\xi+t(\alpha_k)}\times(\eta_{r+1}^L)^{\xi+t(\alpha_{r+1})}, \prod_{k=1}^{r}(\eta_k^U)^{\xi+t(\alpha_k)}\times(\eta_{r+1}^U)^{\xi+t(\alpha_{r+1})}\right],\left[\prod_{k=1}^{r}(\nu_k^L)^{\xi+t(\alpha_k)}\times(\nu_{r+1}^L)^{\xi+t(\alpha_{r+1})}, \prod_{k=1}^{r}(\nu_k^U)^{\xi+t(\alpha_k)}\right.\\ &\left.\left.\times(\nu_{r+1}^U)^{\xi+t(\alpha_{r+1})}\right]\right\}\\ &=\left\{\left[\sqrt[q]{1-\prod_{k=1}^{r+1}(1-(\mu_k^L)^q)^{\xi+t(\alpha_k)}}, \sqrt[q]{1-\prod_{k=1}^{r+1}(1-(\mu_k^U)^q)^{\xi+t(\alpha_k)}}\right],\left[\prod_{k=1}^{r+1}(\eta_k^L)^{\xi+t(\alpha_k)}, \prod_{k=1}^{r+1}(\eta_k^U)^{\xi+t(\alpha_k)}\right],\right.\\ &\left.\left[\prod_{k=1}^{r+1}(\nu_k^L)^{\xi+t(\alpha_k)}, \prod_{k=1}^{r+1}(\nu_k^U)^{\xi+t(\alpha_k)}\right]\right\}\end{aligned}$$

Therefore, equation (11) holds. Furthermore, we can obtain

$$\overset{n}{\underset{k=1}{\oplus}} \frac{\xi + t(\alpha_k)}{\sum_{k=1}^n(\xi + t(\alpha_k))}\alpha_k = \left\{\left[\sqrt[q]{1 - \prod_{k=1}^{n}(1 - (\mu_k^L)^q)^{\frac{\xi+t(\alpha_k)}{\sum_{k=1}^n(\xi+t(\alpha_k))}}}, \sqrt[q]{1 - \prod_{k=1}^{n}(1 - (\mu_k^U)^q)^{\frac{\xi+t(\alpha_k)}{\sum_{k=1}^n(\xi+t(\alpha_k))}}}\right],\right.$$

$$\left.\left[\prod_{k=1}^{n}(\eta_k^L)^{\frac{\xi+t(\alpha_k)}{\sum_{k=1}^n(\xi+t(\alpha_k))}}, \prod_{k=1}^{n}(\eta_k^U)^{\frac{\xi+t(\alpha_k)}{\sum_{k=1}^n(\xi+t(\alpha_k))}}\right], \left[\prod_{k=1}^{n}(\nu_k^L)^{\frac{\xi+t(\alpha_k)}{\sum_{k=1}^n(\xi+t(\alpha_k))}}, \prod_{k=1}^{n}(\nu_k^U)^{\frac{\xi+t(\alpha_k)}{\sum_{k=1}^n(\xi+t(\alpha_k))}}\right]\right\}.$$

In short, equation (10) holds. Since $\mu_k^L \le \mu_k^U$, $\eta_k^L \le \eta_k^U$, and $\nu_k^L \le \nu_k^U$, there must be

$$\sqrt[q]{1 - \prod_{k=1}^{n}(1 - (\mu_k^L)^q)^{\frac{\xi+t(\alpha_k)}{\sum_{k=1}^n(\xi+t(\alpha_k))}}} \le \sqrt[q]{1 - \prod_{k=1}^{n}(1 - (\mu_k^U)^q)^{\frac{\xi+t(\alpha_k)}{\sum_{k=1}^n(\xi+t(\alpha_k))}}}$$

$$\prod_{k=1}^{n}(\eta_k^L)^{\frac{\xi+t(\alpha_k)}{\sum_{k=1}^n(\xi+t(\alpha_k))}} \le \prod_{k=1}^{n}(\eta_k^U)^{\frac{\xi+t(\alpha_k)}{\sum_{k=1}^n(\xi+t(\alpha_k))}}$$

$$\prod_{k=1}^{n}(\nu_k^L)^{\frac{\xi+t(\alpha_k)}{\sum_{k=1}^n(\xi+t(\alpha_k))}} \le \prod_{k=1}^{n}(\nu_k^U)^{\frac{\xi+t(\alpha_k)}{\sum_{k=1}^n(\xi+t(\alpha_k))}}$$

In addition, we obtain

$$\left(\sqrt[q]{1 - \prod_{k=1}^{n}(1 - (\mu_k^U)^q)^{\frac{\xi+t(\alpha_k)}{\sum_{k=1}^n(\xi+t(\alpha_k))}}}\right)^q + \prod_{k=1}^{n}(\eta_k^U)^{\frac{\xi+t(\alpha_k)}{\sum_{k=1}^n(\xi+t(\alpha_k))}q} + \prod_{k=1}^{n}(\nu_k^U)^{\frac{\xi+t(\alpha_k)}{\sum_{k=1}^n(\xi+t(\alpha_k))}q} \le 1.$$

Hence, the calculation result of IVTSFEPA operator is still an IVTSFN. The proof for IVTSFEPA operator meeting Theorem 1 is similar to the proof for IVTSFEPG operator meeting Theorem 1, which is omitted in this study. □

**Property 2.** (Idempotence) Let $\alpha_1, \alpha_2, \ldots, \alpha_n$ be a set of IVTSFNs with $\alpha_1 = \alpha_2 = \ldots = \alpha_n = \alpha$, then

$$\text{IVTSFEPA}^{\xi}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \alpha, \tag{12}$$

$$\text{IVTSFEPG}^{\xi}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \alpha. \tag{13}$$

**Property 3.** (Boundedness) Let $\alpha_1, \alpha_2, \ldots, \alpha_n$ be a set of IVTSFNs. Then, we have $\alpha^- = ([\min_k\mu_k^L, \min_k\mu_k^U], [\max_k\eta_k^L, \max_k\eta_k^U], [\max_k\nu_k^L, \max_k\nu_k^U])$ and $\alpha^+ = ([\max_k\mu_k^L, \max_k\mu_k^U], [\min_k\eta_k^L, \min_k\eta_k^U], [\min_k\nu_k^L, \min_k\nu_k^U])$. There exists

$$\alpha^- \le \text{IVTSFEPA}^{\xi}(\alpha_1, \alpha_2, \ldots, \alpha_n) \le \alpha^+, \tag{14}$$

$$\alpha^- \le \text{IVTSFEPG}^{\xi}(\alpha_1, \alpha_2, \ldots, \alpha_n) \le \alpha^+. \tag{15}$$

**Property 4.** Let $\alpha_1, \alpha_2, \ldots, \alpha_n$ be a set of IVTSFNs, and $\alpha'_1, \alpha'_2, \ldots, \alpha'_n$ be any replacement of $\alpha_1, \alpha_2, \ldots, \alpha_n$. Then, we have

$$\text{IVTSFEPA}^{\xi}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \text{IVTSFEPA}^{\xi}(\alpha'_1, \alpha'_2, \ldots, \alpha'_n), \tag{16}$$

$$\text{IVTSFEPG}^{\xi}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \text{IVTSFEPG}^{\xi}(\alpha'_1, \alpha'_2, \ldots, \alpha'_n). \tag{17}$$

**Property 5.** (Monotonicity) Let the two sets of IVTSFNs be $\alpha_1, \alpha_2, \ldots, \alpha_n$ and $\beta_1, \beta_2, \ldots, \beta_n$, respectively, with $\alpha_k \le \beta_k (k = 1, 2, \ldots, n)$. Then, we have

$$\text{IVTSFEPA}^{\xi}(\alpha_1, \alpha_2, \ldots, \alpha_n) \le \text{IVTSFEPA}^{\xi}(\beta_1, \beta_2, \ldots, \beta_n), \tag{18}$$

$$\text{IVTSFEPG}^{\xi}(\alpha_1, \alpha_2, \ldots, \alpha_n) \le \text{IVTSFEPG}^{\xi}(\beta_1, \beta_2, \ldots, \beta_n). \tag{19}$$

### 3.2 IVTSFEPWA and IVTSFEPWG operators

In practice, different IVTSFNs may be assigned to different weights, and the IVTSFEPA and IVTSFEPG operators do not take into account the weights of IVTSFNs. Therefore, this sub-section proposes IVTSFEPWA and IVTSFEPWG operators.

**Definition 8.** Let a set of IVTSFNs be $\alpha_1, \alpha_2,\ldots, \alpha_n$, and its weight vector is $w = (w_1, w_2,\ldots,w_n)^T$ with $0 \le w_k \le 1$ and $\sum_{k=1}^{n} w_k = 1$. Then,

(1) The IVTSFEPWA operator is expressed as:

$$\begin{aligned}\mathrm{IVTSFEPWA}^{\xi}(\alpha_1, \alpha_2,\ldots, \alpha_n) &= \mathop{\oplus}_{k=1}^{n} \frac{w_k(\xi + t(\alpha_k))}{\sum_{k=1}^{n} w_k(\xi + t(\alpha_k))}\alpha_k \\ &= \left\{\left[\sqrt[q]{1 - \prod_{k=1}^{n}(1 - (\mu_k^L)^q)^{\frac{w_k(\xi+t(\alpha_k))}{\sum_{k=1}^{n} w_k(\xi+t(\alpha_k))}}}, \sqrt[q]{1 - \prod_{k=1}^{n}(1 - (\mu_k^U)^q)^{\frac{w_k(\xi+t(\alpha_k))}{\sum_{k=1}^{n} w_k(\xi+t(\alpha_k))}}}\right],\left[\right.\right. \\ &\qquad \left.\prod_{k=1}^{n}(\eta_k^L)^{\frac{w_k(\xi+t(\alpha_k))}{\sum_{k=1}^{n} w_k(\xi+t(\alpha_k))}}, \prod_{k=1}^{n}(\eta_k^U)^{\frac{w_k(\xi+t(\alpha_k))}{\sum_{k=1}^{n} w_k(\xi+t(\alpha_k))}}\right], \\ &\qquad \left.\left[\prod_{k=1}^{n}(\nu_k^L)^{\frac{w_k(\xi+t(\alpha_k))}{\sum_{k=1}^{n} w_k(\xi+t(\alpha_k))}}, \prod_{k=1}^{n}(\nu_k^U)^{\frac{w_k(\xi+t(\alpha_k))}{\sum_{k=1}^{n} w_k(\xi+t(\alpha_k))}}\right]\right\}.\end{aligned} \tag{20}$$

(2) The IVTSFEPWG operator is expressed as:

$$\begin{aligned}\mathrm{IVTSFEPWG}^{\xi}(\alpha_1, \alpha_2,\ldots, \alpha_n) &= \mathop{\otimes}_{k=1}^{n} (\alpha_k)^{\frac{w_k(\xi+t(\alpha_k))}{\sum_{k=1}^{n} w_k(\xi+t(\alpha_k))}} \\ &= \left\{\left[\prod_{k=1}^{n}(\mu_i^L)^{\frac{w_k(\xi+t(\alpha_k))}{\sum_{k=1}^{n} w_k(\xi+t(\alpha_k))}}, \prod_{k=1}^{n}(\mu_k^U)^{\frac{w_k(\xi+t(\alpha_k))}{\sum_{k=1}^{n} w_k(\xi+t(\alpha_k))}}\right],\left[\right.\right. \\ &\qquad \left.\sqrt[q]{1 - \prod_{k=1}^{n}(1 - (\eta_k^L)^q)^{\frac{w_k(\xi+t(\alpha_k))}{\sum_{k=1}^{n} w_k(\xi+t(\alpha_k))}}}, \sqrt[q]{1 - \prod_{k=1}^{n}(1 - (\eta_k^U)^q)^{\frac{w_k(\xi+t(\alpha_k))}{\sum_{k=1}^{n} w_k(\xi+t(\alpha_k))}}}\right], \\ &\qquad \left.\left[\sqrt[q]{1 - \prod_{k=1}^{n}(1 - (\nu_k^L)^q)^{\frac{w_k(\xi+t(\alpha_k))}{\sum_{k=1}^{n} w_k(\xi+t(\alpha_k))}}}, \sqrt[q]{1 - \prod_{k=1}^{n}(1 - (\nu_k^U)^q)^{\frac{w_k(\xi+t(\alpha_k))}{\sum_{k=1}^{n} w_k(\xi+t(\alpha_k))}}}\right]\right\}.\end{aligned} \tag{21}$$

Clearly, IVTSFEPWA and IVTSFEPWG operators still satisfy Theorem 1, as well as properties 2–5. Moreover, if $w_1 = w_2 = \ldots= w_n = 1/n$, then IVTSFEPWA and IVTSFEPWG operators reduce to IVTSFEPA and IVTSFEPG operators, respectively.

## 4 MCDM model based on interval-valued T-spherical fuzzy extended power aggregation operators and IVTSFCE

This section proposes a novel MCDM model using interval-valued T-spherical fuzzy extended power aggregation operators and IVTSFCE. First, with the aid of maximizing deviation approach and IVTSFCE, the criteria weights are determined. Moreover, IVTSFEPWA and IVTSFEPWG operators are used to obtain comprehensive evaluation results of alternatives. In the end, the ranking of alternative is determined according to their comprehensive evaluation results.

## 4.1 Problem description

Suppose that an IVTSF-MCDM problem involves $m$ alternatives $X = \{x_i | i = 1, 2, \ldots, m\}$ and $n$ criteria $C = \{c_j | j = 1, 2, \ldots, n\}$. Let $M = (\alpha_{ij})_{m\times n}$ be a decision matrix given by DMs, where the IVTSFN $\alpha_{ij} = ([\mu_{ij}^L, \mu_{ij}^U], [\eta_{ij}^L, \eta_{ij}^U], [v_{ij}^L, v_{ij}^U])$ represents the evaluation of alternative $x_i$ under criterion $c_j$. The decision matrix $M$ can be shown as:

$$M = (\alpha_{ij})_{m\times n} = \begin{array}{c} \\ x_1 \\ x_2 \\ \vdots \\ x_m \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_n \end{array} \\ \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{m1} & \alpha_{m2} & \cdots & \alpha_{mn} \end{bmatrix} \end{array}$$

## 4.2 Determination of criteria weights by maximizing deviation approach and IVTSFCE

In this sub-section, motivated by the maximizing deviation approach (Farrokhizadeh et al. [21]), we develop a maximizing CE to obtain the criteria weights under interval-valued T-spherical fuzzy environments.

First, the CE between the alternative $x_i$ and other alternatives $x_k (k = 1, 2, \ldots, m, k \neq i)$ with respect to the criteria $c_j$ is calculated as follows:

$$\mathrm{DE}_{ik} = \frac{1}{m-1} \sum_{g=1, g\neq i}^{m} \mathrm{DE}(\alpha_{ij}, \alpha_{gj}), \tag{22}$$

where

$$\begin{aligned} \mathrm{CE}(\alpha_{ij}, \alpha_{kj}) = \frac{1}{6\ln 2}\Bigg( & \mu_{ij}^L \ln\left(\frac{2\mu_{ij}^L}{\mu_{ij}^L + \mu_{kj}^L}\right) + (1-\mu_{ij}^L)\ln\left(\frac{2(1-\mu_{ij}^L)}{2-\mu_{ij}^L-\mu_{kj}^L}\right) + \mu_{ij}^U \ln\left(\frac{2\mu_{ij}^U}{\mu_{ij}^U+\mu_{kj}^U}\right) \\ & + (1-\mu_{ij}^U)\ln\left(\frac{2(1-\mu_{ij}^U)}{2-\mu_{ij}^U-\mu_{kj}^U}\right) + \eta_{ij}^L \ln\left(\frac{2\eta_{ij}^L}{\eta_{ij}^L+\eta_{kj}^L}\right) + (1-\eta_{ij}^L)\ln\left(\frac{2(1-\eta_{ij}^L)}{2-\eta_{ij}^L-\eta_{kj}^L}\right) \\ & + \eta_{ij}^U \ln\left(\frac{2\eta_{ij}^U}{\eta_{ij}^U+\eta_{kj}^U}\right) + (1-\eta_{ij}^U)\ln\left(\frac{2(1-\eta_{ij}^U)}{2-\eta_{ij}^U-\eta_{kj}^U}\right) + v_{ij}^L \ln\left(\frac{2v_{ij}^L}{v_{ij}^L+v_{kj}^L}\right) \\ & + (1-v_{ij}^L)\ln\left(\frac{2(1-v_{ij}^L)}{2-v_{ij}^L-v_{kj}^L}\right) + v_1^U \ln\left(\frac{2v_{ij}^U}{v_{ij}^U+v_{kj}^U}\right) + (1-v_{ij}^U)\ln\left(\frac{2(1-v_{ij}^U)}{2-v_{ij}^U-v_{kj}^U}\right)\Bigg). \end{aligned} \tag{23}$$

Next, the overall CE of all alternatives concerning the criteria $c_j$ is calculated as follows:

$$\mathrm{DE}_j = \frac{1}{m-1} \sum_{i=1}^{m} \sum_{g=1, g\neq i}^{m} \mathrm{DE}(\alpha_{ij}, \alpha_{gj}). \tag{24}$$

The weighted CE function is then constructed as follows:

$$\mathrm{DE}(w) = \frac{1}{m-1} \sum_{j=1}^{n} w_j \sum_{i=1}^{m} \sum_{g=1, g\neq i}^{m} \mathrm{DE}(\alpha_{ij}, \alpha_{gj}). \tag{25}$$

Then, the following optimization model is constructed to compute the optimal weight vector of criteria, as follows:

$$\begin{cases} \max \dfrac{1}{m-1}\sum_{j=1}^{n} w_j \sum_{i=1}^{m}\sum_{g=1,g\neq i}^{m} \mathrm{DE}(\alpha_{ij},\alpha_{gj}) \\ 0 \leq w_1, w_2,\ldots,w_n \leq 1 \\ \sum_{j=1}^{n}(w_j)^2 = 1 \end{cases}. \tag{26}$$

The Lagrange function is constructed to obtain the solution of the aforementioned model as follows:

$$L(w,\chi) = \frac{1}{m-1}\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{g=1,g\neq i}^{m} \mathrm{DE}(\alpha_{ij},\alpha_{gj})w_j + \chi\left(\sum_{j=1}^{n}(w_j)^2 - 1\right). \tag{27}$$

The criteria weight vector after solving the Lagrange function is obtained as follows:

$$w_j = \frac{\sum_{i=1}^{m}\sum_{g=1,g\neq i}^{m}\mathrm{DE}(\alpha_{ij},\alpha_{gj})}{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{g=1,g\neq i}^{m}\mathrm{DE}(\alpha_{ij},\alpha_{gj})}. \tag{28}$$

## 4.3 Aggregation of the evaluation results of alternatives by using interval-valued T-spherical fuzzy extended power aggregation operators

In this sub-section, we adopt IVTSFPWA and IVTSFPWG operators to aggregate the evaluation results of alternatives, aiming at obtaining the comprehensive evaluation results of alternatives. The calculation process is as follows:

(a) Calculate the support degrees of criteria values by the following equation:

$$\mathrm{Sup}(\alpha_{ij},\alpha_{ij'}) = 1 - \mathrm{DE}(\alpha_{ij},\alpha_{ij'}), i = 1, 2,\ldots, m; j, j' = 1, 2,\ldots, n, j \neq j'. \tag{29}$$

(b) Calculate the total support degrees of criteria values by the following equation:

$$t(\alpha_{ij}) = \sum_{\substack{j'=1 \\ j'\neq j}}^{n} \mathrm{Sup}(\alpha_{ij},\alpha_{ij'}), i = 1, 2,\ldots, m; j = 1, 2,\ldots, n. \tag{30}$$

(c) Calculate the weight information of criteria values by the following equation:

$$\omega_{ij} = \frac{(\xi + t(\alpha_{ij}))w_j}{\sum_{j=1}^{n}(\xi + t(\alpha_{ij}))w_j}, i = 1, 2,\ldots, m; j = 1, 2,\ldots, n. \tag{31}$$

(d) Utilize IVTSFPWA and IVTSFPWG operators to aggregate the evaluation results of alternatives, as follows:

$$\begin{aligned} \alpha_i^A &= \mathrm{IVTSFEPWA}^{\xi}(\alpha_{i1},\alpha_{i2},\ldots,\alpha_{in}) = \bigoplus_{j=1}^{n}\omega_{ij}\alpha_{ij} \\ &= \left\{\left[\sqrt[q]{1-\prod_{j=1}^{n}(1-(\mu_{ij}^{L})^q)^{\omega_{ij}}}, \sqrt[q]{1-\prod_{j=1}^{n}(1-(\mu_{ij}^{U})^q)^{\omega_{ij}}}\right], \left[\prod_{j=1}^{n}(\eta_{ij}^{L})^{\omega_{ij}}, \prod_{j=1}^{n}(\eta_{ij}^{U})^{\omega_{ij}}\right], \left[\prod_{j=1}^{n}(\nu_{ij}^{L})^{\omega_{ij}}, \prod_{j=1}^{n}(\nu_{ij}^{U})^{\omega_{ij}}\right]\right\}, \end{aligned} \tag{32}$$

$$\begin{aligned} \alpha_i^G &= \mathrm{IVTSFEPWG}^{\xi}(\alpha_{i1},\alpha_{i2},\ldots,\alpha_{in}) = \bigotimes_{j=1}^{n}(\alpha_{ij})^{\omega_{ij}} \\ &= \left\{\left[\prod_{j=1}^{n}(\mu_{ij}^{L})^{\omega_{ij}}, \prod_{j=1}^{n}(\mu_{ij}^{U})^{\omega_{ij}}\right], \left[\sqrt[q]{1-\prod_{j=1}^{n}(1-(\eta_{ij}^{L})^q)^{\omega_{ij}}}, \sqrt[q]{1-\prod_{j=1}^{n}(1-(\eta_{ij}^{U})^q)^{\omega_{ij}}}\right], \right. \\ &\quad \left. \left[\sqrt[q]{1-\prod_{j=1}^{n}(1-(\nu_{ij}^{L})^q)^{\omega_{ij}}}, \sqrt[q]{1-\prod_{j=1}^{n}(1-(\nu_{ij}^{U})^q)^{\omega_{ij}}}\right]\right\}. \end{aligned} \tag{33}$$

(e) Calculate the score values of $a_i \in \{a_i^A, a_i^G\}(i = 1, 2,..., m)$. In general, a higher score value of an alternative indicates that its performance is better.

### 4.4 Process of the proposed IVTSF-MCDM model

According to the former analysis, the procedure of the proposed IVTSF-MCDM model is as follows:

***Stage 1.*** Collect the evaluation results given by DMs.

**Step 1.1:** Determine $m$ alternatives $\{x_i|i = 1, 2,..., m\}$ and $n$ criteria $\{c_j|j = 1, 2, ... ,n\}$.

**Step 1.2:** Ask DMs to provide their assessments of alternatives under criteria, and adopt IVTSFS to express these assessments; thus, obtain the interval-valued T-spherical fuzzy decision matrix $M = (\alpha_{ij})_{m\times n}$.

***Stage 2***. Determine criteria weights by maximizing deviation approach and IVTSFCE.

**Step 2.1:** Compute the overall CE of all alternatives under each criterion through equations (22)–(24).

**Step 2.2:** Construct optimization model with aid of equations (25) and (26).

**Step 2.3:** Calculate the weights of criteria according to equations (27) and (28).

***Stage 3.*** Calculate the comprehensive evaluation results of alternatives using interval-valued T-spherical fuzzy extended power aggregation operators.

**Step 3.1:** Calculate the total support degrees of criteria values based on equations (29) and (30).

**Step 3.2:** Calculate the weight information of criteria values, see equation (31).

**Step 3.3:** Aggregate the criteria values of an alternative to obtain the comprehensive evaluation result of this alternative by utilizing IVTSFEPWA or IVTSFEPWG operator, see equations (32) and (33).

**Step 3.4:** Rank the comprehensive evaluation results of alternatives to obtain the order of them by using the ranking method of IVTSFN.

## 5 Case analysis

### 5.1 Case description

A multinational corporation intends to invest in four countries, namely, *Pakistan* ($x_1$), *Iran* ($x_2$), *The United Arab Emirates* ($x_3$), and *Bangladesh* ($x_4$). Then, the board members of this corporation will assess the economic conditions of the aforementioned four countries, and the corresponding evaluation criteria are as follows:

(1) *Comfort zone* ($c_1$), which indicates the economic condition and the degree of social stability of the investee country.

(2) *Government regulations* ($c_2$), which represents the stability and rationality of relevant policies and regulations of the investee country.

(3) *People's interest* ($c_3$), which is the degree of fit between the interests of the people of the invested country and the investment project.

(4) *Market competition* ($c_4$), which refers to the market competition in the invested country.

### 5.2 Solving the aforementioned case by the proposed method

In this subsection, the proposed method is employed to select the optimal invest country.

***Stage 1.*** Collect the evaluation results given by DMs.

**Step 1.1–1.2**: The board members evaluate four countries according to the aforementioned four evaluation criteria, and the relevant evaluation results are expressed in IVTSFNs. Then, Table 1 shows the assessment results of board members.

**Table 1:** Decision matrix

| | $c_1$ | $c_2$ | $c_3$ | $c_4$ |
|---|---|---|---|---|
| $x_1$ | ([0.3,0.5], [0.4, 0.6], [0.2, 0.6]) | ([0.1, 0.5], [0.3, 0.6], [0.4, 0.7]) | ([0.2, 0.7], [0.4, 0.4], [0.3, 0.5]) | ([0.4, 0.9], [0.5, 0.6], [0.3, 0.4]) |
| $x_2$ | ([0.4, 0.5], [0.2, 0.7], [0.6, 0.7]) | ([0.4, 0.7], [0.3, 0.6], [0.5, 0.8]) | ([0.4, 0.8], [0.2, 0.4], [0.1, 0.6]) | ([0.3, 0.8], [0.4, 0.6], [0.2, 0.2]) |
| $x_3$ | ([0.8, 0.8], [0.1, 0.3], [0.5, 0.6]) | ([0.2, 0.7], [0.1, 0.2], [0.4, 0.8]) | ([0.7, 0.9], [0.3, 0.3], [0.4, 0.8]) | ([0.3, 0.4], [0.4, 0.8], [0.4, 0.4]) |
| $x_4$ | ([0.2, 0.2], [0.5, 0.5], [0.2, 0.6]) | ([0.5, 0.7], [0.3, 0.6], [0.3, 0.3]) | ([0.2, 0.8], [0.3, 0.4], [0.1, 0.3]) | ([0.1, 0.9], [0.3, 0.7], [0.5, 0.6]) |

***Stage 2***. Determine criteria weights by maximizing deviation approach and IVTSFCE.

**Step 2.1:** According to equations (22)–(24), the overall CE of all alternatives under each criterion are computed as:

$\mathrm{DE}_1 = 0.2524$, $\mathrm{DE}_2 = 0.1726$, $\mathrm{DE}_3 = 0.1550$, $\mathrm{DE}_4 = 0.1504$

**Step 2.2–2.3:** With the aid of equations (25)–(28), the weights of criteria are determined, which are as follows:

$w_1 = 0.3456$, $w_2 = 0.2363$, $w_3 = 0.2122$, $w_4 = 0.2059$

***Stage 3.*** Calculate the comprehensive evaluation results of alternatives by using interval-valued T-spherical fuzzy extended power aggregation operators.

**Step 3.1:** The total support degrees of criteria values are calculated based on equations (29) and (30), which are shown in Table 2.

**Step 3.2:** Without losing generality, let $\xi = 1$. Then, the weight information of criteria values is derived by using equation (31). The results are shown in Table 3.

**Step 3.3:** Let $q = 5$. Then, we utilize IVTSFEPWA or IVTSFEPWG operator to aggregate the evaluation results of alternatives.

If adopting the IVTSFEPWA operator, then we have

$$\alpha_1^A = ([0.3132, 0.7343], [0.3912, 0.5503], [0.2789, 0.5509])$$

$$\alpha_2^A = ([0.3867, 0.7231], [0.2538, 0.5805], [0.3137, 0.5414])$$

$$\alpha_3^A = ([0.6960, 0.7926], [0.1671, 0.3318], [0.4323, 0.6298])$$

$$\alpha_4^A = ([0.3781, 0.7686], [0.3572, 0.5335], [0.2292, 0.4383])$$

If adopting the IVTSFEPWG operator, then we have

$$\alpha_1^G = ([0.2253, 0.6057], [0.4179, 0.5766], [0.3235, 0.5990])$$

$$\alpha_2^G = ([0.3772, 0.6589], [0.3098, 0.6265], [0.5113, 0.6938])$$

$$\alpha_3^G = ([0.4598, 0.6917], [0.3038, 0.6002], [0.4458, 0.7165])$$

$$\alpha_4^G = ([0.2159, 0.4942], [0.4155, 0.5837], [0.3726, 0.5363])$$

**Table 2:** Total support degree

| | $c_1$ | $c_2$ | $c_3$ | $c_4$ |
|---|---|---|---|---|
| $x_1$ | 2.9360 | 2.9066 | 2.9448 | 2.8885 |
| $x_2$ | 2.8556 | 2.8881 | 2.8662 | 2.8288 |
| $x_3$ | 2.8188 | 2.7993 | 2.8227 | 2.6898 |
| $x_4$ | 2.7799 | 2.8608 | 2.8560 | 2.8079 |

**Table 3:** Weight information of criteria values

| | $c_1$ | $c_2$ | $c_3$ | $c_4$ |
|---|---|---|---|---|
| $x_1$ | 0.3469 | 0.2354 | 0.2135 | 0.2042 |
| $x_2$ | 0.3452 | 0.2380 | 0.2125 | 0.2042 |
| $x_3$ | 0.3484 | 0.2370 | 0.2141 | 0.2006 |
| $x_4$ | 0.3419 | 0.2388 | 0.2141 | 0.2052 |

**Step 3.4:** The score values of $\alpha_1^A, \alpha_2^A, \alpha_3^A, \alpha_4^A, \alpha_1^G, \alpha_2^G, \alpha_3^G, \alpha_4^G$ are calculated by equation (2), respectively, such as $\mathrm{sc}(\alpha_1^A) = 0.0649$, $\mathrm{sc}(\alpha_2^A) = 0.0613$, $\mathrm{sc}(\alpha_3^A) = 0.1471$, $\mathrm{sc}(\alpha_4^A) = 0.0867$, $\mathrm{sc}(\alpha_1^G) = 0.0235$, $\mathrm{sc}(\alpha_2^G) = 0.0332$, $\mathrm{sc}(\alpha_3^G) = 0.0454$, and $\mathrm{sc}(\alpha_4^G) = 0.0089$.

Thus, according to the aforementioned score values, the order of alternatives is $x_3 > x_4 > x_1 > x_2$ when adopting the IVTSFEPWA operator, whereas the order of alternatives is $x_3 > x_2 > x_1 > x_4$ when adopting the IVTSFEPWG operator.

Obviously, *The United Arab Emirates* ($x_3$) is the best country to invest in when adopting two aggregation operators. However, the order of other countries derived by the IVTSFEPWA operator is inconsistent with that derived by the IVTSFEPWG operator. The reason for this gap is their focus is different. The IVTSFEPWA operator emphasizes the overall level of alternatives, whereas the IVTSFEPWG operator pays attention to the strengths of alternatives in a single criterion.

## 5.3 Sensitivity analysis

According to the proposed method, different values of parameters $\xi$ and $q$ may cause different score values of alternatives, thus affecting the ranking result of alternatives. Therefore, this sub-section calculates the score values of countries with different values of $\xi$ and $q$ to validate their influence on the sorting results based on the aforementioned case.

The parameter $\xi$ represents DMs' preferences on extreme data. If the value of $\xi$ is larger than zero, it means extreme data belonging to error or bias information. On the contrary, it implies that extreme data are very important when the value of $\xi$ is smaller than zero. In case of $q = 5$, the values of $\xi$ are set from interval $[-13, -4] \cup [0, 9]$ in increments of 1 for sensitivity analysis. Figure 1 shows the effect of different $\xi$ values on the score values of countries.

As can be seen from Figure 1, when using the IVTSFEPWA operator with different values of $\xi$, the sorting result of countries is still $x_3 > x_4 > x_1 > x_2$. Meanwhile, if employing the IVTSFEPWG operator with different values of $\xi$, the order of countries is $x_3 > x_2 > x_1 > x_4$. Therefore, we can conclude that parameter $\xi$ does not affect the final decision result.

The parameter $q$ reflects the space range of DMs' beliefs about membership degree, abstinence degree, and non-membership degree. Generally, a larger value of $q$ indicates the larger range of the space of DMs' beliefs. In case of $\xi = 1$, the values of parameter $q$ are set from interval $[1, 20]$ in increments of 1 for calculating the score values of countries, which are shown in Figure 2.

From Figure 2, it can be observed that

(1) when $q \in [1, 3]$ and using the IVTSFEPWA operator, the sorting result is $x_3 > x_4 > x_2 > x_1$;
(2) when $q \in [4, 20]$ and using the IVTSFEPWA operator, the sorting result is $x_3 > x_4 > x_1 > x_2$;
(3) when $q = 1$ and using the IVTSFEPWG operator, the sorting result is $x_3 > x_4 > x_1 > x_2$;
(4) when $q \in [2, 20]$ and using the IVTSFEPWG operator, the sorting result is $x_3 > x_2 > x_1 > x_4$.

Thus, the change of the value of $q$ will have an impact on the final sorting result, but it will not result in the change of the optimal solution.

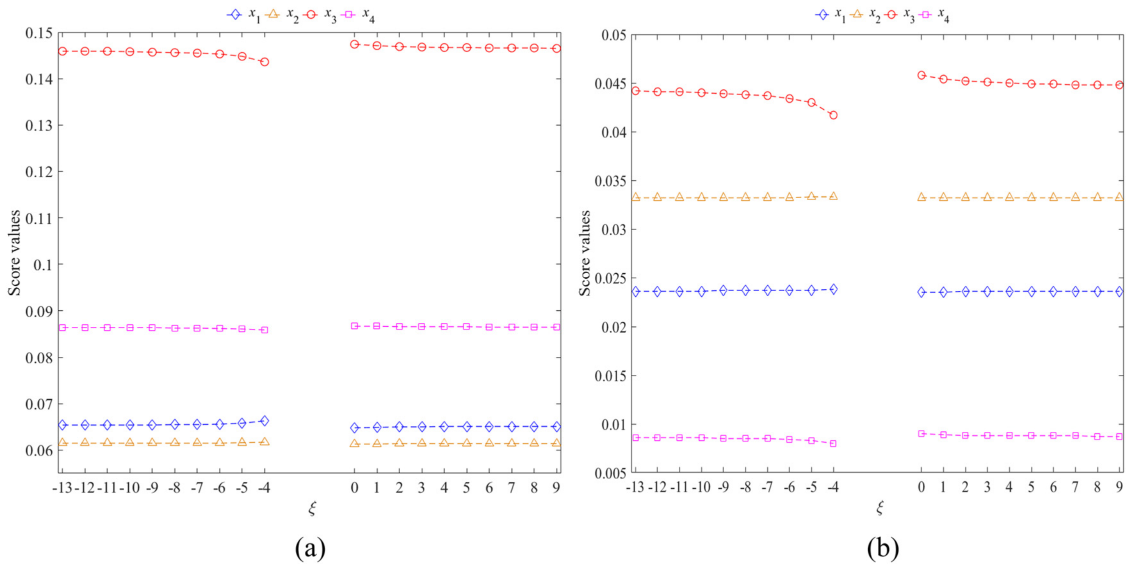


**Figure 1:** Score values of countries with the different values of $\xi$: (a) score values of countries derived by the IVTSEFPWA operator under different $\xi$ and (b) score values of countries derived by the IVTSFEPWG operator under different $\xi$.

## 5.4 Comparative analysis

To demonstrate the advantages of the proposed method, we compare it the following methods, respectively.

(1) The MCDM method-based IVTSFWA or IVTSFWG operator (namely the integrated operation-based method) [7]. This method adopts IVTSFWA and IVTSFWG operator to obtain the total evaluation result of each alternative. Then, the score values of total evaluation results are calculated to rank alternatives.

(2) The IVTSF-MCDM method based on technique for order performance by similarity to ideal solution (TOPSIS) (namely, IVTSF-TOPSIS) [27]. This method depends on the relative closeness of alternatives to sort alternatives.

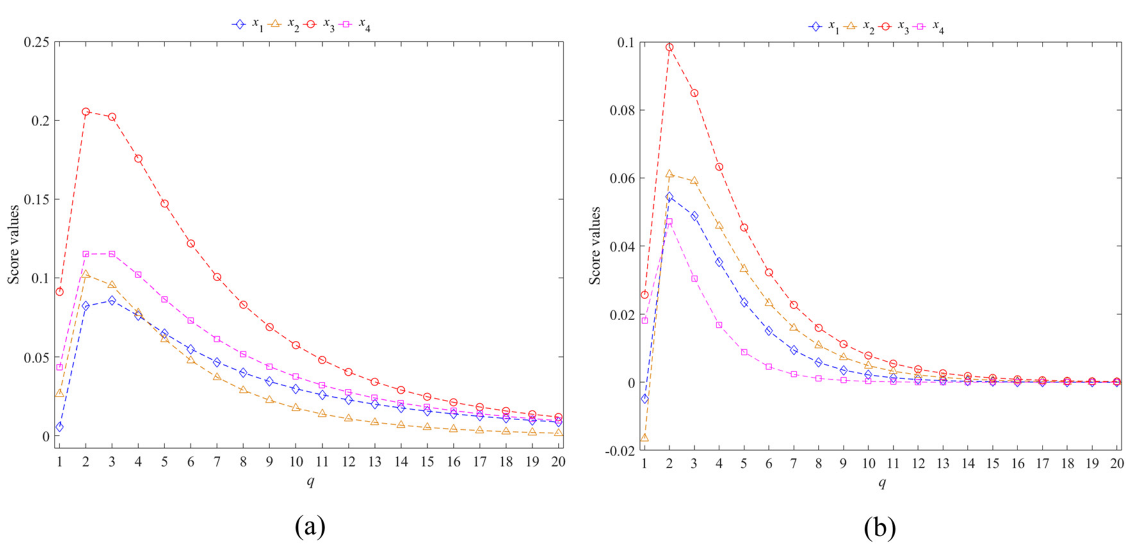


**Figure 2:** Score values of countries with the different values of $q$: (a) score values of countries derived by the IVTSFEPWA operator under different$q$ and (b) score values of countries derived by the IVTSFEPWG operator under different$q$.

**Table 4:** Results obtained by different methods

| Countries | The integrated operation-based method | | | | IVTSF-TOPSIS | | IVTSF-TODIM | | Proposed method | |
|---|---|---|---|---|---|---|---|---|---|---|
| | IVTSFWA Score values | Rank | IVTSFWG Score values | Rank | Relative closeness | Rank | Overall prospect value | Rank | IVTSFEPWA Rank | IVTSFEPWG Rank |
| $x_1$ | 0.1083 | 4 | 0.0520 | 4 | 0.2222 | 4 | 0.0000 | 4 | 3 | 3 |
| $x_2$ | 0.1423 | 3 | 0.0993 | 2 | 0.4897 | 2 | 0.0099 | 3 | 4 | 2 |
| $x_3$ | 0.3846 | 1 | 0.1263 | 1 | 0.7148 | 1 | 1.0000 | 1 | 1 | 1 |
| $x_4$ | 0.1846 | 2 | 0.0589 | 3 | 0.2704 | 3 | 0.2101 | 2 | 2 | 4 |

(3) The IVTSF-MCDM method based on TODIM (an acronym in Portuguese of interactive and MCDM) (namely, IVTSF-TODIM) (Ju et al. [28]). This method relies on the dominance of an alternative over other alternatives to compute the overall prospect value of this alternative that is used to rank alternatives.

For ensuring the rationality of the comparison, all methods use the evaluation information and criteria weight in the aforementioned case. The comparison details and the ranking of countries obtained by the aforementioned three methods are shown in Table 4 and Figure 3.

As shown in Table 4 and Figure 3, the optimal country obtained from all methods is $x_3$, which means the proposed method is valid. Meanwhile, the ranking results of the other countries obtained by the proposed method and other methods are different. The reason for these gaps is as follows:

(1) Both IVTSFWA and IVTSFWG operators simply aggregate the evaluations of alternatives, which ignores the effects of extreme values on the final result. The IVTSFEPWA and IVTSFEPWG operator utilize the parameter $\xi$ to effectively show the impact of extreme values on the ranking of alternatives. Therefore, the calculation results derived by the proposed method are more reasonable than that derived by IVTSFWA and IVTSFWG operators.

(2) The ranking index of IVTSF-TOPSIS and IVTSF-TODIM differs from that of the proposed method. The IVTSF-TOPSIS adopts the relative closeness to rank alternatives, which focus on the utility values of alternatives through reference points. The IVTSF-TODIM employs the overall prospect value to sort alternatives, which depends on the preference, indifference, and incomparability relations among alternatives to determine the priorities of alternatives. However, both IVTSF-TOPSIS and IVTSF-TODIM do not consider too high or

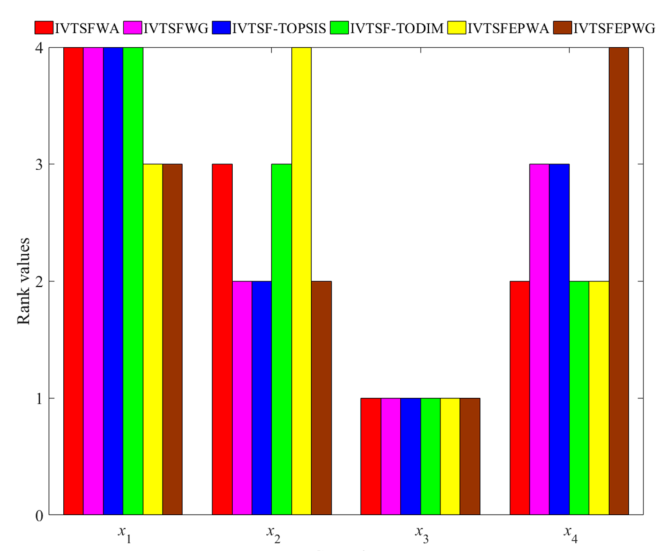


**Figure 3:** Sorting results of different methods.

too low evaluation results may affect the performances of alternatives. Conversely, the proposed method uses the IVTSFEPWA and IVTSFEPWG operator to overcome this shortcoming of IVTSF-TOPSIS and IVTSF-TODIM. Thus, the proposed method can provide more realistic results by comparing with IVTSF-TOPSIS and IVTSF-TODIM.

According to the aforementioned comparative analysis, we can conclude that the proposed method has a main advantage that the impact of extreme data is effectively considered under interval-valued T-spherical fuzzy environment.

## 6 Conclusion

In view of most of the existing IVTSF-MCDM methods that neglect the impacts of extreme data, this study presents a novel MCDM method based on interval-valued T-spherical fuzzy extended power aggregation operator. With the aid of EPA and EPG operators, IVTSFEPA, IVTSFEPG, IVTSFEPWA, and IVTSFEPWG operators are defined to rank alternatives for MCDM problems. Moreover, IVTSFCE and IVTSFSCE are proposed to measure the difference between two IVTSFNs and are used to derive the weights of criteria in MCDM. Final, we prove that the proposed method can reflect the effect of too high or too low evaluation results on the ranking of alternatives by comparing with other IVTSF-MCDM methods.

There are several research directions for future studies. The maximizing deviation approach is used to calculate the objective weights of criteria, and it can be combined with subjective weighting methods to derive the importance of criteria in the future. In addition, the DMs' irrational behaviors also can affect the order of alternatives. Therefore, the IVTSF-MCDM method considering DMs' irrational behaviors is our next work.

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