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# Gray Method for Multiple Attribute Decision Making with Incomplete Weight Information under the Pythagorean Fuzzy Setting

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**Abstract:** In this study, we developed an approach to investigate multiple attribute group decision-making (MAGDM) problems, in which the attribute values take the form of Pythagorean fuzzy numbers whose information about attribute weights is incompletely known. First, the Pythagorean fuzzy Choquet integral geometric operator is utilized to aggregate the given decision information to obtain the overall preference value of each alternative by experts. In order to obtain the weight vector of the criteria, an optimization model based on the basic ideal of the traditional gray relational analysis method is established, and the calculation steps for solving Pythagorean fuzzy MAGDM problems with incompletely known weight information are given. The degree of gray relation between every alternative and positive-ideal solution and negative-ideal solution is calculated. Then, a relative relational degree is defined to determine the ranking order of all alternatives by calculating the degree of gray relation to both the positive-ideal solution and negative-ideal solution simultaneously. Finally, an illustrative example is given to verify the developed approach and to demonstrate its practicality and effectiveness.

**Keywords:** Multiple attribute decision making, gray relational analysis (GRA), Pythagorean fuzzy numbers, incomplete weight information.

# 1 Introduction

To address the issue of difficulties of acquiring sufficient and accurate data for real decision making due to the imprecision and ambiguity of socio-economics, Zadeh introduced the concept of the fuzzy set [39], and it has been used in a wide range of scientific fields. Vahdani et al. [32] proposed an integrated model based on a compromised solution method to solve fuzzy belief multi-objective large-scale non-linear programming problem with block angular structure. The authors developed a new method to transfer each belief decision-making problem into some fuzzy problems. Mojtahedi et al. [24] developed a systematic decision process for identifying and analyzing risks concurrently by applying multiple attribute group decision making (MAGDM) in a fuzzy environment. Because selecting a transport project to invest is an important task, Mohagheghi et al. [22] developed a sustainable transport investment selection method under an interval-valued fuzzy sets setting to address uncertainty. To address the importance of criteria, the authors proposed relative preference relation. Vahdani et al. [31] developed a compromise model to solve the multi-objective large-scale linear programming problems with block angular structure involving fuzzy parameters. After the appearance of a fuzzy set, a series of extensions both in theoretical and practical areas have been presented. Among these extensions, intuitionistic fuzzy set (IFS) [1], initiated by Atanassov, seems to be the most reasonable and acceptable. IFS is a powerful tool to deal with the imprecision and ambiguity of things more comprehensively.

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Since the establishment of IFS, it has been successfully applied in many areas of decision-making problems [2, 3, 17, 29, 33, 34]. Zhang et al. [44] developed a gray relational projection method for multi-attribute decision making (MADM) based on intuitionistic trapezoidal fuzzy number. Liu and Oin [18] developed the linguistic intuitionistic fuzzy Maclaurin symmetric mean operator, weighted linguistic intuitionistic fuzzy Maclaurin symmetric mean operator, linguistic intuitionistic fuzzy dual Maclaurin symmetric mean operator, and weighted linguistic intuitionistic fuzzy dual Maclaurin symmetric mean operator. Ebrahimnejad et al. [8] developed a multi-criteria decision-making (MCDM) model under an interval-valued intuitionistic fuzzy environment to select the best outsourcing provider. Foroozesh et al. [9] proposed the VIKOR (vlsekriterijumska optimizacija i kompromisno resenje, in Serbian) method under a hesitant fuzzy set environment by utilizing the weights of decision makers and the distance measure along with a new index for the ranking process of compromise solution procedure. To decrease the loss for industrial selection problems, Gitinavard et al. [12] developed a soft computing approach based on interval-valued hesitant fuzzy (IVHF) complex proportional assessments and IVHF compromise solution methods with last aggregation. Mousavi et al. [25] purposed a modified compromise ranking method (VIKOR), known as sorting the possible alternatives and determining the compromise solution under IVHF sets for solving group decision-making problems in manufacturing systems. However, in some practical decision-making process, the sum of the membership degree and the non-membership degree to which an alternative satisfying a criterion provided by a decision maker may be >1, but their square sum is  $\le 1$ . To overcome this situation, Yager [35, 36] initiated the concept of Pythagorean fuzzy set (PFS) as a generalization of IFS and characterized by a membership degree and a non-membership degree, which satisfies the condition that their square sum is <1. Zhang and Xu [43] established an extension of TOPSIS to MADM with PFS information. The error to the proof-of-distance measure in Zhang and Xu [43] has been pointed out by Yang et al. [38]. For MADM problems under a Pythagorean fuzzy environment, Yager and Abbasov [37] developed a series of aggregation operators. Peng and Yang [26] discussed their relationship among these aggregation operators and established the superiority and inferiority ranking MAGDM method. Using Einstein operation, Garg [10] generalized Pythagorean fuzzy information aggregation. Gou et al. [13] studied several Pythagorean fuzzy functions and investigated their fundamental properties, such as continuity, derivability, and differentiability, in detail. Zhang [42] put forward a hierarchical qualitative flexible (QUALIFLEX) multi-criteria approach with the closeness index-based ranking methods for multi-criteria Pythagorean fuzzy decision analysis. Zeng et al. [41] explored a hybrid method for Pythagorean fuzzy MCDM. Ren et al. [28] extended an acronym in Portuguese for Interactive Multi-criteria Decision-Making approach to solve the MCDM problems with Pythagorean fuzzy information. Ma and Xu [21] proposed symmetric Pythagorean fuzzy weighted geometric/averaging operators, and investigated the relationships among these operators and those existing ones. Peng and Yang [27] presented a new Pythagorean fuzzy Choquet integral-based multi-attribute border approximation area comparison method for MCGDM. Garg [11] developed the confidence Pythagorean fuzzy weighted averaging operator, confidence Pythagorean fuzzy weighted ordered averaging operator, confidence Pythagorean fuzzy weighted geometric operator, and confidence Pythagorean fuzzy weighted ordered geometric operator, along with their desired properties. Zeng [40] proposed the Pythagorean fuzzy probabilistic ordered weighted average operator and applied it to the MAGDM problem. Liu et al. [20] developed the Pythagorean uncertain linguistic Bonferroni mean operator, as well as the Pythagorean uncertain linguistic partitioned Bonferroni mean operator and its weighted form to solve the MADM problem. Lui et al. [20] proposed the Pythagorean fuzzy uncertain linguistic prioritized weighted averaging aggregation operator and the Pythagorean fuzzy uncertain linguistic prioritized weighted geometric aggregation operators. Mohagheghi et al. [23] developed a last aggregation group decision-making process for weighting and evaluating PFS. The authors developed a method in computing the weight of decision makers. The gray relational analysis (GRA) method was first proposed by Deng [5, 6], and was successfully applied to solve MCDM problems. Khan et al. [15] developed the interval-valued Pythagorean fuzzy TOPSIS based on the Choquet integral. Khan and Abdullah [14] proposed the interval-valued Pythagorean fuzzy GRA method for MADM problems.

In dealing with MADM, there may be a situation where the decision maker may provide the degree of membership and non-membership of a particular attribute in such a way that their sum is >1; then, it is better to use Pythagorean fuzzy information. PFSs in comparison with IFSs provide more flexibility and

power in expressing uncertainty. This enhancement in flexibility and power to express membership degree, non-membership degree, and hesitancy is caused by improving the space in which those degrees can be expressed. Sometimes, the attribute values take the form of Pythagorean fuzzy information and the information about attribute weights is incompletely known because of time stress, lack of data or knowledge, and the expert's limited knowledge about the problem domain. The traditional GRA method [5, 6] and intuitionistic fuzzy GRA method [33, 34] will fail in dealing with the above Pythagorean fuzzy MADM problems with incomplete weight information. How to derive the attribute weights from both the given Pythagorean fuzzy information and incompletely known attribute weight information based on the basic ideal of the traditional GRA method is an interesting and important research topic. Therefore, it is essential to concentrate on this problem. The purpose of this paper is to extend the concept of GRA to develop a methodology for solving MADM problems under a Pythagorean fuzzy environment in which the information about attribute weights is incompletely known and the attribute values take the form of Pythagorean fuzzy numbers (PFNs). To do this, the remainder of the paper is structured as follows.

In Section 2, we briefly review some basic definitions and results about Choquet integral and PFSs. In Section 3, we introduce the GRA method for Pythagorean fuzzy MADM problems with incomplete weight information. In Section 4, we illustrate our proposed algorithmic method with an example. The conclusion is in Section 5.

# 2 Preliminaries

In this section, fuzzy measure, the Choquet integral, and the definition of PFSs are reviewed. Some operations and comparison laws of PFSs, which will be utilized in the latter analysis, are also presented.

## 2.1 Fuzzy Measure and Choquet Integral

In 1974, Sugeno [30] introduced the concept of fuzzy measure (non-additive measure), which only makes a monotonicity instead of being an additivity property. For MADM problems, it does not need assumption that criteria or preferences are independent of one another, and is used as a powerful tool for modeling interaction phenomena in decision making. In the Choquet integral model [4, 7], where criteria can be dependent, a fuzzy measure is used to define a weight on each combination of criteria, thus making it possible to model the interaction existing among criteria. In this subsection, definitions of fuzzy measure,  $\lambda$ -fuzzy measure, discrete Choquet integral, and Pythagorean fuzzy Choquet integral operators are presented.

**Definition 1** ([4]): Let  $X = \{x_1, x_2, \dots, x_n\}$  be a universe of discourse and P(X) be the power set of X. A fuzzy measure  $\mu$  on X is a set function  $\mu$  :  $P(X) \rightarrow [0, 1]$ , satisfying the following conditions:

- (1)  $\mu(\phi) = 0, \mu(X) = 1.$
- (2) If A,  $B \in P(X)$  and  $A \subseteq B$ , then  $\mu(A) \le \mu(B)$ .

Even though it is necessary to add the axiom of continuity when X is infinite, it is enough to consider a finite universal set in actual practice.  $\mu(\{x_1, x_2, \dots, x_n\})$  can be considered as the grade of subjective importance of decision attribute set  $\{x_1, x_2, \dots, x_n\}$ . Thus, with the separate weights of attributes, weights of any combination of attributes can also be defined. Intuitively, we could say the following about any pair of criteria sets  $A, B \in P(X)$ ,  $A \cap B = \phi$ : A and B are considered to be without interaction (or to be independent) if

$$\mu(A \cup B) = \mu(A) + \mu(B),\tag{1}$$

which is called an additive measure. *A* and *B* exhibit a positive synergetic interaction between them (or are complementary) if

$$\mu(A \cup B) > \mu(A) + \mu(B), \tag{2}$$

which is called a super-additive measure. A and B exhibit a negative synergetic interaction between them (or are redundant or substitutive) if

$$\mu(A \cup B) < \mu(A) + \mu(B), \tag{3}$$

which is called a sub-additive measure.

It is difficult to determine the fuzzy measure according to Definition 1; therefore, to confirm a fuzzy measure in MAGDM problems, Sugeno [30] presented the following  $\lambda$ -fuzzy measure:

$$\mu(A \cup B) = \mu(A) + \mu(B) + \lambda \mu(A)\mu(B), \tag{4}$$

 $\lambda \in [-1, \infty), A \cap B = \phi$ . The parameter  $\lambda$  determines interaction between the attributes. In Eq. (4), if  $\lambda = 0$ , the  $\lambda$ -fuzzy measure reduces to simply an additive measure. For negative and positive  $\lambda$ , the  $\lambda$ -fuzzy measure reduces to sub-additive and super-additive measures, respectively. Meanwhile, if all the elements in X are independent, we have

$$\mu(A) = \sum_{i=1}^{n} \mu(\{x_i\}). \tag{5}$$

If *X* is a finite set, then  $\bigcup_{i=1}^{n} \{x_i\} = X$ . The  $\lambda$ -fuzzy measure  $\mu$  satisfies Eq. (6):

$$\mu(X) = \mu(\bigcup_{i=1}^{n} x_i) = \begin{cases} \frac{1}{\lambda} \left( \prod_{i=1}^{n} [1 + \lambda \mu(x_i)] - 1 \right) & \text{if } \lambda \neq 0 \\ \sum_{i=1}^{n} \mu(x_i) & \text{if } \lambda = 0 \end{cases}$$
(6)

where  $x_i \cap x_i = \phi$  for all i, j = 1, 2, ..., n and  $i \neq j$ . It can be noted that  $\mu(x_i)$  for a subset with a single element  $x_i$  is called a fuzzy density and can be denoted as  $\mu_i = \mu(x_i)$ .

Especially for every subset  $A \in P(X)$ , we have

$$\mu(A) = \begin{cases} \frac{1}{\lambda} \left( \prod_{i=1}^{n} [1 + \lambda \mu(x_i)] - 1 \right) & \text{if } \lambda \neq 0 \\ \sum_{i=1}^{n} \mu(x_i) & \text{if } \lambda = 0 \end{cases}$$
 (7)

Based on Eq. (2), the value of  $\lambda$  can be uniquely determined from  $\mu(X) = 1$ , which is equivalent to solving

$$\lambda + 1 = \prod_{i=1}^{n} [1 + \lambda \mu_i]. \tag{8}$$

It should be noted that  $\lambda$  can be uniquely determined by  $\mu(X) = 1$ .

**Definition 2** ([30]): Let f be a positive real-valued function on X and  $\mu$  be a fuzzy measure on X. The discrete Choquet integral of f with respect to  $\mu$  is defined by

$$C_{\mu}(f) = \sum_{i=1}^{n} f_{\sigma(i)}[\mu(A_{\sigma(i)}) - \mu(A_{\sigma(i-1)})], \tag{9}$$

where  $\sigma(i)$  indicates a permutation on X such that  $f_{\sigma(1)} \ge f_{\sigma(2)} \ge ... \ge f_{\sigma(n)}$ ,  $A_{\sigma(i)} = \{1, 2, ..., i\}$ , and  $A_{\sigma(0)} = \phi$ .

It is seen that the discrete Choquet integral is a linear expression up to a reordering of the elements. Moreover, it identifies with the weighted mean (discrete Lebesgue integral) as soon as the fuzzy measure is additive. Moreover, in some conditions, the Choquet integral operator coincides with the ordered weighted average operator.

## 2.2 Pythagorean Fuzzy Sets and Their Operations

In this section, we give some basic definition and operations of PFSs.

**Definition 3** ([35]): Let *X* be a fixed set, then a PFS can be defined as

$$P = \{(x, a_P(x), b_P(x)) | x \in X\}, \tag{10}$$

where the functions  $a_P: X \to [0, 1]$  and  $b_P: X \to [0, 1]$  define the degree of membership and degree of non-membership of the element  $x \in X$  to P, respectively, and for all  $x \in X$  it holds that

$$0 \le a^2(x) + b^2(x) \le 1$$
.

For each PFS *P* and  $x \in X$ ,  $\pi_P(x) = \sqrt{1 - a^2(x) - b^2(x)}$  is said to be the Pythagorean fuzzy index of *x* to *p*.

**Definition 4** ([43]): Let p = (a, b),  $p_1 = (a_1, b_1)$ , and  $p_2 = (a_2, b_2)$  be the three PFNs and  $\delta > 0$ ; then, the following operational laws hold:

$$p_1 \oplus p_2 = \left(\sqrt{a_1^2 + a_2^2 - a_2^2 a_2^2}, b_1 b_2\right).$$
 (11)

$$p_1 \otimes p_2 = \left(a_1 a_2, \sqrt{b_1^2 + b_2^2 - b_1^2 b_2^2}\right).$$
 (12)

$$p^{\delta} = \left(a^{\delta}, \sqrt{1 - \left(1 - b^{2}\right)^{\delta}}\right). \tag{13}$$

$$\delta p = \left(\sqrt{1 - \left(1 - a^2\right)^{\delta}}, b^{\delta}\right). \tag{14}$$

For PFN, Zhang and Xu [43] introduced the score function and accuracy function, and defined a method to compare two PFNs as below.

**Definition 5** ([43]): Let  $p_i = (a_i, b_i)$  (i = 1, 2) be two PFNs, then  $S(p_1) = a_1^2 - b_1^2$  and  $S(p_2) = a_2^2 - b_2^2$  be the scores of  $p_1$  and  $p_2$ , respectively, and  $H(p_1) = a_1^2 + b_1^2$  and  $H(p_2) = a_2^2 + b_2^2$  be the accuracy degrees of  $p_1$  and  $p_2$ , respectively; then, the following holds:

- (1) If  $S(p_1) < S(p_2)$ , then  $p_1 < p_2$ .
- (2) If  $S(p_1) = S(p_2)$ , then we have the following three conditions:
  - (a) If  $H(p_1) = H(p_2)$ , then  $p_1 = p_2$ .
  - (b) If  $H(p_1) < H(p_2)$ , then  $p_1 < p_2$ .
  - (c) If  $H(p_1) > H(p_2)$ , then  $p_1 > p_2$ .

Zhang and Xu [43] presented a Pythagorean fuzzy distance measure for PFNs:

**Definition 6** ([43]): Let  $p_i = (a_i, b_i)$  (i = 1, 2) be two PFNs, then the distance between  $p_1$  and  $p_2$  is defined as follows:

$$d(p_1, p_2) = \frac{1}{2} \left[ \left| a_1^2 - a_2^2 \right| + \left| b_1^2 - b_2^2 \right| + \left| \pi_1^2 - \pi_2^2 \right| \right] ..., \tag{15}$$

where  $\pi_1 = \sqrt{1 - a_1^2 - b_1^2}$  and  $\pi_2 = \sqrt{1 - a_2^2 - b_2^2}$ .

**Definition 7** ([36]): Let  $\Omega$  be the set of all PFNs and  $p_i = (a_i, b_i)$  (i = 1, 2, ..., n) be a collection of PFNs, and let PFWG:  $\Omega^n \to \Omega$ , if

$$PFWG_{w}(p_{1}, p_{2}, ..., p_{n}) = \left( \prod_{i=1}^{n} (a_{i})^{w_{i}}, \sqrt{1 - \prod_{i=1}^{n} (1 - b_{i}^{2})^{w_{i}}} \right), \tag{16}$$

where  $w = (w_1, w_2, ... w_n)^T$  is the weighted vector of  $p_i$  (i = 1, 2, ..., n) with  $w_i \in [0, 1]$  and  $\sum_{i=1}^n w_i = 1$ , then PFWG is called the Pythagorean fuzzy weighted geometric operator.

**Definition 8** ([36]): A Pythagorean fuzzy ordered weighted geometric operator of dimension n is a mapping PFOWG:  $\Omega^n \to \Omega$  that has an associated weighted vector  $w = (w_1, w_2, \dots, w_n)^T$  with  $w_i \in [0, 1], \sum_{i=1}^n w_i = 0$ 1, and is defined to aggregate a collection of PFNs  $p_i = (a_i, b_i)(i = 1, 2, ..., n)$ , according to the following expression:

$$PFOWG_{w}(p_{1}, p_{2}, ..., p_{n}) = \left(\prod_{i=1}^{n} \left(a_{\sigma(i)}\right)^{w_{j}}, \sqrt{1 - \prod_{i=1}^{n} \left(1 - b_{\sigma(i)}^{2}\right)^{w_{i}}}\right), \tag{17}$$

where  $p_{\sigma(i)}$  is the  $i^{\text{th}}$  largest value of  $p_i$ . If  $w = \left(\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n}\right)^T$ , then the Pythagorean fuzzy ordered weighted average operator is reduced to the Pythagorean fuzzy geometric operator.

**Definition 9** ([27]): Let  $p_i = (a_i, b_i)$  (i = 1, 2, ..., n) be a collection of PFNs and  $\lambda$  be a fuzzy measure on X. Based on fuzzy measure, an interval-valued Pythagorean fuzzy Choquet integral geometric (PFCIG) operator of dimension n is a mapping Pythagorean fuzzy Choquet integral average, PFCIG :  $\Omega^n \to \Omega$ , such that

$$PFCIG(p_1, p_2, \dots, p_n) = \left(\prod_{i=1}^n \left(a_{\sigma(i)}\right)^{\lambda\left(A_{\sigma(i)}\right) - \lambda\left(A_{\sigma(i-1)}\right)}, \sqrt{1 - \prod_{i=1}^n \left(1 - b_{\sigma(i)}^2\right)^{\lambda\left(A_{\sigma(i)}\right) - \lambda\left(A_{\sigma(i-1)}\right)}}\right), \quad (18)$$

where  $\{\sigma(1), \sigma(2), \ldots, \sigma(n)\}$  is a permutation of  $\{1, 2, \ldots, 4\}$  such that  $p_{\sigma(1)} \geq p_{\sigma(2)} \geq \ldots \geq p_{\sigma(n)}$  and  $A_{\sigma(k)} = \{x_{\sigma(k)} | j \leq k\}$  for  $k \geq 1$ , and  $A_{\sigma(0)} = \phi$ .

# 3 GRA Method for MADM with Incomplete Weight Information in the Pythagorean Fuzzy Setting

Let  $X = \{X_1, X_2, \dots, X_m\}$  be a discrete set of alternatives and  $A = \{A_1, A_2, \dots, A_n\}$  be the set of attributes,  $w = (w_1, w_2, ..., w_n)$  is the weighting vector of the attributes  $A_j$  (j = 1, 2, ...n), where  $w_j \in [0, 1]$ ,  $\sum_{i=1}^{n} w_i = 1$ . Suppose that the decision makers provide the attribute weight information may be presented in the following forms [16], for  $i \neq j$ :

- 1. A weak ranking:  $\{w_i \geq w_i\}$ ;
- 2. A strict ranking:  $\{w_i w_i \ge \gamma_i(>0)\}$ ;
- 3. A ranking with multiples:  $\{w_i \ge \gamma_i w_i\}$ ,  $0 \le \gamma_i \le 1$ ;
- 4. An interval form:  $\{\lambda_i \leq w_i \leq \lambda_i + \delta_i\}$ ,  $0 \leq \gamma_i \leq \gamma_i + \delta_i \leq 1$ ;
- 5. A ranking of differences:  $\{w_i w_j \ge w_k w_l\}$ , for  $j \ne k \ne l$ .

For convenience, we denote by  $\Delta$  the set of the known information about attribute weights provided by the

Let  $R^k = \left[p_{ij}^{(k)}\right]_{m \times n}$  be a Pythagorean fuzzy decision matrix, provided by decision maker  $d_k$  (k = $1, 2, \ldots, l$ ), as the following form:

$$R^{k} = \left[p_{ij}^{(k)}\right]_{m \times n} = X_{1} \begin{bmatrix} p_{11}^{(k)} & p_{12}^{(k)} & \dots & p_{1n}^{(k)} \\ X_{11} & p_{21}^{(k)} & p_{22}^{(k)} & \dots & p_{2n}^{(k)} \\ \vdots & \vdots & \ddots & \vdots \\ X_{m} & p_{m1}^{(k)} & p_{m2}^{(k)} & \dots & p_{mn}^{(k)} \end{bmatrix},$$

where  $p_{ij}^{(k)} = \left(a_{ij}^{(k)}, b_{ij}^{(k)}\right)$  is a PFN representing the performance rating of the alternative  $x_j \in X$  with respect to the attribute  $A_i \in A$  provided by the decision makers  $d_k$ .

To extend the GRA method in the process of group decision making, we first need to fuse all individual decision matrices into a collective matrix by using the PFCIG operator.

**Step 1:** As for every alternative  $X_i$  (i = 1, 2, ..., m), each expert  $e_k$  (k = 1, 2, ..., r) is invited to express their individual evaluation or preference according to each attribute  $A_j$   $(j=1,2,\ldots,n)$  by a PFN  $p_{ii}^{(k)}=$  $\left(a_{ij}^{(k)},b_{ij}^{(k)}\right)$   $(i=1,2,\ldots,m;j=1,2,\ldots,n;k=1,2,\ldots,r)$  expressed by the experts  $e_k$ . Then, we can obtain a decision-making matrix  $R^k = \left[p_{ij}^{(k)}\right]_{m \times n}$  as follows:

$$A_2 \ A_2 \ \dots \ A_n$$

$$X_1 \begin{bmatrix} p_{11}^{(k)} & p_{12}^{(k)} & \dots & p_{1n}^{(k)} \\ p_{1j}^{(k)} \end{bmatrix}_{m \times n} = X_2 \begin{bmatrix} p_{11}^{(k)} & p_{12}^{(k)} & \dots & p_{1n}^{(k)} \\ p_{21}^{(k)} & p_{22}^{(k)} & \dots & p_{2n}^{(k)} \\ \vdots & \vdots & \ddots & \vdots \\ X_m & p_{m1}^{(k)} & p_{m2}^{(k)} & \dots & p_{mn}^{(k)} \end{bmatrix}.$$

**Step 2:** Confirm the fuzzy density  $\mu_i = \mu(A_i)$  of each expert. According to Eq. (4), parameter  $\lambda_1$  of expert can be determined.

**Step 3:** By Definition 5,  $p_{ij}^k$  is reordered such that  $p_{ij}^{(k)} \ge p_{ij}^{(k-1)}$ . Utilize the Pythagorean fuzzy Choquet integral average operator

$$PFCIG\left(p_{ij}^{(1)}, p_{ij}^{(2)}, ... p_{ij}^{(r)}\right) = \left(\prod_{k=1}^{r} \left(b_{ij}\right)^{\lambda\left(A_{(k)}\right) - \lambda\left(A_{(k-1)}\right)}, \sqrt{1 - \prod_{k=1}^{r} \left(1 - a_{ij}^{2}\right)^{\lambda\left(A_{(k)}\right) - \lambda\left(A_{(k-1)}\right)}}\right), \quad (19)$$

to aggregate all the Pythagorean fuzzy decision matrices  $R^k = \left[p_{ij}^{(k)}\right]_{m \times n} (k=1,2,\ldots,r)$  into a collective Pythagorean fuzzy decision matrix  $R = \left[p_{ij}^{(k)}\right]_{m \times n}$ , where  $p_{ij} = \left(a_{ij}, b_{ij}\right)$   $(i = 1, 2, \ldots, m; j = 1, 2, \ldots,$ 1, 2, ..., n),  $A_{(k)} = \{e_{(k)}, \ldots, e_{(r)}\}$ ,  $A_{(r-1)} = \phi$ , and  $\mu(A_{(k)})$  can be calculated by Eq. (3).

**Step 4:** Let  $H_1$  be a collection of benefit criteria (i.e. the larger  $c_i$ , the greater preference) and  $H_2$  be a collection of cost criteria (i.e. the smaller  $c_i$ , the greater preference). The Pythagorean fuzzy positive-ideal solution (PFPIS), denoted  $p^+ = (p_1^+, p_2^+, \dots, p_n^+)$ , and the Pythagorean fuzzy negative-ideal solution (PFNIS), denoted  $p^- = (p_1^-, p_2^-, \dots, p_n^-)$ , are defined as follows:

$$p^{+} = \{A_{j}, (\max_{i} a_{ij}, \min_{i} b_{ij}) | j \in H_{1}, (\min_{i} a_{ij}, \max_{i} b_{ij}) | j \in H_{2}\}$$
(20)

and

$$p^{-} = \{A_{j}, [(\max_{i} a_{ij}, \min_{i} b_{ij}) | j \in H_{1}, (\min_{i} a_{ij}, \max_{i} b_{ij}) | j \in H_{2}]\},$$
(21)

where  $p^+ = (a_i^+, b_i^+)$  and  $p^- = (a_i^-, b_i^-)(i = 1, 2, ..., n)$ .

**Step 5:** According to Pythagorean fuzzy distance, calculate the distance between the alternative  $X_i$  and the PFPIS  $p^+$  and the distance between the alternative  $X_i$  and the PFNIS  $p^-$ , respectively:

$$d_i(p_i, p^+) = \frac{1}{2} \left[ \left| a_i^2 - a_{ij}^2 \right| + \left| b_i^2 - b_{ij}^2 \right| + \left| \pi_i^2 - \pi_{ij}^2 \right| \right], \tag{22}$$

and construct a Pythagorean fuzzy positive-ideal separation matrix  $D^+$  and Pythagorean fuzzy negative-ideal separation matrix  $D^-$  as follows:

$$D^{+} = \begin{bmatrix} d(p_{11}, p_{1}^{+}) & d(p_{12}, p_{2}^{+}) & \dots & d(p_{1n}, p_{n}^{+}) \\ d(p_{21}, p_{1}^{+}) & d(p_{22}, p_{2}^{+}) & \dots & d(p_{2n}, p_{n}^{+}) \\ \vdots & \vdots & \ddots & \vdots \\ d(p_{m1}, p_{1}^{+}) & d(p_{m2}, p_{2}^{+}) & \dots & d(p_{mn}, p_{n}^{+}) \end{bmatrix}$$

$$(23)$$

and

$$D^{-} = \begin{bmatrix} D_{ij}^{-} \end{bmatrix} = \begin{bmatrix} d(p_{11}, p_{1}^{-}) & d(p_{12}, p_{2}^{-}) & \dots & d(p_{1n}, p_{n}^{-}) \\ d(p_{21}, p_{1}^{-}) & d(p_{22}, p_{2}^{-}) & \dots & d(p_{2n}, p_{n}^{-}) \\ \vdots & \vdots & \ddots & \vdots \\ d(p_{m1}, p_{1}^{-}) & d(p_{m2}, p_{2}^{-}) & \dots & d(p_{mn}, p_{n}^{-}) \end{bmatrix}.$$

$$(24)$$

#### Procedure I

Step 6: Calculate the gray relational coefficient of each alternative from positive-ideal solution (PIS) and negative-ideal solution (NIS) using the following equations, respectively. The gray relational coefficient of each alternative from PIS is given as

$$\zeta_{ij}^{+} = \frac{\min_{1 \le i \le m} \min_{1 \le j \le n} d(p_{ij}, p_{j}^{+}) + \rho \max_{1 \le i \le m} \max_{1 \le j \le n} d(p_{ij}, p_{j}^{+})}{d(p_{ij}, p_{j}^{+}) + \rho \max_{1 \le i \le m} \max_{1 \le j \le n} d(p_{ij}, p_{j}^{+})},$$
(25)

i = 1, 2, ..., m; j = 1, 2, ..., n.

Similarly, the gray relational coefficient of each alternative from NIS is given as

$$\zeta_{ij}^{-} = \frac{\min_{1 \le i \le m} \min_{1 \le j \le n} d(p_{ij}, p_{j}^{-}) + \rho \max_{1 \le i \le m} \max_{1 \le j \le n} d(p_{ij}, p_{j}^{-})}{d(p_{ij}, p_{j}^{-}) + \rho \max_{1 \le i \le m} \max_{1 \le j \le n} d(p_{ij}, p_{j}^{-})}$$
(26)

i=1,2,...,m; j=1,2,...,n, where the identification coefficient is  $\rho=0.5$ .

Step 7: Calculate the degree of gray relational coefficient of each alternative from the PIS and NIS using the following equations, respectively:

$$\zeta_{i}^{+} = \sum_{i=1}^{n} w_{j} \zeta_{ij}^{+}. \tag{27}$$

$$\zeta_{i}^{-} = \sum_{j=1}^{n} w_{j} \zeta_{ij}^{-}. \tag{28}$$

The basic principle of the GRA method is that the chosen alternative should have the largest degree of gray relation from the PIS and the smallest degree of gray relation from the NIS. Obviously, for the weight vector given, the smaller  $\xi_i^-$  and the larger  $\xi_i^+$ , the better alternative  $A_i$  is. However, the information about attribute weights is incompletely known. Thus, in order to get  $\xi_i^-$  and  $\xi_i^+$ , we must first calculate the weight

information. Thus, we can establish the following multiple objective optimization models to calculate the weight information:

$$\min \zeta_{i}^{-} = \sum_{j=1}^{n} w_{j} \zeta_{ij}^{-} \quad i = 1, 2, ..., m$$

$$\max \zeta_{i}^{+} = \sum_{j=1}^{n} w_{j} \zeta_{ij}^{+} \quad i = 1, 2, ..., m$$
subject to:  $w \in H$ .
$$(29)$$

As each alternative is non-inferior, there exists no preference relation on all the alternatives. Then, we may aggregate the above multiple objective optimization models with equal weights into the following single objective optimization model:

(M2) 
$$\begin{cases} \min \zeta_i = \sum_{i=1}^m \sum_{j=1}^n \left(\zeta_{ij}^- - \zeta_{ij}^+\right) w_j \\ \text{subject to: } w \in H. \end{cases}$$
 (30)

By solving the model (M2), we get the optimal solution  $w=(w_1,w_2,\ldots,w_n)$ , which can be used as the weight vector of attributes. Then, we can get  $\zeta_i^+(i=1,\ldots,m)$  and  $\zeta_i^-(i=1,\ldots,m)$  by Eqs. (27) and (28), respectively.

**Step 8:** Calculate the relative relational degree of each alternative from PIS using the following equation:

$$\zeta_i = \frac{\zeta_i^+}{\zeta_i^- + \zeta_i^+}, (i = 1, \dots, m)$$
 (31)

**Step 9:** Rank all the alternatives  $A_i$  (i = 1, 2, ..., m) and select the best one(s) in accordance with  $\zeta_i$  (i = 1, 2, ..., m). If any alternative has the highest  $\zeta_i$  value, then it is the most important alternative.

#### **Procedure II**

**Step 6:** Calculate the gray relational coefficient of each alternative from PIS and NIS using the following equation. The gray relational coefficient of each alternative from PIS is given as

$$\zeta_{ij}^{+} = \frac{\min_{1 \le i \le m} \min_{1 \le j \le n} d(p_{ij}, p_{j}^{+}) + \rho \max_{1 \le i \le m} \max_{1 \le j \le n} d(p_{ij}, p_{j}^{+})}{d(p_{ij}, p_{j}^{+}) + \rho \max_{1 \le i \le m} \max_{1 \le j \le n} d(p_{ij}, p_{j}^{+})},$$
(32)

i = 1, 2, ..., m; j = 1, 2, ..., n.

**Step 7: Case a:** If the information about attribute weights is incompletely known, in order to get the  $\xi_i^+$ , we must first calculate the weight information. The gray relational coefficient between PIS and itself is  $(1, 1, \ldots, 1)$ , so the comprehensive gray relational coefficient deviation sum is

$$d_i(w) = \sum_{i=1}^n \left(1 - \zeta_{ij}^+\right) w_j. \tag{33}$$

Therefore, we can establish the following multiple objective optimization models to calculate the weight information:

(M3) 
$$\begin{cases} \min d_i(w) = \sum_{j=1}^n \left(1 - \zeta_{ij}^+\right) w_j \\ \text{subject to: } w \in H. \end{cases}$$
 (34)

As each alternative is non-inferior, there exists no preference relation on all the alternatives. Then, we may aggregate the above multiple objective optimization models with equal weights into the following single objective optimization model:

(M4) 
$$\begin{cases} \min d_i(w) = \sum_{i=1}^m d_i(w) = \sum_{j=1}^n \left(1 - \zeta_{ij}^+\right) w_j \\ \text{subject to: } \sum_{j=1}^m w_j = 1. \end{cases}$$
 (35)

By solving the model (M4), we get the optimal solution  $w = (w_1, w_2, \dots, w_n)$ , which can be used as the weight vector of attributes. Then, we can get  $\zeta_i^+$  ( $i=1,2,\ldots,m$ ) by Eq. (27).

Case b: If the information about attribute weights is completely unknown, we can establish another multiple objective programming model as follows:

$$(M5) \begin{cases} \min d_i(w) = \sum_{j=1}^n \left[ \left( 1 - \zeta_{ij}^+ \right) w_j \right]^2 \\ \text{subject to: } \sum_{j=1}^m w_j = 1. \end{cases}$$

$$(36)$$

Similarly, we may aggregate the above multiple objective optimization models with equal weights into the following single objective optimization model:

$$(M6) \begin{cases} \min d(w) = \sum_{i=1}^{n} d_i(w) = \sum_{i=1}^{m} \sum_{j=1}^{n} \left[ \left( 1 - \zeta_{ij}^{+} \right) w_j \right]^2 \\ \text{subject to: } \sum_{j=1}^{m} w_j = 1. \end{cases}$$
(37)

To solve this model, we construct the Lagrange function:

$$L(w,\theta) = \sum_{i=1}^{m} \sum_{j=1}^{n} \left[ \left( 1 - \zeta_{ij}^{+} \right) w_{j} \right]^{2} + 2\theta \left( \sum_{j=1}^{n} w_{j} - 1 \right), \tag{38}$$

where  $\theta$  is the Lagrange multiplier. The partial derivatives of L with respect to  $w_i$  and  $\theta$  are computed as follows:

$$\frac{\partial L}{\partial w_j} = 2\sum_{i=1}^m \left(1 - \zeta_{ij}^+\right)^2 w_j + 2\theta = 0$$

$$\Rightarrow \sum_{i=1}^{m} \left(1 - \zeta_{ij}^{+}\right)^{2} w_{j} + \theta = 0.$$

$$\tag{39}$$

$$w_{j} = \frac{-\theta}{\sum_{i=1}^{m} \left(1 - \zeta_{ij}^{+}\right)^{2}} \tag{40}$$

$$\frac{\partial L}{\partial \theta} = 2 \left( \sum_{j=1}^{n} w_j - 1 \right) = 0$$

$$\Rightarrow \sum_{j=1}^{n} w_j = 1. \tag{41}$$

Putting Eq. (40) in Eq. (41), we get

$$\theta = -\left[\sum_{j=1}^{n} \left[\sum_{j=1}^{n} \left(1 - \zeta_{ij}^{+}\right)^{2}\right]^{-1}\right]^{-1}$$
(42)

Combining Eqs. (40) and (42), we get

$$w_{j} = \frac{\left[\sum_{j=1}^{n} \left[\sum_{j=1}^{n} \left(1 - \zeta_{ij}^{+}\right)^{2}\right]^{-1}\right]^{-1}}{\sum_{i=1}^{m} \left(1 - \zeta_{ij}^{+}\right)^{2}}.$$
(43)

Then, we can get  $\zeta_i^+$   $(i=1,\ldots,m)$  by Eq. (27).

**Step 8:** Rank all the alternatives  $X_i$  (i = 1, 2, ..., m) and select the best one(s) in accordance with  $\zeta_i^+$  (i = 1, ..., m). If any alternative has the highest  $\zeta_i^+$  value, then it is the most important alternative.

# **4 Illustrative Examples**

In this section, we shall present a numerical example with Pythagorean fuzzy information in order to illustrate the method developed in this paper.

**Example 1:** Suppose there is a panel with five possible emerging technology enterprises  $X_i$  (i = 1, 2, 3, 4, 5) to select. There are three experts and four attributes are selected to evaluate the five possible emerging technology enterprises:

- 1.  $A_1$  is the technical advancement.
- 2.  $A_2$  is the potential market risk.
- 3.  $A_3$  is the industrialization infrastructure, human resource, and financial conditions.
- 4. A<sub>4</sub> is the employment creation and the development of science and technology.

The three experts' own opinions regarding the results obtained with each emerging technology enterprise are shown in Tables 1–3.

Assume that the information about attribute weights, given by experts, is partly known:  $\Delta = \{0.2 \le w_1 \le 0.25, 0.15 \le w_2 \le 0.2, 0.28 \le w_3 \le 0.32, 0.35 \le w_4 \le 0.4\}$   $w_j \ge 0$ , j = 1, 2, 3, 4,  $\sum_{i=1}^4 w_i = 1$ . Then, we utilize the developed approach to get the most desirable alternative(s).

**Table 1:** Pythagorean Fuzzy Decision Matrix  $R^{(1)}$ .

	<b>A</b> <sub>1</sub>	A <sub>2</sub>	<b>A</b> <sub>3</sub>	A <sub>4</sub>
$\overline{X_1}$	p(0.5, 0.8)	p(0.8, 0.4)	p(0.4, 0.9)	p(0.7, 0.6)
$X_2$	p(0.7, 0.6)	p(0.3, 0.9)	p(0.6, 0.7)	p(0.6, 0.8)
$X_3$	p(0.6, 0.7)	p(0.8, 0.5)	p(0.2, 0.9)	p(0.7, 0.5)
$X_4$	p(0.9, 0.3)	p(0.6, 0.8)	p(0.8, 0.5)	p(0.3, 0.9)
$X_5$	p(0.6, 0.8)	p(0.8, 0.3)	p(0.3, 0.9)	p(0.6, 0.7)

**Table 2:** Pythagorean Fuzzy Decision Matrix  $R^{(2)}$ .

	<b>A</b> <sub>1</sub>	A <sub>2</sub>	<b>A</b> <sub>3</sub>	A <sub>4</sub>
$X_1$	p(0.5, 0.7)	p(0.7, 0.4)	p(0.3, 0.9)	p(0.7, 0.8)
$X_2$	p(0.8, 0.4)	p(0.6, 0.7)	p(0.5, 0.7)	p(0.5, 0.8)
$X_3$	p(0.3, 0.9)	p(0.8, 0.6)	p(0.6, 0.7)	p(0.9, 0.4)
$\chi_4$	p(0.6, 0.5)	p(0.3, 0.9)	p(0.8, 0.6)	p(0.5, 0.7)
$X_5$	p(0.8, 0.3)	p(0.7, 0.6)	p(0.4, 0.8)	p(0.8, 0.5)

**Table 3:** Pythagorean Fuzzy Decision Matrix  $R^{(3)}$ .

	A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>	A <sub>4</sub>
<i>X</i> <sub>1</sub>	p(0.8, 0.6)	p(0.4, 0.7)	p(0.9, 0.4)	p(0.5, 0.7)
$X_2$	p(0.3, 0.9)	p(0.7, 0.5)	p(0.6, 0.5)	p(0.4, 0.9)
$X_3$	p(0.7, 0.5)	p(0.8, 0.4)	p(0.4, 0.7)	p(0.8, 0.6)
$\chi_4$	p(0.8, 0.5)	p(0.6, 0.7)	p(0.7, 0.6)	p(0.8, 0.5)
$X_5$	p(0.6, 0.5)	p(0.9, 0.4)	p(0.8, 0.4)	p(0.3, 0.9)

**Step 2:** We first determine the fuzzy density of each decision maker and its  $\lambda$  parameter. Suppose that  $\mu(A_1) = 0.30$ ,  $\mu(A_2) = 0.40$ ,  $\mu(A_3) = 0.50$ . Then,  $\lambda$  of expert can be determined as  $\lambda = -0.45$ . By Eq. (6), we have  $\mu(A_1, A_2) = 0.65$ ,  $\mu(A_1, A_3) = 0.73$ ,  $\mu(A_2, A_3) = 0.81$ , and  $\mu(A_1, A_2, A_3) = 1$ .

**Step 3:** According to Definition 5,  $p_{ii}^{(k)}$  is reordered such that  $p_{ii}^{(k)} \geq p_{ii}^{(k-1)}$ . Then, utilize the PFCIG operator

$$PFCIG(p_1, p_2, p_3) = \left( \prod_{k=1}^{3} (a_{\sigma(k)})^{\lambda(A_{\sigma(k)}) - \lambda(A_{\sigma(k-1)})}, \sqrt{1 - \prod_{k=1}^{3} (1 - b_{\sigma(k)}^2)^{\lambda(A_{\sigma(k)}) - \lambda(A_{\sigma(k-1)})}} \right)$$

to aggregate all the Pythagorean fuzzy decision matrices  $R^{(k)} = \left[p_{ij}^{(k)}\right]_{m \times n}$  into a collective Pythagorean fuzzy decision matrix  $R = [p_{ij}]_{m \times n}$ , as below.

Step 4: Utilizing Eqs. (20) and (21), we get the PIS and NIS, respectively, as

$$p^{+} = \{\langle 0.7668, 0.4530 \rangle, \langle 0.8185, 0.4125 \rangle, \langle 0.7635, 0.6165 \rangle, \langle 0.8176, 0.5144 \rangle\}$$
$$p^{-} = \{\langle 0.5375, 0.7254 \rangle, \langle 0.4976, 0.7996 \rangle, \langle 0.4124, 0.7598 \rangle, \langle 0.4884, 0.8439 \rangle\}$$

Step 5: Utilize Eqs. (23) and (24) to get the positive-ideal separation matrix and negative-ideal separation matrix, respectively, as follows:

$$D^{+} = \begin{bmatrix} D_{ij}^{+} \end{bmatrix} = \begin{bmatrix} 0.2601 & 0.2776 & 0.2747 & 0.2654 \\ 0.3283 & 0.3472 & 0.2567 & 0.4476 \\ 0.3210 & 0.0694 & 0.4129 & 0 \\ 0 & 0.4692 & 0 & 0.3390 \\ 0.1424 & 0 & 0.2961 & 0.3895 \end{bmatrix}.$$

$$D^{-} = \begin{bmatrix} 0.1112 & 0.3735 & 0.1614 & 0.1822 \\ 0.0197 & 0.1520 & 0.1957 & 0 \\ 0 & 0.3999 & 0 & 0.4476 \\ 0.3210 & 0 & 0.4129 & 0.2248 \\ 0.1786 & 0.4692 & 0.1168 & 0.1334 \end{bmatrix}.$$

**Step 6:** Utilizing Eqs. (25) and (26), we get the gray relational coefficient matrices in which each alternative is calculated from PIS and NIS as follows:

$$\begin{bmatrix} \zeta_{ij}^+ \end{bmatrix} = \begin{bmatrix} 0.4742 & 0.5634 & 0.4606 & 0.4692 \\ 0.4168 & 0.4032 & 0.4775 & 0.3439 \\ 0.4223 & 0.7717 & 0.3623 & 1.0000 \\ 1.0000 & 0.3697 & 1.0000 & 0.4090 \\ 0.6223 & 1.0000 & 0.4421 & 0.3759 \end{bmatrix}.$$

$$\begin{bmatrix} \zeta_{ij}^- \end{bmatrix} = \begin{bmatrix} 0.6784 & 0.3858 & 0.2038 & 0.6514 \\ 0.9225 & 0.6068 & 0.5452 & 1.0000 \\ 1.0000 & 0.3697 & 1.0000 & 0.3439 \\ 0.4223 & 1.0000 & 0.3623 & 0.5107 \\ 0.5678 & 0.3333 & 0.6676 & 0.6375 \end{bmatrix}.$$

**Step 7:** Utilize the model (M2) to establish the following single-objective programming model:  $\{\min \xi(w) = 0\}$  $-0.6554w_1 + 0.4124w_2 - 0.0364w_3 - 0.5455w_4$ .

Solving this model, we get the weight vector of attributes:

$$w = (0.22, 0.15, 0.28, 0.35).$$

Then, we can get the degree of gray relational coefficient of each alternative from PIS and NIS:

$$\xi_1^+ = 0.482022, \xi_2^+ = 0.406241, \xi_3^+ = 0.660105, \xi_4^+ = 0.698605, \xi_5^+ = 0.542259,$$
  
 $\xi_1^- = 0.492172, \xi_2^- = 0.796626, \xi_3^- = 0.675820, \xi_4^- = 0.523095, \xi_5^- = 0.584964.$ 

**Step 8:** Utilizing Eq. (29), we get the relative relational degree of each alternative from PIS as follows:

$$\xi_1 = 0.4948, \xi_2 = 0.3377, \xi_3 = 0.4941, \xi_4 = 0.5718, \text{ and } \xi_5 = 0.4811.$$

**Step 9:** According to the relative relational degree, the ranking order of the five alternatives is  $X_4 > X_3 >$  $X_5 > X_1 > X_2$ , and thus the most desirable alternative is  $X_4$ .

In the following example, we will use Procedure II to get the gray relation coefficient matrix and optimal weight of the attributes.

**Example 2:** Suppose that a Hawaii company in Pakistan desire to hire a radiofrequency modeling engineer. After the initial screening, four candidates (i.e. alternatives),  $X_1$ ,  $X_2$ ,  $X_3$  and  $X_4$ , remain for further evaluation. In order to select the most suitable candidate, the decision maker takes into account the following five attributes:

- 1. Emotional steadiness  $(A_1)$ :
- 2. Oral communication skill  $(A_2)$ ;
- 3. Personality  $(A_3)$ ;
- 4. Past experience  $(A_4)$ ;
- 5. Self-confidence  $(A_5)$ .

Assume that the four decision makers  $d_i$  (i = 1, 2, 3, 4) provide his/her preference information on candidates with regard to attributes by using PFNs, as shown in Tables 4–7.

Then, we utilize the approach developed to get the most desirable alternative(s), which involves the following two cases.

**Table 4:** Pythagorean Fuzzy Decision by Decision Maker  $d_1$ .

	<b>A</b> <sub>1</sub>	<b>A</b> <sub>2</sub>	<b>A</b> <sub>3</sub>	A <sub>4</sub>	A <sub>5</sub>
X <sub>1</sub>	p(0.8, 0.6)	p(0.7, 0.5)	p(0.6, 0.5)	p(0.9, 0.4)	p(0.7, 0.6)
$X_2$	p(0.6, 0.5)	p(0.8, 0.4)	p(0.7, 0.6)	p(0.6, 0.5)	p(0.9, 0.4)
$X_3$	p(0.9, 0.3)	p(0.6, 0.5)	p(0.8, 0.4)	p(0.7, 0.4)	p(0.8, 0.3)
$X_4$	p(0.7, 0.4)	p(0.9, 0.4)	p(0.6, 0.5)	p(0.8, 0.5)	p(0.7, 0.5)

**Table 5:** Pythagorean Fuzzy Decision by Decision Maker  $d_2$ .

	<b>A</b> <sub>1</sub>	<b>A</b> <sub>2</sub>	<b>A</b> <sub>3</sub>	A <sub>4</sub>	A <sub>5</sub>
$\overline{X_1}$	p(0.7, 0.6)	p(0.6, 0.5)	p(0.8, 0.6)	p(0.7, 0.5)	p(0.9, 0.4)
$X_2$	p(0.8, 0.4)	p(0.7, 0.4)	p(0.6, 0.5)	p(0.8, 0.3)	p(0.6, 0.5)
$X_3$	p(0.6, 0.5)	p(0.9, 0.3)	p(0.7, 0.4)	p(0.9, 0.4)	p(0.7, 0.6)
$X_4$	p(0.9, 0.4)	p(0.8, 0.5)	p(0.9, 0.3)	p(0.6, 0.5)	p(0.8, 0.5)

**Table 6:** Pythagorean Fuzzy Decision by Decision Maker  $d_3$ .

	$A_1$	$A_2$	<b>A</b> <sub>3</sub>	<b>A</b> 4	<b>A</b> <sub>5</sub>
<i>X</i> <sub>1</sub>	p(0.6, 0.5)	p(0.8, 0.5)	p(0.7, 0.6)	p(0.8, 0.4)	p(0.6, 0.5)
$X_2$	p(0.7, 0.6)	p(0.9, 0.3)	p(0.8, 0.3)	p(0.7, 0.6)	p(0.8, 0.4)
$X_3$	p(0.8, 0.4)	p(0.7, 0.6)	p(0.6, 0.5)	p(0.8, 0.6)	p(0.7, 0.4)
$X_4$	p(0.9, 0.3)	p(0.6, 0.5)	p(0.9, 0.4)	p(0.9, 0.3)	p(0.9, 0.4)

**Table 7:** Pythagorean Fuzzy Decision by Decision Maker  $d_4$ .

	<b>A</b> <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>	<b>A</b> 4	A <sub>5</sub>
<i>X</i> <sub>1</sub>	p(0.9, 0.4)	p(0.7, 0.6)	p(0.8, 0.5)	p(0.7, 0.4)	p(0.8, 0.6)
$X_2$	p(0.8, 0.5)	p(0.9, 0.4)	p(0.7, 0.4)	p(0.8, 0.5)	p(0.9, 0.2)
<i>X</i> <sub>3</sub>	p(0.7, 0.4)	p(0.8, 0.6)	p(0.8, 0.3)	p(0.8, 0.4)	p(0.7, 0.5)
$X_4$	p(0.6, 0.5)	p(0.8, 0.4)	p(0.9, 0.2)	p(0.7, 0.5)	p(0.8, 0.4)

Case a: Assume that the information about attribute weight given by decision makers is partially known, i.e.

$$\Delta = \left\{ egin{aligned} 0.15 \leq w_1 \leq 0.22, 0.18 \leq w_2 \leq 0.24, 0.20 \leq w_3 \leq 0.30, 0.15 \leq w_4 \leq 0.20, \\ 0.14 \leq w_5 \leq 0.19; w_j \geq 0, j = 1, 2, 3, 4, 5, \sum_{j=1}^5 w_j = 1 \end{aligned} 
ight\}.$$

Then, we utilize the developed approach to get the most desirable alternative(s).

**Step 2:** We first determine the fuzzy density of each decision maker and its  $\lambda$  parameter. Suppose that  $\mu(A_1) = 0.40$ ,  $\mu(A_2) = 0.30$ ,  $\mu(A_3) = 0.20$ , and  $\mu(A_4) = 0.50$ . Then  $\lambda$  of expert can be determined as  $\lambda = 0.40$ -0.65. By Eq. (6), we have

 $\mu(A_1, A_2) = 0.62$ ,  $\mu(A_1, A_3) = 0.55$ ,  $\mu(A_1, A_4) = 0.77$ ,  $\mu(A_2, A_3) = 0.46$ ,  $\mu(A_2, A_4) = 0.70$ ,  $\mu(A_3, A_4) = 0.63, \ \mu(A_1, A_2, A_3) = 0.74, \ \mu(A_1, A_2, A_4) = 0.92, \ \mu(A_1, A_3, A_4) = 0.87, \ \mu(A_2, A_3, A_4) = 0.87$ 0.81, and  $\mu(A_1, A_2, A_3, A_4) = 1$ .

**Step 3:** According to Definition 5,  $p_{ij}^{(k)}$  is reordered such that  $p_{ij}^{(k)} \ge p_{ij}^{(k-1)}$ . Then, utilize the PFCIG operator

$$PFCIG(p_1, p_2, p_3, p_4) = \left( \prod_{k=1}^{4} \left( a_{\sigma(k)} \right)^{\lambda \left( A_{\sigma(k)} \right) - \lambda \left( A_{\sigma(k-1)} \right)}, \sqrt{1 - \prod_{k=1}^{4} \left( 1 - b_{\sigma(k)}^2 \right)^{\lambda \left( A_{\sigma(k)} \right) - \lambda \left( A_{\sigma(k-1)} \right)}} \right)$$

to aggregate all the Pythagorean fuzzy decision matrices  $R^{(k)} = \left[p_{ij}^{(k)}\right]_{m \times n}$  into a collective Pythagorean fuzzy decision matrix  $R = [p_{ij}]_{m \times n}$  (see Table 8).

Step 4: Utilizing Eq. (20), we get the PIS and NIS, respectively, as

$$p^+ = \{p(0.8128, 0.5074), p(0.8469, 0.3827), p(0.8333, 0.3270), p(0.8080, 0.4298), p(0.8438, 0.3393)\}.$$

**Step 5:** Utilize Eq. (23) to get the positive-ideal separation matrix as follows:

$$D^{+} = \begin{bmatrix} D_{ij}^{+} \end{bmatrix} = \begin{bmatrix} 0 & 0.2206 & 0.1791 & 0.0416 & 0.1816 \\ 0.1235 & 0 & 0.1978 & 0.0957 & 0 \\ 0.1736 & 0.1205 & 0.1072 & 0 & 0.1668 \\ 0.1770 & 0.0457 & 0 & 0.0813 & 0.0794 \end{bmatrix}.$$

Step 6: Utilizing Eq. (25), we get the gray relational coefficient matrices in which each alternative is calculated from PIS as follows:

$$\left[\zeta_{ij}^{+}\right] = \begin{bmatrix} 1.0000 & 0.3333 & 0.3811 & 0.7261 & 0.3779 \\ 0.4718 & 1.0000 & 0.3580 & 0.5354 & 1.0000 \\ 0.3885 & 0.4779 & 0.5071 & 1.0000 & 0.3981 \\ 0.3839 & 0.7071 & 1.0000 & 0.5757 & 0.5814 \end{bmatrix}.$$

**Step 7:** Utilize the model (M4) to establish the following single-objective programming model:  $\{\min \xi(w) = 0\}$  $1.7558w_1 + 1.4817w_2 + 1.7538w_3 + 1.1628w_4 + 1.6426w_5$ .

Solving this model, we get the weight vector of attributes:

$$w = (0.15, 0.24, 0.2, 0.19, 0.22).$$

Then, we can get the degree of gray relational coefficient of each alternative from PIS:

$$\zeta_1^+ = 0.5273, \zeta_2^+ = 0.7041, \zeta_3^+ = 0.5520, \zeta_4^+ = 0.6646.$$

**Step 8:** According to the relative relational degree, the ranking order of the four alternatives is  $X_2 > X_4 >$  $X_3 > X_1$ , and thus the most desirable alternative is  $X_2$ .

Case b: Step 7: If the information about the attribute weights is completely unknown, we utilize another approach developed to get the most desirable alternative(s). Utilize Eq. (43) to get the weight vector of attributes:

$$w = (0.1518, 0.1952, 0.1509, 0.3327, 0.1695)^{T}$$
.

Table 8: Collective Pythagorean Fuzzy Decision Matrix.

	<b>A</b> <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>	A4	A <sub>5</sub>
<i>X</i> <sub>1</sub>	p(0.8128, 0.5074)	p(0.7047, 0.5358)	p(0.7464, 0.5348)	p(0.7897, 0.4151)	p(0.7865, 0.5447)
$X_2$	p(0.7464, 0.4873)	p(0.8469, 0.3827)	p(0.70470.4605)	p(0.7464, 0.4679)	p(0.8438, 0.3393)
$X_3$	p(0.7740, 0.3813)	p(0.7734, 0.5167)	p(0.7663, 0.3655)	p(0.8080, 0.4298)	p(0.7384, 0.4389)
$X_4$	p(0.7549, 0.4138)	p(0.8195, 0.4262)	p(0.8333, 0.2370)	p(0.7560, 0.4695)	p(0.7986, 0.4410)

Then, we can get the degree of gray relational coefficient of each alternative from PIS:

$$\zeta_1^+ = 0.5800, \zeta_2^+ = 0.6685, \zeta_3^+ = 0.6290, \zeta_4^+ = 0.6373.$$

**Step 8:** According to the relative relational degree, the ranking order of the four alternatives is  $X_2 > X_4 >$  $X_3 > X_1$ , and thus the most desirable alternative is  $X_2$ .

### 4.1 Comparative Analysis

In order to verify the validity and effectiveness of the proposed approach, a comparative study is conducted using the Pythagorean fuzzy TOPSIS [44] and GRA method for intuitionistic fuzzy sets [37], to the same illustrative example.

#### 4.1.1 Comparison Analysis with the Pythagorean Fuzzy TOPSIS

These two approaches are valid for solving MCDM problems. The GRA proposed by Deng [5, 6] is suggested as a tool for implementing a multiple-criteria performance scheme, which is used to identify solutions from a finite set of alternatives. GRA is an impact evaluation model that can measure the degree of similarity or difference between two sequences based on the relation. TOPSIS method aims at choosing the alternative with the shortest distance from the PIS and the farthest distance from the NIS.

According to Zhang et al. [44], the first step is to identify the Pythagorean fuzzy PIS and the Pythagorean fuzzy NIS of the decision matrix (see Table 9), which are

$$p^+ = \{(0.7668, 0.4530), (0.8185, 0.4125), (0.7635, 0.6165), (0.8176, 0.5144)\}$$
  
 $p^- = \{(0.5375, 0.7254), (0.4976, 0.7996), (0.4124, 0.7598), (0.4884, 0.8439)\}$ 

The next step is to calculate the distance between the alternatives  $X_i$  and the Pythagorean fuzzy PIS and Pythagorean fuzzy NIS, respectively, in each matrix by

$$d_i(X_i, p^+) = \sum_{j=1}^4 w_j d(p_j, p^+)$$
 (44)

$$d_i(X_i, p^-) = \sum_{j=1}^4 w_j d(p_j, p^-)$$
 (45)

In the last stag of Pythagorean fuzzy TOPSIS, calculate the relative relational degree of each alternative from PIS using Eq. (29) as

$$\zeta_i = \frac{\zeta_i^+}{\zeta_i^- + \zeta_i^+}, (i = 1, \dots, m).$$
 (46)

Table 9: Collective Pythagorean Fuzzy Decision Matrix R.

	$A_1$	<b>A</b> <sub>2</sub>	<b>A</b> <sub>3</sub>	A4
<i>X</i> <sub>1</sub>	p(0.6325, 0.6821)	p(0.6264, 0.5156)	p(0.5552, 0.7749)	p(0.6392, 0.7280)
$X_2$	p(0.5489, 0.7304)	p(0.5681, 0.6981)	p(0.5712, 0.6177)	p(0.4884, 0.8439)
<i>X</i> <sub>3</sub>	p(0.5375, 0.7254)	p(0.8000, 0.4894)	p(0.4124, 0.7598)	p(0.8176, 0.5144)
$X_4$	p(0.7668, 0.4530)	p(0.4976, 0.7996)	p(0.7635, 0.6165)	p(0.5740, 0.6981)
$X_5$	p(0.6732, 0.5896)	p(0.8185, 0.4125)	p(0.5356, 0.7162)	p(0.5282, 0.7608)

Then, for the same weight of the decision makers, we can get the overall relative closeness index  $\zeta_i$  of each alternative. The bigger is  $\zeta_i$ , the better the alternatives and the most desirable alternative is  $\zeta_2$ , as given

$$\zeta_1 = 0.5865, \zeta_2 = 0.8115, \zeta_3 = 0.4758, \zeta_4 = 0.4164, \zeta_5 = 0.5700.$$

Thus, we know that the ranking results of the alternatives obtained by the Pythagorean fuzzy GRA approach and the Pythagorean fuzzy TOPSIS approach are distinctly different, where the optimal choice derived by the Pythagorean fuzzy GRA approach is  $X_4$ , but the most desirable alternative obtained by the Pythagorean fuzzy TOPSIS approach is X2. Essentially, these two approaches are discrepant at the consideration of the decision makers' behaviors. The Pythagorean fuzzy TOPSIS approach can only be used in the situations where the decision makers are entirely rational. However, in practice, for the incomplete information or some other factors, the decision makers usually cannot provide accurate preferences. In other words, the decision makers are not rational to some degree. The Pythagorean fuzzy GRA approach can reasonably depict the decision makers' behaviors under risk, and thus it may deal with the above issue effectively.

#### 4.1.2 Comparison Analysis with the Intuitionistic Fuzzy GRA Relation

IFNs and the PFNs can portray uncertain things from the membership degrees and the non-membership degrees. They provide an effective tool to express the indeterminacy of the decision makers. On the one hand, as mentioned before, the PFNs and IFNs can be transferred by the fuzzy intervals, which are more scientific and practical for human beings to handle uncertain decision-making problems. On the other hand, by judging things from the good, bad, and hesitant aspects, these two kinds of fuzzy numbers can reflect the cognitions of the decision makers precisely. However, unlike the PFNs, the IFNs are not usable in some cases. The IFNs must satisfy that the sum of the membership degree and the non-membership degree belongs to [0, 1]. Thus, in our case analysis, there exist some numbers that IFNs cannot deal with. For example, in the decision matrix, the first decision maker gives the membership degree and the non-membership degree of the alternative  $X_1$  with respect to attribute  $X_1$  as 0.5 and 0.8, respectively. Then, the sum of 0.5 and 0.8 is >1. However, this order is valid in PFNs for  $0.5^2 + 0.8^2 = 0.89$ . In summary, the PFNs have stronger ability to process information in MCDM.

To conclude from the above, the discussion has delivered the advantages of the proposed method. In evaluation with fuzzy set and IFS-based methods, using PFS has increased the space in which decision makers can define their level of agreement, disagreement, and hesitation. One of the advantages of the developed method is using multiple objective optimization models. This method offers the model with the ability to address the decision preference in order to allocate attribute weights more reasonably. Moreover, it has provided a way to apply the objective information that shows membership, non-membership, and hesitation degrees in PFS assessments.

## 5 Conclusion

The traditional GRA methods are generally suitable for dealing with MAGDM problems in which the information takes the form of numerical values, yet they will fail in dealing with MAGDM problems in which the information takes the form of Pythagorean fuzzy information. In this paper, we establish multiple objective optimization models based on the basic ideal of the traditional GRA. To determine the attribute, we proposed the optimization models based on the GRA. We proposed two procedures for MAGDM based on GRA under a Pythagorean fuzzy environment. In the first procedure, we utilized the PFCIG operator to fuse all the individual matrices. Then, based on the traditional GRA method, calculation steps for solving Pythagorean fuzzy MAGDM problems with incomplete information are given. Following the calculating steps, we first calculated the positive ideal separation matrix and the negative-ideal separation matrix. Then, the degree of gray relation between every alternative and PIS and NIS is calculated. A relative relational degree is then defined to

determine the ranking order of all alternatives by calculating the degree of gray relation to both the PIS and NIS simultaneously. In the second procedure, we first utilized the PFCIG operator to aggregate all the matrices. In the calculation steps of the traditional GRA, we first calculated the positive ideal separation matrix, Then, we calculated the degree of GRA relation between every alternative and PIS. Furthermore, to verify and demonstrate the practicality and effectiveness of the proposed method, two illustrative examples are given. Finally, we compare our proposed approach to the existing ones.

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