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A Novel Weakest t-norm based Fuzzy Fault Tree Analysis Through Qualitative Data Processing and Its Application in System Reliability Evaluation

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Abstract: The quantification of the fuzzy fault tree analysis (FFTA) is based on fuzzy arithmetic operations. It is well known that the weakest t-norm (T_W)-based fuzzy arithmetic operations have some advantages. The T_W-based fuzzy arithmetic operations provide fuzzy results with less fuzziness and preserve the shape of fuzzy numbers. The purpose of this study is to develop a T_W-based fuzzy fault tree analysis (TBFFTA) to assess system reliability when only qualitative data such as expert opinions or decisions are available and described in linguistic terms. The developed TBFFTA applies T_W-based fuzzy arithmetic operations to evaluate the lower bound, best estimate, and upper bound top event probability of a system fault tree, where occurrence possibilities of basic events are characterized by triangular fuzzy membership functions. To demonstrate the applicability and feasibility of TBFFTA, a case study has been performed. The computed results have been compared with results analyzed by existing fuzzy approach. The comparative study concludes that TBFFTA reduces fuzzy spreads (uncertainty interval) and provides more exact fuzzy results.

Keywords: Fuzzy failure possibility, fault tree analysis, weakest t-norm, fuzzy number.

1 Introduction

Reliability and safety issues are two key aspects of complex engineering systems. Many researchers have developed various methods to analyze system reliability. Fault tree analysis (FTA) is one of those developed methods and widely used techniques to analyze the risks related to system safety and reliability. FTA refers to the analysis of the system logic model provided by the fault tree. In FTA, the occurrence probability of the top event depends upon the occurrence probabilities of basic events [44]. FTA can be either qualitative or quantitative or both. Qualitative analysis does not require the assignment of probability values to basic events defined on the system logic. An important aspect of qualitative analysis is to determine the minimal cut sets (MCSs) for the top event using Boolean logic. An MCS is the smallest set of primary failures that may cause system failure. After obtaining MCSs, quantitative FTA can be performed to estimate the top event probability by using the likelihood occurrence possibilities of basic events and Boolean algebras [44]. In quantitative sense, system reliability is a complement value of top event occurrence probability. To evaluate system reliability, the exact failure data of system components are essential. Sometimes, it is not possible to extract the exact failure data of components for every engineering system and rough data can be utilized for system reliability analysis, which could lead to uncertainty [1]. In the probability evaluation of a basic event, there may arise two types of uncertainties, i.e. alleatory and epistemic uncertainties [1, 8, 14]. Alleatory uncertainty is derived due to data randomness, and these uncertainties can be represented by probability distributions. Epistemic uncertainty is derived due to lack of knowledge, imprecise or incomplete information. To incorporate epistemic uncertainty, fuzzy approach has been successfully employed [5, 9, 15, 19, 21, 22, 48]. In literature, conventional FTA has been effectively integrated with the fuzzy set theory. A number of researchers have developed fuzzy fault tree analysis (FFTA) [13, 42] and its extensions [23, 24] for analyzing system reliability under an uncertain environment.

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In FFTA, when probability distributions of the basic events are unknown, the reliability characteristics of basic events have been evaluated using qualitative data such as expert opinions or judgements described in natural languages/linguistic terms [7, 29]. These linguistic terms are quantified by using the membership functions of fuzzy numbers and represent those expert judgments mathematically [3, 27]. The reliability of various engineering systems has also been analyzed using FFTA. Tanaka et al. [42] used the fuzzy set theory in FTA to evaluate the top event fuzzy probability, where fuzzy probabilities of basic events were represented as trapezoidal fuzzy numbers. Yuhua and Datao [49] presented FFTA to estimate the failure probability of oil and gas transmission pipelines. FFTA has also been applied to assess the failures of a building construction [31] and a bridge construction [32]. Ferdous et al. [6] proposed a computer-aided FFTA method. Tyagi et al. [43] applied FFTA in reliability analysis of an electric power transformer. Recently, the failure probability of fire and explosion in crude oil tanks has been evaluated using FFTA [45]. Rajakarunakaran et al. [38] applied FFTA for risk evaluation of an LPG refuelling station. FFTA has been applied to analyze the reliabilities of safety instrumented systems [40], a shipping accident [2], a chlor-alkali industry [39], a spread mooring system [29], and nuclear power plant safety systems [33, 34] and also to evaluate risk in petrochemical industries [25, 41]. Recently, Purba et al. [36, 37] proposed a fuzzy probability based fault tree analysis (FPFTA) approach to propagate and to quantify the overall epistemic uncertainties by using a simple fuzzy multiplication rule and a fuzzy complement rule, where likelihood occurrences probabilities of basic events are represented by triangular fuzzy numbers (TFNs). In FPFTA, the obtained occurrence possibilities of a top event are approximated TFNs.

The objective of this study is to quantify epistemic uncertainty in FFTA and to obtain exact results with less uncertainty. It is well known that the T_W-based fuzzy arithmetic operations preserve the shape of fuzzy numbers, effectively reduce uncertainty, and provide more exact results [10–12, 18, 20, 26]. Therefore, in this paper, a TBFFTA approach is developed to propagate and quantify epistemic uncertainties in a more exact way. To demonstrate the applicability and feasibility of the proposed TBFFTA, a case study has been performed and obtained results are then compared to the results generated by the existing approaches. The rest of the paper is organized as follows. In Section 2, the concepts and definitions implemented in the proposed TBFFTA are described. Section 3 describes the quantification processes of the proposed methodology in details. An illustrative case study with result discussions and comparison is given in Section 4 to show the effectiveness and applicability of the proposed approach. Finally, conclusions and further studies are given in Section 5.

2 Some Basic Concepts of Fuzzy Set Theory

This section provides some basic definitions including fuzzy sets, TFN, linguistic term, and fuzzy possibility. It also presents T_W-based fuzzy arithmetic operations on TFNs.

2.1 Definitions

2.1.1 Fuzzy Sets

Let X be the universal set. Then a fuzzy set $\tilde{A}$ defined on X is expressed as a set of ordered pairs $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x)\rangle : x \in X\}$, where $\mu_{\tilde{A}} : X \to [0, 1]$ is a membership function. The value $\mu_{\tilde{A}}(x)$ represents the degree to which x belongs to $\tilde{A}$.

A fuzzy set $\tilde{A}$ defined on R (real line) is called a fuzzy number [4] if it possesses at least the following conditions:

(i) $\tilde{A}$ must be a normal fuzzy set, i.e. there exists $x_0 \in R$ such that $\mu_{\tilde{A}}(x_0) = 1$.

(ii) $\tilde{A}$ must be a convex fuzzy set, i.e. for every $x_1,\ x_2 \in R$ and $\lambda \in [0,\ 1]$,

$$\mu_{\tilde{A}}(\lambda x_1 + (1 - \lambda x_2)) \geq \min(\mu_{\tilde{A}}(x_1),\ \mu_{\tilde{A}}(x_2)).$$

(iii) The support $S(\tilde{A})$ of $\tilde{A}$ must be bounded, where $S(\tilde{A}) = \{x \in X : \mu_{\tilde{A}}(x) > 0\}$.

A fuzzy number $\tilde{A}$ with the following membership function $\mu_{\tilde{A}}$ is called LR-type fuzzy number [4, 17].

$$\mu_{\tilde{A}}(x) = \begin{cases} L\left(\dfrac{m-x}{\alpha}\right), & m-\alpha \le x \le m, \\ R\left(\dfrac{x-m}{\beta}\right), & m \le x \le m+\beta, \\ 0, & \text{otherwise} \end{cases}$$

where $m \in R$ is the mean value and $\alpha, \beta > 0$ are the left and right spreads of $\tilde{A}$, respectively. The functions L and R are non-increasing and continuous and defined from [0, 1] to [0, 1], satisfying $L(0) = R(0) = 1$, $L(1) = R(1) = 0$. An LR-type fuzzy number $\tilde{A}$ is denoted by $\tilde{A} = (m, \alpha, \beta)_{LR}$.

An LR-type fuzzy number is called TFN if $L(x) = R(x) = \max(0, 1 - x)$. It is denoted by $\tilde{A} = (m, \alpha, \beta)$, and its membership function $\mu_{\tilde{A}}$ is defined in (1).

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-(m-\alpha)}{\alpha}, & m-\alpha < x \le m, \\ \dfrac{(m+\beta)-x}{\beta}, & m < x \le m+\beta, \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

If both left and right spreads are equal, i.e. $\alpha = \beta$, then TFN is called a symmetric TFN and is denoted by (m, α).

2.1.2 Triangular Norm

A triangular norm (t-norm) T is a binary function defined on [0, 1], i.e. $T : [0, 1]^2 \to [0, 1]$, such that it is associative, commutative, non-decreasing, and $T(x, 1) = x$ for each $x \in [0, 1]$. Mathematically, the most important triangular norms [4] are as follows:

$$\text{Algebraic product}: \quad T_P(x, y) = xy$$

$$\text{Standard intersection}: \quad T_M(x, y) = \min(x, y)$$

$$\text{Bounded difference}: \quad T_L(x, y) = \max(0,\ x + y - 1)$$

$$\text{Weakest t-norm } (\mathrm{T_W}): \quad T_W(x, y) = \begin{cases} x, & \text{if } y = 1, \\ y, & \text{if } x = 1, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

The present research applies $\mathrm{T_W}$ due to its shape-preserving and fuzziness-reducing characteristics within uncertain environment [10–12, 18, 20, 26].

2.1.3 Linguistic Terms and Fuzzy Failure Possibilities

The safety and reliability of a system can be evaluated by using the quantitative historical failure data of its components. If quantitative failure data are improper or become inadequate, then only qualitative data such as expert opinions, which are described in linguistic terms, can be used to assess safety and reliability of the system. Seven qualitative linguistic terms have been defined based on the collected component failure data from nuclear power plant operating experiences [33, 34, 36]. These seven linguistic terms are shown in Table 3. Fuzzy numbers are used to represent linguistic terms quantitatively. Any shape of membership function of the fuzzy number could be applied to represent the fuzzy failure possibilities for system reliability analysis but should be modeled based on the nature of the problem at hand. Meanwhile, trapezoidal and triangular fuzzy numbers have been confirmed to be a sound practical alternative to reflect uncertainties and fuzziness of human justifications [7].

2.2 The T_w-based Arithmetic Operations on Triangular Fuzzy Numbers

The T_w-based fuzzy arithmetic operations have some obvious advantages: the calculation is drastically simplified and obtained results are more exact with less uncertainty. The T_w-based addition and multiplication preserve the shape of fuzzy numbers, in particular, they preserve the triangular fuzzy numbers [10–12, 18, 20, 26].

Let $\tilde{A} = (a, \alpha_A, \beta_A)$ and $\tilde{B} = (b, \alpha_B, \beta_B)$ be any two TFNs. The five main T_w-based fuzzy arithmetic operations on TFNs $\tilde{A}$ and $\tilde{B}$ are summarized as follows:

$$\text{Addition}: \quad \tilde{A} \oplus \tilde{B} = (a, \alpha_A, \beta_A) \oplus (b, \alpha_B, \beta_B) = \big(a + b,\ \max(\alpha_A, \alpha_B),\ \max(\beta_A, \beta_B)\big) \tag{3}$$

$$\text{Subtraction}: \quad \tilde{A} \ominus \tilde{B} = (a,\ \alpha_A, \beta_A) \ominus (b, \alpha_B, \beta_B) = \big(a - b,\ \max(\alpha_A, \beta_B),\ \max(\beta_A, \alpha_B)\big) \tag{4}$$

$$\text{Multiplication}: \quad \tilde{A} \otimes \tilde{B} = (a, \alpha_A, \beta_A) \otimes (b, \alpha_B, \beta_B)$$
$$= \begin{cases} \big(ab, \max(\alpha_A b, \alpha_B a), \max(\beta_A b, \beta_B a)\big), & \text{if } a, b > 0 \\ \big(ab, \max(-\beta_A b, -\beta_B a), \max(-\alpha_A b, -\alpha_B a)\big), & \text{if } a, b < 0 \\ \big(ab, \max(\alpha_A b, -\beta_B a), \max(\beta_A b, -\alpha_B a)\big), & \text{if } a < 0, b > 0 \\ \big(ab, \max(-\beta_A b, \alpha_B a), \max(-\alpha_A b, \beta_B a)\big), & \text{if } a > 0, b < 0 \\ (0, -\beta_A b, -\alpha_A b), & \text{if } a = 0, b < 0 \\ (0, 0, 0) & \text{if } a = 0, b = 0 \end{cases} \tag{5}$$

$$\text{Scalar multiplication}: \quad \lambda\tilde{A} = (\lambda a, \lambda\alpha_A, \lambda\beta_A) \quad \text{where } \lambda \in R,\ \lambda > 0 \tag{6}$$

$$\text{Compliment}: 1 \ominus \tilde{A} = (1 - a,\ \beta_A,\ \alpha_A) \tag{7}$$

3 The Proposed TBFFTA

The main idea behind TBFFTA is to use the fuzzy representation of the available occurrence possibilities of basic events to quantify the lower bound, best estimate, and upper bound top event failure probability.

3.1 Process of Defining Lower Bound, Best Estimate, and Upper Bound Fuzzy Possibilities for Basic Events

The qualitative failure data of basic events are available as linguistic terms. These linguistic terms can be quantified by using the membership functions of fuzzy numbers. After that, the fuzzy possibilities of basic events can be obtained in the prescribed format. There are different methods available to obtain fuzzy possibilities such as expert knowledge elicitation or $3\sigma\sigma$ expression [28]. In this paper, the triangular form of a fuzzy number is used to represent the occurrence possibilities of the basic events. Also expert knowledge elicitation method is used to obtain the fuzzy possibilities of basic events.

3.1.1 Evaluation of Expert Opinions and Fuzzification

The objective of this subsection is to obtain a set of qualitative data representing the occurrence possibilities of basic events. For this, a set of m experts $\{E_1, E_2, ...E_m\}$ is provided to make their decisions about the occurrence possibilities of n basic events $\{b_1, b_2, \ldots, b_n\}$ from the fault tree. First, the experts will subjectively evaluate the failure possibility of the components and then make decisions about different basic events based on their expertise, working experience, and knowledge about the system. As experts are human beings, different experts may have different opinions about the components/system. In a real-world scenario, the opinion of an expert with higher experience and expertise should be given higher priority. To facilitate this, a weighting factor can be used to define the relative quality of the expert opinions. For complex systems, it is not possible for the experts to provide the exact numerical values for the failure possibility of components; instead, they provide their opinions as linguistic terms. For instance, *"failure possibility of a component"* can be considered as a linguistic variable consisting of linguistic terms like *very low, low, reasonable low, moderate, reasonable high, high, very high*. Once an expert gives his/her opinion about the occurrence possibility of an event as linguistic term, then this opinion can be mapped to corresponding quantitative data in the form of a membership function of fuzzy number.

3.1.2 Aggregation of Experts Opinions to Obtain Best Estimate, Lower Bound, and Upper Bound Fuzzy Possibilities of Basic Events

As different experts may have different opinions about the same basic event as per his/her experience and ability in the relevant field, keeping in mind the end goal to achieve agreement among experts' conflicted views, the fuzzy numbers assigned by different experts should be aggregated to a single one. In this subsection, a consistency aggregation method [16, 45, 46] has been used for aggregation to evaluate a best estimate, a lower bound, and an upper bound fuzzy possibility for each basic event.

Best estimate fuzzy possibility evaluations of basic events:

The best estimate fuzzy possibilities of all basic events can be evaluated in the following steps:

Step 1. Similarity measures

The similarity $s(A_i, A_j)$ between the opinions A_i and A_j of experts E_i and E_j, respectively, can be calculated as:

$$s(A_i, A_j) = \begin{cases} EV_i/EV_j, & EV_i \le EV_j \\ EV_j/EV_i, & EV_j \le EV_i \end{cases}, \tag{8}$$

where $s(A_i, A_j) \in [0,\ 1]$ is the similarity measure function, A_i and A_j are two standard fuzzy numbers, and EV_i and EV_j represent the expectancy evaluation for A_i and A_j. The EV of a TFN $\tilde{A} = (a,\ \alpha_A,\ \beta_A)$ is defined as follows.

$$EV(\tilde{A}) = a + \frac{\beta_A - \alpha_A}{4} \tag{9}$$

If there are m experts, then a matrix known as *similarity matrix* (SM) can be obtained in the following form:

$$SM = \begin{bmatrix} 1 & s_{12} & s_{13} & \cdots & s_{1m} \\ s_{21} & 1 & s_{23} & \cdots & s_{2m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ s_{m1} & s_{m2} & s_{m3} & s_{m4} & 1 \end{bmatrix}, \tag{10}$$

where $s_{ij} = s(A_i, A_j)$. if $i = j$ then $s_{ij} = 1$.

Step 2. Average agreement degree

The average agreement degree $AAD(E_i)$ for each of the experts is obtained as follows:

$$AAD(E_i) = \frac{1}{m-1} \sum_{\substack{j=1 \\ j \ne i}}^{m} s_{ij},\ i = 1, 2, \ldots, m \tag{11}$$

Step 3. Relative agreement degree

The relative agreement degree $RAD(E_i)$ for all experts is obtained as follows:

$$RAD(E_i) = \frac{AAD(E_i)}{\sum_{i=1}^{m} AAD(E_i)}, i = 1, 2, \ldots, m \tag{12}$$

Step 4. Weighting factor calculation

Based on professional positions, years of working experience, and educational qualifications, weighting scores for all experts are defined in Table 4. As a result, when we select m experts, each of them may have a different weighting score. For example, if we choose a professor with a PhD degree and 20 years of work experience, then his/her weighting score would be 15 ($5 + 5 + 5 = 15$). So the weighting factor for each of the experts is calculated as follows:

$$WF(E_i) = \frac{WS(E_i)}{\sum_{i=1}^{m} WS(E_i)}, i = 1, 2, \ldots, m, \tag{13}$$

where $WS(E_i)$ is the weighting score of expert E_i and $WF(E_i)$is the weighting factor of expert E_i.

Step 5. Aggregation weight calculation

The aggregation weight (w_i) of each expert E_i is the combination of the $RAD(E_i)$ and the weighting factor $WF(E_i)$ of expert E_i.

$$w_i = r.WF(E_i) + (1 - r).RAD(E_i), \tag{14}$$

where r $(0 \leq r \leq 1)$ is a relaxation factor that represents the importance of $WF(E_i)$ over $RAD(E_i)$. If r is zero, then no importance is paid on $WF(E_i)$; on the other hand, if r is 1, then no importance is paid to $RAD(E_i)$.

Step 6. Aggregation of experts' opinions

$$\tilde{p}_j^M = \sum_{i=1}^{m} w_i \otimes \tilde{p}_{ij}, \quad j = 1, 2, \ldots, n, \tag{15}$$

where $\tilde{p}_j^M$ is the aggregated best estimate fuzzy possibility of basic event b_j, $j = 1, 2, \ldots, n$ and $\tilde{p}_{ij}$ is the fuzzy possibility of basic event b_j assigned by expert E_i.

Lower bound fuzzy possibility evaluations of basic events:

The lower bound fuzzy possibility $\tilde{p}_j^L$ of basic event $b_j$is evaluated as follows:

$$\tilde{p}_j^L = \min\{\tilde{p}_{1j}, \tilde{p}_{2j}, \ldots, \tilde{p}_{mj}\} = \min\{\tilde{p}_{ij}\}_{i=1}^{m}, \tag{16}$$

where $\tilde{p}_{ij}$ is the fuzzy possibility of basic event $b_j$assigned by expert E_i.

Upper bound fuzzy failure possibility evaluations of basic events:

The upper bound fuzzy possibility $\tilde{p}_j^U$ of basic event $b_j$is evaluated as follows:

$$\tilde{p}_j^U = \max\{\tilde{p}_{1j}, \tilde{p}_{2j}, \ldots, \tilde{p}_{mj}\} = \max\{\tilde{p}_{ij}\}_{i=1}^{m}, \tag{17}$$

where $\tilde{p}_{ij}$ is the fuzzy possibility of basic event $b_j$assigned by expert E_i.

Finally, the output of this subsection is a matrix of fuzzy possibilities of all basic events b_j, $j = 1, 2, \ldots, n$, which is denoted and defined in (18).

$$\begin{bmatrix} \tilde{p}_1^L & \tilde{p}_1^M & \tilde{p}_1^U \\ \tilde{p}_2^L & \tilde{p}_2^M & \tilde{p}_2^U \\ \vdots & \vdots & \vdots \\ \tilde{p}_n^L & \tilde{p}_n^M & \tilde{p}_n^U \end{bmatrix} \tag{18}$$

3.2 Evaluation of Best Estimate, Lower Bound, and Upper Bound Fuzzy Possibilities of Top Event by Using T_w-based Fuzzy Arithmetic Operations

After obtaining the fuzzy possibilities of all basic events, these values can be used to evaluate the top event fuzzy possibilities. The T_w-based fuzzy arithmetic operations are used to evaluate the fuzzy possibilities of the MCSs and the same for top event fuzzy possibilities.

A set of MCSs of a system fault can be defined as $S = \{C_i : \; i = 1, 2, \ldots, m\}$, where C_i is the i^{th} MCS of order k and defined as $C_i = b_1 . b_2 \ldots b_k$.

Let the fuzzy possibility $\tilde{p}_j^Z$ of event $b_j : \; j = 1, 2, \ldots, n$ be represented by TFN $\left(a_j^Z, \; \alpha_j^Z, \; \beta_j^Z\right)$, then the fuzzy possibility $\tilde{p}_{C_i}^Z$ of the MCS C_i is estimated using the following expressions:

$$\begin{aligned} \tilde{p}_{C_i}^Z &= AND_{fuzzy}\left(\tilde{p}_1^Z, \tilde{p}_2^Z, \ldots, \tilde{p}_k^Z\right) = \tilde{p}_1^Z \otimes \tilde{p}_2^Z \otimes \ldots \otimes \tilde{p}_k^Z \\ &= \left(\prod_{j=1}^{k} a_j^Z, \; \max_{1\le i\le k}\left(\alpha_i^Z \prod_{\substack{j=1\\ j\ne i}}^{k} a_j^Z\right), \; \max_{1\le i\le k}\left(\beta_i^Z \prod_{\substack{j=1\\ j\ne i}}^{k} a_j^Z\right)\right) \end{aligned} \tag{19}$$

$$\begin{aligned} \tilde{p}_{C_i}^Z &= OR_{Fuzzy}\left(\tilde{p}_1^Z, \tilde{p}_2^Z, \ldots, \tilde{p}_k^Z\right) = 1\Theta \left(1\Theta\, \tilde{p}_1^Z\right) \otimes \left(1\Theta\, \tilde{p}_2^Z\right) \otimes \ldots \otimes \left(1\Theta\, \tilde{p}_k^Z\right) \\ &= \left(1 - \prod_{j=1}^{k}\left(1 - a_j^Z\right), \; \max_{1\le i\le k}\left(\alpha_i^Z \prod_{\substack{j=1\\ j\ne i}}^{k}\left(1 - a_j^Z\right)\right), \; \max_{1\le i\le k}\left(\beta_i^Z \prod_{\substack{j=1\\ j\ne i}}^{k}\left(1 - a_j^Z\right)\right)\right), \end{aligned} \tag{20}$$

where superscript Z is a general identifier for superscript M, L, and U representing the best estimate, lower bound, and upper bound values respectively.

The generated matrix of fuzzy possibilities of all MCSs can be defined as follows:

$$\begin{bmatrix} \tilde{p}_{C_1}^L & \tilde{p}_{C_1}^M & \tilde{p}_{C_1}^U \\ \tilde{p}_{C_2}^L & \tilde{p}_{C_2}^M & \tilde{p}_{C_2}^U \\ \vdots & \vdots & \vdots \\ \tilde{p}_{C_m}^L & \tilde{p}_{C_m}^M & \tilde{p}_{C_m}^U \end{bmatrix} \tag{21}$$

Hence, the fuzzy possibility $\tilde{p}_T^Z$ of the top event can be calculated using the following equation:

$$\tilde{p}_T^Z = 1\Theta\left(1\Theta\, \tilde{p}_{C_1}^Z\right) \otimes \left(1\Theta\, \tilde{p}_{C_2}^Z\right) \otimes \ldots \otimes \left(1\Theta\, \tilde{p}_{C_m}^Z\right) \tag{22}$$

The set of fuzzy possibilities representing the lower bound, best estimate, and upper bound likelihood occurrences of the top event are as follows:

$$p_T = \left\{\tilde{p}_T^L, \; \tilde{p}_T^M, \; \tilde{p}_T^U\right\} \tag{23}$$

3.3 Defuzzify Top Event Fuzzy Possibilities

The process of converting fuzzy numbers into a single scalar quantity is called defuzzification. Purba et al. [35] proposed an area defuzzification technique (ADT) for nuclear power plant probabilistic safety assessment, which involves experts' qualitative judgments. The ADT for TFN $(m, \alpha_{\tilde{A}}, \beta_{\tilde{A}})$ can be defined as follows:

$$ADT(\tilde{A}) = \frac{1}{18}\left(6m - 4\alpha_{\tilde{A}} + \beta_{\tilde{A}}\right) \tag{24}$$

In this subsection, the top event fuzzy possibilities are defuzzified into a score using the ADT defined in (24). The lower bound $\tilde{p}_T^L$, best estimate $\tilde{p}_T^M$, and upper bound $\tilde{p}_T^U$ fuzzy probabilities given in (23) are converted to a lower bound score Sc^L, a best estimate score Sc^M, and an upper bound score Sc^U, respectively, as denoted in (25) using ADT.

$$Sc = \left\{ Sc^L,\ Sc^M,\ Sc^U \right\} \tag{25}$$

3.4 Convert Crisp Possibility Score Into Top Event Probability

This section generates a set of top event probabilities from the set of scores given in (28). Each member of the set in (26) is generated by inserting its corresponding score into a logarithmic function defined in (27) [30].

$$P = \left\{ P^L,\ P^M,\ P^U \right\} \tag{26}$$

$$\text{where} \quad P^Z = \begin{cases} \dfrac{1}{10^{\left(\left[\frac{1-Sc^Z}{Sc^Z}\right]^{\frac{1}{3}} \times 2.301\right)}}, & Sc^Z \neq 0 \\ 0, & Sc^Z = 0 \end{cases} \tag{27}$$

Superscript Z is a general identifier for superscript M, L, and U representing the best estimate, lower bound, and upper bound values respectively. From (26), it can be interpreted that the uncertainty range of the top event probability is between the lower bound probability and the upper bound probability. Meanwhile, the best estimate probability represents the most relevant reliability value of the system fault tree being evaluated.

4 Case Study

To show how the proposed weakest t-norm-based FPFTA can be used to quantify epistemic uncertainties in FTA, we use the case study of Group 1 of the U.S. Combustion Engineering Reactor Protection System (CERPS) [47]. The failure probability of Group 1 of the U.S. CERPS is evaluated using the proposed FPFTA, and obtained results are investigated to validate and confirm its feasibility.

4.1 System FTA

The U.S. CERPS is a complex control system comprising several components to produce a safe shutdown of the nuclear reactor during some sudden happening. The CERPS components can be roughly divided into four segments, i.e. four channels, six trip matrices, trip breakers/relays/contactors, and a group of control rods. Each channel includes bi-stables and instrumentation to measure plant parameters. Out of six trip matrices, one is sufficient to trip the reactor trip switchgear. The trip breakers/contactors remove the power to the control element assembly drive mechanisms to allow gravity to insert the control rod assembly into the reactor core and control rods de-energized on successful CERPS actuation. The U.S. CERPS immediately terminates nuclear reaction by inserting control rod clusters into the reactor core to eliminate heat generation. The integrity of the fuel and the reactor coolant pressure boundary is maintained with the help of other safety systems. In this study, it is not possible to show the fault tree of U.S. CERPS due to its complexity; it was studied in Ref. [47]. The simplified diagram of the U.S. CERPS Group 1 is shown in Figure 1. The basic events of U.S. CERPS fault tree are furnished in Table 1. The likelihood occurrences of those basic events will be evaluated by means of experts based totally on their qualitative failure possibilities. In Table 1, basic events with the same component types are identified by one identifier. For example, the trip units CE1-CBI-FF-PA, CE1-CBI-FF-PB, CE1-CBI-FF-PC, and CE1-CBI-FF-PD are implemented in four different channels A, B, C, and D, respectively. All these four basic events are given identifier b_1. This simplification is also carried out to basic events b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{16}, b_{17}, b_{21}, b_{22}, and b_{24}. The MCSs were also been developed to quantify the CERPS fault tree and shown in Table 2.

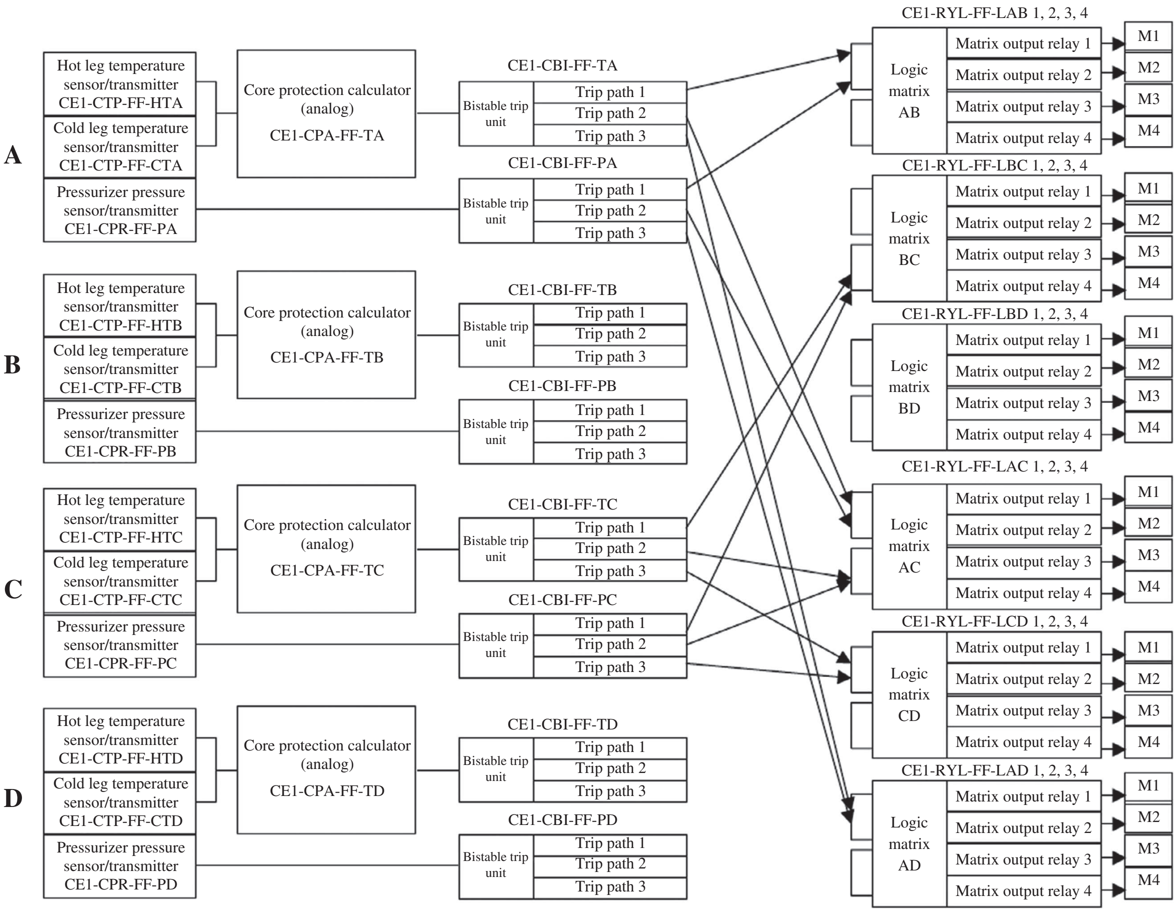

Figure 1: Simplified Diagram of the U.S. CERPS Group 1 [47].

4.2 Quantitative Evaluation of U.S. CERPS Group 1 Using Proposed TBFFTA

In this subsection, to illustrate the proposed TBFFTA to evaluate system reliability, TFNs were used to represent the occurrence possibilities of basic events. The fault tree of U.S. CERPS has 26 basic events, which are shown in Table 1. The fuzzy possibilities of these basic events $\{b_1, b_2, b_3, \ldots, b_{26}\}$ are required to be generated by using expert's linguistic opinions. For this evaluation, a group of six credible experts $\{E_1, E_2, E_3, E_4, E_5, E_6\}$ has been selected. In order to obtain the experts' opinions about the occurrence possibilities of the basic events as linguistic terms, seven levels of qualitative linguistic terms, i.e. very low (VL), low (L), reasonable low (RL), moderate (M), reasonable high (RH), high (H), and very high (VH), are defined and tabulated in Table 3. The weighting scores and weighting factors of the chosen experts are calculated by using Table 4 and tabulated in Table 5. The opinions of different experts are listed as linguistic terms in Table 6. The TFNs corresponding to linguistic terms are obtained using the methodology shown in Ref. [38] and tabulated in Table 7.

As different experts are chosen from different backgrounds and experiences, they may have widely varying opinions from one basic event to another. These variations are accounted by the weighting process so there is no need to discount particular values. It is therefore necessary to aggregate the results to obtain an agreement among the conflicted views of the experts. Using the methodology shown in Section 3.1.2, the experts' opinions are aggregated to obtain a single consensus about the occurrence possibilities of basic events. These aggregated results are shown in Table 8.

As the fuzzy possibilities of the basic events are obtained in the triangular fuzzy form, therefore, the MCSs can be quantified in triangular fuzzy form using the fuzzy possibility values from Table 8 and the fuzzy operators defined in Section 3.2. The results of quantification of the MCSs have been tabulated in Table 9.

Table 1: Basic Events of the U.S. CERPS Fault Tree [47].

Id.	Name	Description
b_1	CE1-CBI-FF-PA,B,C,D	Channel trip unit (bi-stable) fails to trip at its pressure set point
b_2	CE1-CBI-FF-TA,B,C,D	Channel trip unit (bi-stable) fails to trip at its temperature set point
b_3	CE1-CPA-FF-TA,B,C,D	Channel analog core protection calculator fails to send a signal to the trip unit
b_4	CE1-CPR-FF-PA,B,C,D	Channel reactor vessel pressure sensor/transmitter fails to detect a high pressure and sends a signal to the trip unit
b_5	CE1-CTP-FF-C(H)TA,B,C,D	Channel reactor vessel temperature/transmitter (cold or hot leg) fails to detect a low level and sends a signal to the trip unit
b_6	CE1-MSW-FF-MT1,2	Manual scram switch fails to operate upon demand
b_7	CE1-RYL-FF-LA,B,C,D–1,2,3,4	Channel logic relay fails to de-energize upon demand
b_8	CE1-CBI-CF-P(T)2OF3TM	Common cause failure specific 2 of 3 bi-stables associated with either a pressure (P) or temperature (T) signal (T&M)
b_9	CE1-CBI-CF-P(T)3OF4	Common cause failure specific 3 of 4 bi-stables associated with either a pressure (P) or temperature (T) signal
b_{10}	CE1-CBI-CF-4OF6TM	Common cause failure specific 4 of 6 bi-stables (T&M)
b_{11}	CE1-CBI-CF-6OF8	Common cause failure specific 6 of 8 bi-stables
b_{12}	CE1-CPA-CF-T2OF3TM	Common cause failure 2 of 3 analog core protection calculators (T&M)
b_{13}	CE1-CPA-CF-T3OF4	Common cause failure 3 of 4 analog core protection calculators
b_{14}	CE1-CPR-CF-P2OF3TM	Common cause failure 2 of 3 pressure sensor/transmitters (T&M)
b_{15}	CE1-CPR-CF-P3OF4	Common cause failure 3 of 4 pressure sensor/transmitters
b_{16}	CE1-CTP-CF-C(H)T2OF3TM	Common cause failure 2 of 3 temperature sensor/transmitters (T&M)
b_{17}	CE1-CTP-CF-C(H)T3OF4	Common cause failure 3 of 4 temperature sensor/transmitters
b_{18}	CE1-ROD-CF-RODS	Common cause failure 20% or more CRD/rods fail to insert
b_{19}	CE1-RYL-CF-LM6OF12TM	Common cause failure specific 6 of 12 logic relays (T&M)
b_{20}	CE1-RYL-CF-LM12OF24	Common cause failure specific 12 of 24 logic relays
b_{21}	CE1-RYL-CF-1,2,3,4LM3OF3TM	Common cause failure 3 of 3 logic relays (T&M)
b_{22}	CE1-RYL-CF-1,2,3,4LM6OF6	Common cause failure 6 of 6 logic relays
b_{23}	CE1-RYT-CF-TR2OF4	Common cause failure 2 of 4 trip relays
b_{24}	CE1-RYT-FF-ICM1,2,3,4	Trip system trip relay fails to de-energize upon demand
b_{25}	/CE1-RPS-TM-CHA	Channel A through D bypassed because of testing or maintenance
b_{26}	CE1-RPS-TM-CHA	Complement of /CE1-RPS-TM-CHA

Using equation (22) and the fuzzy possibilities of the MCSs from Table 9, the lower bound, best estimate, and upper bound fuzzy possibilities of the top event (U.S. CERPS fault) are computed, which are also TFNs. The set of computed top event fuzzy possibilities representing the lower bound, the best estimate, and the upper bound fuzzy possibilities of the U.S. CERPS fault tree is given by (28).

$$p_T = \{(0.12604, 0.16100, 0.19595), (0.45402, 0.48220, 0.51038), (0.78792, 0.80160, 0.81528)\} \quad (28)$$

The obtained lower bound, best estimate, and upper bound top event fuzzy possibilities can be mapped to a crisp score by applying the ADT defined in (24). A best estimate score, a lower bound score, and an upper bound score calculated by ADT are given, respectively, as follows:

$$Sc^L = \frac{1}{18}(4 \times 0.12604 + 0.16100 + 0.19595) = 0.04784$$

$$Sc^M = \frac{1}{18}(4 \times 0.45402 + 0.48220 + 0.51038) = 0.15604$$

$$Sc^U = \frac{1}{18}(4 \times 0.78792 + 0.80160 + 0.81528) = 0.26492$$

Therefore, the set of these three scores of the U.S. CERPS fault tree is given by (29).

$$Sc = \{0.04784, 0.15604, 0.26492\} \quad (29)$$

Table 2: The MCSs of the CERPS Group 1 Fault Tree [47].

MCS Id	MCSs
mcs_1	CE1-RYT-CF-TR2OF4
mcs_2	CE1-ROD-CF-RODS
mcs_3	CE1-CBI-CF-6OF8 * /CE1-RPS-TM-CHA
mcs_4	/CE1-RPS-TM-CHA * CE1-RYL-CF-LM12OF24
mcs_5	CE1-CBI-CF-4OF6TM * CE1-RPS-TM-CHA
mcs_6	CE1-RYT-FF-ICM1 * CE1-RYT-FF-ICM2
mcs_7	CE1-RYT-FF-ICM3 * CE1-RYT-FF-ICM4
mcs_8	CE1-RPS-TM-CHA * CE1-RYL-CF-LM6OF12TM
mcs_9	CE1-CBI-CF-P3OF4 * CE1-CPA-CF-T3OF4 * /CE1-RPS-TM-CHA
mcs_{10}	CE1-CPA-CF-T3OF4 * CE1-CPR-CF-P3OF4 * /CE1-RPS-TM-CHA
mcs_{11}	CE1-CBI-CF-P3OF4 * CE1-CTP-CF-CT3OF4 * /CE1-RPS-TM-CHA
mcs_{12}	CE1-CBI-CF-P3OF4 * CE1-CTP-CF-HT3OF4 * /CE1-RPS-TM-CHA
mcs_{13}	CE1-CBI-CF-P3OF4 * CE1-CBI-CF-T3OF4 * /CE1-RPS-TM-CHA
mcs_{14}	CE1-CPA-CF-T2OF3TM * CE1-CPR-CF-P2OF3TM * CE1-RPS-TM-CHA
mcs_{15}	/CE1-RPS-TM-CHA * CE1-RYL-CF-1LM6OF6 * CE1-RYT-FF-ICM2
mcs_{16}	/CE1-RPS-TM-CHA * CE1-RYL-CF-2LM6OF6 * CE1-RYT-FF-ICM1
mcs_{17}	/CE1-RPS-TM-CHA * CE1-RYL-CF-3LM6OF6 * CE1-RYT-FF-ICM4
mcs_{18}	/CE1-RPS-TM-CHA * CE1-RYL-CF-4LM6OF6 * CE1-RYT-FF-ICM3
mcs_{19}	CE1-CPR-CF-P3OF4 * CE1-CTP-CF-CT3OF4 * /CE1-RPS-TM-CHA
mcs_{20}	CE1-CPR-CF-P3OF4 * CE1-CTP-CF-HT3OF4 * /CE1-RPS-TM-CHA
mcs_{21}	CE1-CBI-CF-T3OF4 * CE1-CPR-CF-P3OF4 * /CE1-RPS-TM-CHA
mcs_{22}	CE1-CPA-FF-TB * CE1-CPA-FF-TC * CE1-CPR-CF-P2OF3TM * CE1-RPS-TM-CHA
mcs_{23}	CE1-CPA-FF-TB * CE1-CPA-FF-TD * CE1-CPR-CF-P2OF3TM * CE1-RPS-TM-CHA
mcs_{24}	CE1-CPA-FF-TC * CE1-CPA-FF-TD * CE1-CPR-CF-P2OF3TM * CE1-RPS-TM-CHA

Table 3: The Failure Possibilities/Linguistic terms and Their Likelihood Values [36, 37].

A set of seven failure possibilities	Likelihood occurrences
Very low (VL)	$<1.0E-08$
Low (L)	$1.0E-08$ – $1.0E-07$
Reasonable low (RL)	$1.0E-07$ – $1.0E-06$
Moderate (M)	$1.0E-06$ – $1.0E-05$
Reasonable high (RH)	$1.0E-05$ – $1.0E-04$
High (H)	$1.0E-04$ – $1.0E-03$
Very high (VH)	$1.0E-03$

Table 4: Weighting scores for different experts [38].

Constitution	Classification	Score
Professional position	Professor, GM/DGM, chief engineer, director	5
	Assistant professor, manager, factory inspector	4
	Engineer, supervisors	3
	Foreman, technician, graduate apprentice	2
	Operator	1
Professional experience (years)	≥ 20	5
	15 to 19	4
	10 to 14	3
	5 to 9	2
	<5	1
Educational or technical qualification	PhD or M. Tech.	5
	MSc or B. Tech.	4
	Diploma or BSc	3
	ITI	2
	Secondary school	1

Table 5: Weighting Factors for Six Experts.

Expert	Professional position	Experience (years)	Educational qualification	Weighting score	Weighting factor
E_1	Professor	≥ 20	PhD	15	0.241935
E_2	Assistant Professor	10 to 14	PhD	12	0.193548
E_3	Engineer	5 to 9	M.Tech	10	0.161290
E_4	Manager	15 to 19	M.Sc	12	0.193548
E_5	Operator	<5	Diploma	5	0.080645
E_6	Technician	5 to 9	B.Tech	8	0.129032

Table 6: Expert Opinions on the Basic Events of the CERPS Group 1 Fault Tree [36, 37].

Basic events	E_1	E_2	E_3	E_4	E_5	E_6
b_1	RL	M	RL	M	RL	M
b_2	RL	RL	RL	L	L	L
b_3	VL	L	RL	L	RL	VL
b_4	M	M	L	RL	RL	M
b_5	M	M	RL	RL	M	M
b_6	M	RL	M	M	L	L
b_7	RL	M	M	RL	M	RL
b_8	RL	RL	RL	L	RL	M
b_9	VL	RL	L	RL	L	L
b_{10}	L	L	L	VL	VL	L
b_{11}	L	VL	L	VL	L	L
b_{12}	L	RL	L	RL	L	L
b_{13}	M	M	M	M	RL	L
b_{14}	L	RL	VL	L	L	VL
b_{15}	L	VL	L	L	L	VL
b_{16}	RL	L	M	L	M	RL
b_{17}	L	L	RL	L	RL	M
b_{18}	VL	L	VL	VL	VL	L
b_{19}	L	VL	L	VL	L	VL
b_{20}	L	VL	L	VL	VL	L
b_{21}	L	VL	L	VL	L	L
b_{22}	L	L	VL	L	L	L
b_{23}	L	VL	L	VL	L	VL
b_{24}	RL	L	RL	L	L	RL
b_{25}	M	RH	M	RL	M	RH

Table 7: Linguistic terms with Conversion Scales [36, 37].

Linguistic variables	Triangular fuzzy numbers	LR-form of triangular fuzzy numbers (m, αA)
Very low (VL)	(0.00, 0.04, 0.08)	(0.04, 0.04)
Low (L)	(0.07, 0.13, 0.19)	(0.13, 0.06)
Reasonable low (RL)	(0.17, 0.27, 0.37)	(0.27, 0.1)
Moderate (M)	(0.35, 0.50, 0.65)	(0.5, 0.15)
Reasonable high (RH)	(0.63, 0.73, 0.83)	(0.73, 0.1)
High (H)	(0.81, 0.87, 0.93)	(0.87, 0.06)
Very high (VH)	(0.92, 0.96, 1.00)	(0.96, 0.04)

The calculated scores are substituted into (27) to get the lower bound probability, the best estimate probability, and the upper bound probability of the top event, respectively, which are listed in (30).

$$P = \{5.81\mathrm{E}{-07},\ \ 9.14\mathrm{E}{-05}, 5.84\mathrm{E}{-04}\} \tag{30}$$

Table 8: The Best Estimate, Lower Bound, and Upper Bound Fuzzy Possibilities of Basic Events by Aggregation of Expert Opinions.

Basic event	Aggregated fuzzy failure possibilities (m, αA) of basic events		
	Lower bound	Best estimate	Upper bound
b_1	(0.27, 0.1)	(0.38685, 0.12540)	(0.5, 0.15)
b_2	(0.13, 0.06)	(0.20677, 0.08194)	(0.27, 0.1)
b_3	(0.04, 0.04)	(0.14093, 0.06499)	(0.27, 0.1)
b_4	(0.13, 0.06)	(0.38098, 0.12289)	(0.5, 0.15)
b_5	(0.27, 0.1)	(0.42709, 0.13415)	(0.5, 0.15)
b_6	(0.13, 0.06)	(0.36933, 0.11926)	(0.5, 0.15)
b_7	(0.27, 0.1)	(0.37758, 0.12339)	(0.5, 0.15)
b_8	(0.13, 0.06)	(0.27735, 0.10017)	(0.5, 0.15)
b_9	(0.04, 0.04)	(0.16521, 0.07097)	(0.27, 0.1)
b_{10}	(0.04, 0.04)	(0.10705, 0.05490)	(0.13, 0.06)
b_{11}	(0.04, 0.04)	(0.10197, 0.05377)	(0.13, 0.06)
b_{12}	(0.13, 0.06)	(0.17597, 0.07313)	(0.27, 0.1)
b_{13}	(0.13, 0.06)	(0.43663, 0.13523)	(0.5, 0.15)
b_{14}	(0.04, 0.04)	(0.12623, 0.06053)	(0.27, 0.1)
b_{15}	(0.04, 0.04)	(0.10487, 0.05442)	(0.13, 0.06)
b_{16}	(0.13, 0.06)	(0.28613, 0.10019)	(0.5, 0.15)
b_{17}	(0.13, 0.06)	(0.21435, 0.08225)	(0.5, 0.15)
b_{18}	(0.04, 0.04)	(0.06513, 0.04558)	(0.13, 0.06)
b_{19}	(0.04, 0.04)	(0.08427, 0.04984)	(0.13, 0.06)
b_{20}	(0.04, 0.04)	(0.08645, 0.05032)	(0.13, 0.06)
b_{21}	(0.04, 0.04)	(0.10197, 0.05377)	(0.13, 0.06)
b_{22}	(0.04, 0.04)	(0.11974, 0.05772)	(0.13, 0.06)
b_{23}	(0.04, 0.04)	(0.08427, 0.04984)	(0.13, 0.06)
b_{24}	(0.13, 0.06)	(0.20226, 0.08065)	(0.27, 0.1)
b_{25}	(0.27, 0.1)	(0.53950, 0.12610)	(0.73, 0.1)
b_{26}	(0.27, 0.1)	(0.46050, 0.12610)	(0.73, 0.1)

Table 9: The Best Estimate, Lower Bound, and Upper Bound Fuzzy Possibilities of the MCSs of the CERPS Group 1 Fault Tree.

MCSs	The fuzzy possibilities (m, αA) of the MCSs		
	Lower bound	Best estimate	Upper bound
mcs_1	(0.04, 0.04)	(0.08427, 0.04984)	(0.13, 0.06)
mcs_2	(0.04, 0.04)	(0.06513, 0.04558)	(0.13, 0.06)
mcs_3	(0.0108, 0.0108)	(0.05501, 0.02901)	(0.0949, 0.0438)
mcs_4	(0.0108, 0.0108)	(0.04664, 0.02715)	(0.0949, 0.0438)
mcs_5	(0.0108, 0.0108)	(0.04930, 0.02528)	(0.0949, 0.0438)
mcs_6	(0.0169, 0.0078)	(0.04091, 0.01631)	(0.0729, 0.027)
mcs_7	(0.0169, 0.0078)	(0.04091, 0.01631)	(0.0729, 0.027)
mcs_8	(0.0108, 0.0108)	(0.03881, 0.02295)	(0.0949, 0.0438)
mcs_9	(0.0014, 0.0014)	(0.03892, 0.01672)	(0.09855, 0.0365)
mcs_{10}	(0.0014, 0.0014)	(0.02470, 0.01282)	(0.04745, 0.0219)
mcs_{11}	(0.0014, 0.0014)	(0.01910, 0.00821)	(0.09855, 0.0365)
mcs_{12}	(0.0014, 0.0014)	(0.01910, 0.00821)	(0.09855, 0.0365)
mcs_{13}	(0.00043, 0.00043)	(0.01472, 0.00633)	(0.05322, 0.01971)
mcs_{14}	(0.0014, 0.001404)	(0.01663, 0.00798)	(0.09855, 0.0365)
mcs_{15}	(0.0014, 0.001404)	(0.01307, 0.00630)	(0.02562, 0.01183)
mcs_{16}	(0.0014, 0.001404)	(0.01307, 0.00630)	(0.02562, 0.01183)
mcs_{17}	(0.0014, 0.001404)	(0.01307, 0.00630)	(0.02562, 0.01183)
mcs_{18}	(0.0014, 0.001404)	(0.01307, 0.00630)	(0.02562, 0.01183)
mcs_{19}	(0.0014, 0.001404)	(0.01213, 0.00629)	(0.04745, 0.02190)
mcs_{20}	(0.0014, 0.001404)	(0.01213, 0.00629)	(0.04745, 0.02190)
mcs_{21}	(0.00043, 0.00043)	(0.00935, 0.00485)	(0.02562, 0.01183)
mcs_{22}	(0.00002, 0.00002)	(0.00115, 0.00055)	(0.01437, 0.00532)
mcs_{23}	(0.00002, 0.00002)	(0.00115, 0.00055)	(0.01437, 0.00532)
mcs_{24}	(0.00002, 0.00002)	(0.00115, 0.00055)	(0.01437, 0.00532)

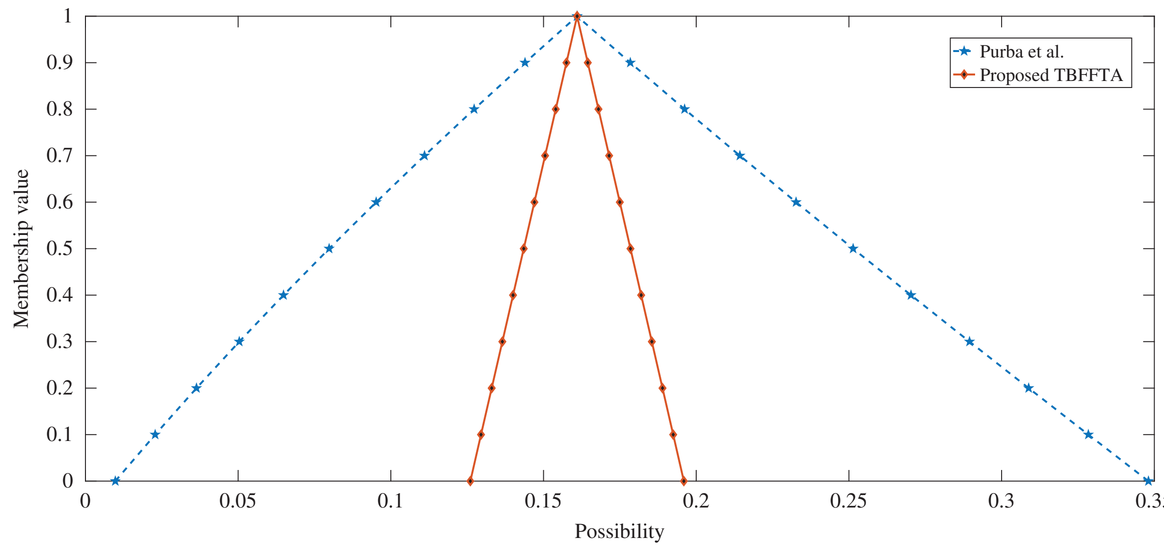

Figure 2: The Lower Bound Fuzzy Possibility of the Top Event.

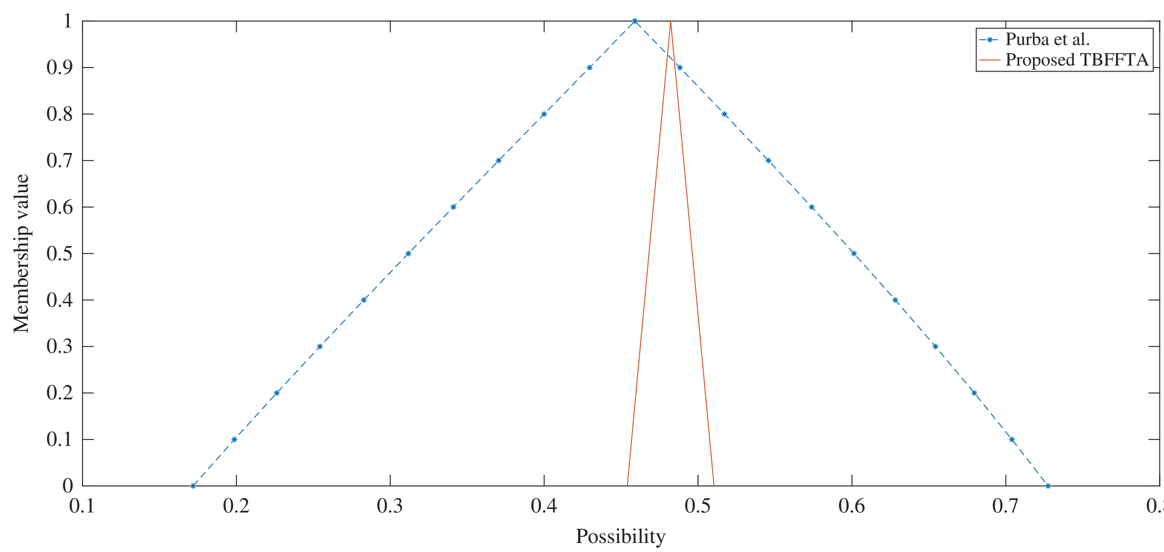

Figure 3: The Best Estimate Fuzzy Possibility of the Top Event.

The set of top probabilities means that the failure probability of the U.S. CERPS lies between 5.81E−07 and 5.84E−04. Also, in this set, the best estimate probability that describes the most likely failure probability of the U.S. CERPS is 9.14E−05.

4.3 Discussions and comparison

– *The proposed approach preserves the shape of fuzzy numbers, i.e. gives exact solutions for fuzzy possibilities of the top event.*
  Purba et al. [36] used simple fuzzy arithmetic operations on TFNs in FPFTA and got approximate solutions for fuzzy failure possibilities as approximated TFNs. To get the exact solution, this study applies T_W-based fuzzy arithmetic operations and evaluates the lower bound, the best estimate, and the upper bound of top event fuzzy possibility as TFNs. The proposed TBFFTA preserves the shape of TFN for top event fuzzy possibilities. The results are plotted in Figures 2–4. Shape-preserving property provides a good possibility of controlling the growth of uncertainty during calculations.

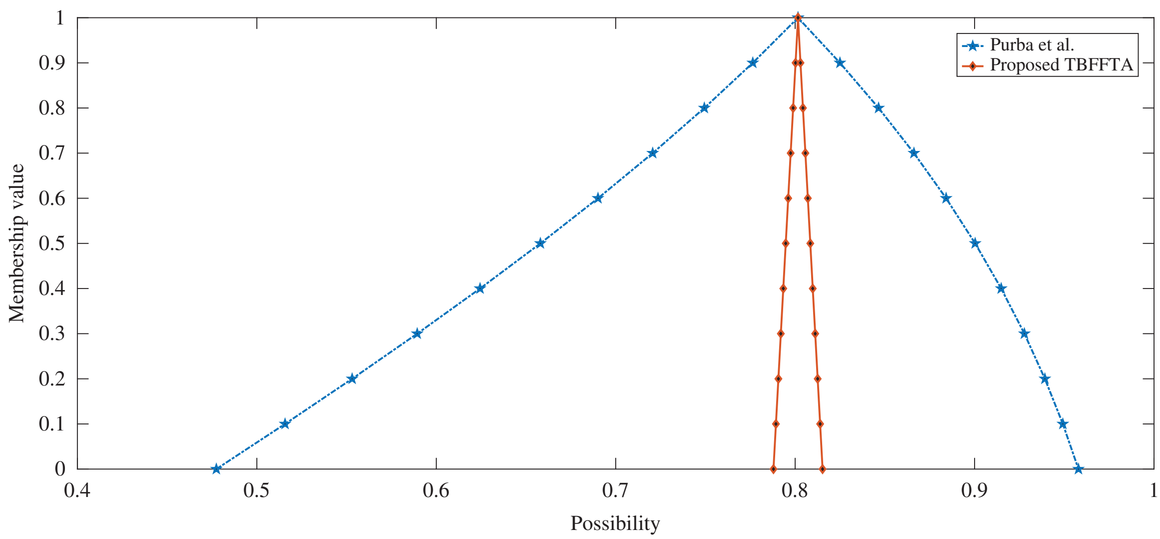

Figure 4: The Upper Bound Fuzzy Possibility of the Top Event.

Table 10: The Best Estimate, Lower Bound, and Upper Bound Fuzzy Possibilities of Top Event CERPS Group 1 Fault Computed by Different Approaches.

Approaches	**Fuzzy possibilities of top event (m − α_A, m, m + β_A)**		
	Lower bound	**Best estimate**	**Upper bound**
Purba et al. method [36]	(0.0098, 0.1770, 0.3811)	(0.1760, 0.4710, 0.7427)	(0.4562, 0.7910, 0.9560)
Proposed method	(0.12604, 0.16100, 0.19595)	(0.45402 , 0.48220, 0.51038)	(0.78792, 0.80160, 0.81528)

– *The proposed approach gives results with reduced range of prediction of top event fuzzy failure possibilities.* The proposed TBFFTA gives reduced range of prediction of the lower bound, the best estimate, and the upper bound of top event fuzzy possibility in comparison to existing FPFTA, i.e. fuzzy spreads (uncertainty interval) are reduced [36, 37]. The results are tabulated in Table 10 and plotted in Figures 2–4.

5 Conclusion

In this research, a new TBFFTA has been proposed to quantify the epistemic uncertainty that occurred in basic event reliability evaluations. The main advantage of the proposed approach is that using the proposed TBFFTA, uncertainty range decreases, i.e. fuzzy spreads are reduced and the obtained results are more exact, while using the existing FPFTA, the results are approximately due to the approximate product of TFNs. The failure probability of the U.S. CERPS Group 1 has been evaluated to demonstrate the performance of the proposed TBFFTA. The results confirm that the proposed approach can be feasible and more effective to quantify epistemic uncertainties in FTA when basic events do not have probability distributions and described in fuzzy probabilities.

In the future, this research work still needs to extend by looking at how the system unreliability estimated by the proposed approach be affected by the different choices of membership functions, weighting scores, expert opinions, etc.

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