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# Proximal Support Vector Machine-Based Hybrid Approach for Edge Detection in Noisy Images

https://doi.org/10.1515/jisys-2017-0566 Received November 1, 2017; previously published online March 27, 2019.

**Abstract:** We propose a novel edge detector in the presence of Gaussian noise with the use of proximal support vector machine (PSVM). The edges of a noisy image are detected using a two-stage architecture: smoothing of image is first performed using regularized anisotropic diffusion, followed by the classification using PSVM, termed as regularized anisotropic diffusion-based PSVM (RAD-PSVM) method. In this process, a feature vector is formed for a pixel using the denoised coefficient's class and the local orientations to detect edges in all possible directions in images. From the experiments, conducted on both synthetic and benchmark images, it is observed that our RAD-PSVM approach outperforms the other state-of-the-art edge detection approaches, both qualitatively and quantitatively.

**Keywords:** Edge detection, proximal support vector machine, structure tensor, Hessian matrix, anisotropic diffusion, regularization.

# 1 Introduction

In images, the edges confer rapid change in image brightness distribution and play a significant role by extracting the rich information from images, making the interpretation easier by filtering out the unwanted features and providing significant information of the image content. Detection of edges is a low-level operation and is preliminary used in many applications such as computer vision, target recognition, image compression, pattern recognition, etc. The primary objective of edge detection techniques is to locate sharp changes or discontinuities in texture, surface color, depth, and orientation in a scene [15]. Constitutively, the important property of a good edge detector algorithm is that it should preserve edges such as curves, corners, and lines by extracting the important features from an image. Although the classical edge detection algorithms can obtain satisfactory results in clean or noiseless data, they usually give relatively poor performance in the presence of degradation or noise. This is due to the similar characteristics of noise and edges in terms of gradient information. Therefore, it is worthy to study and develop an efficient edge detector that is less sensitive to noise, is robust, and can extract true edges from noisy images.

#### 1.1 Related Work

A number of algorithms are available to identify abrupt changes in pixel intensity. In general, there are different classes of edge detection methods, such as derivative based [15, 31], Gaussian based [5], and statistical based [4, 14, 19, 22]. The primary aim of various classical methods that are based on derivative information, such as Sobel, Prewitt, or Roberts filters, is to detect edges in the image using the maximum along the gradient

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direction [15]. These gradient-based edge detectors generate a gradient map by approximating the first-order derivative of the image intensity surface. Besides the gradient operators, Laplacian operators or second-order derivatives are also commonly used for edge detection [15, 25]. These derivative-based edge detectors seem to perform well in the case of clean images. However, as the masks are always fixed in these approaches, they are sensitive to noise and need more computational time. As a result, the performance becomes inversely proportional to the noise level [12]. Further, to overcome the issue of sensitivity to noise, a very important as well as interesting explanation was given by Canny [5]. Basically, it is an optimal step edge detector that includes the concepts of good localization of edges, good detection, and only a single response for an edge. After that, many approaches based on the Canny algorithm were proposed and are available in the literature for edge detection in images [8, 9, 30]. Alternatively, several statistical-based edge detection algorithms have also been proposed by some authors [14, 19, 22]. The major pitfalls of these algorithms lie in the following facts: they (i) are highly sensitive to noise, (ii) result in discontinuous edges due to failure in differentiating between edges and textures in the image, and (iii) are computationally expensive. Moreover, transform-based approaches such as the wavelet transform method [32], neural network method [2, 35], fuzzy method [7, 17], genetic algorithm [11], and mathematical morphological method [21] were also introduced for edge detection. Unfortunately, these transform-based methods produce jagged and broken edges, and are sometimes less efficient than classical edge detectors. Therefore, it has been found that pre-processing of the noisy image using some filtering methods is essential [3]. For this, motivated by the Canny filter, linear techniques based on partial derivatives [18, 34] paved the initial foundation of noise reduction techniques. However, their fragile effectiveness in noise reduction gives rise to non-linear techniques [6, 23, 27, 28].

In the last decade, the support vector machine (SVM) learning algorithms have also been widely used for edge detection, data classification, pattern recognition, and object detection [33]. Due to their minimized structure risk framework, SVM methods are better than earlier methods [10, 26, 36]. SVM is formulated as a convex quadratic programming problem (QPP) that leads to a global optimal solution. However, the solution of the QPP requires high computational time  $O(m^3)$ , where m is the number of training points. To overcome this, Mangasarian and Wild [24] proposed a much faster and simpler version of standard SVM, the proximal SVM (PSVM). In PSVM, data samples are classified based on their proximity to one of the two parallel surfaces that have as far as possible distance from each other. The PSVM formulation leads to a strongly convex QPP, which requires less computational time in comparison to SVM, which is  $O(n^3)$ , where n is the number of features. PSVM is faster than standard SVM because it requires solving only the system of linear equations while SVM needs to solve a quadratic system.

## 1.2 Contributions

This study proposes a new two-stage edge detection architecture termed regularized anisotropic diffusionbased PSVM (RAD-PSVM), which combines a blend of derivative information (structure tensor and Hessian matrix) with PSVM. For noise reduction, we are using a regularized anisotropic filtering method with an advanced iterative solver [16]. After noise reduction, derivative information is employed to extract the local features of an image. For this, the following two matrices are widely used: (i) Hessian matrix and (ii) structure tensor. Both structure tensor and Hessian matrix explain the local shape orientations, characteristics, and curvature of structures. The major advantage of structure tensor over Hessian matrix is that it is more efficient in extracting continuous directions and has non-negative eigenvalues. Further, PSVM is trained using a new edge descriptor that uses first- as well as second-order derivatives. The given approach has been compared with other edge detectors, viz., Canny, Sobel, and PSVM. The experimental results suggest the superiority of the proposed approach for edge detection.

The rest of the paper is organized as follows. In Section 2, we briefly describe the advantages of SVM and PSVM. The proposed algorithm for edge detection is given in Section 3. In Section 4, we evaluate the experimental results. Finally, we conclude the work in Section 5.

## 2 SVM and PSVM

#### 2.1 SVM

SVM is a statistical approach-based machine learning method that is developed for binary classification problems. To classify m data samples in the n-dimensional real space  $R^n$ , a two-group classification problem is considered by SVM and a data sample matrix A is formed, where data sample  $a_j$  is the  $j^{th}$  row of A. A sample  $a_j$  is labeled as  $l_j \in \{1, -1\}$  or a diagonal matrix L with diagonal elements  $l_j$ . A data sample can be classified as

$$a_j. \varpi + \beta \ge 1 \quad l_j = +1,$$
 (1)

$$a_j.\varpi + \beta \le -1 \quad l_j = -1. \tag{2}$$

The optimal decision surface  $f(\varpi, \beta)$  is defined as

$$a.\varpi + \beta = 0, \quad \varpi \in \mathbb{R}^n, \quad \beta \in \mathbb{R},$$
 (3)

where  $\varpi$  is called weight vector and  $\beta$  is known as bias. The optimal decision surface is determined by maximizing  $\frac{1}{||w||}$  and can be given as

minimize 
$$\frac{1}{2}||\varpi||^2 + \nu \sum_{j=1}^m \zeta_j$$
 subject to: 
$$l_j(a_j.\varpi + \beta) \ge 1 - \zeta_j, \quad \forall_j, \quad \zeta_j \ge 0, \quad \forall j,$$

where  $\zeta_j$  (j = 1, ..., m) are non-negative slack variables and  $\nu$  is the regularization term. The solution of the above QPP [Eq. (4)] can be obtained by using any standard QPP solver.

#### 2.2 PSVM

Mangasarian and Wild [24] proposed a faster version of standard SVM, in a more general framework of the regularization network, called PSVM. In PSVM, the formulation of standard SVMs [Eq. (4)] is altered in two ways. First, the inequality constraints are replaced by equality constraints. Then, both weight vector  $\boldsymbol{\varpi}$  and bias  $\boldsymbol{\beta}$  are regularized instead of  $\boldsymbol{\varpi}$  alone, which leads to optimization problem in  $R^{(n+1)}$ , as follows:

minimize 
$$\frac{1}{2}(\varpi^T\varpi + \beta^2) + \frac{1}{2}\nu \sum_{j=1}^m \zeta_j^2$$
 subject to: 
$$l_i(a_i.\varpi + \beta) = 1 - \zeta_i.$$
 (5)

The planes  $a_j \cdot \varpi + \beta = \pm 1$  are called proximal planes and the data samples are clustered around them. The first term of the objective function  $(\varpi^T \varpi + \beta^2)$  is the reciprocal of the 2-norm distance squared between proximal planes. Equation (5) is a QPP with equality constraints with additional property of being strongly convex. The Karush–Kuhn–Tucker system with necessary and sufficient optimal conditions can be used to solve Eq. (5). The Lagrangian form of Eq. (5) is written as follows:

$$L(\varpi, \beta, \zeta, \lambda) = \frac{1}{2} (\varpi^T \varpi + \beta^2) + \frac{\nu}{2} \sum_{j=1}^m \zeta_j^2$$
$$- \sum_{j=1}^m \lambda_j (l_j (a_j \cdot \varpi + \beta) - 1 + \zeta_j). \tag{6}$$

Here,  $\lambda_j \in R$ , j = 1, 2, ..., m are Lagrange multipliers for each constraint [Eq. (5)]. Setting the gradients of the Lagrangian [Eq. (6)] equal to zero, with respect to  $(\varpi, \beta, \zeta, \lambda)$ , gives the following expressions for  $(\varpi, \beta, \zeta, \lambda)$ :

$$\varpi = \sum_{i=1}^{m} a_i l_i \lambda_i, \quad \beta = \sum_{j=1}^{m} l_j \lambda_j, \quad \zeta_j = \frac{\lambda_j}{\nu}.$$
(7)

Substituting these expressions of  $\varpi$ ,  $\beta$ , and  $\zeta_i$  in the following equation:

$$l_i(a_i \cdot \overline{w} + \beta) - 1 + \zeta_i = 0. \tag{8}$$

Now, solve the above equation for  $\lambda$  and substitute  $\lambda$  in the first two expressions of Eq. (7), to determine the optimal  $\varpi$  and  $\beta$  as

$$[\boldsymbol{\varpi} \quad \boldsymbol{\beta}] = \frac{I}{\nu} + M'M^{-1}M'Le, \tag{9}$$

where  $M = [A \quad e]$ .

# 3 Proposed Algorithm

SVM-based edge-finding approaches combined with derivative information have been employed in the theory of edge detection. In this work, a PSVM-based hybrid edge detection system is developed due to its simple and fast classification ability, and the regularized anisotropic filter is found to be very efficient for the removal of additive Gaussian noise in digital images.

## 3.1 Noise Reduction

The scale space-based noise reduction technique proposed by Witkin produces coarser-resolution denoised images by convolving the original noisy image with a Gaussian kernel. This model is theoretically considered as a linear diffusion equation, and filters the noise present in an image but also blurs the image textures and edges. To overcome the limitations of isotropic diffusion, an anisotropic diffusion model has been proposed by Perona and Malik in 1990. The key advantage of the Perona-Malik (PM) model is that it can be discretized on an image plane with variable diffusivity function [28]. However, due to the ill-posedness of the PM model, various regularizations have been proposed. Hence, to have clear edges and texture information from noisy images, denoising has been performed using regularized anisotropic diffusion filters [6, 16] and can be given as

$$U_{i,j}^{n+1} = U_{i,j}^{n} + \delta t \sum_{k \in N(i,j)} c_k(||\nabla U_{i,j}^{n}||) \cdot \nabla U_{k_n},$$
(10)

where  $U_{i,j}^0 = f_{i,j}$  is a noisy image,  $U_{i,j}$  is a denoised image,  $\nabla$  is the gradient operator,  $\delta t$  is the time step associated with the denoising process duration, and N(i,j) is the neighborhood of the pixel (i,j). The diffusion coefficient  $c_k(.)$  is a non-negative function of the magnitude of local gradient in the direction of neighborhood k of the pixel (i,j). This diffusion function is liable to diffuse as well as to preserve edges during the restoration of a noisy image and given as follows:

$$c(||\nabla U||) = \frac{1}{1 + \frac{||\nabla U_{\xi}||^2}{k^2}}.$$
 (11)

Here  $U_{\xi} = G_{\xi} * U$ ;  $G_{\xi}$  is a two-dimensional Gaussian kernel and k > 0 is served as the soft threshold that controls the rate of the diffusion for the employed diffusion process. This diffusion function obeys some properties as c(0) = 1,  $c(s) \to 0$  as  $s \to \infty$ .

## 3.2 Edge Detection

Traditional edge detection algorithms use either first-order derivative or second-order derivative information [13, 20]. As both values are important in locating the edge position, we have used a new edge descriptor using a blend of structure tensor and Hessian matrix of image:

$$S(U) = \varphi * (\phi(\sigma)ST(U) + \xi(\sigma)H(U)^{2}), \tag{12}$$

where ST, H, and  $\varphi$  are structure tensor, Hessian matrix, and Gaussian kernel, respectively. To make Hessian matrix a positive matrix, a square operation has been employed on Hessian matrix. Also, for edge descriptor, a weighted combination of structure tensor and Hessian matrix has been taken by multiplying with  $\phi(\sigma)$  and  $\xi(\sigma)$ , respectively. To ensure the characteristics of invariance, linearity under rescaling, rotation, and translation, the convolution product with a Gaussian kernel has been used. This method consists in structure tensor calculation using first-order derivatives and Hessian matrix calculation using second-order derivatives. A decision is then made on whether the pixel is an edge or not.

In this study, for each pixel, a vector is formed using the difference between the pixel under consideration and the neighborhood pixels. This results in an eight-component vector for each pixel except boundary pixels, as the differences cannot be calculated. The resultant vectors are used for the training purpose of PSVM. The edges used in the training process are diagonal, vertical, and horizontal.

In PSVM form, the image intensity plane *U* can be represented as

$$f(U) = \sum_{i=1}^{m} K(U, U_i) \varpi + \beta, \tag{13}$$

where *U* is the denoised image obtained by Eq. (10) and  $\varpi$  and  $\beta$  are the solutions obtained by Eq. (9). The trained PSVM [Eq. (13)] gives us a value corresponding to each pixel of an image. The obtained values give the probability of being an edge or not.

## 3.3 Implementation

In this section, we implement the proposed RAD-PSVM system and perform the experiments on various images. This system takes a noisy image as an input and processes it. In the first step, restoring the image is the key objective to have clear objects in the given noisy image. Several algorithms are used in the literature to perform denoising. To improve the signal-to-noise ratio and provide a better description of edges, the regularized anisotropic diffusion technique has been used in this paper. After restoration, features are extracted using the neighborhood information of the pixels.

#### 3.3.1 Training Phase

PSVM is trained over the extracted feature vectors. One of the advantages of linear PSVM is that there is only one penalty parameter ( $\nu$ ) to tune, which affects the performance of PSVM. In our experiment, a five-fold cross-validation (CV) method is used to choose  $\nu$ , which has been extensively used. The parameter  $\nu$  with the maximum accuracy for in-sample data is chosen. After conducting the five-fold CV method for in-sample data, the optimal values of v have been found. For this, input pixels are classified into a set of two classes. If a pixel is involved in the edge, it is classified as 1; otherwise, it is classified as 0 (i.e. non-edge class).

#### 3.3.2 Testing Phase

The edges in the test images have been detected using a trained RAD-PSVM model. The test images are shown in Figure 1. Similar to the training phase, a feature vector for the restored test image has been formed for each pixel by considering the neighborhood. Further, edge detection has been performed using the trained model. The algorithmic representation of our RAD-PSVM model is organized as follows (see Algorithm 1):



Figure 1: Test Images.

Algorithm 1: Algorithm of the RAD-PSVM model for edge detection

- Denoise all the training and testing images using the discussed regularized anisotropic filter. **Training Phase:**
- 2: Extract the edge label and feature vectors of denoised training images.
- 3: The RAD-PSVM model is trained over all of the above extracted features.
- The regularization parameter of RAD-PSVM is computed using the five-fold CV technique. **Testing Phase:**
- 5: Feature vectors are obtained from the denoised test images.
- Edge detection is performed using the trained RAD-PSVM model. All pixels of the testing image have been classified into edge and non-edge categories.
- 7: End.

# 4 Results and Discussion

The experimental study has been carried out to explore the effectiveness and adaptability of the proposed edge detection algorithm using gray scale images. For this, we have considered some natural and synthetic test images, as shown in Figure 1. The effectiveness of the present edge detection algorithm is compared with the some existing classical algorithms like the Sobel and Canny algorithms. Also, we have compared the results of the proposed RAD-PSVM edge detector with the results obtained by the PSVM approach. The threshold parameter (k) of the considered regularized anisotropic diffusion model is tweaked manually to obtain the best performance level. Moreover, linear PSVM has only one penalty parameter to tune, which may affect the performance of PSVM. In our experimental study, to choose the value of the penalty parameter, the five-fold CV method is used, which has been extensively used in conjunction with SVM. The value of the parameter that gives the maximum accuracy over the training data is chosen. To conduct the experiments for Canny and Sobel edge detectors, we have used edge function in MATLAB as edge(image, method) with standard thresholds chosen by MATLAB automatically. To evaluate the performance of the proposed RAD-PSVM under a noisy condition, identically and independently distributed random Gaussian noise with mean zero and different standard deviations [e.g.  $\sigma \in (5, 30)$ ] are added to degrade the images.

Figure 2 shows a comparison between our RAD-PSVM, PSVM, and classical approaches on a synthetic "square" image under different noisy environments. Rows 3 and 4 show the results of the PSVM edge detector and the proposed RAD-PSVM, respectively, whereas rows 1 and 2 depict the quality of edges using the Sobel and Canny approaches. Column 1 shows a clean image. Columns 2–7 show the evaluation of different edge detectors at noise levels 5, 10, 15, 20, 25, and 30, respectively. We can see the impact of the proposed RAD-PSVM edge detector from the edge map in row 4, especially in the noisy environment. When the noise level

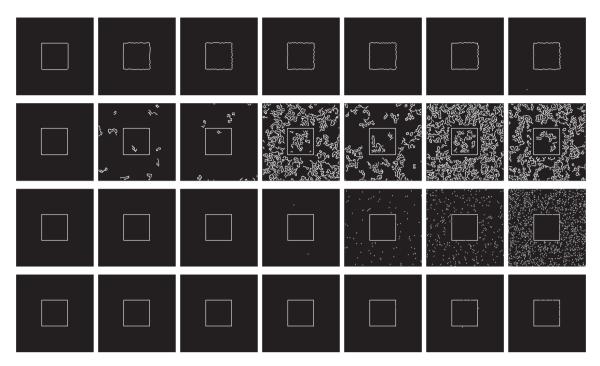


Figure 2: Comparison between our RAD-PSVM, PSVM, and classical approaches on a synthetic "square" image under different noisy environments.

Rows 1, 2, 3, and 4 show the edge map of the synthetic square image, obtained by the Sobel, Canny, PSVM, and the proposed RAD-PSVM edge detectors, respectively. Column 1 is for the clean image, whereas in columns 2-7, the added noise is 5, 10, 15, 20, 25, and 30 dB, respectively.

is low, the PSVM edge detector produces efficient results; however, the quality of edges decreases rapidly as the noise level increases, as shown by the edge map of row 3. This is due to the absence of a data-smoothing filter for high-frequency signals. Further, the Sobel edge detector gives better results than the Canny method, presented in the first and second rows, respectively. It is easy to conclude that the proposed RAD-PSVM outperforms PSVM as well as both classical approaches in most of the cases.

Now, in addition to the visual comparison of the detected edge maps for the "square" image, we have used some different quantitative metrics to objectively evaluate our algorithm, defined as follows:

- Figure of merit (FOM) [1], proposed by Abdou and Pratt in 1978, uses Euclidean distance to compare two images:

FOM = 
$$\frac{1}{\max(n_i, n_d)} \sum_{j=1}^{n_d} \frac{1}{(1 + \phi d_j^2)}$$
,

where  $d_i$  is the distance between the assumed edge pixel and the nearest ideal edge point,  $\phi$  is a constant (penalty), and  $n_d$  and  $n_i$  are the total numbers of found and ideal edge points, respectively. A higher value of FOM suggests that the detected edge map is closer to the ideal edge map.

The Jaccard index (JI) and dice coefficient (DC) [29] are used to evaluate the similarity between sample sets, and can be defined as

$$JI = \frac{|SO \cap GT|}{|SO \cup GT|},$$

$$DC = 2 * \frac{|SO \cap GT|}{|SO| + |GT|},$$

where SO and GT are the segmented and ground truth edge map, respectively. The values for both measures lie between 0 and 1 for no similarity and similarity between the images.

Hence, to get a clearer view, we compare the FOM results obtained by RAD-PSVM with those obtained by the other edge detectors using a line graph (see Figure 3). It can be noted from the graph that the present approach outperforms the other edge detectors. Similar to visual analysis, the graph of FOM values also suggests that the performance of the proposed RAD-PSVM and PSVM edge detectors are rather similar for low-level noise, but the proposed RAD-PSVM outshines PSVM as the noise increases.

Apart from the FOM value, we have also calculated the JI and DC for the edge map of the "square" image for different noise levels and approaches, as shown in Table 1. For ease of comparison, we have highlighted the higher values of JI and DC in each case. The higher values of both quantitative measures depict that the current method is better able to detect edges in comparison to the other discussed approaches. As the noise level increases, the edge maps obtained by the PSVM and Canny detectors first generate edge distortion with spurious features and then their metric JI and DC decrease, sharply. Meanwhile, the edge map generated by the proposed RAD-PSVM approach is closer to its original shape.

Figure 4 depicts the performance of different edge detection algorithms on various benchmark images, degraded with additive Gaussian noise of mean zero and standard deviations ( $\sigma$ ) = 10. In this figure, the results obtained by the proposed RAD-PSVM edge detector are shown in column 4, while the regularized anisotropic filtering is applied. Column 3 shows the results obtained using the PSVM edge detection algorithm. Columns 1 and 2 represent the results of the Sobel and Canny detectors, respectively. The edge maps obtained by the Sobel detector lose some fine structures of the image, whereas some false features were detected by the PSVM and Canny edge detectors (as shown in columns 2 and 3 of Figure 4). Moreover, the visual quality differences between the results obtained using Canny and the proposed RAD-PSVM method

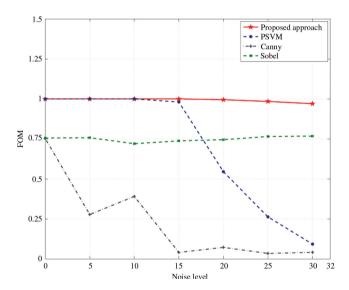


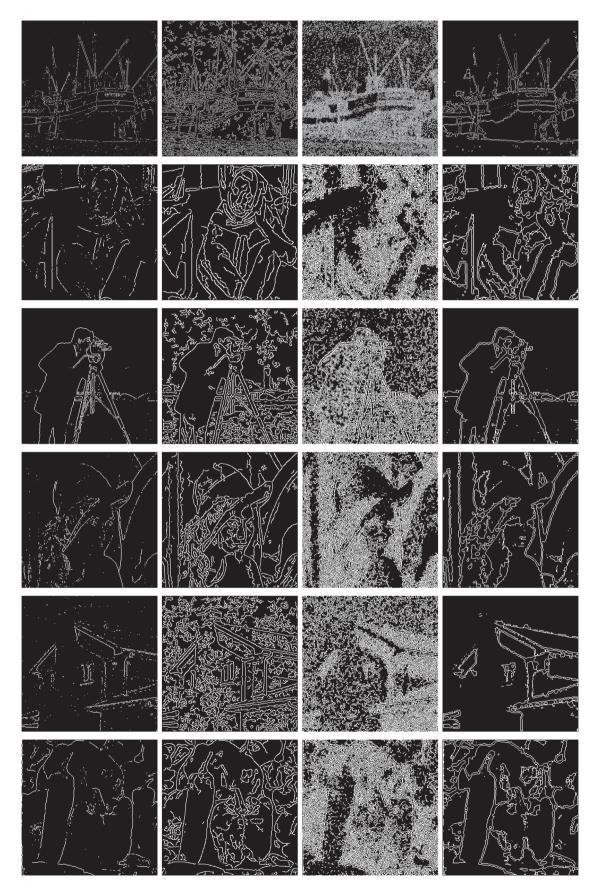
Figure 3: FOM for Different Edge Detectors.

**Table 1:** Comparison of Jaccard Index and Dice Coefficient for Various Schemes for "Square" Test Image with Clean and Noisy Images, Degraded by Additive Gaussian Noise of Zero Mean and Different Standard Deviations ( $\sigma$ ).

Noise level	Jaccard index				Dice coefficient			
	Sobel	Canny	PSVM	Proposed	Sobel	Canny	PSVM	Proposed
Clean (no noise)	0.3423	0.3422	1.0000	1.0000	0.5100	0.5099	1.0000	1.0000
$\sigma = 5$	0.3468	0.1577	1.0000	1.0000	0.5150	0.2725	1.0000	1.0000
$\sigma=10$	0.2821	0.2089	1.0000	1.0000	0.4400	0.3456	1.0000	1.0000
$\sigma = 15$	0.3115	0.0244	0.9804	1.0000	0.4750	0.0476	0.9901	1.0000
$\sigma = 20$	0.3245	0.0441	0.5408	0.9950	0.4900	0.0844	0.7019	0.997
$\sigma = 25$	0.3605	0.0200	0.2572	0.9706	0.5300	0.0392	0.4092	0.985
$\sigma = 30$	0.3767	0.0247	0.0873	0.9700	0.5473	0.0482	0.1606	0.985



Figure 4: Columns 1, 2, 3, and 4 show the edge images obtained by Sobel, Canny, PSVM, and the proposed RAD-PSVM method, respectively. The added Gaussian noise level is 10 dB.



**Figure 5:** Columns 1, 2, 3, and 4 show the edge images obtained by Sobel, Canny, PSVM, and the proposed RAD-PSVM method, respectively. The added Gaussian noise level is 20 dB.

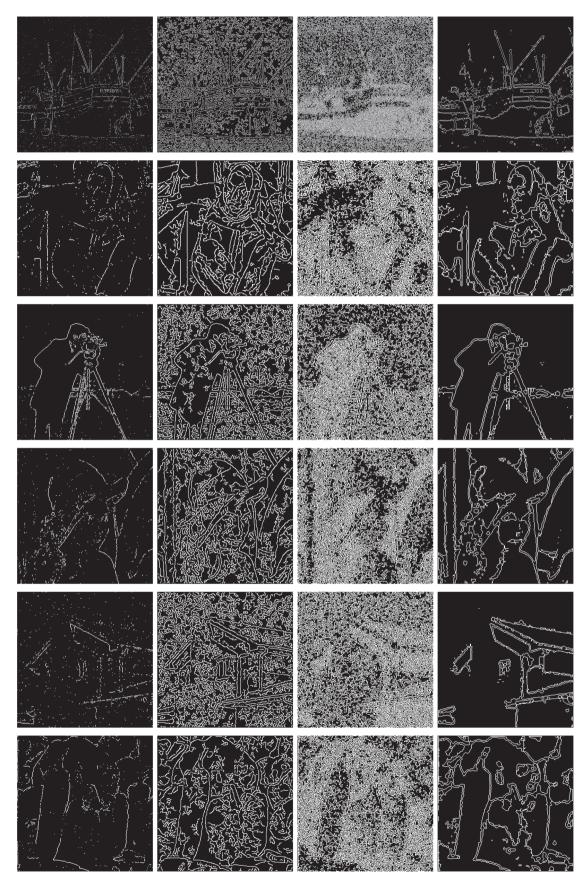


Figure 6: Columns 1, 2, 3, and 4 show the edge images obtained by Sobel, Canny, PSVM, and the proposed RAD-PSVM method, respectively. The added Gaussian noise level is 30 dB.

clearly indicate that the performance of the proposed edge detector is comparable to that of the Canny

Further, similar observations can also be made for noise levels of  $\sigma = 20$  and 30 (see Figures 5 and 6). These figures exhort the marginal gap between the quality of edges of RAD-PSVM and classical approaches (e.g. Sobel and Canny detectors). The edge map obtained by the Canny edge detector produces false edges in the homogeneous region and also blurs the edges due to noise, as shown in column 2 of Figures 5 and 6. Meanwhile, the other conventional edge detector, Sobel, leaves some isolated points of the noise particle in the edge map. Compared with the discussed classical approaches, it is easy to observe that the edge map captured by the proposed RAD-PSVM approach reflects the exact location of edges without generating false edges in the homogeneous area of the image.

Additionally, it is worth mentioning that although our proposed model detects edges very well in case of noisy images, the quality of detected edges can be improved by using some suitable post-processing techniques for edge thinning.

## 5 Conclusion

In this study, a new hybrid approach (RAD-PSVM) for edge detection in noisy images corrupted by Gaussian noise has been proposed using PSVM with the combination of structure tensor and Hessian matrix of the image. In the proposed RAD-PSVM edge detector, the non-linear diffusion filter is first adopted to obtain the noiseless gradient map by removing the noise and then edges are detected in all possible directions using PSVM. To test the effectiveness of the model, experiments on a variety of images have been performed. Furthermore, a comparison is made with the classical methods, which showed that the proposed RAD-PSVM is effective in detecting edges in images under noisy environments. Also, quantitative measures, namely FOM, JI, and DC, showed that the results obtained using the proposed approach are significantly higher than those obtained by the other methods considered in this study. The present RAD-PSVM model outperformed the other methods in terms of quantitative as well as qualitative results. Moreover, an experimental study on a variety of natural images degraded with different levels of noise revealed that the RAD-PSVM method is more efficient for low signal-to-noise ratio conditions and is able to preserve the significant features of a processed image. In addition, the application of the present RAD-PSVM approach reduces the computational complexity and produces better results irrespective of noise levels.

While the proposed edge detection algorithm can be easily adopted to detect the edges in noisy images, we expect a direct integration of edge detection and thinning to be superior. Therefore, future works will include the development of such a joint algorithm and investigation of its performance on different real-life images.

# **Bibliography**

- [1] I. E. Abdou and W. K. Pratt, Quantitative design and evaluation of enhancement/thresholding edge detectors, Proc. IEEE 67 (1979), 753-763.
- [2] I. N. Aizenberg, N. N. Aizenberg and J. Vandewalle, Precise edge detection: representation by Boolean functions, implementation on the CNN, in: Fifth IEEE International Workshop on Cellular Neural Networks and Their Applications Proceedings, pp. 301-306, IEEE, London, UK, 1998.
- [3] G. Aubert and P. Kornprobst, Mathematical Problems in Image Processing: Partial Differential Equations and the Calculus of Variations, vol. 147, Springer Science & Business Media, Berlin, Germany, 2006.
- [4] A. C. Bovik, T. S. Huang and D. C. Munson Jr, Nonparametric tests for edge detection in noise, *Pattern Recognit.* 19 (1986),
- [5] J. Canny, A computational approach to edge detection, IEEE Trans. Pattern Anal. Mach. Intell. (1986), PAMI-8, 679-698.
- [6] F. Catté, P.-L. Lions, J.-M. Morel and T. Coll, Image selective smoothing and edge detection by nonlinear diffusion, SIAM J. Numer. Anal. 29 (1992), 182-193.
- [7] E. De Micheli, B. Caprile, P. Ottonello and V. Torre, Localization and noise in edge detection, IEEE Trans. Pattern Anal. Mach. Intell. 11 (1989), 1106-1117.

- [8] R. Deriche, Optimal edge detection using recursive filtering, Int. J. Comput. Vis. 2 (1987), 167-187.
- [9] R. Deriche, Fast algorithms for low-level vision, IEEE Trans. Pattern Anal. Mach. Intell. 12 (1990), 78-87.
- [10] H. Gómez-Moreno, S. Maldonado-Bascón and F. López-Ferreras, Edge detection in noisy images using the support vector machines, in: Connectionist Models of Neurons, Learning Processes, and Artificial Intelligence, pp. 685-692, Springer, Berlin, Heidelberg, 2001.
- [11] M. Gudmundsson, E. A. El-Kwae and M. R. Kabuka, Edge detection in medical images using a genetic algorithm, IEEE Trans. Med. Imaging 17 (1998), 469-474.
- [12] R. M. Haralick and J. S. Lee, Context dependent edge detection, in: Proceedings CVPR'88, Computer Society Conference on Computer Vision and Pattern Recognition, pp. 223-228, IEEE, Ann Arbor, MI, USA, 1988.
- [13] C. Harris and M. Stephens, A combined corner and edge detector, in: Alvey Vision Conference, vol. 15, p. 50, Citeseer, Manchester, UK, 1988.
- [14] Z. Hou and T. Koh, Robust edge detection, Pattern Recognit. 36 (2003), 2083-2091.
- [15] A. K. Jain, Fundamentals of Digital Image Processing, Prentice-Hall Inc., Englewood Cliffs, NJ, 1989.
- [16] S. K. Jain, R. K. Ray and A. Bhavsar, Iterative solvers for image denoising with diffusion models: a comparative study, Comput. Math. Appl. 70 (2015), 191-211.
- [17] R. Katoch and R. K. Bhogal, Edge detection using fuzzy logic (fuzzy Sobel, fuzzy template, and fuzzy inference system), in: Intelligent Communication, Control and Devices, pp. 741-752, Springer, Singapore, 2018.
- [18] J. J. Koenderink, The structure of images, Biol. Cybernet. 50 (1984), 363-370.
- [19] I. Kokkinos, Boundary detection using f-measure-, filter- and feature- (F3) boost, in: European Conference on Computer Vision, pp. 650-663, Springer, Berlin, Heidelberg, 2010.
- [20] U. Köthe, Edge and junction detection with an improved structure tensor, in: Pattern Recognition, pp. 25-32, Springer, Berlin, Heidelberg, 2003.
- [21] J. Lee, R. Haralick and L. Shapiro, Morphologic edge detection, IEEE J. Robot. Autom. 3 (1987), 142-156.
- [22] D. H. Lim and S. J. Jang, Comparison of two-sample tests for edge detection in noisy images, J. R. Stat. Soc. Ser. D Stat. 51 (2002), 21-30.
- [23] R. Malladi and J. A. Sethian, Image processing: flows under min/max curvature and mean curvature, Graph. Models Image Process. 58 (1996), 127-141.
- [24] O. L. Mangasarian and E. W. Wild, Proximal support vector machine classifiers, in: Proceedings KDD-2001: Knowledge Discovery and Data Mining, pp. 77-86, San Francisco, CA, USA, 2001.
- [25] D. Marr and E. Hildreth, Theory of edge detection, Proc. R. Soc. Lond. B 207 (1980), 187-217.
- [26] K.-R. Müller, S. Mika, G. Rätsch, K. Tsuda and B. Schölkopf, An introduction to kernel-based learning algorithms, IEEE Trans. Neural Netw. 12 (2001), 181-201.
- [27] S. Osher and L. I. Rudin, Feature-oriented image enhancement using shock filters, SIAM J. Numer. Anal. 27 (1990), 919-940.
- [28] P. Perona and J. Malik, Scale-space and edge detection using anisotropic diffusion, IEEE Trans. Pattern Anal. Mach. Intell. 12 (1990), 629-639.
- [29] D. W. Shattuck, G. Prasad, M. Mirza, K. L. Narr and A. W. Toga, Online resource for validation of brain segmentation methods, Neurolmage 45 (2009), 431-439.
- [30] J. Shen and S. Castan, An optimal linear operator for step edge detection, CVGIP Graph. Models Image Process. 54 (1992),
- [31] I. Sobel, Neighborhood coding of binary images for fast contour following and general binary array processing, Comput. Graphics Image Process. 8 (1978), 127-135.
- [32] G. K. Srivastava, R. Verma, R. Mahrishi and S. Rajesh, A novel wavelet edge detection algorithm for noisy images, in: International Conference on Ultra Modern Telecommunications & Workshops, ICUMT'09, pp. 1-8, IEEE, St. Petersburg, Russia, 2009.
- [33] V. Vapnik, The Nature of Statistical Learning Theory, Springer Science & Business Media, Berlin, Germany, 2013.
- [34] A. P. Witkin, Scale-space filtering: a new approach to multi-scale description, in: IEEE International Conference on Acoustics, Speech, and Signal Processing, ICASSP'84, vol. 9, pp. 150-153, IEEE, San Diego, CA, USA, 1984.
- [35] H.-J. Yang and D.-Q. Liang, A new method of edge detection based on information measure and neural network, Acta Electron. Sin. 29 (2001), 51-53.
- [36] S. Zheng, J. Liu and J. W. Tian, A new efficient SVM-based edge detection method, Pattern Recognit. Lett. 25 (2004), 1143-1154.

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