Research Article

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# Hessenberg factorization and firework algorithms for optimized data hiding in digital images

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**Abstract:** Data hiding and watermarking are considered one of the most important topics in cyber security. This article proposes an optimized method for embedding a watermark image in a cover medium (color image). First, the color of the image is separated into three components (RGB). Consequently, the discrete wavelet transform is applied to each component to obtain four bands (high–high, high–low, low–high, and low–low), resulting in 12 bands in total. By omitting the low–low band from each component, a new square matrix is formed from the rest bands to be used for the hiding process after adding keys to it. These keys are generated using a hybrid approach, combining two chaotic functions, namely Gaussian and exponential maps. The embedding matrix is divided into square blocks with a specific length, each of which is converted using Hessenberg transform into two matrices, $P$ and $H$. For each block, a certain location within the $H$-matrix is used for embedding a secret value; the updated blocks are assembled, and the reverse process is performed. An optimization method is applied, through the application of the firework algorithm, on the set of the initial values that generate keys. Using an optimization procedure to obtain keys requires performing lowest possible change rate in an image and maintain the quality of the image. To analyze and test the efficiency of the proposed method, mean-square error (MSE) and peak signal-to-noise ratio (PSNR) measurements are calculated. Furthermore, the robustness of the watermark is computed by applying several attacks. The experimental results show that the value of the MSE is reduced by about 0.01 while the value of the PSNR is increased by about 1.25 on average. Moreover, the proposed method achieved a high-retrieval rate in comparison with the non-optimization approach.

**Keywords:** data hiding, watermarking, Hessenberg factorization, firework algorithms

## 1 Introduction

With the increased amounts of digital information on the web emerged the need to provide a way to preserve the intellectual property rights of individuals and organizations. Digital watermarking systems provide the necessary tools to protect copyrights, which attracted many researchers' attention due to

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increasing multimedia developments [1–3]. The digital image watermarking systems could be classified into two types, namely spatial and frequency domains. Each type has its characteristics. The former type includes those approaches that directly adjust the values of pixels in the cover images. The techniques under this type are characterized by being simple, fast, and embedding large data. However, they fail to persevere against various types of attacks, such as noise addition, median filtering, sharpening, and blurring. On the other hand, the latter type involves those watermarking techniques that embed data by modifying the coefficients of the transformed digital images. The advantages of these techniques are their robustness in terms of persisting against various security attacks. However, they are not problem free, requiring more computational power [4,5]. Several algorithms fall under the frequency domain type. The most used algorithms have singular value decomposition (SVD), discrete cosine transform (DCT), and discrete wavelet transform (DWT), just to name a few [2,4,6–8].

Wavelet analysis is a technique for separating the information in an image into approximation and detailed sub-signals [9]. Wavelet analysis is calculated using filter banks. Filters are divided into two categories. A high-pass filter preserves high-frequency information while obliterating low-frequency information, and a low-pass filter: high-frequency information is obliterated while low-frequency data are preserved. DWT can be easily combined with linear algebra; therefore, it is considered one of the best candidates compared to the other transformation methods. The literature shows the effectiveness of using the matrix decomposition schemes in the field of digital image watermarking. The eigenvalue decomposition matrices were used in the study presented by Kokabifar et al. [10], in which the authors suggested several digital image watermarking systems. The idea behind their suggested methods was to transform the cover and watermark images into the normal matrices space, whereas the watermarked images are obtained by performing the spectral decompositions. Although the achieved results show high-quality embedding, their persistence against various types of attack was not considered.

The Hessenberg matrix decomposition (HMD) was used for color image watermarking in Su's [11] work. The authors suggested an embedding process that modifies each element in the matrix of the color watermark image. In addition, the watermark extraction process of their proposed system does not require the original watermark and cover images. Moreover, the use of HMD showed a great improvement in the performance of the proposed watermarking system, as the experiential results indicated. A multiple decomposition watermarking system was proposed by Thajeel et al. [12]. The authors utilized the Arnold transform to increase the image watermarking security. Furthermore, several transforms were applied on the cover image, such as Schur decomposition, DCT, and Slantlet transform. The final step includes the scrambled watermark images. The quality of image watermarking was promising, as the experimental results illustrated. Liu et al. [13] introduced a hybrid decomposition watermarking system that combines DWT, SVD, and HMD. The DWT has first applied to the cover image. The resulted matrix is transformed using HMD, whereas the watermark image is obtained using SVD to be included in the cover image. Good quality embedding results were achieved for different image watermarking sizes. However, the time complexity of combining these methods is high.

In this article, an optimized digital image watermarking system is proposed. The suggested system depends on the use of DWT and Hessenberg Factorization. Furthermore, to enhance the performance of the embedding process, some generated keys are used. These keys are generated using chaotic hybrid maps, and their initial values are optimized using the firework algorithm (FWA). Consequently, the use of optimized keys will reduce the distortion in the covered image. The proposed method could be used in any watermarking-based applications, such as copyright protection.

The rest of the article is organized such that the fundamentals of Hessenberg factorization and FWAs are presented in Section 2. The proposed watermarking method is presented in Section 3. The results of implementing the proposed method are enumerated in Section 4. Finally, Section 5 draws the research conclusion and provides future work.

# 2 Methods and materials

This section reviews the main methods that are used in the proposed watermarking scheme.

## 2.1 Hessenberg factorization

A Hessenberg matrix is a 2D array that has been introduced by Karl Hessenberg, which is almost triangular in terms of linear algebra. It could be either lower or upper. The values of the elements above the first sub-diagonal in the former matrix are zeros. Likewise, the elements below the first sub-diagonal in the latter matrix are also zeros [14]. It can be noticed that many linear algebra procedures can be effectively applied on triangular matrices with substantially less computational power required. This advantage could be extended to include Hessenberg matrices. Therefore, if a typical complete matrix is hard to convert into a triangular to manipulate it in terms of computational power effectively, the next best alternative is to convert it into a Hessenberg matrix. In general, any matrix can be transformed into a Hessenberg matrix. This conversion can be done in a finite number of steps that do not require high computational power [15,16]. One of the conversion procedures is the shifting QR factorization, which is an iterative algorithm that can be employed to transform any matrix into a Hessenberg matrix.

## 2.2 FWA

An FWA is a swarm-based approach that simulates the explosion of fireworks at night. It produces sparks around each firework [17], as illustrated in Figure 1. Moreover, it can be used as a local search since each firework that sparks around some positions in the range is called amplitude. The global optimum can be reached from the cooperation of evolved populations. FWA is distinguished from other swarm and population-based methods by its distributed parallelism implementation, providing diversity, simplicity, and easy extension [18].

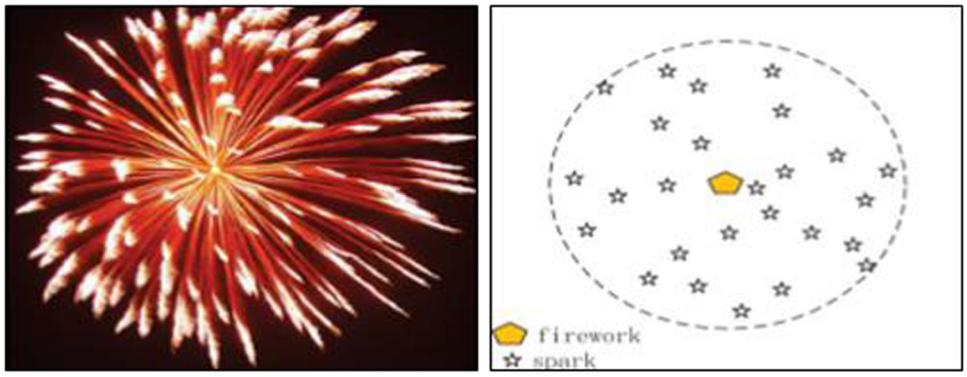


**Figure 1:** Firework search algorithm.

The general procedure of FWA can be described as follows [19]: it first randomly generates an initial population that contains N fireworks. It then computes their fitness values. The number of sparks and the explosion (explosion amplitude) range are determined by evaluating each firework. The new populations were obtained from exploding the previous fireworks and applying a local search on each one. The amplitude makes sure that the space between the local and global optimum is balanced. As a result, the diversity is maintained, and the chances of escaping from local minima are increased. Furthermore, FWA applies a mutation operator, which is known as Gaussian mutation, and a selection scheme that iteratively chooses a subset of the whole population [20]. The details of each step in FWA are explained as follows:

- Explosion operator: An explosion operator is applied to select multiple sparks from each firework in fireworks. This operator can be calculated by the following equation:

$$S_i = m \times \frac{y_{\max} - f(x_i) + \varepsilon}{\sum_{i=1}^{N}(y_{\max} - f(x_i)) + \varepsilon}, \tag{1}$$

where $S_i$ denotes the number of sparks, each firework contains, $m$ represents the total number of solutions in a firework, $N$ denotes the total number of sparks, $y_{\max}$ is the value of the worst fitness in the fireworks, $f(x_i)$ is the fitness value of $x_i$, and $\varepsilon$ is a small value to avoid division by zero. The amplitude of explosion $A_i$ is in the following equation:

$$A_i = \widehat{A} \times \frac{f(x_i) - y_{\min} + \varepsilon}{\sum_{i=1}^{N}(f(x_i) - y_{\min}) + \varepsilon}, \tag{2}$$

where $\hat{A}$ is the sum of all amplitudes and $\varepsilon$ is a small value to avoid division by zero.

To specify the displacement on every dimension in a firework, the aforementioned parameters are utilized as described in the following equation:

$$x_i^k = x_i^k + U(-A_i,\ A_i), \tag{3}$$

where $U(-A_i,\ A_i)$ is a random number in the uniform range of the amplitude $A_i$.

- Mutation operator: The mutation operator is applied to the current individual. Let $x_i^k$ be the position of the current solution, where $i$ belongs to the interval (1 to N), and k represents the current dimension. The neighbors of this position, which are called sparks of Gaussian explosion, are computed by the following equation:

$$x_i^k = x_i^k \times \mathrm{RG}(1, 1), \tag{4}$$

where RG (1, 1) is a random number obtained from the Gaussian distribution.

- Mapping rule: This rule guarantees that all solutions in a population are kept in an accepted domain. It makes sure that the resulting solutions after applying any FWA operation will be within the set of feasible solutions. This rule makes use of the modular operation as stated in the following equation:

$$x_i^k = X_{\mathrm{LBound},k} + x_i^k \mathrm{AMod}(X_{\mathrm{UBound},k} - X_{\mathrm{LBound},k}), \tag{5}$$

where $x_i^k$ represents the positions of sparks, which are laying outside the boundaries, whereas $X_{\mathrm{UBound}}$ and $X_{\mathrm{LBound}}$ are the boundary limits of the position of a spark, and "AMod" is the arithmetic modular [21].

- Selection method: The selection method might be required to calculate the distance between solutions. To serve this goal, the Euclidean distance measurement could be utilized to find the closest solution, as depicted in the following equation:

$$R(x_i) = \sum_{i=1}^{K} d(x_i, x_j) = \sum_{i=1}^{K} x_i - x_j, \tag{6}$$

where $d$ is the distance, which may be calculated using the Euclidean distance rule, between the two given individuals $x_i$ and $x_j$. Sparks, which are obtained from performing the explosion or mutation operators, could be included in this distance.

One of the approaches that FWA could use for selecting a new population is the classical Roulette Wheel procedure. It depends on $P(x_i)$, which can be computed by the following equation:

$$P(x_i) = \frac{R(x_i)}{\sum_{j \in k} R(x_j)}. \tag{7}$$

The resulting generation's diversity can be increased due to maintaining solutions with high distances to each other.

# 3 The proposed watermarking method

The general outlines of the proposed method are illustrated in Figure 2. It includes several steps for hiding a watermark in a color image [22]. The cover image is split into three color bands, and then, the DWT transform is performed.

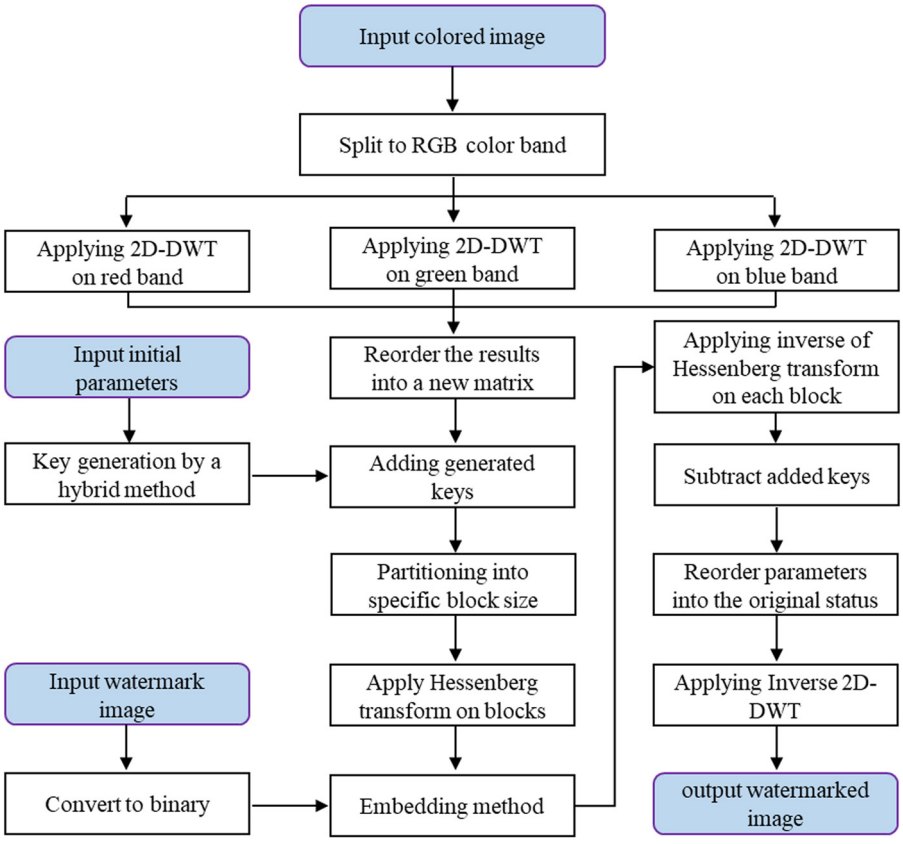


**Figure 2:** The watermarking method.

The next step involves reordering DWT parameters into a new embedding matrix (EM), and then a generated key is added to this matrix. The resulted matrix is divided into specific block sizes. The next subsections explain in detail each step. The optimization process is applied to reduce the percentage of change in the cover image. The cover image is tested with the watermark image (secret image) to find the best set of generated numbers added to the DWT embedding matrix (transformation process). This operation is done by FWA. A set of keys is initially generated, and then, these keys are changed by the operations associated with the algorithm that controls the generation of new keys. Each firework contains several sparks, as presented in the equations. From each solution, a set of sparks is generated by performing the rotation operation for these solutions, as shown in Figure 3.

## 3.1 Split to RGB color band

In this step, a color image is split into three color bands (RGB) [23], as shown in Figure 4. Each of these bands is used in the hiding procedure.

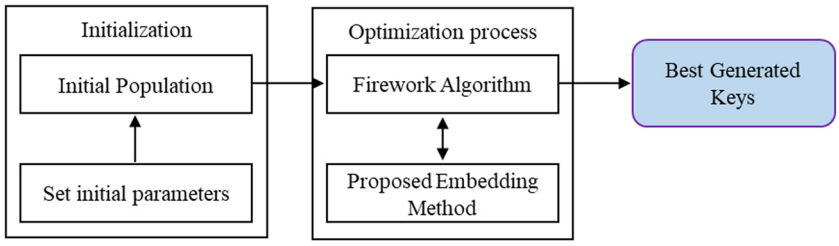


**Figure 3:** The optimization stage of the FWA.

**Figure 4:** The splitting of the color image.

## 3.2 Applying 2D-DWT

In this step, the 2D-DWT procedure is applied to the three-color bands. The result of this conversion will be (high–high, high–low, low–high, and low–low) for each color band, as shown in Figure 5.

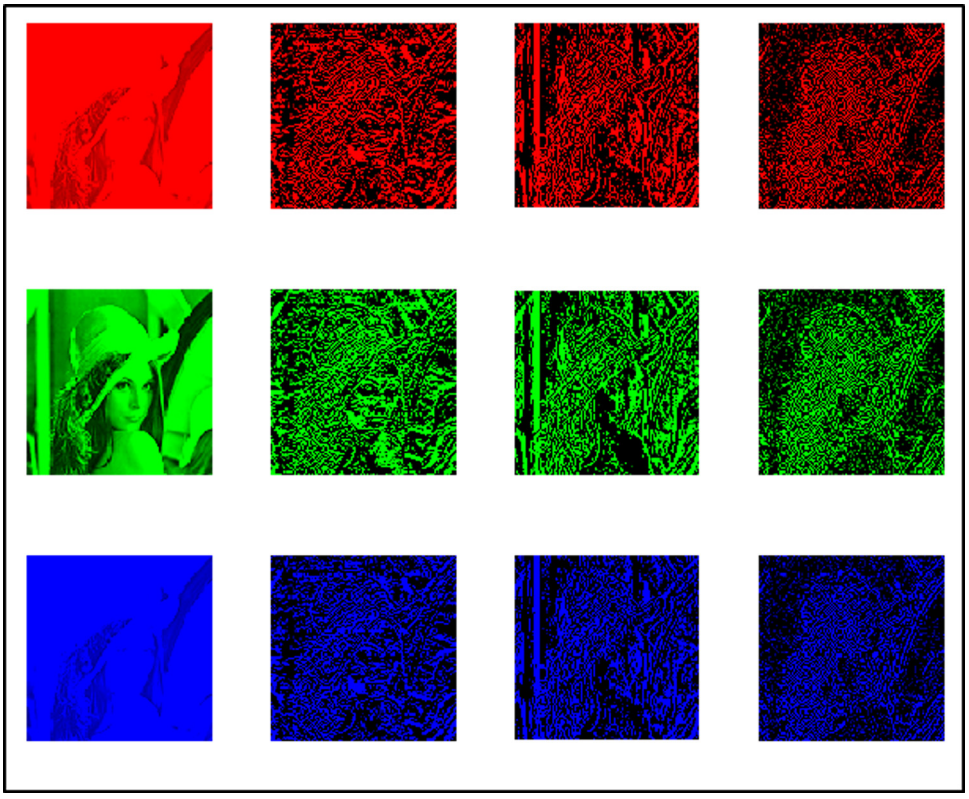

**Figure 5:** The 2D-DWT of the three-color band.

## 3.3 Reordering DWT parameters

After the conversion process, a new matrix (embedding matrix) is formed by taking all the conversion parameters except the low–low band, which retains the basic characteristics of the image. As a result, the new matrix consists of the merge of the three conversion parameters high–high, low–high, and high–low for the red band as a first row, the three conversion parameters of the green band as a second row, and the three conversion parameters of blue as a third row, as shown in Figure 6 [24–28]. The minimum value in the embedding matrix is found, and its absolute value is added to all parameters to process the negative values, i.e., all values will be positive.

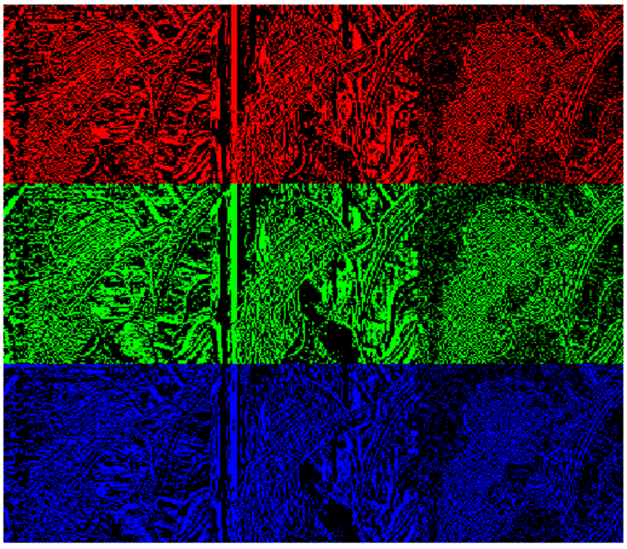

**Figure 6:** Embedding matrix.

## 3.4 Key generation

Two chaotic map methods, namely Gaussian map and exponential map, are used for the key generation process to produce numbers that will be added to the embedding matrix. The generated numbers are equal to the number of parameters in the embedding matrix. The initial parameters of these chaotic map functions are carefully selected later through the optimization process. The parameters of FWA are as follows: population size ($N = 100$), $y_{\min} = 3$, $y_{\max} = 50$, $A_i = 40$, and mutation spark number $x_i^k = 5$.

## 3.5 Partitioning embedding matrix

The embedding matrix is partitioned into blocks with a specific size, such as $4 \times 4$. Each block contains secret information (secret bit). The Hessenberg transform is applied on each block in the next stage, as shown in Figure 7.

## 3.6 Applying Hessenberg transform

Hessenberg transformation (factorization) is applied to each block that was partitioned in the previous step. The result of applying this transformation will be two matrices called $P$-matrix and $H$-matrix, as shown in Table 1. The embedding process will only use $H$-matrix for embedding.

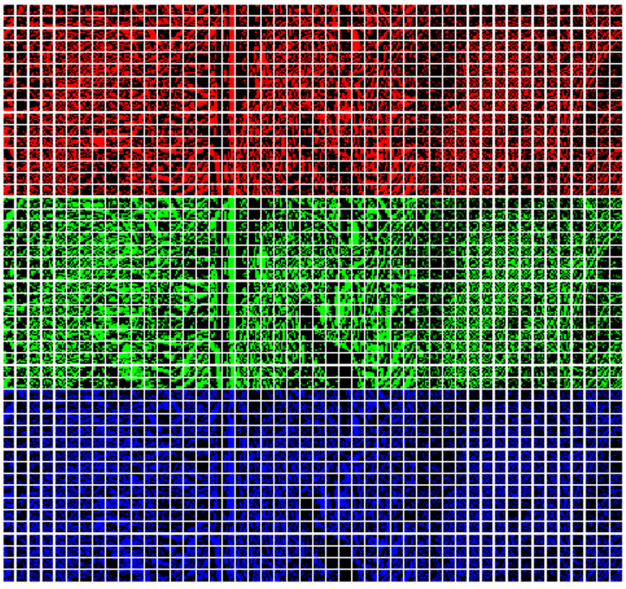

**Figure 7:** Embedding matrix partitioning.

**Table 1:** Hessenberg factorization example

| Block number samples | | | | | *P*-matrix | | | *H*-matrix | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 162.000 | 176.000 | 176.500 | 178.000 | 1.000 | 0.000 | 0.000 | 0.000 | 162.000 | −301.253 | 44.346 | −33.049 |
| 157.500 | 128.000 | 127.500 | 125.000 | 0.000 | −0.420 | 0.803 | −0.423 | −375.364 | 602.987 | −89.929 | 70.669 |
| 231.500 | 235.500 | 235.500 | 233.500 | 0.000 | −0.617 | −0.594 | −0.516 | 0.000 | 46.591 | −6.587 | 4.526 |
| 250.000 | 234.500 | 234.500 | 235.000 | 0.000 | −0.666 | 0.044 | 0.745 | 0.000 | 0.000 | −0.291 | 2.101 |

## 3.7 Embedding method

The embedding method involves converting the secret information (watermark image) into a vector of binary numbers [29,30]. Each binary number (bit) is embedded into one block using the following steps: first, specify one position in $H$-matrix such as (4,4); second, get the sign of numbers; third, get the integer value of the number; and fourth, get the digit after the floating point and check if it is odd or even. If it is odd and the secret bit is one or even and the secret bit is zero, do nothing. The change is applied when the secret bit is one and the selected digit is even. This is modified to be odd by adding one to it in the same option when the secret bit is zero, and the selected digit is odd. It will be modified by subtracting one from it to be even.

## 3.8 The inverse of Hessenberg transform

The inverse Hessenberg transform is applied on the $P$-matrix and the modified $H$-matrix by multiplying the $P$-matrix with the modified $H$-matrix, and the result is multiplied by the transpose of the $P$-matrix.

## 3.9 Reorder modifying parameters

The modified embedding matrix is first subtracted from the added key (generated numbers). The minimum value that was added to all blocks is subtracted before applying the inverse DWT (IDWT). The matrix is portioned into $3 \times 3$ sub-matrices representing the DWT parameters for each color band. The three sub-

matrices are in the first row for the red band. The three sub-matrices are in the second row for the green band, and the three sub-matrices are in the third row for the blue band, respectively. The IDWT is applied on the low–low parameters and the three modified high–low, low–high, and high–high parameters and produced a red-color band. Similarly, the same procedure is applied to the other parameters to the produced green-color band and blue-color band.

### 3.10 Extraction of secret information

To perform the extraction process, the same steps in embedding are applied. It is started with splitting the image into three color bands and applying 2D-DWT on the three color bands. It then reorders parameters into the embedding matrix, adds minimum value, and adds the generated numbers. It is followed by partitioning the embedding matrix into blocks $(4 \times 4)$ and applying Hessenberg transform on the block that gets the digit after the floating point. Finally, checking if it is odd for secret bit one and even for secret bit zero and collecting secret bit into array represent watermark image.

## 4 Experiment and results

The experimental results analyze the performance of the proposed algorithm by comparing it with the non-optimization hiding approach. Six standard images, namely house, car, Lenna, peppers, woman, and Baboon, were used as cover images. In addition, six binary images were used as a logo for watermarking, as shown in Figure 8.

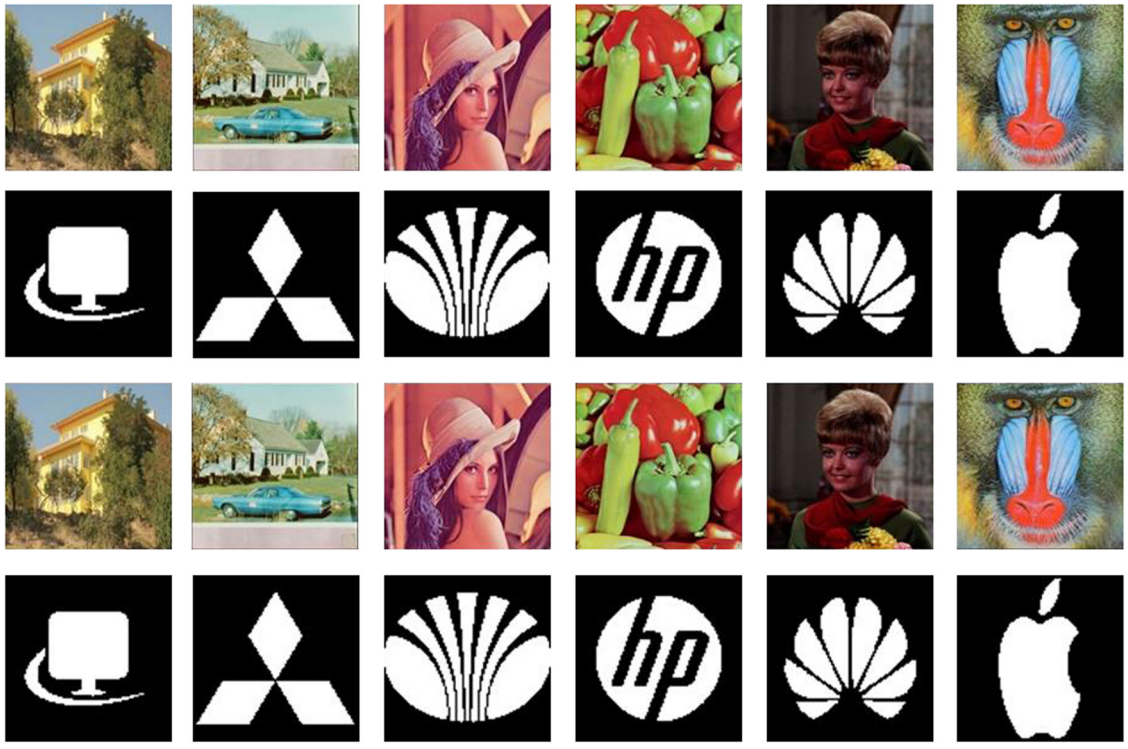

**Figure 8:** Images and watermark sets.

To test the effect of the optimization, the FWA process was used to reduce the change in the cover image by keeping the blocks without changing as much as possible. This test was applied on all cover images with all secret images. Table 2 shows the optimization process's changing rate on the cover images (six images) with all embedding watermark images (six watermarks).

**Table 2:** The changing rate in the cover image

| Wmk → Image | 1 | | 2 | | 3 | | 4 | | 5 | | 6 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Per Chg | Use FWA | Per Chg | Use FWA | Per Chg | Use FWA | Per Chg | Use FWA | Per Chg | Use FWA | Per Chg | Use FWA |
| 1 | 0.54 | 0.44 | 0.49 | 0.38 | 0.48 | 0.38 | 0.52 | 0.42 | 0.44 | 0.36 | 0.57 | 0.45 |
| 2 | 0.58 | 0.47 | 0.47 | 0.37 | 0.49 | 0.38 | 0.52 | 0.43 | 0.45 | 0.36 | 0.60 | 0.47 |
| 3 | 0.54 | 0.44 | 0.55 | 0.43 | 0.57 | 0.45 | 0.53 | 0.44 | 0.59 | 0.48 | 0.56 | 0.44 |
| 4 | 0.58 | 0.47 | 0.59 | 0.47 | 0.51 | 0.40 | 0.52 | 0.43 | 0.49 | 0.39 | 0.59 | 0.46 |
| 5 | 0.52 | 0.42 | 0.47 | 0.37 | 0.55 | 0.43 | 0.49 | 0.40 | 0.50 | 0.40 | 0.49 | 0.39 |
| 6 | 0.59 | 0.47 | 0.50 | 0.40 | 0.50 | 0.39 | 0.53 | 0.43 | 0.50 | 0.40 | 0.58 | 0.46 |
| Average | 0.56 | 0.45 | 0.51 | 0.40 | 0.52 | 0.40 | 0.52 | 0.42 | 0.49 | 0.40 | 0.56 | 0.45 |

Table 2 shows the percentage of change in different cover images. It can be noticed that the proposed method provides an improvement between 9 and 12% over the non-optimization method.

Two measurements are applied to the cover images after embedding information to examine the quality of embedding. These measurements are MSE and PSNR, which are used to find the degradation in quality. A small MSE value means good quality, whereas a high PSNR value means a good-quality index [30].

**Table 3:** Images and watermark sets

| Image | MSE | | PSNR | |
|---|---|---|---|---|
| | Non-optimization method | FWA method | Non-optimization method | FWA method |
| 1 | 0.04719 | 0.03539 | 37.32736 | 38.57674 |
| 2 | 0.03637 | 0.02728 | 38.45764 | 39.70703 |
| 3 | 0.03638 | 0.02728 | 38.45690 | 39.70629 |
| 4 | 0.05154 | 0.03865 | 36.94413 | 38.19351 |
| 5 | 0.03748 | 0.02811 | 38.32698 | 39.57637 |
| 6 | 0.03600 | 0.02700 | 38.50226 | 39.75165 |
| Average | 0.04083 | 0.03062 | 38.00254 | 39.25193 |

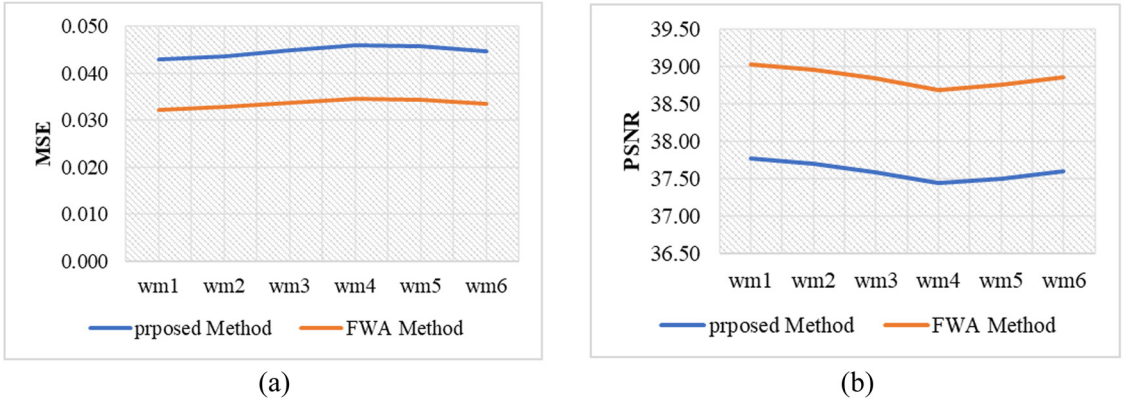


**Figure 9:** Images and watermarks comparative results. (a) The MSE results. (b) The PSNR results.

**Table 4:** Watermark attacks on proposed methods

| Watermark image | 1 | | 2 | | 3 | | 4 | | 5 | | 6 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Attack | Non-optimization method | FWA method | Non-optimization method | FWA method | Non-optimization method | FWA method | Non-optimization method | FWA method | Non-optimization method | FWA method | Non-optimization method | FWA method |
| JPG compression | 96.23 | 98.12 | 96.23 | 98.12 | 96.23 | 98.12 | 96.23 | 98.12 | 96.24 | 98.12 | 96.23 | 98.11 |
| Scaling | 98.31 | 99.01 | 98.31 | 99.00 | 98.31 | 99.00 | 98.31 | 99.00 | 98.31 | 99.00 | 98.32 | 99.01 |
| Rotation | 95.54 | 97.62 | 95.54 | 97.62 | 95.54 | 97.62 | 95.54 | 97.62 | 95.54 | 97.62 | 95.54 | 97.62 |
| Gaussian noise | 94.85 | 95.24 | 94.84 | 95.24 | 94.85 | 95.25 | 94.85 | 95.25 | 94.85 | 95.25 | 94.85 | 95.24 |
| Histogram equalization | 92.77 | 94.85 | 92.77 | 94.84 | 92.77 | 94.85 | 92.77 | 94.85 | 92.77 | 94.84 | 92.76 | 94.84 |
| Image adjustment | 93.16 | 93.96 | 93.17 | 93.96 | 93.17 | 93.95 | 93.17 | 93.95 | 93.17 | 93.95 | 93.17 | 93.95 |

The recorded results show that the use of optimization increases the quality of images as the change rate in the cover medium is reduced. Although the results of the non-optimization method are promising, the FWA-based method is even better in terms of MSE and PSNR, as shown in Table 3. The value of the MSE is reduced by about 0.01, and the value of the PSNR is increased by about 1.25 on average, as shown in Figure 9.

Table 4 shows the retrieval percentages after applying different types of attacks, namely JPG compression, scaling, rotation, Gaussian noise, histogram equalization, and image adjustment.

The results of the non-optimization method are promising; nevertheless, the FWA-based method provides better results in persisting against all the tested attacks. The improvement percentages of the FWA-based method over the non-optimization method in terms of persisting all attacks are listed in Table 5.

**Table 5:** Improving percentages of FWA-based method over non-optimization method

| Attack | Image 1 | Image 2 | Image 3 | Image 4 | Image 5 | Image 6 |
|---|---|---|---|---|---|---|
| JPG compression | 0.020 | 0.020 | 0.020 | 0.020 | 0.020 | 0.020 |
| Scaling | 0.007 | 0.007 | 0.007 | 0.007 | 0.007 | 0.007 |
| Rotation | 0.022 | 0.022 | 0.022 | 0.022 | 0.022 | 0.022 |
| Gaussian noise | 0.004 | 0.004 | 0.004 | 0.004 | 0.004 | 0.004 |
| Histogram equalization | 0.022 | 0.022 | 0.022 | 0.022 | 0.022 | 0.022 |
| Image adjustment | 0.009 | 0.008 | 0.008 | 0.008 | 0.008 | 0.008 |

Table 5 shows the percentage of improvement in retrieval secret image when adding the optimization stage using the FWA algorithm. The table shows the enhancement results at different rates as compared with the traditional method. It is for all the six images used in the examination. This percentage is according to the types of possible attacks, such as JPG compression, scaling, rotation, Gaussian noise, histogram equalization, and image adjustment.

# 5 Conclusion

Data hiding and watermarking are highly challenging issues in cyber security that attracted many researchers. This article proposed an optimized method for embedding a watermark in a host image. The DWT is applied, and four bands are obtained. A new square matrix includes all bands except the low–low band and some keys. These keys are generated using a hybrid approach, combining two chaotic functions, namely Gaussian and exponential maps. The FWA is used, as an optimization method, to select the best set of the initial values for the generated keys. Using an optimization procedure is to obtain keys that require the lowest possible change rate in the cover and increase the image quality scale. The embedding process involves the use of the Hessenberg transform, which generates two matrices, $P$ and $H$. A certain location within the $H$-matrix for each block is used for embedding a secret value. The same operations are performed in reverse order to retrieve the original image. The MSE and PSNR measurements are utilized to evaluate the overall performance of the proposed method. In addition, the robustness of the watermark is computed by applying several attacks. The experimental results show that the proposed system obtained a high-retrieval rate compared to the non-optimization approach. Although the recorded results in this article were promising, there is room for further improvements. One possible improvement includes examining other matrix decomposition techniques, such as SVD, and performing a comparison between them to determine which one is the best. Similarly, selecting different chaotic-based key generators can also be considered. In addition, it will be interesting to test the performance of the suggested method of another type of multimedia, such as video.

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