Research Article

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On characterizing solution for multi-objective fractional two-stage solid transportation problem under fuzzy environment

https://doi.org/10.1515/jisys-2020-0095
received October 03, 2020; accepted March 13, 2021

Abstract: This article attempts to study cost minimizing multi-objective fractional solid transportation problem with fuzzy cost coefficients $\tilde{c}_{ijk}^{r}$, fuzzy supply quantities $\tilde{a}_i$, fuzzy demands $\tilde{b}_j$, and/or fuzzy conveyances $\tilde{e}_k$. The fuzzy efficient concept is introduced in which the crisp efficient solution is extended. A necessary and sufficient condition for the solution is established. Fuzzy geometric programming approach is applied to solve the crisp problem by defining membership function so as to obtain the optimal compromise solution of a multi-objective two-stage problem. A linear membership function for the objective function is defined. The stability set of the first kind is defined and determined. A numerical example is given for illustration and to check the validity of the proposed approach.

Keywords: solid transportation problem, multi-objective, fuzzy number, fuzzy efficient solution, fuzzy programming, optimality, parametric study

1 Introduction

Solid transportation problem (STP) is a generalization of the well-known classical transportation problem (TP), where three item properties are taken into account in the constraint set of the STP (namely, supply, demand, and mode of transportation or conveyance) instead of two constraints (source and destination). The STP was first proposed by Shell [1] in his work by introducing the distribution of a product by some properties. Later many researchers discussed the STP in different aspects. Haley [2] introduced a solution procedure for STP as an extension of the modified distribution method. Patel and Tripathy [3] investigated a computationally superior method for an STP with mixed constraints. Bit et al. [4] applied fuzzy programming approach to solve the multi-objective STP with real-life applications. Vejda [5] developed an algorithm for a multi-index TP, which is the extension of the distribution modification method. The zero-point method for finding the optimal solution of TP was introduced by Pandian and Natarajan [6]. Pandian and Anuradha [7] developed an efficient methodology to determine the optimal solution of STP with the help of the principle of zero-point method.

Fuzzy sets theory was first introduced by Zadeh [8]. Dubois and Prade [9] extended the use of algebraic operations on real numbers to fuzzy numbers. Jimenez and Verdegay [10] applied two ways under

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uncertainty for STP: interval and fuzzy STP. Orlovski [11] formulated general multi-objective non-linear programming problems with fuzzy parameters. Sakawa and Yano [12] introduced the concept of α-pareto optimality of fuzzy parametric programs. Recently, Das et al. [13] introduced an STP with mixed type of constraints under different environment: crisp, fuzzy, and intuitionistic fuzzy. Baidya et al. [14] introduced a new concept safety factor in a TP and also considered an STP with imprecise unit cost, sources, destinations, and capacities of conveyances represented by triangular and trapezoidal fuzzy numbers. Kundu et al. [15] studied multi-objective STP under different uncertain environment, in which the unit transportation costs are represented as fuzzy, random, and hybrid variables, respectively. Numerous researchers presented their work on STP by introducing new method, for example, Sinha et al. [16], Aggarwal and Gupta [17], Sinha et al. [16], etc. addressed a novel concept regarding the TP where they maximized the profit and minimized the transporting time subject to constraints. They considered all the parameters as trapezoidal interval type-2 fuzzy numbers. Aggarwal and Gupta [17] introduced a new ranking system for signed distance of intuitionistic fuzzy numbers and formulated an STP in intuitionistic environment to compute initial basic feasible solution. Acharya et al. [18] applied an interactive fuzzy goal programming approach for solving multi-objective generalized STP. Sobana and Anuradha [19] used the α-cut under imprecise environment, and they proposed a new algorithm to find an optimal solution for STP. Singh et al. [20] formulated a general model of the multi-objective STP with some random parameters and they proposed a solution method by using the chance-constraint programming technique to solve the model of multi-objective STP. Kumar et al. [21] proposed a new computing procedure for solving fuzzy Pythagorean TP, where they extended the interval basic feasible solution, then existing optimality method to obtain the cost of transportation. Khalifa et al. [22] investigated a neutrosophic programming using lexicographic order to determine the optimal solution. Arqub and Al-Smadi [23] presented the fractional differential equation and solved by using the fuzzy approach.

Fractional programming (FP) is considered as one of the various applications on non-linear programming, and it is applicable in numerous fields such as finance, economic, financial and corporate planning, and health care. Normally, the minimization or maximization of objective functions such as return on investment, return/risk, time/cost, or output/input under a limitation of constraints are some other examples of the applications of FP. Charnes and Cooper [24] introduced the linear fractional programming (LFP). Tantawy [25] investigated an iterative method using the conjugate gradient projection method for solving LFP problems. Stanojevic and Stanojevic [26] applied the efficiency test introduced by Lotfi et al. (2010) to the proposed two procedures for deriving weakly and strongly efficient solutions in multi-objective LFP problems. They started from any feasible solution and introduced its applications in the multi-criteria decision-making process. Das and Mandal [27] addressed an efficient approach for solving a class of single-stage constraint LFP problems, based on the transformation of the objective value and the constraints also. Dutta and Kumar [28] presented an application of FP approach to inventory control problem. Simi and Talukder [29] introduced a new method for solving LFP problem. In their work, they first transformed the LFP into linear programming and hence solved this problem algebraically using the duality concept. Rubi and Pitam [30] proposed an iterative fuzzy approach for solving LFP.

In this research article, the cost minimizing fuzzy multi-objective fractional STP is studied under uncertainty. Fuzzy programming approach is applied to solve the corresponding crisp problem and hence the notions of solvability set and the stability set of the first kind are defined and characterized.

The rest of the article is organized as follows: in Section 2, multi-objective two-stage fuzzy STP is formulated. Section 3 proposes a solution procedure for solving the problem. Section 4 provides a numerical example to illustrate the efficiency of the solution procedure. Finally, some concluding remarks are reported in Section 5.

2 Problem formulation and solution concepts

Let $\tilde{p}_{ijk}^{r}$ and $\tilde{q}_{ijk}^{r}$ be the coefficients of the objective functions, $\widetilde{a}_i$ be the availability of the product at the source i, $\tilde{b}_j$ be the minimum requirement at the destination j, and $\tilde{e}_k$ be the conveyance. All of $\tilde{p}_{ijk}^{r}$, $\tilde{q}_{ijk}^{r}$ $\widetilde{a}_i$, $\tilde{b}_j$,

and $\tilde{e}_k$ are represented as triangular fuzzy numbers. $\tilde{Z}_r(x) = \{\tilde{Z}_1(x),\ \tilde{Z}_2(x),\ \ldots,\ \tilde{Z}_K(x)\}$ is a vector r objective function and the subscript on both Z_r, $\tilde{p}^r_{ijk}$, $\tilde{q}^r_{ijk}$ identified the number of objectives ($r = 1,\ 2,\ldots,\ K$). Without loss of generality, it is assumed that:

$$\tilde{p}^r_{ijk}, \tilde{q}^r_{ijk} \succ 0, \tilde{a}_i \succ 0; \quad \forall i, \widetilde{b_j} \succ 0;\ \forall j, \tilde{e}_k \succ 0.$$

The problem can be formulated as:

$$\text{Min}\ \tilde{Z}_r(x,\ \tilde{p}^r,\ \tilde{q}^r) = \text{Min}\left[\frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\tilde{p}^r_{ijk}x_{ijk}}{\sum_{i=1}^{m}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\tilde{q}^r_{ijk}x_{ijk}}\right] = \text{Min}\ \frac{\tilde{f}_r(x,\ \tilde{p}^r)}{\tilde{g}_r(x,\ \tilde{q}^r)}, \quad r = 1,\ 2,\ldots,\ K. \tag{1a}$$

Subject to

$$\tilde{G} = \left\{ \begin{array}{l} x \in \mathbb{R}^{m\times n\times l} :\ \sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} = \widetilde{a_i},\ i = \overline{1, m},\ \ \sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} = \widetilde{b_j},\ \ j = \overline{1, n}\ ; \\ \sum_{k=1}^{l} x_{ijk} = \tilde{e}_k;\ x_{ijk} \geq 0;\ i = \overline{1, m};\ j = \overline{1,\ n};\ k = \overline{1, l};\ \ \sum_{i=1}^{m}\widetilde{a_i} = \sum_{j=1}^{n}\widetilde{b_j} = \sum_{k=1}^{l}\tilde{e}_k \end{array} \right\}. \tag{1b}$$

It is assumed that the feasible region $\tilde{G}$ is compact and all of $\tilde{p}^r_{ijk}, \tilde{q}^r_{ijk}, \tilde{a}_i, \widetilde{b_j}, \tilde{e}_k$ are triangular fuzzy numbers.

Definition 1. (Kaufmann and Gupta [31]) The α-level set of fuzzy number $\tilde{a}$ is defined as the ordinary set $(\tilde{a})_\alpha$ for which the degree of their membership function exceeds the level $\alpha \in [0, 1]$:

$$(\tilde{a})_\alpha = \{a \in \mathbb{R}^m :\ \mu_{\tilde{a}}(a) \geq \alpha, i = 1,\ 2,\ldots,\ m\}.$$

Alternatively, defining the interval of confidence at level α, the triangular fuzzy number is characterized as:

$$\tilde{A}_\alpha = [(q - p)\alpha + p,\ -(r - s)\alpha + r]; \text{ for all } \alpha \in [0, 1].$$

Definition 2. A feasible solution vector $x^0 \in \tilde{G}$ (feasible domain) is called the fuzzy feasible solution of problems (1a and 1b)–(2) if and only if there is no X such that

$$\frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\tilde{p}^r_{ijk}x_{ijk}}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\tilde{q}^r_{ijk}x_{ijk}} \leq \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\tilde{p}^r_{ijk}x^0_{ijk}}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\tilde{q}^r_{ijk}x^0_{ijk}}; \quad \forall r$$

and

$$\frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\tilde{p}^r_{ijk}x_{ijk}}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\tilde{q}^r_{ijk}x_{ijk}} \neq \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\tilde{p}^r_{ijk}x^0_{ijk}}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\tilde{q}^r_{ijk}x^0_{ijk}}$$

for some r, $r = 1, 2,\ldots, K$.

Definition 3. A fuzzy feasible solution $x^\star \in \tilde{G}$ is said to be fuzzy efficient solution of problem (1), if and only if $x^\star \in \tilde{G}$ and $\tilde{Z}_r(x^\star) \leq \wedge_{x\in\tilde{F}}\tilde{Z}_r(x)$, where $\tilde{F}$ denotes the set of all fuzzy efficient solutions and $\wedge$ is the minimum.

For a certain degree of α, the non-fuzzy form of problem (1) is as follows:

$$\text{Min}\ Z_r(x,\ p^r,\ q^r)_\alpha = \text{Min}\left[\frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}p^r_{ijk}x_{ijk}}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}q^r_{ijk}x_{ijk}}\right] = \text{Min}\ \frac{f_r(x, p^r)}{g_r(x,\ q^r)}, \quad r = \overline{1, K}.$$

Subject to

$$\begin{array}{l} X \in G(a,\ b,\ e),\ p^r_{ijk} \in (\tilde{p}^r_{ijk})_\alpha,\ q^r_{ijk} \in (\tilde{q}^r_{ijk})_\alpha \quad i = \overline{1, m};\ j = \overline{1,\ n};\ k = \overline{1,\ l};\ r = \overline{1,\ K}; \\ a_i \in (\tilde{a}_i)_\alpha,\ i = \overline{1,\ m};\ b_j \in (\tilde{b}_j)_\alpha, j = \overline{1, n};\ e_k \in (\tilde{e}_k)_\alpha, k = \overline{1,\ l}, \end{array} \tag{2}$$

$$x_{ijk} \geq 0;\ i = \overline{1,\ m};\ j = \overline{1,\ n};\ k = \overline{1,\ l}$$
$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = \sum_{k=1}^{l} e_k.$$

Definition 4. A point $\hat{X}(\hat{p},\ \hat{q}) \in G(\hat{a},\ \hat{b},\ \hat{e})$ is called an α-parametric efficient solution of problem (2) if and only if there is no $X(\hat{p},\ \hat{q}) \in G(\hat{a},\ \hat{b},\ \hat{e})$ such that

$$\frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\hat{p}_{ijk}^{r}x_{ijk}}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\hat{p}_{ijk}^{r}x_{ijk}} \leq \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\hat{p}_{ijk}^{r}\hat{x}_{ijk}^{0}}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\hat{q}_{ijk}^{r}\hat{x}_{ijk}^{0}};\quad \forall i,\ j,\ k,$$

and

$$\frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\hat{p}_{ijk}^{r}x_{ijk}}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\hat{q}_{ijk}^{r}x_{ijk}} \neq \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\hat{p}_{ijk}^{r}\hat{x}_{ijk}^{0}}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\hat{q}_{ijk}^{r}\hat{x}_{ijk}^{0}}$$

for some r.

Theorem 1. *A point $x^{\star}(p^{\star}, q^{\star}) \in G(a^{\star}, b^{\star}, e^{\star})$ is an α-fuzzy efficient solution of problem* (1) *if and only if for $p_{ijk}^{r} \in (\tilde{p}_{ijk}^{r})_{\alpha}$, $q_{ijk}^{r} \in (\tilde{q}_{ijk}^{r})_{\alpha}$, $a_i \in (\tilde{a}_i)_{\alpha}$, $b_j \in (\tilde{b}_j)_{\alpha}$, $e_k \in (\tilde{e}_k)_{\alpha}$, $X^{\star}(p^{\star}, q^{\star}) \in G(a^{\star}, b^{\star}, e^{\star})$ is an α-parametric efficient solution of problem* (2).

Proof. (Necessity) Let $x^{\star}(p^{\star}, q^{\star}) \in G(a^{\star}, b^{\star}, e^{\star})$ be an α-fuzzy efficient solution to problem (1) and not an α-parametric efficient solution of problem (2), then there exist $\bar{x}(p^{\star}, q^{\star}) \in G(a^{\star}, b^{\star}, e^{\star})$ for $p_{ijk}^{r} \in (\tilde{p}_{ijk}^{r})_{\alpha}$, $q_{ijk}^{r} \in (\tilde{q}_{ijk}^{r})_{\alpha}$, $a_i \in (\tilde{a}_i)_{\alpha}$, $b_j \in (\tilde{b}_j)_{\alpha}$, $e_k \in (\tilde{e}_k)_{\alpha}$ such that $\tilde{Z}_r(\bar{x}, \tilde{p}^{r\star}, \tilde{q}^{r\star}) \leq \tilde{Z}_r(x^{\star}, \tilde{p}^{r\star}, \tilde{q}^{r\star})$, for $r = 1, 2, \ldots, K$, and $\tilde{Z}_r(\bar{x}, \tilde{p}^{r\star}, \tilde{q}^{r\star}) \neq \tilde{Z}_r(x^{\star}, \tilde{p}^{r\star}, \tilde{q}^{r\star})$ for some r.

This leads to

$$\begin{aligned}
&\mu_{(\tilde{p},\,\tilde{q})}\{(p, q) \in \mathbb{R}^{K(m\times n\times l)} : \tilde{Z}_1(\bar{x}, \tilde{p}^{\star 1}, \tilde{q}^{\star 1})\\
&\quad \leq \tilde{Z}_1(x^{\star}, \tilde{p}^{\star 1}, \tilde{q}^{\star 1}), \ldots, \tilde{Z}_{r-1}(\bar{x}, \tilde{p}^{\star r-1}, \tilde{q}^{\star r-1})\\
&\quad \leq \tilde{Z}_{r-1}(x^{\star}, \tilde{p}^{\star r-1}, \tilde{q}^{\star r-1}), \tilde{Z}_r(\bar{x}, \tilde{p}^{\star r}, \tilde{q}^{\star 1})\\
&\quad \leq \tilde{Z}_r(x^{\star}, \tilde{p}^{\star r}, \tilde{q}^{\star r}), \tilde{Z}_{r+1}(\bar{x}, \tilde{p}^{\star r+1}, \tilde{q}^{\star r+1})\\
&\quad \leq \tilde{Z}_{r+1}(x^{\star}, \tilde{p}^{\star r+1}, \tilde{q}^{\star r+1}), \ldots, \tilde{Z}_K(\bar{x}, \tilde{p}^{\star K}, \tilde{q}^{\star K})\\
&\quad \leq \tilde{Z}_K(x^{\star}, \tilde{p}^{\star K}, \tilde{q}^{\star K})\}\\
&\quad \geq \alpha, \alpha \in [0, 1],
\end{aligned}$$

and with strict inequality holds for at least one r, which is contradiction. $x^{\star}(p^{\star}, q^{\star}) \in G(a^{\star}, b^{\star}, e^{\star})$ is an α-fuzzy efficient solution to problem (1), then $x^{\star}(p^{\star}, q^{\star}) \in G(a^{\star}, b^{\star}, e^{\star})$ is an α-parametric efficient solution of problem (2).

Sufficiency: Let $x^{\star}(p^{\star}, q^{\star}) \in G(a^{\star}, b^{\star}, e^{\star})$ be an α-parametric efficient solution of problem (2), but not an α-fuzzy efficient solution to problem (1). Then there exist $\overline{\bar{x}}(p^{\star}, q^{\star}) \in G(a^{\star}, b^{\star}, e^{\star})$ such that

$$\begin{aligned}
&\mu_{(\tilde{p},\,\tilde{q})}\{(p, q) \in \mathbb{R}^{K(m\times n\times l)} : \tilde{Z}_1(\overline{\bar{x}}, \tilde{p}^{\star 1}, \tilde{q}^{\star 1})\\
&\quad \leq \tilde{Z}_1(x^{\star}, \tilde{p}^{\star 1}, \tilde{q}^{\star 1}), \ldots, \tilde{Z}_{r-1}(\overline{\bar{x}}, \tilde{p}^{\star r-1}, \tilde{q}^{\star r-1})\\
&\quad \leq \tilde{Z}_{r-1}(x^{\star}, \tilde{p}^{\star r-1}, \tilde{q}^{\star r-1}), \tilde{Z}_r(\overline{\bar{x}}, \tilde{p}^{\star r}, \tilde{q}^{\star 1})\\
&\quad \leq \tilde{Z}_r(x^{\star}, \tilde{p}^{\star r}, \tilde{q}^{\star r}), \tilde{Z}_{r+1}(\overline{\bar{x}}, \tilde{p}^{\star r+1}, \tilde{q}^{\star r+1})\\
&\quad \leq \tilde{Z}_{r+1}(x^{\star}, \tilde{p}^{\star r+1}, \tilde{q}^{\star r+1}), \ldots, \tilde{Z}_K(\overline{\bar{x}}, \tilde{p}^{\star K}, \tilde{q}^{\star K})\\
&\quad \leq \tilde{Z}_K(x^{\star}, \tilde{p}^{\star K}, \tilde{q}^{\star K})\}\\
&\quad \geq \alpha, \alpha \in [0, 1].
\end{aligned}$$

From the continuity and convexity of the membership function, we get

$$\begin{aligned}\tilde{Z}_1(\overline{x}, \tilde{p}^{\star 1}, \tilde{q}^{\star 1}) &\le \tilde{Z}_1(x^\star, \tilde{p}^{\star 1}, \tilde{q}^{\star 1}), \ldots, \tilde{Z}_{r-1}(\overline{x}, \tilde{p}^{\star r-1}, \tilde{q}^{\star r-1})\\ &\le \tilde{Z}_{r-1}(x^\star, \tilde{p}^{\star r-1}, \tilde{q}^{\star r-1}), \tilde{Z}_r(\overline{x}, \tilde{p}^{\star r}, \tilde{q}^{\star 1})\\ &\le \tilde{Z}_r(x^\star, \tilde{p}^{\star r}, \tilde{q}^{\star r}), \tilde{Z}_{r+1}(\overline{x}, \tilde{p}^{\star r+1}, \tilde{q}^{\star r+1})\\ &\le \tilde{Z}_{r+1}(x^\star, \tilde{p}^{\star r+1}, \tilde{q}^{\star r+1}), \ldots, \tilde{Z}_K(\overline{x}, \tilde{p}^{\star K}, \tilde{q}^{\star K})\\ &\le \tilde{Z}_K(x^\star, \tilde{p}^{\star K}, \tilde{q}^{\star K}),\end{aligned}$$

for $i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n;\ k = 1, 2, \ldots, l$, which is a contradiction. □

By the transformation $y_{ijk} = tx_{ijk},\ i = \overline{1, m};\ j = \overline{1, n};\ k = \overline{1, l}$, problem (3) is equivalent to the following problem:

$\mathrm{Max}_r\{tf_1(y_{ijk}/t),\ tf_2(y_{ijk}/t), \ldots,\ tf_K(y_{ijk}/t)\}$.

Subject to

$$\begin{aligned}&y \in G(a, b, e), p^r_{ijk} \in (\tilde{p}^r_{ijk})_\alpha, q^r_{ijk} \in (\tilde{q}^r_{ijk})_\alpha \quad i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n;\ k = 1, 2, \ldots, l; r = 1, 2, \ldots, K;\\ &a_i \in (\tilde{a}_i)_\alpha,\ \ i = 1, 2, \ldots, m;\ b_j \in (\tilde{b}_j)_\alpha,\ \ j = 1, 2, \ldots, n;\ e_k \in (\tilde{e}_k)_\alpha,\quad k = 1, 2, \ldots, l,\\ &tg_r(y_{ijk}/t, q^r) \le 1, y_{ijk} \ge 0;\ i = 1, 2, \ldots;\ j = 1, 2, \ldots, n;\ k = 1, 2, \ldots, l; y_{ijk} \ge 0, t > 0,\\ &\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = \sum_{k=1}^{l} e_k.\end{aligned} \tag{3}$$

The membership function of each objective function can be constructed as:

$$\mu_r\left(tf_r\left(\frac{y_{ijk}}{t}, p^r\right)\right) = \begin{cases} 0, & tf_1\left(\dfrac{y_{ijk}}{t}, p^r\right) \le \underline{N}_r \\ \dfrac{tf_r\left(\frac{y_{ijk}}{t}, p^r\right) - \underline{N}_r}{\bar{N}_r - \underline{N}_r}, & \underline{N}_{r_r} > tf_1\left(\dfrac{y_{ijk}}{t}, p^r\right) < \bar{N}_r \\ 0, & tf_1\left(\dfrac{y_{ijk}}{t}, p^r\right) \ge \bar{N}_r. \end{cases} \tag{4}$$

For each $r = \overline{1, K}$, applying Zadeh's min operator [8], problem (3) reduces to the following model (5).

Max δ

Subject to

$$\begin{aligned}&\delta \le \mu_r\left(t\ f_r\left(\frac{y_{ijk}}{t}, p^r\right)\right), \quad r = 1, 2, \ldots, K\\ &\sum_{j=1}^{n}\sum_{k=1}^{l} y_{ijk} = a_i, \quad i = 1, 2, \ldots, m,\\ &\sum_{i=1}^{m}\sum_{k=1}^{l} y_{ijk} = b_j, \quad j = 1, 2, \ldots, n,\\ &\sum_{i=1}^{m}\sum_{j=1}^{n} y_{ijk} = e_i, \quad k = 1, 2, \ldots, l, 0 \le \delta \le 1,\\ &p^r_{ijk} \in (\tilde{p}^r_{ijk})_\alpha, q^r_{ijk} \in (\tilde{q}^r_{ijk})_\alpha, a_i \in (\tilde{a}_i)_\alpha, b_j \in (\tilde{b}_j)_\alpha, e_k \in (\tilde{e}_k)_\alpha,\\ &tg_r\left(\frac{y_{ijk}}{t}, q^r\right) \le 1, \quad t > 0,\\ &\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = \sum_{k=1}^{l} e_k, y_{ijk} \ge 0; \quad \forall i, j, k.\end{aligned} \tag{5}$$

It clear that the constraints in (6) may be reduced into the following form:

$$\delta(\bar{N}_r - \underline{N}_r) \le (\bar{N}_r - f_r(y_{ijk}/t)),$$
$$\delta(\bar{N}_r - \underline{N}_r) + f_r(y_{ijk}/t) \le \bar{N}_r,$$
$$\frac{\delta(\bar{N}_r - \underline{N}_r)}{\bar{N}_r} + \left(\frac{1}{\bar{N}_r}\right) f_r(y_{ijk}/t) \le 1.$$

Model (5) can be rewritten as in the equivalent form as in Model (6):

Max δ

Subject to

$$G = \begin{cases} \delta \le \mu_r\left(t\ f_r\left(\dfrac{y_{ijk}}{t}, p^r_{ijk}\right)\right), & r = 1, 2, \ldots, K \\ \displaystyle\sum_{j=1}^{n}\sum_{k=1}^{l} y_{ijk} = a_i, & i = 1, 2, \ldots, m, \\ \displaystyle\sum_{i=1}^{m}\sum_{k=1}^{l} y_{ijk} = b_j, & j = 1, 2, \ldots, n, \\ \displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n} y_{ijk} = e_i, & k = 1, 2, \ldots, l, 0 \le \delta \le 1, \\ tg_r\left(\dfrac{y_{ijk}}{t}, q^r_{ijk}\right) \le 1, t > 0, \\ M^{r-}_{ijk} \le p^r_{ijk} \le M^{r}_{ijk}{}^{+}, H^{r}_{ijk}{}^{-} \le q^r_{ijk} \le H^{r+}_{ijk} \\ H^-_i \le a_i \le H^+_i, H^-_j \le b_j \le H^+_j, H^-_k \le e_k \le H^+_k, \\ \displaystyle\sum_{i=1}^{m} \breve{a}_i = \sum_{j=1}^{n} \breve{b}_j = \sum_{k=1}^{l} \breve{e}_k \quad y_{ijk} \ge 0; \forall i, j, k. \end{cases} \tag{6}$$

Here, in Model (6), $M^{r-}_{ijk}, H^{r-}_{ijk}, H^-_i, H^-_j, H^-_k$ are the lower bounds, $M^{r+}_{ijk}, H^{r+}_{ijk}, H^+_i, H^+_j, H^+_k$ are the upper bounds, and G is the set of all constraints.

3 Solution procedure

The steps of the solution procedure for solving the STP can be summarized as follows:

Step 1: Calculate the individual minimum and maximum of each objective function subject to the given constraints so as to determine the lower and upper bounds of the objectives Z_r using the variable transformation method.

Step 2: Using the variable transformation method, problem (2) can be converted into problem (3).

Step 3: Determine the membership function as in (4).

Step 4: By introducing an auxiliary variable δ, problem (5) is equivalent to the following classical linear programming (6).

Step 5: Solve problem (6) using any software package (say, MATLAB), to obtain the optimal compromise solution.

Step 6: Combining stage I and stage II to obtain the optimal solution for the two-stage problem.

Step 7: Determine $S(\hat{x}, \hat{p}, \hat{q}, \hat{a}, \hat{b}, \hat{e})$ by applying the following condition:

$$\gamma_r(\hat{p}^r - d_{2r}) = 0, \quad r = 1, 2, \ldots, K;$$
$$\eta_r(d_{1r} - \hat{p}^r) = 0, \quad r = \overline{1, K};$$
$$\zeta_r(\hat{q}^r - g_{2r}) = 0, \quad r = 1, 2, \ldots, K;$$
$$\xi_r(g_{1r} - \hat{q}^r) = 0, \quad r = \overline{1, K};$$

$$
\begin{aligned}
&x_i(\hat{a}_i - h_{2i}), \quad i = \overline{1, m};\\
&\rho_i(h_{1i} - \hat{a}_i), \quad i = \overline{1, m};\\
&\varrho_j(\hat{b}_j - u_{2j}), \quad j = \overline{1, n};\\
&\sigma_j(u_{1j} - \hat{b}_j), \quad j = \overline{1, n};\\
&\tau_k(\hat{e}_k - v_{2k}), \quad k = \overline{1, l};\\
&\pi_k(v_{1k} - \hat{e}_k), \quad k = \overline{1, l};
\end{aligned}
$$

$\gamma_r,\ \eta_r,\ \zeta_r,\ \xi_r \geq 0,\ r = \overline{1, K}; \varpi_i,\ \rho_i \geq 0,\ i = \overline{1, m}\ ;\ \varrho_j,\ \sigma_j,\ j \geq 0,\ j = \overline{1, n}; \tau_k, \pi_k \geq 0,\ k = \overline{1, l}.$

Here, $[d_{1r}, d_{2r}] = [p^{rC}, p^{rR}]$ and $[g_{1r}, g_{2r}] = [q^{rC}, q^{rR}],\ [h_{1i}, h_{2i}] \in L_\alpha(a_i),\ i = \overline{1, m};\quad [u_{1j}, u_{2j}] \in L_\alpha(b_j),\ j = \overline{1, m}; [v_{1k}, v_{2k}] \in L_\alpha(e_k),\ k = \overline{1, l}.$

Consider the following three cases:

(i) $\gamma_r > 0,\ r \in J_1 \subset \{1,\ 2,\ldots,\ K\};\ \gamma_r = 0,\ r \notin J_1;\ \varpi_i > 0,\ i \in J_2 \subset \{1,\ 2,\ldots m\},\ \varpi_i = 0,\ i \notin J_2;\ \varrho_j > 0, j \in J_3 \subset \{1,\ 2,\ldots,\ n\}, \varrho_j = 0,\ j \notin J_3;\ \gamma_r = 0,\ r \notin J_5; \varpi_i > 0, i \in J_6 \subset \{1, 2, \ldots m\},\ \varpi_i = 0,\ i \notin J_6;\quad \tau_k > 0,\ k \in J_4 \subset \{1,\ 2,\ldots,\ k\}, \tau_k = 0,\ k \notin J_4; \gamma_r > 0, r \in J_5 \subset \{1,\ 2,\ldots,\ K\},$
$\varrho_j > 0, j \in J_7 \subset \{1,\ 2,\ldots,\ n\}, \varrho_j = 0, j \notin J_7; \tau_k > 0, k \in J_8 \subset \{1,\ 2,\ldots, k\}, \tau_k = 0,\ k \notin J_8\ .$

Let N be the set of all proper subsets of $\{1,\ 2,\ldots,\ K\}$. Then, we obtain

$$
\begin{aligned}
&S_{J_1,\ldots,J_8}(\hat{x}, \hat{p}, \hat{q}, \hat{a}, \hat{b}, \hat{e})\\
&= \left\{ \begin{array}{c} (d_1, d_2; g_1, g_2; h_1, h_2; u_1, u_2; v_1, v_2) \in \mathbb{R}^{4(K\times m\times n\times l)} : \hat{p}^r = d_{2r}, r \in J_1, d_{2r} \geq \hat{p}^r, i \notin J_1;\\ d_{1r} = \hat{p}^r, r \in J_2, d_{1r} \leq \hat{p}^r, i \notin J_2;\ \hat{a}_i = h_{2i}, i \in J_3, h_{2i} \geq \hat{a}_i, i \notin J_3;\\ h_{1i} = \hat{a}_i, r \in J_4, h_{1i} \leq \hat{a}_i, i \notin J_4, \hat{b}_j = u_{2j}, j \in J_5, u_{2j} \leq \hat{b}_j, j \notin J_5\ u_{1j} = \hat{b}_j, j \in J_6;\\ u_{1j} \leq \hat{b}_j, j \notin J_6;\ \hat{e}_k = v_{2k}, j \in J_7, v_{2k} \leq \hat{e}_k, j \notin J_7;\ v_{1k} = \hat{e}_k, j \in J_8, v_{1k} \leq \hat{e}_k, j \notin J_8 \end{array} \right\}.
\end{aligned}
$$

Hence,

$$S_1(\hat{x}, \hat{p}, \hat{q}, \hat{a}, \hat{b}, \hat{e}) = \bigcup_{J_1,\ldots,J_8} S_{J_1,\ldots,J_8}(\hat{x}, \hat{p}, \hat{q}, \hat{a}, \hat{b}, \hat{e})$$

(ii) $\gamma_r,\ \zeta_r; \eta_r,\ \xi_r; = 0,\ r = \overline{1, K}$. Then, we have

$$S_2(\hat{x}, \hat{p}, \hat{q}) = \left\{ \begin{array}{c} (d_1, d_2;\ g_1, g_2) \in \mathbb{R}^{4K} :\ d_{2r} \geq \hat{p}^r, r = \overline{1, K};\\ d_{1r} = \hat{p}^r, r \in J_2, d_{1r} \leq \hat{p}^r, r = \overline{1, K}; g_{2r} \geq \hat{q}^r;\\ g_{1r} \leq \hat{q}^r, r = \overline{1, K} \end{array} \right\},$$

(iii) $\gamma_r,\ \zeta_r; \eta_r,\ \xi_r > 0,\ r = \overline{1, K}$. Then, we have

$$S_3(\hat{x}, \hat{p}, \hat{q}) = \left\{ \begin{array}{c} (d_1, d_2;\ g_1, g_2\ ) \in \mathbb{R}^{4K} :\ d_{2r} = \hat{p}^r, r = \overline{1, K};\\ d_{1r} = \hat{p}^r, r \in J_2, d_{1r} = \hat{p}^r, r = \overline{1, K};\ g_{2r} = \hat{q}^r;\\ g_{1r} = \hat{q}^r, r = \overline{1, K}. \end{array} \right\}$$

Thus, we have

$$S(\hat{x}, \hat{p}, \hat{q}) = \bigcup_{w=1}^{3} S_w(\hat{x}, \hat{p}, \hat{q}).$$

Step 8: Stop.

4 Numerical example

Consider the following multi-objective two-stage cost minimizing STP with supplies, demands, and conveyances represented by triangular fuzzy numbers as:

- Supplies: $\tilde{a}_1 = (3,\ 5,\ 7)$, $\tilde{a}_2 = (4,\ 7,\ 9)$, $\tilde{a}_2 = (4,\ 6,\ 8)$.
- Demands: $\tilde{b}_1 = (9,\ 12,\ 14)$, $\tilde{b}_2 = (14,\ 17,\ 19)$, $\tilde{b}_3 = (16,\ 19,\ 22)$.
- Conveyances: $\tilde{e}_1 = (13,\ 15,\ 18)$, $\tilde{e}_2 = (15,\ 18,\ 20)$, $\tilde{e}_3 = (16,\ 17,\ 21)$.
- Penalties:

$$\tilde{s}^1_{ijk} = \begin{bmatrix} (6\ ,7,8) & (4,10,15) & (8,11,18) \\ (5,13,24) & (1,12,14) & (2,9,20) \\ (7,13,19) & (11,17,20) & (\ 11,12,18) \end{bmatrix},$$

$$\tilde{t}^1_{ijk} = \begin{bmatrix} (1,4,9) & (1,2,5) & (2,5,8) \\ (2,4,9) & (0,6,8) & (4,7,9) \\ (4,8,12) & (4,7,9) & (8,9,12) \end{bmatrix},$$

$$s^2_{ijk} = \begin{bmatrix} (3,12,15) & (6,7,9) & (4,7,10) \\ (1,6,11) & (3,9,11) & (2,6,8) \\ (1,2,4) & (5,7,12) & (1,3,9) \end{bmatrix},$$

$$\tilde{t}^2_{ijk} = \begin{bmatrix} (0,2,4) & (0,6,8) & (2,4,9) \\ (2,5,9) & (4,9,13) & (4,9,19) \\ (8,12,16) & (7,9,12) & (4,6,8) \end{bmatrix},$$

$$\tilde{s}^3_{ijk} = \begin{bmatrix} (2,4,6) & (3,4,6) & (4,8,9) \\ (2,3,5) & (1,5,6) & (3,6,9) \\ (8,9,10) & (3,8,9) & (5,7,11) \end{bmatrix},$$

$$\tilde{t}^3_{ijk} = \begin{bmatrix} (0,1,4) & (0,2,8) & (7,9,12) \\ (6,8,10) & (4,9,13) & (5,8,18) \\ (9,13,28) & (12,20,27) & (5,10,15) \end{bmatrix}.$$

- At $\alpha = 0.8$, we get
- Supplies: $4.6 \le a_1 \le 5.4,\ 6.4 \le a_2 \le 7.4,\ 5.6 \le a_3 \le 6.4$;
- Demands: $11.4 \le b_1 \le 12.4,\ 16.4 \le b_2 \le 17.4,\ 16.8 \le b_3 \le 17.8$;
- Conveyances: $14.6 \le e_1 \le 15.6,\ 17.4 \le e_2 \le 18.4,\ 16.8 \le e_3 \le 17.8$.

Stage I

Steps 1–3:

$$a_1 = 3,\ \ a_2 = 5,\ \ a_3 = 4;\ b_1 = 4,\ \ b_2 = 3,\ \ b_3 = 5;\ e_1 = 5,\ \ e_2 = 3,\ \ e_3 = 4.$$

Now, by solving $Z_1^R(x)$ with respect to the given constraints

$$\text{Min } Z_1^R = \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\frac{\tilde{s}^1_{ijk}}{\tilde{t}^1_{ijk}}$$
$$= \frac{8x_{111} + 15x_{121} + 18x_{131} + 24x_{211} + 14x_{221} + 20x_{231} + 19x_{311} + 20x_{321} + 18x_{331}}{9x_{111} + 5x_{121} + 8x_{131} + 9x_{211} + 8x_{221} + 9x_{231} + 12x_{311} + 9x_{321} + 12x_{331}}$$

Subject to

$$\begin{aligned} &x_{111} + x_{121} + x_{131} + x_{122} + x_{132} + x_{133} + x_{123} = 3, \\ &x_{211} + x_{212} + x_{213} + x_{222} + x_{223} + x_{233} + x_{231} = 5, \\ &x_{311} + x_{312} + x_{313} + x_{321} + x_{331} + x_{332} + x_{333} = 4, \\ &x_{111} + x_{211} + x_{311} + x_{212} + x_{213} + x_{312} + x_{313} = 4, \end{aligned} \tag{7}$$

$$
\begin{aligned}
&x_{121} + x_{221} + x_{321} + x_{221} + x_{222} + x_{223} + x_{323} = 3,\\
&x_{131} + x_{231} + x_{331} + x_{132} + x_{133} + x_{233} + x_{333} = 5\\
&x_{111} + x_{121} + x_{131} + x_{221} + x_{231} + x_{331} + x_{321} = 5,\\
&x_{112} + x_{122} + x_{222} + x_{212} + x_{312} + x_{322} + x_{332} = 3,\\
&x_{113} + x_{123} + x_{133} + x_{213} + x_{223} + x_{313} + x_{333} = 4,\\
&x_{ijk} \geq 0, i = j = k = 1, 2, 3.
\end{aligned}
$$

To solve problem (7), we used MATLAB R2020a, which is operated on a computer with the specifications. CPU: Intel_Core_i3-9100F_3.60 GHz; Memory: 16 GB DDR4 dual-channel RAM; and Operating system: Windows 10.

The solution of problem (7) is as follows:

$$Z_1^R = 0.0134328 \quad \text{at } x_{111} = x_{332} = x_{113} = 3,\ x_{221} = 1.5, x_{331} = x_{333} = 0.5, x_{211} = 1, x_{233} = 4.$$

By solving $Z_2^R(x)$ with respect to the given constraints

$$
\begin{aligned}
\text{Min } Z_2^R &= \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\frac{\tilde{s}_{ijk}^2}{\tilde{t}_{ijk}^2}\\
&= \frac{15x_{112} + 9x_{122} + 10x_{132} + 11x_{212} + 11x_{222} + 8x_{232} + 4x_{312} + 12x_{322} + 9x_{332}}{4x_{112} + 8x_{122} + 9x_{132} + 9x_{212} + 13x_{222} + 19x_{232} + 16x_{312} + 12x_{322} + 8x_{332}}.
\end{aligned}
$$

Subject to

Constraints in (7). (8)

The solution is $Z_2^R = 0.00443787$ at $x_{312} = 3,\ x_{131} = x_{223} = 1.234568, x_{123} = x_{323} = 1.765432,\ x_{231} = 3.765432, x_{311} = x_{113} = 0.08984136,\ x_{313} = 0.9101586.$

Solving $Z_3^R(x)$ with respect to the given constraints

$$
\begin{aligned}
\text{Min } Z_3^R &= \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\frac{\tilde{s}_{ijk}^3}{\tilde{t}_{ijk}^3}\\
&= \frac{6x_{113} + 6x_{123} + 9x_{133} + 5x_{213} + 6x_{223} + 9x_{233} + 10x_{313} + 9x_{323} + 11x_{333}}{4x_{113} + 8x_{123} + 12x_{133} + 10x_{213} + 13x_{223} + 18x_{233} + 28x_{313} + 27x_{323} + 15x_{333}}.
\end{aligned}
$$

Subject to

Constraints in (7). (9)

The solution is $Z_3^R = 0.3471503$ at $x_{313} = 4,\ x_{323} = x_{122} = 3,\ x_{231} = 5, x_{331} = x_{333} = 0.5,\ x_{211} = 1,\ x_{233} = 4.$
In the same way, by solving we have Model (10) as follows:

$$
\begin{aligned}
\text{Min } Z_1^C &= \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\frac{\tilde{s}_{ijk}^1}{\tilde{t}_{ijk}^1}\\
&= \frac{7x_{111} + 10x_{121} + 11x_{131} + 13x_{211} + 12x_{221} + 9x_{231} + 13x_{311} + 17x_{321} + 12x_{331}}{4x_{111} + 2x_{121} + 5x_{131} + 4x_{211} + 6x_{221} + 7x_{231} + 8x_{311} + 7x_{321} + 9x_{331}}.
\end{aligned}
$$

Subject to

Constraints in (7). (10)

The solution is $Z_1^C = 1.285714$, with $x_{231} = 5,\ x_{122} = 0.9396557,$

$$x_{123} = x_{321} = 2.060344,\ \ x_{313} = 1.939656,\ \ x_{323} = 3.$$

By solving $Z_2^C(x)$ with respect to the given constraints

$$\begin{aligned}\text{Min } Z_2^C &= \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\frac{\tilde{s}_{ijk}^2}{\tilde{t}_{ijk}^2}\\ &= \frac{12x_{112}+7x_{122}+7x_{132}+6x_{212}+9x_{222}+6x_{232}+2x_{312}+7x_{322}+3x_{332}}{2x_{112}+6x_{122}+4x_{132}+5x_{212}+9x_{222}+9x_{232}+2x_{312}+9x_{322}+6x_{332}}.\end{aligned}$$

Subject to

Constraints in (9). (11)

The solution is given by

$$Z_2^C = 0.6666667, \quad \text{with } x_{132}=2,\ x_{232}=0.2230347\times 10^{11},\ x_{322}=0.5,\ x_{213}=3,\ x_{231}=x_{331}=x_{221}=1.5.$$

Solving $Z_3^C(x)$ with respect to the given constraints

$$\begin{aligned}\text{Min } Z_3^C &= \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\frac{\tilde{s}_{ijk}^3}{\tilde{t}_{ijk}^3}\\ &= \frac{4x_{113}+4x_{123}+8x_{133}+3x_{213}+5x_{223}+6x_{233}+9x_{313}+8x_{323}+7x_{333}}{x_{113}+2x_{123}+9x_{133}+8x_{213}+9x_{223}+8x_{233}+13x_{313}+20x_{323}+10x_{333}}.\end{aligned}$$

Subject to

Constraints in (7). (12)

The solution is given by

$$Z_3^C(x) = 0.3750000, \quad \text{with } x_{213}=4,\ x_{131}=x_{132}=x_{221}=1.5,\ x_{231}=x_{331}=1,\ x_{332}=3,$$

i.e.,

$$\begin{aligned}&0.0134328 \le Z_1(x) \le 1.285714,\\ &0.00443787 \le Z_2(x) \le 0.6666667,\\ &0.3471503 \le Z_3(x) \le 0.3750000.\end{aligned}$$

The membership function for $Z_1(x)$, $Z_2(x)$, and $Z_3(x)$ are as follows:

$$\begin{aligned}\mu_1(Z_1) &= \frac{1.285714 - Z_1}{1.285714 - 0.0134328} = \frac{1.285714 - Z_1}{1.2722812},\\ \mu_2(Z_2) &= \frac{0.6666667 - Z_2}{0.6666667 - 0.00443787} = \frac{0.6666667 - Z_2}{0.2228797},\end{aligned}$$

and

$$\mu_3(Z_3) = \frac{0.3750000 - Z_3}{0.3750000 - 0.3471503} = \frac{0.3750000 - Z_3}{0.0278497}.$$

Step 4: Let us solve the following mathematical problem.

Max δ

Subject to

$$\begin{aligned}&\left(\begin{aligned}&7y_{111}+10y_{121}+11y_{131}+13y_{211}+12y_{221}\\ &+9y_{231}+13y_{311}+17y_{321}+12y_{331}+1.2722812\delta\end{aligned}\right) \ge 1.285714,\\ &\left(\begin{aligned}&12y_{112}+7y_{122}+7y_{132}+6y_{212}+9y_{222}\\ &+6y_{232}+2y_{312}+7y_{322}+3y_{332}+0.2228797\delta\end{aligned}\right) \ge 0.6666667,\\ &\left(\begin{aligned}&4y_{113}+4y_{123}+8y_{133}+3y_{213}+5y_{223}\\ &+6y_{233}+9y_{313}+8y_{323}+7y_{333}+0.0278497\delta\end{aligned}\right) \ge 0.3750000,\end{aligned} \tag{13}$$

$$
\begin{aligned}
&4y_{111} + 2y_{121} + 5y_{131} + 4y_{211} + 6y_{221} + 7y_{231} + 8y_{311} + 7y_{321} + 9y_{331} \le 1,\\
&2y_{112} + 6y_{122} + 4y_{132} + 5y_{212} + 9y_{222} + 9y_{232} + 2y_{312} + 9y_{322} + 6y_{332} \le 1,\\
&y_{113} + 2y_{123} + 9y_{133} + 8y_{213} + 9y_{223} + 8y_{233} + 13y_{313} + 20y_{323} + 10y_{333} \le 1,\\
&y_{111} + y_{121} + y_{131} + y_{122} + y_{132} + y_{133} + y_{123} - 3t = 0,\\
&y_{211} + y_{212} + y_{213} + y_{222} + y_{223} + y_{233} + y_{231} - 5t = 0,\\
&y_{311} + y_{312} + y_{313} + y_{321} + y_{331} + y_{332} + y_{333} - 4t = 0,\\
&y_{111} + y_{211} + y_{311} + y_{212} + y_{213} + y_{312} + y_{313} - 4t = 0,\\
&y_{121} + y_{221} + y_{321} + y_{221} + y_{222} + y_{223} + y_{323} - 3t = 0,\\
&y_{131} + y_{231} + y_{331} + y_{132} + y_{133} + y_{233} + y_{333} - 5t = 0\\
&y_{111} + y_{121} + y_{131} + y_{221} + y_{231} + y_{331} + y_{321} - 5t = 0,\\
&y_{112} + y_{122} + y_{222} + y_{212} + y_{312} + y_{322} + y_{332} - 3t = 0,\\
&y_{113} + y_{123} + y_{133} + y_{213} + y_{223} + y_{313} + y_{333} - 4t = 0,\\
&y_{ijk} \ge 0,\ i = j = k = 1, 2, 3;\ t \ge 0,\ 0 < \delta \le 1.
\end{aligned}
$$

Step 5: Using MATALAB package, the solution of problem (13) is as follows:

$$x_{121} = x_{112} = x_{132} = x_{222} = 1.5,\ x_{231} = 3.5,\ x_{232} = 0.6702,\ x_{313} = 4,$$

and the overall satisfaction $\delta = 1$. Thus, $Z_1 = 2.375$, $Z_2 = 1.61298$, and $Z_3 = 0.69231$.

Stage II

Step 1:

Let us take the following data:

$$a_1 = 2,\quad a_2 = 3,\quad a_3 = 3;\quad b_1 = 2,\quad b_2 = 1,\quad b_3 = 2;\quad e_1 = 2,\quad e_2 = 1,\quad e_3 = 3.$$

Now, by solving $Z_1^R(x)$ with respect to the given constraints

$$
\begin{aligned}
\text{Min } Z_1^R &= \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\frac{\tilde{s}_{ijk}^1}{\tilde{t}_{ijk}^1}\\
&= \frac{8x_{111} + 15x_{121} + 18x_{131} + 24x_{211} + 14x_{221} + 20x_{231} + 19x_{311} + 20x_{321} + 18x_{331}}{9x_{111} + 5x_{121} + 8x_{131} + 9x_{211} + 8x_{221} + 9x_{231} + 12x_{311} + 9x_{321} + 12x_{331}}.
\end{aligned}
$$

Subject to

$$
\begin{aligned}
&x_{111} + x_{121} + x_{131} + x_{122} + x_{132} + x_{133} + x_{123} = 2,\\
&x_{211} + x_{212} + x_{213} + x_{222} + x_{223} + x_{233} + x_{231} = 3,\\
&x_{311} + x_{312} + x_{313} + x_{321} + x_{331} + x_{332} + x_{333} = 3,\\
&x_{111} + x_{211} + x_{311} + x_{212} + x_{213} + x_{312} + x_{313} = 2,\\
&x_{121} + x_{221} + x_{321} + x_{221} + x_{222} + x_{223} + x_{323} = 1,\\
&x_{131} + x_{231} + x_{331} + x_{132} + x_{133} + x_{233} + x_{333} = 2\\
&x_{111} + x_{121} + x_{131} + x_{221} + x_{231} + x_{331} + x_{321} = 2,\\
&x_{112} + x_{122} + x_{222} + x_{212} + x_{312} + x_{322} + x_{332} = 1,\\
&x_{113} + x_{123} + x_{133} + x_{213} + x_{223} + x_{313} + x_{333} = 3,\\
&x_{ijk} \ge 0,\ i = j = k = 1, 2, 3.
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
\text{Min } Z_2^R &= \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\frac{\tilde{s}_{ijk}^2}{\tilde{t}_{ijk}^2}\\
&= \frac{15x_{112} + 9x_{122} + 10x_{132} + 11x_{212} + 11x_{222} + 8x_{232} + 4x_{312} + 12x_{322} + 9x_{332}}{4x_{112} + 8x_{122} + 9x_{132} + 9x_{212} + 13x_{222} + 19x_{232} + 16x_{312} + 12x_{322} + 8x_{332}}.
\end{aligned}
$$

Subject to

Constraints in (14). (15)

Also,

$$\text{Min } Z_3^R = \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\frac{\tilde{s}_{ijk}^3}{\tilde{t}_{ijk}^3} = \frac{6x_{113} + 6x_{123} + 9x_{133} + 5x_{213} + 6x_{223} + 9x_{233} + 10x_{313} + 9x_{323} + 11x_{333}}{4x_{113} + 8x_{123} + 12x_{133} + 10x_{213} + 13x_{223} + 18x_{233} + 28x_{313} + 27x_{323} + 15x_{333}}.$$

Subject to

Constraints in (14). (16)

We have, $Z_1^R = 1.857143$, $Z_2^R = 0.4210923$, and $Z_3^R = 0.5000000$.
In the same way, solving

$$\text{Min } Z_1^C = \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\frac{\tilde{s}_{ijk}^1}{\tilde{t}_{ijk}^1} = \frac{7x_{111} + 10x_{121} + 11x_{131} + 13x_{211} + 12x_{221} + 9x_{231} + 13x_{311} + 17x_{321} + 12x_{331}}{4x_{111} + 2x_{121} + 5x_{131} + 4x_{211} + 6x_{221} + 7x_{231} + 8x_{311} + 7x_{321} + 9x_{331}}.$$

Subject to

Constraints in (14). (17)

$$\text{Min } Z_2^C = \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\frac{\tilde{s}_{ijk}^2}{\tilde{t}_{ijk}^2} = \frac{12x_{112} + 7x_{122} + 7x_{132} + 6x_{212} + 9x_{222} + 6x_{232} + 2x_{312} + 7x_{322} + 3x_{332}}{2x_{112} + 6x_{122} + 4x_{132} + 5x_{212} + 9x_{222} + 9x_{232} + 2x_{312} + 9x_{322} + 6x_{332}}.$$

Subject to

Constraints in (14), (18)

and

$$\text{Min } Z_3^C = \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\frac{\tilde{s}_{ijk}^3}{\tilde{t}_{ijk}^3} = \frac{4x_{113} + 4x_{123} + 8x_{133} + 3x_{213} + 5x_{223} + 6x_{233} + 9x_{313} + 8x_{323} + 7x_{333}}{x_{113} + 2x_{123} + 9x_{133} + 8x_{213} + 9x_{223} + 8x_{233} + 13x_{313} + 20x_{323} + 10x_{333}}.$$

Subject to

Constraints in (14). (19)

We obtain the following solution:

$$\text{Min } Z_1^C = 1.466667,\ Z_2^C = 0.5000000,\ \text{and}\ Z_3^C = 0.8500000,$$

i.e.,

$$1.466667 \le Z_1(x) \le 1.857143, \quad 0.4210923 \le Z_2(x) \le 0.5000000, \quad 0.5000000 \le Z_3(x) \le 0.8500000.$$

The membership function for $Z_1(x)$, $Z_2(x)$, and $Z_3(x)$ are as follows:

$$\mu_1(Z_1) = \frac{1.857143 - Z_1}{1.857143 - 1.466667} = \frac{1.857143 - Z_1}{0.390476},$$
$$\mu_2(Z_2) = \frac{0.5000000 - Z_2}{0.5000000 - 0.4210923} = \frac{0.5000000 - Z_2}{0.0789077},$$

and

$$\mu_3(Z_3) = \frac{0.8500000 - Z_3}{0.8500000 - 0.5000000} = \frac{0.8500000 - Z_3}{0.35}.$$

Step 2: Let us solve the following problem:

Max δ

Subject to

$$\begin{aligned}
&\left(\begin{array}{l} 7y_{111} + 10y_{121} + 11y_{131} + 13y_{211} + 12y_{221} \\ \quad + 9y_{231} + 13y_{311} + 17y_{321} + 12y_{331} + 1.2722812\delta \end{array}\right) \geq 1.285714,\\
&\left(\begin{array}{l} 12y_{112} + 7y_{122} + 7y_{132} + 6y_{212} + 9y_{222} \\ \quad + 6y_{232} + 2y_{312} + 7y_{322} + 3y_{332} + 0.2228797\delta \end{array}\right) \geq 0.6666667,\\
&\left(\begin{array}{l} 4y_{113} + 4y_{123} + 8y_{133} + 3y_{213} + 5y_{223} \\ \quad + 6y_{233} + 9y_{313} + 8y_{323} + 7y_{333} + 0.0278497\delta \end{array}\right) \geq 0.3750000,\\
&4y_{111} + 2y_{121} + 5y_{131} + 4y_{211} + 6y_{221} + 7y_{231} + 8y_{311} + 7y_{321} + 9y_{331} \leq 1,\\
&2y_{112} + 6y_{122} + 4y_{132} + 5y_{212} + 9y_{222} + 9y_{232} + 2y_{312} + 9y_{322} + 6y_{332} \leq 1,\\
&y_{113} + 2y_{123} + 9y_{133} + 8y_{213} + 9y_{223} + 8y_{233} + 13y_{313} + 20y_{323} + 10y_{333} \leq 1,\\
&y_{111} + y_{121} + y_{131} + y_{122} + y_{132} + y_{133} + y_{123} - 2t = 0,\\
&y_{211} + y_{212} + y_{213} + y_{222} + y_{223} + y_{233} + y_{231} - 3t = 0,\\
&y_{311} + y_{312} + y_{313} + y_{321} + y_{331} + y_{332} + y_{333} - 3t = 0,\\
&y_{111} + y_{211} + y_{311} + y_{212} + y_{213} + y_{312} + y_{313} - 2t = 0,\\
&y_{121} + y_{221} + y_{321} + y_{221} + y_{222} + y_{223} + y_{323} - t = 0,\\
&y_{131} + y_{231} + y_{331} + y_{132} + y_{133} + y_{233} + y_{333} - 2t = 0\\
&y_{111} + y_{121} + y_{131} + y_{221} + y_{231} + y_{331} + y_{321} - 2t = 0,\\
&y_{112} + y_{122} + y_{222} + y_{212} + y_{312} + y_{322} + y_{332} - t = 0,\\
&y_{113} + y_{123} + y_{133} + y_{213} + y_{223} + y_{313} + y_{333} - 3t = 0,\\
&y_{ijk} \geq 0, i = j = k = 1, 2, 3;\ t \geq 0, 0 < \delta \leq 1.
\end{aligned} \tag{20}$$

Step 5: The solution is $x_{211} = x_{231} = x_{321} = x_{332} = x_{233} = x_{313} = 1.5$, $x_{232} = 4.4$, $x_{132} = 2$, and the overall satisfaction $\delta = 1$. Thus, $Z_1 = 20.8947$, $Z_2 = 0.64474$, and $Z_3 = 0.69231$.

By combining stage I and stage II, the optimal values of the objectives are $Z_1 = 2.375 + 20.8947 = 23.2697$, $Z_2 = 1.61298 + 0.64474 = 2.25772$, and $Z_3 = 0.69231 + 0.69231 = 1.38462$.

Step 6: The stability set S can be determined as

$$\begin{aligned}
&\gamma_1^{11}(7 - d_{21}^{11}) = 0, \quad \gamma_1^{12}(10 - d_{21}^{12}) = 0, \quad \gamma_1^{13}(11 - d_{21}^{13}) = 0,\\
&\gamma_1^{21}(13 - d_{21}^{21}) = 0, \quad \gamma_1^{22}(2 - d_{21}^{22}) = 0, \quad \gamma_1^{23}(9 - d_{21}^{23}) = 0,\\
&\gamma_1^{31}(13 - d_{21}^{31}) = 0, \quad \gamma_1^{32}(17 - d_{21}^{32}) = 0, \quad \gamma_1^{33}(12 - d_{21}^{33}) = 0,\\
&\gamma_2^{11}(12 - d_{22}^{11}) = 0, \quad \gamma_2^{12}(7 - d_{22}^{12}) = 0, \quad \gamma_2^{13}(7 - d_{22}^{13}) = 0,\\
&\gamma_2^{21}(6 - d_{22}^{21}) = 0, \quad \gamma_1^{22}(9 - d_{22}^{22}) = 0, \quad \gamma_2^{23}(6 - d_{22}^{23}) = 0,\\
&\gamma_2^{31}(2 - d_{22}^{31}) = 0, \quad \gamma_2^{32}(7 - d_{22}^{32}) = 0, \quad \gamma_2^{33}(3 - d_{22}^{33}) = 0,\\
&\gamma_3^{11}(4 - d_{23}^{11}) = 0, \quad \gamma_3^{12}(4 - d_{23}^{12}) = 0, \quad \gamma_3^{13}(8 - d_{23}^{13}) = 0,\\
&\gamma_3^{21}(3 - d_{23}^{21}) = 0, \quad \gamma_3^{22}(5 - d_{23}^{22}) = 0, \quad \gamma_3^{23}(6 - d_{23}^{23}) = 0,\\
&\gamma_3^{31}(9 - d_{23}^{31}) = 0, \quad \gamma_3^{32}(8 - d_{23}^{32}) = 0, \quad \gamma_3^{33}(7 - d_{23}^{33}) = 0,\\
&\zeta_1^{11}(4 - d_{21}^{11}) = 0, \quad \zeta_1^{12}(2 - d_{21}^{12}) = 0, \quad \zeta_1^{13}(5 - d_{21}^{13}) = 0,\\
&\zeta_1^{21}(4 - d_{21}^{21}) = 0, \quad \zeta_1^{22}(6 - d_{21}^{22}) = 0, \quad \zeta_1^{23}(7 - d_{21}^{23}) = 0,\\
&\zeta_1^{31}(8 - d_{21}^{31}) = 0, \quad \zeta_1^{32}(7 - d_{21}^{32}) = 0, \quad \zeta_1^{33}(9 - d_{21}^{33}) = 0,\\
&\zeta_2^{11}(12 - d_{22}^{11}) = 0, \quad \zeta_2^{12}(7 - d_{22}^{12}) = 0, \quad \zeta_2^{13}(7 - d_{22}^{13}) = 0,\\
&\zeta_2^{21}(6 - d_{22}^{21}) = 0, \quad \zeta_1^{22}(9 - d_{22}^{22}) = 0, \quad \zeta_2^{23}(6 - d_{22}^{23}) = 0,
\end{aligned} \tag{21}$$

$$\zeta_2^{31}(2 - d_{22}^{31}) = 0, \quad \zeta_2^{32}(7 - d_{22}^{32}) = 0, \quad \zeta_2^{33}(3 - d_{22}^{33}) = 0,$$
$$\zeta_3^{11}(1 - d_{22}^{11}) = 0, \quad \zeta_3^{12}(2 - d_{22}^{12}) = 0, \quad \zeta_3^{13}(9 - d_{22}^{13}) = 0,$$
$$\zeta_3^{21}(8 - d_{22}^{21}) = 0, \quad \zeta_3^{22}(9 - d_{22}^{22}) = 0, \quad \zeta_3^{23}(8 - d_{22}^{23}) = 0,$$
$$\zeta_3^{31}(13 - d_{22}^{31}) = 0, \quad \zeta_3^{32}(20 - d_{22}^{32}) = 0, \quad \zeta_3^{33}(10 - d_{22}^{33}) = 0,$$
$$\varpi_1(5 - h_{21}^1) = 0, \quad \varpi_2(6 - h_{21}^2) = 0, \quad \varpi_3(9 - h_{21}^3) = 0,$$
$$\varrho_1(6 - u_{21}^1) = 0, \quad \varrho_2(11.9 - u_{21}^2) = 0, \quad \varrho_3(16.9 - u_{21}^3) = 0,$$
$$\tau_1(15.1 - v_{21}^1) = 0, \quad \tau_2(17.9 - v_{21}^2) = 0, \quad \tau_3(17.3 - v_{21}^3) = 0,$$
$$\gamma_1^{11}, \gamma_1^{12}, \ldots, \gamma_1^{33} \geq 0; \quad \gamma_2^{11}, \ldots, \gamma_2^{33} \geq 0; \quad \gamma_3^{11}, \ldots, \gamma_3^{33} \geq 0 \geq 0;$$
$$\zeta_1^{11}, \zeta_1^{12}, \ldots, \zeta_1^{33} \geq 0; \quad \zeta_2^{11}, \ldots, \zeta_2^{33} \geq 0; \quad \zeta_3^{11}, \ldots, \zeta_3^{33} \geq 0;$$
$$\varpi_1, \varpi_1, \varpi_1 \geq 0; \quad \varrho_1, \varrho_2, \varrho_3 \geq 0; \quad \tau_1, \tau_2, \tau_3 \geq 0.$$

We have $J_1 \subset \{1, 2, 3\}$. For $J_1 = \varnothing, \gamma_1^{11}, \gamma_1^{12}, \ldots, \gamma_1^{33} = 0; \gamma_2^{11}, \ldots, \gamma_2^{33} = 0; \gamma_3^{11}, \ldots, \gamma_3^{33} = 0;$ $J_1 \subset \{1, 2, 3, 4, 5, 6, 7, 8\}$. For $J_1 = \varnothing, \gamma_1^{11}, \gamma_1^{12}, \ldots, \gamma_1^{33} = 0; \gamma_2^{11}, \ldots, \gamma_2^{33} = 0; \gamma_3^{11}, \ldots, \gamma_3^{33} = 0; \zeta_1^{11}, \zeta_1^{12}, \ldots, \zeta_1^{33} = 0; \zeta_2^{11}, \ldots, \zeta_2^{33} = 0; \zeta_3^{11}, \ldots, \zeta_3^{33} = 0$. Then

$$S_{J_1} = \left\{ \begin{array}{l} (d_2, h_2, u_2, v_2) \in \mathbb{R}^3 : d_{21}^{11} \geq 7, d_{21}^{12} \geq 10, d_{21}^{13} \geq 11, d_{21}^{21} \geq 13, d_{21}^{22} \geq 12, d_{21}^{23} \geq 9, \\ d_{21}^{31} \geq 13, d_{21}^{32} \geq 17, d_{21}^{33} \geq 12, : d_{22}^{11} \geq 12, d_{22}^{12} \geq 7, d_{22}^{13} \geq 7, \\ d_{22}^{21} \geq 6\, d_{22}^{22} \geq 9, d_{22}^{23} \geq 6, d_{22}^{21} \geq 2, d_{22}^{22} \geq 7, d_{22}^{23} \geq 3, d_{23}^{11} \geq 4, d_{23}^{12} \geq 4, \\ d_{23}^{13} \geq 8, d_{23}^{21} \geq 3, d_{23}^{22} \geq 6, d_{23}^{23} \geq 9, d_{23}^{21} \geq 7, d_{23}^{22} \geq 8, d_{23}^{23} \geq 7, a_1 \geq 5, a_2 \geq 6, \\ a_3 \geq 9, b_1 \geq 6, b_2 \geq 11.9, b_3 \geq 16.9, e_1 \geq 15.1, e_2 \geq 17.9, e_3 \geq 17.3. \end{array} \right\}$$

For $J_2 = \{1, 3, 4, 5, 6, 7, 8\}$. Then

$$S_{J_2} = \left\{ \begin{array}{l} (d_2, h_2, u_2, v_2) \in \mathbb{R}^3 : d_{21}^{11} = 7, d_{21}^{12} = 10, d_{21}^{13} = 11, d_{21}^{21} = 13, d_{21}^{22} = 12, d_{21}^{23} = 9, \\ d_{21}^{31} = 13, d_{21}^{32} = 17, d_{21}^{33} = 12, : d_{22}^{11} \geq 12, d_{22}^{12} \geq 7, d_{22}^{13} \geq 7, \\ d_{22}^{21} \geq 6\, d_{22}^{22} \geq 9, d_{22}^{23} \geq 6, d_{22}^{21} \geq 2, d_{22}^{22} \geq 7, d_{22}^{23} \geq 3, d_{23}^{11} = 4, d_{23}^{12} = 4, \\ d_{23}^{13} = 8, d_{23}^{21} = 3, d_{23}^{22} = 6, d_{23}^{23} = 9, d_{23}^{21} = 7, d_{23}^{22} = 8, d_{23}^{23} = 7, a_1 = 5, a_2 = 6, \\ a_3 = 9, b_1 = 6, b_2 = 11.9, b_3 = 16.9, e_1 = 15.1, e_2 = 17.9, e_3 = 17.3. \end{array} \right\}$$

For $J_3 = \{1, 2, 4, 5, 6, 7, 8\}$. Then

$$S_{J_3} = \left\{ \begin{array}{l} (d_2, h_2, u_2, v_2) \in \mathbb{R}^3 : d_{21}^{11} = 7, d_{21}^{12} = 10, d_{21}^{13} = 11, d_{21}^{21} = 13, d_{21}^{22} = 12, d_{21}^{23} = 9, \\ d_{21}^{31} = 13, d_{21}^{32} = 17, d_{21}^{33} = 12, : d_{22}^{11} \geq 12, d_{22}^{12} \geq 7, d_{22}^{13} \geq 7, \\ d_{22}^{21} \geq 6\, d_{22}^{22} \geq 9, d_{22}^{23} \geq 6, d_{22}^{21} \geq 2, d_{22}^{22} \geq 7, d_{22}^{23} \geq 3, d_{23}^{11} \geq 4, d_{23}^{12} \geq 4, \\ d_{23}^{13} \geq 8, d_{23}^{21} \geq 3, d_{23}^{22} \geq 6, d_{23}^{23} \geq 9, d_{23}^{21} \geq 7, d_{23}^{22} \geq 8, d_{23}^{23} \geq 7, a_1 = 5, a_2 = 6, \\ a_3 = 9, b_1 = 6, b_2 = 11.9, b_3 = 16.9, e_1 = 15.1, e_2 = 17.9, e_3 = 17.3. \end{array} \right\}$$

In view of this, we obtain

$$S = \bigcup_{q=1}^{3} S_{J_q}. \tag{22}$$

The objective function value obtained with the proposed approach is better compared with that obtained by Radhakrishnan and Anukokila [32].

5 Concluding remarks

Solid fractional TP has wide application in supply chain and logistics so as to reduce the cost. In this article, a two-stage cost minimizing fuzzy STP with multi-objective constraints has been studied. Fractional fuzzy

geometric programming approach has been applied to determine the optimal compromise solution for a multi-objective two-stage fuzzy STPs in which sources' availabilities and destination's demands are triangular fuzzy numbers, and membership function for the objective functions has been defined rather than the crisp value provides more information for the decision-maker. MATALB software has been used to find out the optimal compromise solution. This approach provides an easy and simple analyst mathematical programming problem.

Acknowledgments: The authors would like to thank the Editor-in-Chief and anonymous referees for their suggestive and helpful comments that have led to an improvement both in the quality and clarity of the article, and to the Dean of VIT Bhopal University as well as Qassim University.

Conflict of interest: The author declares no conflicts of interest.

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