Research Article

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Intelligent multiple-attributes decision support for classroom teaching quality evaluation in dance aesthetic education based on the GRA and information entropy

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Abstract: Dance education in colleges and universities is the most important means of inheriting dance skills, cultivating dance talents, and promoting the prosperity and development of dance art. In the new era, the country's emphasis on "aesthetic education" has provided fertile policy soil for the development of dance majors in universities. Based on the spiritual and cultural needs of the people and the development needs of the national dance art, it is of great urgency for colleges and universities to explore the future oriented Chinese dance higher education and dance creation. Dance education and dance creation are closely linked and interdependent. In the process of Dance education, dance creation inspiration is stimulated. Dance creation and innovation inject new soul into Dance education. College Dance education should combine the two organically to promote the high-quality development of Chinese dance art. The classroom teaching quality evaluation in dance aesthetic education is classical multiple-attributes decision-making (MADM). The probabilistic hesitancy fuzzy sets (PHFSs) are used as a tool for characterizing uncertain information during the classroom teaching quality evaluation in dance aesthetic education. In this paper, we extend the classical grey relational analysis (GRA) method to the probabilistic hesitancy fuzzy MADM with unknown weight information. Firstly, the basic concept, comparative formula and Hamming distance of PHFSs are introduced. Then, the information entropy is used to compute the attribute weights based on the expected values and deviation degree. Then, probabilistic hesitancy fuzzy GRA (PHF-GRA) method is built for MADM under PHFSs. Finally, a practical case study for classroom teaching quality evaluation in dance aesthetic education is designed to validate the proposed method and some comparative studies are also designed to verify the applicability.

Keywords: multiple attributes decision making, probabilistic hesitant fuzzy sets, grey relational analysis method, information entropy, classroom teaching quality evaluation

1 Introduction

As a professional course, dance teaching has significant practical significance in promoting the all-round development of college students [1,2]. On the one hand, physical beauty is the main way to improve personal appearance. With the popularization of the Internet and smartphones, most students have developed bad habits of playing with computers and looking down at their phones for a long time, resulting in problems such as hunchback and forward leaning of the neck [3–5]. Through dance practice, not only can students' behavioral habits be improved, promoting a positive improvement in their personal posture, but it is also beneficial

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for improving their physical coordination, sensitivity, and softness, thereby comprehensively enhancing their personal temperament. On the other hand, physical training has strong practicality [6,7]. The study of dance courses can enable students to master certain theoretical knowledge and dance skills, and apply them to practical life, cultivate excellent training patterns, and promote significant improvement in physical fitness. At the same time, physical training can also help students better understand themselves, enhance self-confidence, and enhance personal charm. The current situation of dance teaching in universities mainly includes the following aspects [8,9]. One is the simple teaching environment. Physical training involves a large number of complex and difficult dance movements, so there are high requirements for relevant equipment and training venues. Only by having a comfortable learning environment can dance teaching activities be carried out smoothly and safely [10,11]. However, most universities do not attach enough importance to physical training, and dance training rooms are relatively simple and lack professional training equipment, which cannot meet the teaching needs of dance courses in ordinary universities today [12,13]. Secondly, the teaching content is single. At present, dance courses in Chinese universities mainly focus on public fitness knowledge, with insufficient specialization. In addition, many teachers in dance teaching mainly teach based on course content and personal training experience, and then teach to students through explanations and demonstrations [14]. This teaching method lacks professionalism and comprehensiveness, and is not in line with the demands of modern education development, making it difficult to stimulate students' enthusiasm for learning. Thirdly, the faculty is weak [15,16]. Compared to other dance courses, the focus of dance training is relatively low, and the professional faculty is weak. Most dance teachers are part-time teachers from other projects [17,18]. However, non-professional teachers are unable to meet dance teaching standards in terms of dance movements or educational concepts, nor can they provide professional guidance to students, which is not conducive to the development of dance teaching in universities. In response to the current teaching situation, corresponding reforms are needed in dance teaching in universities [19,20]. Firstly, we need to improve the venue equipment. Dance training cannot do without complete and professional teaching equipment, which is a prerequisite requirement for dance teaching reform. Therefore, universities need to provide specialized dance training rooms and add professional equipment such as joysticks, wall mirrors, multimedia teaching equipment, and audio facilities [21,22]. A good teaching environment is conducive to stimulating students' enthusiasm for physical training and actively increasing the actual training volume, thereby effectively improving students' body coordination and motor proficiency. Secondly, we need to enrich the teaching content [23–25]. On the one hand, dance teaching content should abandon the inherent "Empiricism," and teachers should select appropriate training content for students according to their personal characteristics to stimulate their dance creation and arrangement ability; on the other hand, teachers can appropriately add children's dance, Folk dance, campus group dance and other diversified dance training programs in the teaching content to expand students' artistic vision and promote their comprehensive ability. Thirdly, we need to strengthen the teaching staff [26–28]. College dance teachers need to continuously provide professional training, strengthen their personal dance professional knowledge and skills through professional books, online resources, and other means, in order to improve the level of dance teaching. In addition, dance teachers need to strengthen their information-based teaching abilities, learn to apply multimedia devices, and make the course content more vivid and intuitive, making it easier for students to learn [29,30].

As an important branch of Operations research and decision science, multiple attributes decision-making (MADM) is often used to deal with decision-making problems in discrete decision space [31–36]. It can be specifically described as a decision-making method in which decision-makers compare the comprehensive evaluation information of various options under several attributes, and then obtain the best option among known multiple alternative options or obtain the ranking of the advantages and disadvantages of the options, when the alternative options are known [37–40]. Because multi-attribute decision-making method can deal with both qualitative and quantitative information, as well as accurate and fuzzy information, it is widely used in military, engineering, economic and other fields, such as supply chain management, cadre selection, investment bidding, student performance evaluation, etc. [41–46]. Nowadays, MADM is still a hot research topic for scholars in various fields [47–52]. The classroom teaching quality evaluation in dance aesthetic education is classical MADM. Recently, the grey relational analysis (GRA) technique [53–57] has been utilized to put up with MADM issues. The probabilistic hesitancy fuzzy sets (PHFSs) [58] are employed as a technique for characterizing uncertain information during the classroom teaching quality evaluation in dance aesthetic education. Until now, no or few techniques have been constructed on entropy technique [59] and GRA technique under PHFSs. Therefore, a novel probabilistic hesitancy fuzzy GRA (PHF-GRA) technique is founded to manage the MADM under PHFSs based on the entropy technique [59]. Thus, in this paper, the PHF-GRA technique is implemented to solve the MADM under PHFSs. Finally, a numerical case study for the classroom teaching quality evaluation in dance aesthetic education and several comparative analysis is implemented to validate the proposed PHF-GRA technique. The motivations and main contribution of this study are implemented: (1) the GRA is implemented under PHFSs; (2) the information entropy is implemented to compute the attribute weights based on the expected values and deviation degree; (3) the PHF-GRA method is built for MADM under PHFSs; (4) a practical case study for classroom teaching quality evaluation in dance aesthetic education is designed to validate the proposed method and some comparative studies are also designed to verify the applicability.

In order to do so, the layout of this paper is as follows. Section 2 gives the basic introduction of PHFS. In Section 3, the model of PHF-GRA is then applied to MADM with information entropy weight. A numerical example for classroom teaching quality evaluation in dance aesthetic education is given in Section 4. In the last section, we drawn a conclusion and list the expect of future work.

2 Basic knowledge

Xu and Zhou [58] set up the PHFSs based on the HFSs [60–65].

Definition 1. [58]. Let θ be a fixed collection, a PHFSs on Θ is designed:

$$\mathrm{VH} = \{\langle \theta, vh_\theta(vp_\theta)\rangle | \theta \in \Theta\}, \tag{1}$$

where $vh_\theta(vp_\theta)$ is a set of values, and $vh_\theta(vp_\theta) \in [0,1]$ denoting the possible membership of the element $\theta \in \Theta$ to the set VH, vh_θ is called a hesitant fuzzy values (HFVs), vp_θ is a group of probabilities connected with vh_θ.

For convenience, $vh(vp)$ is named as the probabilistic hesitant fuzzy number (PHFNE) which are named as $vh(vp) = \{vh^{(v\phi)}(vp^{(v\phi)})|\ v\phi = 1, 2, \cdots, \#vh(vp)\}$, where $vp^{(v\phi)}$ is the probability values of the $vh^{(v\phi)}$, $\#vh(vp)$ is the number of membership, and $\sum_{v\phi=1}^{\#vh(vp)} vp^{(v\phi)} = 1$.

Definition 2. [58]. $vh_1(vp) = \{vh_1^{(v\phi)}(vp_1^{(v\phi)})|\ v\phi = 1, 2, \cdots, \#vh_1(vp)\}$ and $vh_2(vp) = \{vh_2^{(v\phi)}(vp_2^{(v\phi)})|\ v\phi = 1, 2, \cdots, \#vh_2(vp)\}$ be two PHFSs, $\#vh_1(vp)$ and $\#vh_2(vp)$ are the length of $vh_1(vp)$ and $vh_2(vp)$. If $\#vh_1(vp) > \#vh_2(vp)$, then add $\#vh_1(vp) - \#vh_2(vp)$ HFVs to $vh_2(vp)$. The added HFVs are the smallest HFVs in $vh_2(vp)$ and the probabilities are zero.

Definition 3. [58]. For the $vh_1(vp) = \{vh_1^{(v\phi)}(vp_1^{(v\phi)})|\ v\phi = 1, 2, \cdots, \#vh_1(vp)\}$, the expected values $\mathrm{EV}(vh_1(vp))$ and deviation degree $\mathrm{DD}(vh_1(vp))$ is defined:

$$\mathrm{EV}(vh_1(vp)) = \sum_{v\phi=1}^{\#vh_1(vp)} (vh_1^{(v\phi)} \times vp_1^{(v\phi)}), \tag{2}$$

$$\mathrm{DD}(vh_1(vp)) = \sqrt{\sum_{v\phi=1}^{\#vh_1(vp)} (vh_1^{(v\phi)} \times vp_1^{(v\phi)} - \mathrm{EV}(vh_1(vp)))^2}. \tag{3}$$

By equations (2) and (3), the order between two PHFEs is defined: (1) if $\mathrm{EV}(vh_1(vp)) > \mathrm{EV}(vh_2(vp))$, $vh_1(vp) > vh_2(vp)$; (2) if $\mathrm{EV}(vh_1(vp)) = \mathrm{EV}(vh_2(vp))$, if $\mathrm{DD}(vh_1(vp)) = \mathrm{DD}(vh_2(vp))$, then $vh_1(vp) = vh_2(vp)$; if $\mathrm{DD}(vh_1(vp)) < \mathrm{DD}(vh_2(vp))$, then, $vh_1(vp) > vh_2(vp)$.

Definition 4. [58]. Let $vh_1(vp) = \{vh_1^{(v\phi)}(vp_1^{(v\phi)})|\ v\phi = 1, 2, \cdots, \#vh_1(vp)\}$ and $vh_2(vp) = \{vh_2^{(v\phi)}(vp_2^{(v\phi)})|\ v\phi = 1, 2, \cdots, \#vh_2(vp)\}$ be two PHFEs with $\#vh_1(vp) = \#vh_2(vp) = \#vh(vp)$, then Hamming distance $\mathrm{HD}(vh_1(vp), vh_2(vp))$ is designed:

$$\mathrm{HD}(vh_1(vp), vh_2(vp)) = \frac{\sum_{v\phi=1}^{\#vh(vp)} |vh_1^{(v\phi)} \times vp_1^{(v\phi)} - vh_2^{(v\phi)} \times vp_2^{(v\phi)}|}{\#vh(vp)}. \tag{4}$$

3 PHF-GRA method for MAGDM with PHFSs

Then, PHF-QUALIFLEX is built to solve the MADM problems, where the decision-making information are depicted through PHFSs. The mathematical notations are used to depict the MADM issues under PHFSs. Let $\mathrm{VL} = \{\mathrm{VL}_1, \mathrm{VL}_2, \cdots, \mathrm{VL}_m\}$ be alternatives, and attributes $\mathrm{VT} = \{\mathrm{VT}_1, \mathrm{VT}_2, \cdots, \mathrm{VT}_n\}$ with weight vector $vw = (vw_1, vw_2, \cdots, vw_n)$, where $vw_j \in [0, 1]$, $\sum_{j=1}^{n} vw_j = 1$. Suppose that decision information is assessed and denoted as PHFSs $vh_{ij}(vp) = \{vh_{ij}^{(v\phi)}(vp_{ij}^{(v\phi)})|v\phi = 1, 2, \cdots, \#vh_{ij}(vp)\}$ $(i = 1, 2, \cdots, m,\ \ j = 1, 2, \cdots, n)$.

Then, PHF-GRA method is defined to cope with MADM issues under PHFSs and entropy weight. The detailed decision algorithms are designed as follows:

Step 1. Build the PHF-matrix $\mathrm{VH} = [vh_{ij}(vp)]_{m\times n}$:

$$\mathrm{VH} = [vh_{ij}(vp)]_{m\times n} = \begin{array}{c} \\ \mathrm{VL}_1 \\ \mathrm{VL}_2 \\ \vdots \\ \mathrm{VL}_m \end{array} \begin{bmatrix} \mathrm{VT}_1 & \mathrm{VT}_2 & \dots & \mathrm{VT}_n \\ vh_{11}(vp) & vh_{12}(vp) & \dots & vh_{1n}(vp) \\ vh_{21}(vp) & vh_{22}(vp) & \dots & vh_{2n}(vp) \\ \vdots & \vdots & \vdots & \vdots \\ vh_{m1}(vp) & vh_{m2}(vp) & \dots & vh_{mn}(vp) \end{bmatrix} \tag{5}$$

$$vh_{ij}(vp) = \{vh_{ij}^{(v\phi)}(vp_{ij}^{(v\phi)})|v\phi = 1, 2, \cdots, \#vh_{ij}(vp)\}. \tag{6}$$

Step 2. Standardize the matrix $\mathrm{VH} = [vh_{ij}(vp)]_{m\times n}$ into $\mathrm{NVH} = [nvh_{ij}(nvp)]_{m\times n}$.

$$\begin{cases} nvh_{ij}(nvp) = vh_{ij}(vp_{ij}) & \text{when the attribute is positive} \\ nvh_{ij}(nvp) = \{1 - vh_{ij}(vp_{ij})\} & \text{when the attribute is negative.} \end{cases} \tag{7}$$

Step 3. Compute the attributes weight by information entropy.

Entropy [59] is a conventional tool to derive weight. Firstly, the normalized values $nnh_{ij}(nnp)$ is obtained:

$$nnh_{ij}(nnp) = \frac{\mathrm{DD}(nvh_{ij}(nvp))/\mathrm{EV}(nvh_{ij}(nvp))}{\sum_{i=1}^{m}(\mathrm{DD}(nvh_{ij}(nvp))/\mathrm{EV}(nvh_{ij}(nvp)))}. \tag{8}$$

Then, the PHFS Shannon entropy $\mathrm{PHFSSE} = (\mathrm{PHFSSE}_1, \mathrm{PHFSSE}_2, \cdots, \mathrm{PHFSSE}_n)$ is obtained by equation (8):

$$\mathrm{PHFSSE}_j = -\frac{1}{\ln m}\sum_{i=1}^{m} \frac{\frac{\mathrm{DD}(nvh_{ij}(nvp))}{\mathrm{EV}(nvh_{ij}(nvp))}}{\sum_{i=1}^{m}\frac{\mathrm{DD}(nvh_{ij}(nvp))}{\mathrm{EV}(nvh_{ij}(nvp))}} \ln \frac{\frac{\mathrm{DD}(nvh_{ij}(nvp))}{\mathrm{EV}(nvh_{ij}(nvp))}}{\sum_{i=1}^{m}\frac{\mathrm{DD}(nvh_{ij}(nvp))}{\mathrm{EV}(nvh_{ij}(nvp))}} \tag{9}$$

and

$$\frac{\frac{\mathrm{DD}(nvh_{ij}(nvp))}{\mathrm{EV}(nvh_{ij}(nvp))}}{\sum_{i=1}^{m}\frac{\mathrm{DD}(nvh_{ij}(nvp))}{\mathrm{EV}(nvh_{ij}(nvp))}} \ln \frac{\frac{\mathrm{DD}(nvh_{ij}(nvp))}{\mathrm{EV}(nvh_{ij}(nvp))}}{\sum_{i=1}^{m}\frac{\mathrm{DD}(nvh_{ij}(nvp))}{\mathrm{EV}(nvh_{ij}(nvp))}} = 0$$

if

$$\frac{\mathrm{DD}(nvh_{ij}(nvp))/\mathrm{EV}(nvh_{ij}(nvp))}{\sum_{i=1}^{m}(\mathrm{DD}(nvh_{ij}(nvp))/\mathrm{EV}(nvh_{ij}(nvp)))} = 0.$$

Then, the weights $vw = (vw_1, vw_2, \cdots, vw_n)$ is obtained:

$$vw_j = \frac{1 - \mathrm{PHFSSE}_j}{\sum_{j=1}^{n}(1 - \mathrm{PHFSSE}_j)}, \quad j = 1, 2, \cdots, n. \tag{10}$$

Step 4. Define the PHFPIS and PHFNIS:

$$\mathrm{PHFPIS} = (\mathrm{PHFPIS}_1, \mathrm{PHFPIS}_2, \cdots, \mathrm{PHFPIS}_n), \tag{11}$$

$$\mathrm{PHFNIS} = (\mathrm{PHFNIS}_1, \mathrm{PHFNIS}_2, \cdots, \mathrm{PHFNIS}_n), \tag{12}$$

where

$$\mathrm{PHFPIS}_j = \{nvh_j^{(v\phi+)}(nvp_j^{(v\phi+)})|v\phi = 1, 2, \cdots, \#nvh_{ij}(nvp)\}, \tag{13}$$

$$\mathrm{PHFNIS}_j = \{nvh_j^{(v\phi-)}(nvp_j^{(v\phi-)})|v\phi = 1, 2, \cdots, \#nvh_{ij}(nvp)\}, \tag{14}$$

$$\mathrm{EV}(\mathrm{PHFPIS}_j) = \{\max_i \mathrm{EV}(nvh_{ij}(nvp))\}, \tag{15}$$

$$\mathrm{EV}(\mathrm{PHFNIS}_j) = \{\min_i \mathrm{EV}(nvh_{ij}(nvp))\}. \tag{16}$$

Step 5. compute the grey relational coefficient of all alternatives from PHFPIS and PHFNIS.

$$\mathrm{PHFPIS}(\xi_{ij}) = \frac{\rho \max_{1\le i\le m} \mathrm{HD}(nvh_{ij}(nvp), \mathrm{PHFPIS}_j)}{\mathrm{HD}(nh_{ij}(nvp), \mathrm{PHFPIS}_j) + \rho \max_{1\le i\le m} \mathrm{HD}(nvh_{ij}(nvp), \mathrm{PHFPIS}_j)}, \quad i = 1, 2, \cdots, m,\ j \in 1, 2, \cdots, n \tag{17}$$

$$\mathrm{PHFNIS}(\xi_{ij}) = \frac{\rho \max_{1\le i\le m} \mathrm{HD}(nh_{ij}(nvp), \mathrm{PHFNIS}_j)}{\mathrm{HD}(nh_{ij}(nvp), \mathrm{PHFNIS}_j) + \rho \max_{1\le i\le m} \mathrm{HD}(nh_{ij}(nvp), \mathrm{PHFNIS}_j)}, \quad i = 1, 2, \cdots, m,\ j \in 1, 2, \cdots, n. \tag{18}$$

Suppose that the identification coefficient is $\rho = 0.5$.

$$\mathrm{HD}(nvh_{ij}(nvp), \mathrm{PLPIS}_j) = \frac{\sum_{v\phi=1}^{\#nvh_{ij}(nvp)}|nvh_{ij}^{(v\phi)} \times nvp_{ij}^{(v\phi)} - nvh_j^{(v\phi+)} \times nvp_j^{(v\phi+)}|}{\#nvh_{ij}(nvp)}, \tag{19}$$

$$\mathrm{HD}(nvh_{ij}(nvp), \mathrm{PLNIS}_j) = \frac{\sum_{v\phi=1}^{\#nvh_{ij}(nvp)}|nvh_{ij}^{(v\phi)} \times nvp_{ij}^{(v\phi)} - nvh_j^{(v\phi-)} \times nvp_j^{(v\phi-)}|}{\#nvh_{ij}(nvp)}. \tag{20}$$

Step 6. Calculating the degree of grey relational coefficient of all alternatives from PHFPIS and PHFNIS:

$$\mathrm{PHFPIS}(\xi_i) = \sum_{j=1}^{n} vw_j \mathrm{PHFPIS}(\xi_{ij}), \quad i = 1, 2, \cdots, m \tag{21}$$

$$\mathrm{PHFNIS}(\xi_i) = \sum_{j=1}^{n} vw_j \mathrm{PHFNIS}(\xi_{ij}), \quad i = 1, 2, \cdots, m. \tag{22}$$

The fundamental idea of GRA method is that the optimal alternative is supposed to possess the "largest degree of grey relational coefficient" from PHFPIS and "smallest degree of grey relational coefficient" from PHFNIS. Obviously, the larger $\mathrm{PHFPIS}(\xi_i)$ along with smaller $\mathrm{PHFNIS}(\xi_i)$, the better alternative VL_i is.

Step 7. Derive the PHF relative relational degree (PHFRRD) of the alternatives from PHFPIS.

$$\mathrm{PHFRRD}(\xi_i) = \frac{\mathrm{PHFPIS}(\xi_i)}{\mathrm{PHFPIS}(\xi_i) + \mathrm{PHFNIS}(\xi_i)}, \quad i = 1, 2, \cdots, m \tag{23}$$

Step 8. According to PHFRRD(ξ_i), the sorting order of all possible alternatives can be obtained. If any alternative has the largest PHFRRD(ξ_i), then, it is optimal choice.

4 Case study and comparative analysis

4.1 Case study

Music and dance have artistic commonalities. In the teaching process of dance majors in universities, teachers not only need to cultivate students' dance skills, but also need to carry out teaching work to improve students' music literacy, promote students' precise control of stage music rhythm, achieve the coordination between dance performances and beautiful music, and further strengthen the artistic effect of dance performances. Based on this, the article will conduct research on the specific improvement paths based on the importance of improving the music literacy of dance majors in universities. First of all, college Dance education and creation should integrate artistic resources and aesthetic resources. Dance art is not only a performance at the technical level, but the injection of aesthetic resources will shape the soul of dance art. Aesthetics resources gather aesthetics, psychology, sociology, philosophy and other humanities and social disciplines, and are the cornerstone of Dance education and dance art creation. The current work of Dance education in colleges and universities, whether teachers or students, is relatively inadequate in the humanities and social sciences. It is limited to learning dance movements and related theoretical knowledge, which leads to the lack of deep understanding of dance art between teachers and students, and it is difficult to create excellent works. Therefore, in promoting the education and creation of dance art in universities, attention should be paid to the integration of artistic and aesthetic resources. While continuously improving basic dance skills, attention should be paid to the cultural core and ideological depth, in order to endow dance art with life and energy. First, we should strengthen interdisciplinary learning. In the dance art teaching system, we should strengthen the popularization of literature, history, art, psychology, philosophy, sociology and other multi-disciplinary basic knowledge, deepen students' cultural accumulation and improve their aesthetic taste and artistic taste through reading classic works, special reading seminars and other forms. Secondly, it is necessary to cultivate students' good attitude towards life and their pursuit of truth, goodness, and beauty. Dance art performance is not only a display of dance posture and skills, but also a display of the performer's personal appearance, attitude towards life, and image and temperament. Therefore, it is necessary to cultivate students' good temperament and cultivation, ambitious pursuit of ideals, and positive and upward attitude towards life, in order to enhance the aesthetic experience of dance art works in the "moistening and silent" environment. Dance art education and creation should emphasize cultural accumulation, cultivate students' aesthetic literacy, and enable them to achieve an artistic cultivation of internal and external integration. Secondly, college Dance education and creation should combine artistic resources with technical resources. The rapid development of Internet technology has provided a driving force for the reform of Dance education in colleges and universities. Dance art education in colleges and universities should rely on modern information and communication technology and take the Internet as the carrier to promote the transformation of Dance education and creation from the traditional two-dimensional plane teaching mode to three-dimensional or even four-dimensional. However, current dance teaching in universities still relies more on traditional teaching models, with students mainly receiving knowledge passively in one direction and lacking interactive teaching and learning. In the era of "Internet plus education," dance teaching and creation in colleges and universities should actively accept Internet thinking, use modern teaching tools, and improve the interactivity of teaching process, the mobility of teaching resources, and the autonomy of students' learning. Firstly, universities should create conditions to create smart classrooms using modern tools such as computers and multimedia, 3D naked eye videos, and two-way video conferencing, in order to enhance the experience and effectiveness of the teaching process. Secondly, the comprehensive popularization of the Internet and smart phones has laid a material foundation for college Dance education and creation. Teachers and students can use smart devices to provide exhibition space and design means for college dance teaching. For example, dance works can be disseminated through short video platforms to stimulate inspiration for dance creation among teachers and students. Social tools such as webcast, Tencent conference and WeChat live broadcast have also created many new learning scenes, which can be used by college Dance education to greatly realize the sharing of educational resources, the dissemination of educational information and the display of educational achievements. The classroom teaching quality evaluation in dance aesthetic education is classical MAGDM. The model of PHF-GRA is then applied to a numerical example for classroom teaching quality evaluation in dance aesthetic education. Six evaluation principles are given to evaluate the five dance colleges $VL_i (i = 1, 2, 3, 4, 5)$ as follows: VT_1 is dance teaching contents. VT_2 is the dance teaching method. VT_3 is the student satisfaction. VT_4 is the social influence. VT_5 is the peer recognition. VT_6 is the dance teaching achievements.

Then, the PHF-GRA method is applied to cope with a numerical example for classroom teaching quality evaluation in dance aesthetic education.

Step 1. Build the PHF-matrix $VH = [vh_{ij}(vp)]_{5\times4}$ (Table 1).

Step 2. Standardize the matrix $VH = [vh_{ij}(vp)]_{5\times4}$ into $NVH = [nvh_{ij}(nvp)]_{5\times4}$ (Table 2).

Step 3. Compute the weight values (Table 3).

Table 1: Decision matrix $VH = [vh_{ij}(vp)]_{5\times4}$

Alternative	VT₁	VVT₂
VL_1	{0.32 (0.5), 0.14 (0.2), 0.27 (0.3)}	{0.23 (0.2), 0.36 (0.3), 0.43 (0.5)}
VL_2	{0.15 (0.2), 0.26 (0.5), 0.42 (0.3)}	{0.37 (0.7), 0.28 (0.3)}
VL_3	{0.49 (0.5), 0.23 (0.5)}	{0.23 (0.5), 0.39 (0.5)}
VL_4	{0.72 (1)}	{0.65 (0.7), 0.73 (0.3)}
VL_5	{0.37 (0.5), 0.24 (0.2), 0.18 (0.3)}	{0.36 (0.2), 0.43 (0.3), 0.26 (0.5)}
Alternative	**VT_3**	**VT_4**
VL_1	{0.82 (0.7), 0.36 (0.3)}	{0.47 (1)}
VL_2	{0.32 (0.2), 0.21 (0.5), 0.16 (0.3)}	{0.26 (0.2), 0.34 (0.5), 0.16 (0.3)}
VL_3	{0.19 (0.2), 0.23 (0.5), 0.36 (0.3)}	{0.53 (0.5), 0.19 (0.5)}
VL_4	{0.62 (0.5), 0.54 (0.5)}	{0.76 (0.7), 0.52 (0.3)}
VL_5	{0.38 (1)}	{0.38 (0.5), 0.27 (0.5)}
Alternative	**VT_5**	**VT_6**
VL_1	{0.42 (0.2), 0.31 (0.3), 0.14 (0.5)}	{0.29 (0.7), 0.32 (0.3)}
VL_2	{0.26 (0.5), 0.35 (0.5)}	{0.37 (0.5), 0.24 (0.5)}
VL_3	{0.18 (0.7), 0.34 (0.3)}	{0.46 (1)}
VL_4	{0.54 (0.2), 0.62 (0.8)}	{0.62 (0.2), 0.73 (0.8)}
VL_5	{0.28 (0.5), 0.39 (0.2), 0.16 (0.3)}	{0.37 (0.2), 0.25 (0.5), 0.43 (0.3)}

Step 4. Determine the PHFPIS and PHFNIS (Table 4):

Step 5. Compute the grey relational coefficient from PHFPIS and PHFNIS (Tables 5 and 6).

Step 6. Compute the weighted grey relational coefficient from PHFPIS and PHFNIS (Tables 7 and 8).

Step 7. Obtain the degree of grey relational coefficient (Table 9):

Step 8. Calculate the PHFRRD from PHFPIS (Table 10).

Step 9. According to the PHFRRD$(\xi_i)(i = 1, 2, 3, 4, 5)$, the order is: $VL_4 > VL_1 > VL_3 > VL_2 > VL_5$ and the optimal dance colleges is VL_4.

Table 2: The NVH = $[nvh_{ij}(nvp)]_{5\times4}$

Alternative	VT$_1$	VT$_2$
VL$_1$	{0.14 (0.2), 0.27 (0.3), 0.32 (0.5)}	{0.23 (0.2), 0.36 (0.3), 0.43 (0.5)}
VL$_2$	{0.15 (0.2), 0.26 (0.5), 0.42 (0.3)}	{0.28 (0.0), 0.28 (0.3), 0.37 (0.7)}
VL$_3$	{0.23 (0.0), 0.23 (0.5), 0.49 (0.5)}	{0.23 (0.0), 0.23 (0.5), 0.39 (0.5)}
VL$_4$	{0.72 (0.0), 0.72 (0.0), 0.72 (1.0)}	{0.65 (0.0), 0.65 (0.7), 0.73 (0.3)}
VL$_5$	{0.18 (0.3), 0.24 (0.2), 0.37(0.5)}	{0.26 (0.5), 0.36 (0.2), 0.43 (0.3)}
Alternative	**VT$_3$**	**VT$_4$**
VL$_1$	{0.36 (0.0), 0.36 (0.3), 0.82 (0.7)}	{0.47 (0.0), 0.47 (0.0), 0.47 (1.0)}
VL$_2$	{0.16 (0.3), 0.21 (0.5), 0.32 (0.2)}	{0.16 (0.3), 0.26 (0.2), 0.34 (0.5)}
VL$_3$	{0.19 (0.2), 0.23 (0.5), 0.36 (0.3)}	{0.19 (0.0), 0.19 (0.5), 0.53 (0.5)}
VL$_4$	{0.54 (0.0), 0.54 (0.5), 0.62 (0.5)}	{0.52 (0.0), 0.52 (0.3), 0.76 (0.7)}
VL$_5$	{0.38 (0.0), 0.38 (0.0), 0.38 (1.o)}	{0.27 (0.0), 0.27 (0.5), 0.38 (0.5)}
Alternative	**VT$_5$**	**VT$_6$**
VL$_1$	{0.14 (0.5), 0.31 (0.3), 0.42 (0.2)}	{0.29 (0.0), 0.29 (0.7), 0.32 (0.3)}
VL$_2$	{0.26 (0.0), 0.26 (0.5), 0.35 (0.5)}	{0.24 (0.0), 0.24 (0.5), 0.37 (0.5)}
VL$_3$	{0.18 (0.0), 0.18 (0.7), 0.34 (0.3}	{0.46 (0.0), 0.46 (0.0), 0.46 (1,0)}
VL$_4$	{0.54 (0.0), 0.54 (0.2), 0.62 (0.8)}	{0.62 (0.0), 0.62 (0.2), 0.73 (0.8)}
VL$_5$	{0.16 (0.3), 0.28 (0.5), 0.39 (0.2)}	{0.25 (0.5), 0.37 (0.2), 0.43 (0.3)}

Table 3: The weight values

Attributes	AT$_1$	AT$_2$	AT$_3$
Weight information	0.1611	0.1432	0.1598
Attributes	**AT$_1$**	**AT$_2$**	**AT$_3$**
Weight information	0.1857	0.1951	0.1551

Table 4: The PHFPIS and PHFNIS

Alternative	VT$_1$	VT$_2$
PHFPIS	{0.72 (0.0), 0.72 (0.0), 0.72 (1.0)}	{0.65 (0.0), 0.65 (0.7), 0.73 (0.3)}
PHFNIS	{0.14 (0.2), 0.27 (0.3), 0.32 (0.5)}	{0.23 (0.0), 0.23 (0.5), 0.39 (0.5)}
Alternative	**VT$_3$**	**VT$_4$**
PHFPIS	{0.36 (0.0), 0.36 (0.3), 0.82 (0.7)}	{0.52 (0.0), 0.52 (0.3), 0.76 (0.7)}
PHFNIS	{0.16 (0.3), 0.21 (0.5), 0.32 (0.2)}	{0.16 (0.3), 0.26 (0.2), 0.34 (0.5)}
Alternative	**VT$_5$**	**VT$_6$**
PHFPIS	{0.54 (0.0), 0.54 (0.2), 0.62 (0.8)}	{0.62 (0.0), 0.62 (0.2), 0.73 (0.8)}
PHFNIS	{0.14 (0.5), 0.31 (0.3), 0.42 (0.2)}	{0.24 (0.0), 0.24 (0.5), 0.37 (0.5)}

4.2 Comparative analysis

In this section, we use the same data to solve the problem using other methods to verify the validity to prove this method's applicability. Thus, the PHF-GRA is compared with PHF-TODIM method [66], PHF-TOPSIS method

Table 5: The grey relational coefficient from PHFPIS

Alternatives	VT_1	VT_2	VT_3
VL_1	0.3299	0.8994	1.0000
VL_2	0.4714	0.8306	0.4714
VL_3	0.4458	0.4122	0.8752
VL_4	1.0000	1.0000	0.9571
VL_5	0.8862	0.4853	0.4458
Alternatives	VT_4	VT_5	VT_4
VL_1	0.9717	0.5068	0.4492
VL_2	0.8216	0.4624	0.3658
VL_3	0.4033	0.8662	0.6272
VL_4	1.0000	1.0000	1.0000
VL_5	0.4763	0.4368	0.4763

Table 6: The grey relational coefficient from PHFNIS

Alternatives	VT_1	VT_2	VT_3
VL_1	1.0000	0.3571	0.6463
VL_2	0.5379	0.4023	1.0000
VL_3	0.9717	1.0000	0.3441
VL_4	0.3441	0.3710	0.3209
VL_5	0.3209	0.9717	0.9717
Alternatives	VT_1	VT_2	VT_3
VL_1	0.3463	1.0000	0.6355
VL_2	1.0000	0.3915	1.0000
VL_3	0.9609	0.4728	0.3333
VL_4	0.3333	0.3601	0.3100
VL_5	0.3100	0.9609	0.9609

Table 7: The weighted grey relational coefficient from PHFPIS

Alternatives	VT_1	VT_2	VT_3
VL_1	0.0531	0.1288	0.1598
VL_2	0.0759	0.1189	0.0753
VL_3	0.0718	0.0590	0.1399
VL_4	0.1611	0.1432	0.1529
VL_5	0.1428	0.0695	0.0712
Alternatives	VT_4	VT_5	VT_4
VL_1	0.1804	0.0989	0.0697
VL_2	0.1526	0.0902	0.0567
VL_3	0.0749	0.1690	0.0973
VL_4	0.1857	0.1951	0.1551
VL_5	0.0884	0.0852	0.0739

[67], PHF- COPRAS method [68], generalized probabilistic hesitant fuzzy Bonferroni mean (GPHFBM) operator [69], PHFWA operator [58] and PHFWG operator [58]. The order is listed in Table 11.

Table 8: The weighted grey relational coefficient from PHFNIS

Alternatives	VT_1	VT_2	VT_3
VL_1	0.1611	0.0511	0.1033
VL_2	0.0867	0.0576	0.1598
VL_3	0.1565	0.1432	0.0550
VL_4	0.0554	0.0531	0.0513
VL_5	0.0517	0.1391	0.1553
Alternatives	**VT_1**	**VT_2**	**VT_3**
VL_1	0.0643	0.1951	0.0986
VL_2	0.1857	0.0764	0.1551
VL_3	0.1784	0.0922	0.0517
VL_4	0.0619	0.0703	0.0481
VL_5	0.0576	0.1875	0.1490

Table 9: $\text{PHFPIS}(\xi_i)$ and $\text{PHFNIS}(\xi_i)$

Alternatives	$\text{PHFPIS}(\xi_i)$	$\text{PHFNIS}(\xi_i)$
VL_1	0.6907	0.6735
VL_2	0.5697	0.7212
VL_3	0.6119	0.6771
VL_4	0.9931	0.3401
VL_5	0.5310	0.7402

Table 10: The PHFRRD from PHFPIS

Alternatives	VL_1	VL_2	VL_3	VL_4	VL_5
$\text{PHFRRD}(\xi_i)$	0.5063	0.4413	0.4747	0.7449	0.4177

Table 11: The rank of different models

Method	The order	The best solution
PHF-TODIM method [66]	$VL_4 > VL_1 > VL_2 > VL_3 > VL_5$	VL_4
PHF-TOPSIS method [67]	$VL_4 > VL_1 > VL_3 > VL_2 > VL_5$	VL_4
PHF-COPRAS method [68]	$VL_4 > VL_1 > VL_3 > VL_2 > VL_5$	VL_4
GPHFBM operator [69]	$VL_4 > VL_1 > VL_3 > VL_2 > VL_5$	VL_4
PHFWA operator [58]	$VL_4 > VL_1 > VL_3 > VL_2 > VL_5$	VL_4
PHFWG operator [58]	$VL_4 > VL_1 > VL_2 > VL_3 > VL_5$	VL_4
PHF-GRA method	$VL_4 > VL_1 > VL_3 > VL_2 > VL_5$	VL_4

In light with the WS coefficients [70,71], the WS coefficient between the PHF-TODIM method [66], PHF-TOPSIS method [67], PHF- COPRAS method [68], GPHFBM operator [69], PHFWA operator [58], PHFWG operator [58] and the proposed PHF-GRA technique is 0.8542, 1.0000, 1.0000, 1.0000, 1.0000, 0.8542, respectively. The WS coefficient shows the order of the proposed PHF-GRA technique are same to the order of PHF-TOPSIS method [67], PHF- COPRAS method [68], GPHFBM operator [69], PHFWA operator [58]; the WS coefficient shows the order of the proposed PHF-GRA technique are slightly different to the order of the PHF-TODIM method [66] and PHFWG operator [58]. Furthermore, the main reason for this slight difference is that PHF-

TODIM method [66] emphasize the psychological behavior of DMs, while PHFWG operator [58] emphasize the individual's influence. From the above detailed analysis, it could be seen that these four methods have the same optimal choice. This verifies the PHF-GRA technique is reasonable and effective.

5 Conclusions

New media technology has had a huge impact on the traditional teaching mode of dance art in universities. In the new era, it is necessary to actively utilize new concepts and technologies to innovate teaching modes and explore artistic creation ideas that are more in line with the needs of the times. The book briefly discusses the theory of Dance education and dance teaching mode, which has certain guiding significance for colleges and universities to promote the construction of related majors and personnel training. The administrators and educators of dance major in colleges and universities should constantly self-examine, self-innovate and learn from others to constantly push the cause of Chinese dance art to a new peak. The classroom teaching quality evaluation in dance aesthetic education is classical MADM. In this paper, we extend the classical GRA method to the probabilistic hesitancy fuzzy MADM with unknown weight information. Firstly, the basic concept, comparative formula and Hamming distance of PHFSs are introduced. Then, the information entropy is used to compute the attribute weights based on the expected values and deviation degree. The PHF-GRA method is built for MADM under PHFSs. Finally, a practical case study for classroom teaching quality evaluation in dance aesthetic education is designed to validate the proposed method and some comparative studies are also designed to verify the applicability. The main contributions of this study are implemented: (1) the GRA is implemented under PHFSs; (2) the information entropy is implemented to compute the attribute weights based on the expected values and deviation degree; (3) the PHF-GRA method is built for MADM under PHFSs; (4) a practical case study for classroom teaching quality evaluation in dance aesthetic education is designed to validate the proposed method and some comparative studies are also designed to verify the applicability.

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