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Trapezoidal Linguistic Cubic Fuzzy TOPSIS Method and Application in a Group Decision Making Program

https://doi.org/10.1515/jisys-2017-0560
Received October 30, 2017; previously published online March 20, 2019.

Abstract: The aim of this paper is to define some new operation laws for the trapezoidal linguistic cubic fuzzy number and Hamming distance. Furthermore, we define and use the trapezoidal linguistic cubic fuzzy TOPSIS method to solve the multi criteria decision making (MCDM) method. The new ranking method for trapezoidal linguistic cubic fuzzy numbers (TrLCFNs) are used to rank the alternatives. Finally, an illustrative example is given to verify and prove the practicality and effectiveness of the proposed method.

Keywords: Trapezoidal linguistic cubic fuzzy number, Trapezoidal linguistic cubic fuzzy TOPSIS method, MCDM, Numerical application.

1 Introduction

Multiple attribute decision making (MADM) issues are the essential research used in choice hypotheses. As the question assets are fuzzy and uncertain, the complicated properties in the choice issues are not consistently expressed as fresh numbers and some of them additionally can be discerned by fuzzy numbers, e.g. interval number, linguistic variable, IFN and so on. In such conditions, the acquisition office can exhibit an essential part of the cost decrease and provider determination is a standout amongst the most active elements of achieveing the best administration [13]. In a typical multi criteria decision making (MCDM) problem, different and normally contrary criteria are quickly set up to settle on a choice. A wide-ranging review and organization of the MCDM approach for vendor selection were carried out in [5]. In reality, the rating models of choices, and also rank weights of criteria, regularly have different sorts of ambiguity, imprecision or subjectiveness and one cannot always utilize the conventional basic leadership frameworks for these issues. Typically, no inimitable optimal solution exists for such problems and it is necessary to use a decision maker's performance to differentiate between solutions. MCDM has been an active area of research since the 1970s. Different methods have been offered by many researchers, including the analytic hierarchy process (AHP), the technique for order preference by similarity to ideal solution (TOPSIS) and MCDM. One of the most broadly used multi-criteria decision exploration methods is the TOPSIS method, which was proposed by Hwang and Yoon in 1981 [15], and extended by Yoon in 1987 [45], as well as by Hwang et al. in 1993 [16]. In the TOPSIS method, the optimal alternative is nearest to the positive ideal solution (PIS) and the farthest from the negative ideal solution (NIS). It controls inaccurate and imprecise data, mostly modeling social judgments. Numerous fuzzy TOPSIS methods and applications have been established since the 1990s, e.g. for supplier selection [35, 44], finance [2, 29], the power industry [48, 50], and negotiation problems [28]. In our study, we discuss a

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fuzzy extension of the TOPSIS method as presented by Chen [3]. The TOPSIS developed by Hwang and Yoon is a practical technique to solve MCDM problems.

Xu et al. [41] proposed a new aggregation method to solve a heterogeneous multiple attribute group decision-making (MAGDM) problem which involves real numbers, interval numbers, triangular fuzzy numbers (TFNs), trapezoidal fuzzy numbers (TrFNs), linguistic values and Atanassov's intuitionistic fuzzy numbers (AIFNs). Li et al. [19] proposed a method which can effectively avoid the failure caused by the use of inconsistent decision information and provides a decision-making idea for the case where "the truth be held in minority". Krishankumar et al. [18] presented a new two-tier decision-making framework with linguistic preferences for scientific decision making. Dwivedi et al. [7] proposed a generalized fuzzy TOPSIS method as a versatile evaluation model. The model is suitable for different types of fuzzy or interval-valued numbers, with or without subjective weights of criteria being defined by evaluators. Li et al. [20] developed a prospect value determination method based on multiple reference points (mRPs) under a trapezoidal intuitionistic fuzzy environment.

Furthermore, Wang and Li [34] proposed intuitionistic linguistic sets, intuitionistic linguistic numbers, intuitionistic two semantics and the Hamming distance between two intuitionistic two-semantics, and rank the alternatives by calculating the comprehensive membership degree to the ideal solution for each alternative. So, Chen and Liu [4] proposed the linguistic intuitionistic fuzzy numbers (LIFNs) which can combine linguistic variable (LV) and intuitionistic fuzzy numbers (IFNs). Li et al. [21] proposed some new operational laws including subtraction, division, then the entropy of LIFNs and the extended VIKOR method were proposed to deal with uncertain information with LIFNs. Liu and Wang [26] proposed some improved linguistic intuitionistic operational laws to overcome the weaknesses in linguistic operations. In addition, some aggregation operators for LIFNs were developed for different goals [23–25]. The intuitionistic linguistic variables are more correct for fast fuzzy data than the uncertain linguistic variables. The linguistic variables are relaxed to deal with qualitative data, in general, of necessity we allocate a linguistic set which comprises some linguistic terms, but in the applied application, we might have the choice for a linguistic term from the linguistic set to accurately express the assessment data for an impartial assessment. We can complete it using the indeterminate linguistic variables, but this is not accurate. We can then use the intuitionistic linguistic variables to increase the accuracy, by selecting a linguistic term, which is neighboring the evaluation data, from the linguistic set. Then we give a membership degree and a non-membership degree to this linguistic term. This creates an intuitionistic linguistic number. For example, we can appraise the appearence of a car using the linguistic set $S = (s_1, s_2, s_3, s_4, s_5, s_6, s_7) =$ {extremely poor, very poor, poor, slightly poor, fair, slightly good, good, very good, extremely good}. We may reflect the performance estimate result as better than "good" (s_5) and lower than "very good" (s_6), however, we can use the uncertain linguistic number $[s_5, s_6]$ to express this evaluation result, but this is not accurate, because it merely provides a range. In this situation, we can use an intuitionistic linguistic number to evaluate the car; firstly, we can give a linguistic term (s_6), then we give the membership degree (0.8) and non-membership degree (0) to s_6. So, the intuitionistic linguistic number is s_6, (0.8, 0.0). Of course, if we use a linguistic term (s_5), it will not fully fast the assessment data, because the membership degree will be 1 and non-membership degree is 0. If we use linguistic term (s_7), the membership degree to s_7 may be 0.7 and non-membership degree to s_7 may be 0.1, etc. So, we think the intuitionistic linguistic variables can express the assessment data more accurate than the uncertain linguistic variables. Yue [47] developed a new methodology for GDM problems in an intuitionistic fuzzy environment. In this model, the weights of decision makers are determined by using an extended TOPSIS technique. Ou et al. [27] proposed the linguistic intuitionistic fuzzy set TOPSIS method for LMCDMs, compared with the traditional TOPSIS method, which is different from the positive ideal solution, the negative ideal solution and the relative closeness degrees of alternatives, in addition, we designed an algorithm to finish the linguistic intuitionistic fuzzy set TOPSIS method for LMCDMs. Ren et al. [12] presented the positive (optimistic) and negative (pessimistic) information of each criterion provided by each decision maker and aggregate these by using weights of decision makers to obtain the hesitant fuzzy linguistic positive and negative ideal solutions. Pei et al. [51] developed the fuzzy linguistic multiset TOPSIS method for linguistic decision making, the method mainly consists of transformation, aggregation and exploitation phases.

Xu [38] developed a system based on the uncertain linguistic ordered weighted averaging (ULOWA) and the uncertain linguistic hybrid aggregation (ULHA) operators. Xu et al. [39] developed some new geometric aggregation operators, such as the intuitionistic fuzzy weighted geometric (IFWG) operator, the intuitionistic fuzzy ordered weighted geometric (IFOWG) operator, and the intuitionistic fuzzy hybrid geometric (IFHG) operator.

Dong et al. [6] introduced overcoming the drawback of using the single real number to represent membership degree and non-membership degree for intuitionistic linguistic set (ILS) and the concept of the interval-valued intuitionistic linguistic set (IVILS) through representing the membership degree and non-membership degree with intervals for ILS. Wan et al. [30] developed the construct of a novel bi-objective interval-valued intuitionistic fuzzy (IVIF) mathematical programming of minimizing the inconsistency index and meanwhile maximizing the consistency index, which solved by the technically developed linear goal programming approach. Wan et al. [31] developed the fuzzy mathematical programming method for solving heterogeneous multiattribute decision-making problems based on the linear programming technique for multidimensional analysis of preference. Wan et al. [33] derived the PIS, NIS and the criteria weights simultaneously, a new four-objective hesitant fuzzy mathematical programming model by minimizing the hesitant fuzzy positive ideal group inconsistency index (HFPGICI) and hesitant fuzzy negative ideal group inconsistency index (HFNGICI) as well as maximizing the HFPGCI and HFNGCI. Wan et al. [32] proposed a new general method to aggregate the attribute value vector into interval-valued intuitionistic fuzzy numbers (IVIFNs) under a heterogeneous MAGDM environment utilizing the relative closeness in the technique for order preference by its similarity to the ideal solution (TOPSIS). Xu et al. [40] developed as crisp numbers, intervals, intuitionistic fuzzy sets (IFSs), linguistic variables and hesitant fuzzy sets (HFSs).

Cubic sets introduced by Jun et al. [17], are the generalizations of fuzzy sets and intuitionistic fuzzy sets, in which there are two representations, one is used for the degree of membership and other is used for the degree of non-membership. The membership function is held in the form of an interval while non-membership is thought to be the normal fuzzy set. A fuzzy number is a quantity whose value is imprecise, rather than exact as is the case with "ordinary" (single-valued) numbers.

Any fuzzy number can be thought of as a function whose domain is a specified set (usually the set of real numbers), and whose range is the span of non-negative real numbers between, and including, 0 and 1000. Each numerical value in the domain is assigned a specific "grade of membership" where 0 represents the smallest possible grade, and 1000 is the largest possible grade. The technique of positive ideal and negative ideal points easily produces satisfactory results which are composed of the overall best criteria values and overall worst criteria values attainable. Linguistic and subjective evaluations take place in questionnaire form. Each linguistic variable has its own numerical value in the predefined scale. In classical trapezoidal linguistic cubic fuzzy numbers, these numerical values are exact numbers whereas in the fuzzy AHP method they are intervals between two numbers with the most likely value. Due to the nature of human beings, linguistic values can change from person to person. In these circumstances, taking the fuzziness into account will provide less risky decisions.

We are interested in GDM situations defined in linguistic contexts, that is, it is assumed that decision makers use linguistic values in the pairwise comparison of alternatives in preference relations. For instance, linguistic values as being "Low" or "High" could be used. It should be pointed out that the linguistic values may be organized in a linear fashion, as there is an apparent linear order among them. In any case, a quantification of the linguistic values is required in order to use them. Finally, we should point out that a joint treatment of the linguistic values, coming from the decision makers involved in the GDM problem, is considered here. On the one hand, this allows us to deal with the linguistic values in a unified fashion. On the other hand, it allows us to reconcile the semantics of the linguistic values in such a way that the individual consistencies are made comparable and, therefore, could be aggregated to arrive at the joint view at the optimization criterion.

Fahmi et al. [11] defined the triangular cubic fuzzy number and operational laws. Fahmi et al. [10] defined the weighted average operator of triangular cubic fuzzy numbers and the Hamming distance of the trapezoidal cubic fuzzy numbers (TCFN). In [8], Fahmi et al. defined the aggregation operators for

triangular cubic linguistic hesitant fuzzy sets which includes the generalized triangular cubic linguistic hesitant fuzzy weighted averaging (GTCLHFWA) operator, the generalized triangular cubic linguistic hesitant fuzzy weighted geometric (GTCHFWG) operator, the generalized triangular cubic linguistic hesitant fuzzy ordered weighted average (GTCLHFOWA) operator, the generalized triangular cubic linguistic hesitant fuzzy ordered weighted geometric (GTCLHFOWG) operator, the generalized triangular cubic linguistic hesitant fuzzy hybrid averaging (GTCLHFHA) operator and the generalized triangular cubic linguistic hesitant fuzzy hybrid geometric (GTCLHFHG) operator. Fahmi et al. [9] proposed the cubic TOPSIS method and cubic gray relation analysis (GRA) method. Finally, the proposed method is used for selection in sol-gel synthesis of titanium carbide nanopowders.

As we have discussed earlier that cubic sets are the generalization of intuitionistic fuzzy sets and a powerful tool to deal with fuzziness. Also, trapezoidal linguistic intuitionistic fuzzy sets are suitable to deal with fuzziness. However, there may be a situation where the decision maker may provide the degree of membership and nonmembership of a particular attribute in such a way that the membership degree is a trapezoidal linguistic interval fuzzy number and the non-membership degree is a trapezoidal linguistic fuzzy number. Therefore, to overcome this shortcoming we generalize the concept of trapezoidal linguistic intuitionistic fuzzy sets and introduce the concept of trapezoidal linguistic cubic fuzzy sets which are very suitable to be used for depicting uncertain or fuzzy information. If we take only one element in the membership degree of the trapezoidal linguistic cubic fuzzy number, i.e. instead of an interval we take a fuzzy number, than we get trapezoidal linguistic intuitionistic fuzzy numbers, similarly if we take the membership degree as a fuzzy number and the non-membership degree as beingequal to zero, than we get trapezoidal linguistic fuzzy numbers. Thus, motivated by the idea proposed by Li et al. [19], in this paper we first proposed a trapezoidal linguistic cubic fuzzy weighted geometric (TrLCFWG) operator and a trapezoidal linguistic cubic fuzzy TOPSIS method for a multi-attribute decision-making problem.

Despite having much related literature on the problem under consideration, the following aspects related to trapezoidal cubic linguistic fuzzy sets and their aggregation operators motivated the researchers to carry out an in-depth inquiry into the current study.

(1) The main advantages of the proposed operators are that these aggregation operators provided more accurate and precious results compared to the mentioned operators.
(2) We generalized the concept of the trapezoidal linguistic cubic fuzzy number, the trapezoidal linguistic intuitionistic fuzzy sets and introduce the concept of the trapezoidal linguistic cubic fuzzy number. If we take only one element in the membership degree of the trapezoidal linguistic cubic fuzzy number, i.e. instead of interval we take a fuzzy number, then we get trapezoidal linguistic intuitionistic fuzzy numbers, similarly if we take the membership degree as a fuzzy number and non-membership degree as being equal to zero, then we get trapezoidal linguistic fuzzy numbers.
(3) The objectives of the study include:
    Proposing a trapezoidal linguistic cubic fuzzy number, operational laws, Hamming distance.
    Establishing a MADM program approach based on the trapezoidal linguistic cubic fuzzy TOPSIS method.
    Providing illustrative examples of the MADM program.
(4) In order to testify to the application of the developed method, we apply the trapezoidal linguistic cubic fuzzy number in the decision making.
(5) The initial decision matrix is composed of LVs. In order to fully consider the randomness and ambiguity of the linguistic term, we convert LVs into the trapezoidal linguistic cubic fuzzy number, and the decision matrix is transformed into the trapezoidal linguistic cubic fuzzy decision matrix.
(6) The operator can fully express the uncertainty of the qualitative concept and trapezoidal linguistic cubic fuzzy operators can capture the interdependencies among any multiple inputs or attributes by a variable parameter. The aggregation operators can take into account the importance of the attributes weights. Nevertheless, sometimes, for some MAGDM problems, the weights of the attributes are important factors for the decision process.

In Section 2, we first introduce some basic definitions of the fuzzy set, cubic set, trapezoidal linguistic cubic fuzzy number, and Hamming distance. In Section 3, we propose a trapezoidal linguistic cubic fuzzy TOPSIS

method and a numerical example. In Section 4, we compare this with a different method. The paper is concluded in Section 5.

2 Preliminaries

Definition 1 ([1]): An Atanassov intuitionistic fuzzy set on H is a set $J = \left\{ \begin{matrix} h, \Gamma(h), \eta(h) \\ : h \in H \end{matrix} \right\}$ where Γ_J and η_j are membership and non-membership function, respectively $\Gamma_J(h) : h[\to 0, 1],\ h \in H \to \Gamma_J(h) \in [0, 1]; \eta_J(h) : h \to [0, 1], h \in H \to \eta_J(h) \in [0, 1]$ and $0 \le \Gamma_J(h) + \eta_J(h) \le 1$ for all $h \in H$. $\pi_J(h) = 1 - \Gamma_J(h) - \eta_J(h)$.

Definition 2 ([17]): Let H be a nonempty set. By a cubic set in H we mean a structure $F = \{h, \alpha(h), \beta(h) : h \in H\}$ in which α is an IVF set in H and β is a fuzzy set in H. A cubic set $F = \{h, \alpha(h), \beta(h) : h \in H\}$ is simply denoted by $F = \langle \alpha, \beta \rangle$. Denote by C^H the collection of all cubic sets in H. A cubic set $F = \langle \alpha, \beta \rangle$ in which $\alpha(h) = 0$ and $\beta(h) = 1$ (resp. $\alpha(h) = 1$ and $\beta(h) = 0$ for all $h \in H$ is denoted by 0 (resp. 1). A cubic set $D = \langle \lambda, \xi \rangle$ in which $\lambda(h) = 0$ and $\xi(h) = 0$ (resp. $\lambda(h) = 1$ and $\xi(h) = 1$) for all $h \in H$ is denoted by 0 (resp. 1).

Definition 3 ([17]): Let H be a non-empty set. A cubic set $F = (\alpha, \beta)$ in H is said to be an internal cubic set if $\alpha^-(h) \le \beta(h) \le \alpha^+(h)$ for all $h \in H$.

Definition 4 ([17]): Let H be a non-empty set. A cubic set $F = (\alpha, \beta)$ in H is said to be an external cubic set if $\beta(h) \notin (\alpha^-(h), \alpha^+(h))$ for all $h \in H$.

Definition 5 ([38]): Let $s = [s_\alpha, s_\beta]$, where $s_\alpha, s_\beta \in S$, s_α and s_β are the lower and the upper limits, respectively, we then call s the uncertain linguistic variable. Let S be the set of all uncertain linguistic variables. Consider any three uncertain linguistic variables $s = [s_\alpha, s_\beta]$, $s_1 = [s_{\alpha_1}, s_{\beta_1}]$ and $s_2 = [s_{\alpha_2}, s_{\beta_2}]$, then their operational laws are defined as

(1) $s_1 \oplus s_2 = [s_{\alpha_1}, s_{\beta_1}] \oplus [s_{\alpha_2}, s_{\beta_2}] = [s_{\alpha_1} \oplus s_{\alpha_2}, s_{\beta_1} \oplus s_{\beta_2}] = [s_{\alpha_1+\alpha_2}, s_{\beta_1+\beta_2}]$,
(2) $\lambda s = \lambda[s_\alpha, s_\beta] = [\lambda s_\alpha, \lambda s_\beta] = [s_{\lambda\alpha}, s_{\lambda\beta}]$, where $\lambda \in [0, 1]$;
(3) $s_1 \oplus s_2 = s_2 \oplus s_1$,
(4) $\lambda(s_1 \oplus s_2) = \lambda s_1 \oplus \lambda s_2$, where $\lambda \in [0, 1]$;
(5) $(\lambda_1 + \lambda_2)s = \lambda_1 s + \lambda_2 s$, where $\lambda_1, \lambda_2 \in [0, 1]$.

2.1 Trapezoidal Linguistic Cubic Fuzzy Number

Definition 6: Let $\tilde{b}$ be the TrLCFN on the set of real numbers, its interval value trapezoidal linguistic fuzzy set is defined as: $\lambda_{\tilde{b}}(h) = \begin{cases} s_\theta, \frac{(h-r)}{(s-r)}[\omega_b^-, \omega_b^+] & r \le h < s \\ [\omega_b^-, \omega_b^+] & s \le h < t \\ s_\theta, \frac{(t-h)}{(u-t)}[\omega_b^-, \omega_b^+] & t \le h < u \\ 0 & \text{otherwise} \end{cases}$ and its trapezoidal linguistic fuzzy set $\Gamma_{\tilde{b}}(h) = \begin{cases} s_\theta, \frac{s-h+(h-r)\eta_{\tilde{b}}}{(r-s)}, & r \le h < s \\ \eta_{\tilde{b}} & s \le h \le t \\ s_\theta, \frac{h-t+(u-h)\eta_{\tilde{b}}}{(u-t)} & t < h \le u \\ 0 & \text{otherwise} \end{cases}$ where $0 \le \lambda_{\tilde{b}}(h) \le 1, 0 \le \Gamma_{\tilde{b}}(h) \le 1$ and r, s, t, u are real numbers. The values of $\omega_{b^-}, \omega_{b^+}$ consequently the maximum values of interval value fuzzy set and $\eta_{\tilde{b}}$ minimum value of the fuzzy set. Then the TrLCFN $\tilde{b}$ basically denoted by $\tilde{b} = \{s_\theta, [(r, s, t, u)]; \langle [\omega_b^-, \omega_b^+], \eta_{\tilde{b}} \rangle\}$. Further, the TrLCFN reduced to a TLCFN. Moreover, if $\omega_b^- = 1, \omega_b^+ = 1$ and $\eta_{\tilde{b}} = 0$, if the TrLCFN $\tilde{b}$ is called a normal TrLCFN denoted as $\tilde{b} = \{s_\theta, [(r, s, t, u)]; [\langle (1, 1) \rangle, (0)]\rangle\}$. Therefore, the TrLCFN considered now can be regarded as generalized TLCFN. Such numbers remand the doubt information in a more flexible approach than normal fuzzy numbers as the values $\omega_b^-, \omega_b^+, \eta_{\tilde{b}} \in [0, 1]$ can be interpreted as the degree of confidence in the quantity characterized by r, s, t, u. Then $\tilde{b}$ is called the TrLCFN.

Example 1: Let $b = \{s_4,\ [0.4,\ 0.6,\ 0.8,\ 0.10];\ \langle[0.12,\ 0.14],\ 0.13\rangle\}$ be the TrLCFN on the set of real numbers, its interval value trapezoidal linguistic fuzzy set is defined as: $\lambda_{\tilde{b}}(h) = \begin{cases} s_4, \frac{(h-0.4)}{(0.6-0.4)}[0.12, 0.14] & 0.4 \le h < 0.6 \\ [0.12, 0.14] & 0.6 \le h < 0.8 \\ s_4, \frac{(0.8-h)}{(0.10-0.8)}[0.12, 0.14] & 0.8 \le h < 0.10 \\ 0 & \text{otherwise} \end{cases}$ and its trapezoidal linguistic fuzzy set $\Gamma_{\tilde{b}}(h) = \begin{cases} s_4, \frac{0.6-h+(h-0.4)(0.13)}{(r-s)}, & 0.4 \le h < 0.6 \\ 0.13 & 0.6 \le h \le 0.8 \\ s_4, \frac{h-0.8+(0.10-h)(0.13)}{(u-t)} & 0.8 < h \le 0.10 \\ 0 & \text{otherwise} \end{cases}$ Then $\tilde{b}$ is TrLCFN.

Definition 7: Let $b = \begin{Bmatrix} s_\theta, \\ [r, \\ s, \\ t, \\ u], \\ \langle[\omega_b^-, \\ \omega_b^+], \\ \eta_{\tilde{b}}\rangle \end{Bmatrix}$, $b_1 = \begin{Bmatrix} s_{\theta_1}, \\ [r_1, \\ s_1, \\ t_1, \\ u_1], \\ \langle[\omega_{b_1}^-, \\ \omega_{b_1}^+], \\ \eta_{\tilde{b}_1}\rangle \end{Bmatrix}$ and $b_2 = \begin{Bmatrix} s_{\theta_2}, \\ [r_2, \\ s_2, \\ t_2, \\ u_2], \\ \langle[\omega_{b_2}^-, \\ \omega_{b_2}^+], \\ \eta_{\tilde{b}_2}\rangle \end{Bmatrix}$ be TrLCFNs and $\lambda \ge 0$. Then $b^c = \{\alpha^c | \alpha \in b\} = s_\theta, [r, s, t, u], \langle[\omega_b^-, \omega_b^+], \eta_{\tilde{b}}\rangle$;

$$b_1 \cup b_2 = \left\{ \begin{matrix} s_{\theta(b_1)\cup\theta(b_2)}, [r_1 \cup r_2, s_1 \cup s_2, t_1 \cup t_2, u_1 \cup u_2], \langle[\omega_{b_1}^- \cup \omega_{b_2}^-, \omega_{b_1}^+ \cup \omega_{b_2}^+], \\ \eta_{\tilde{b}_1} \cap \eta_{\tilde{b}_2}\rangle \end{matrix} \right\};$$

$$b_1 \cap b_2 = \left\{ \begin{matrix} s_{\theta(b_1)\cap\theta(b_2)}, [r_1 \cap r_2, s_1 \cap s_2, t_1 \cap t_2, u_1 \cap u_2], \langle[\omega_{b_1}^- \cap \omega_{b_2}^-, \omega_{b_1}^+ \cap \omega_{b_2}^+], \\ \eta_{\tilde{b}_1} \cup \eta_{\tilde{b}_2}\rangle \end{matrix} \right\};$$

$$b_1 \oplus b_2 = \left\{ \begin{matrix} s_{\theta(b_1)+\theta(b_2)}, [r_1 + r_2 - r_1 r_2, s_1 + s_2 - s_1 s_2, t_1 + t_2 - t_1 t_2, u_1 + u_2 - u_1 u_2], \\ \langle[\omega_{b_1}^- + \omega_{b_2}^- - \omega_{b_1}^- \omega_{b_2}^-, \omega_{b_1}^+ + \omega_{b_2}^+ - \omega_{b_1}^+ \omega_{b_2}^+], \eta_{\tilde{b}_1}\eta_{\tilde{b}_2}\rangle \end{matrix} \right\};$$

$$h_1 \ominus h_2 = \left\{ \begin{matrix} s_{\theta(h_1)-\theta(h_2)}, [r_1 - r_2 + r_1 r_2, s_1 - s_2 + s_1 s_2, t_1 - t_2 + t_1 t_2, u_1 - u_2 + u_1 u_2], \\ \langle[\omega_{b_1}^- - \omega_{b_2}^- + \omega_{b_1}^- \omega_{b_2}^-, \omega_{b_1}^+ - \omega_{b_2}^+ + \omega_{b_1}^+ \omega_{b_2}^+], \eta_{\tilde{b}_1}\eta_{\tilde{b}_2}\rangle \end{matrix} \right\};$$

$$\lambda b = \left\{ \begin{matrix} \{\lambda b | a \in b\} = s_{\theta^\lambda(b)}, [1 - (1 - r)^\lambda], [1 - (1 - s)^\lambda], [1 - (1 - t)^\lambda], \\ [1 - (1 - u)^\lambda]; \langle[1 - (1 - \omega_{\tilde{b}}^-)^\lambda], [1 - (1 - \omega_{\tilde{b}}^+)^\lambda], (\eta_{\tilde{b}})^\lambda\rangle \end{matrix} \right\};$$

$$b^\lambda = \left\{ \{\alpha^\lambda | a \in b\} = [s_{\lambda\times\theta(b)}], [(r)^\lambda, (s)^\lambda, (t)^\lambda, (u)^\lambda], \langle[(\omega_{\tilde{b}}^-)^\lambda, (\omega_{\tilde{b}}^+)^\lambda], 1 - (1 - \eta_{\tilde{b}})^\lambda\rangle \right\}.$$

Example 2: Let $b = \begin{Bmatrix} s_2, [0.1, 0.2, \\ 0.3, 0.4], \\ \langle[0.6, 0.8], \\ 0.7\rangle \end{Bmatrix}$, $b_1 = \begin{Bmatrix} s_4, [0.4, \\ 0.6, 0.8, 0.10], \\ \langle[0.5, 0.7], \\ 0.6\rangle \end{Bmatrix}$ and $b_2 = \begin{Bmatrix} s_5, [0.7, 0.9, \\ 0.11, 0.13], \\ \langle[0.15, 0.17], \\ 0.16\rangle \end{Bmatrix}$ be the three TrLCFNs. Then $b^c = \langle s_2, [0.1, 0.2, 0.3, 0.4], [0.6, 0.8], 0.7\rangle$,

$$b_1 \cup b_2 = \left\{ \begin{matrix} s_{4\cup5}, [0.4 \cup 0.7, 0.6 \cup 0.9, 0.8 \cup 0.11, 0.10 \cup 0.13], \langle[0.5 \cup 0.15, 0.7 \cup 0.17], \\ 0.6 \cap 0.16\rangle \end{matrix} \right\};$$

$$b_1 \cap b_2 = \left\{ \begin{matrix} s_{4\cup5}, [0.4 \cap 0.7, 0.6 \cap 0.9, 0.8 \cap 0.11, 0.10 \cap 0.13], \langle[0.5 \cap 0.15, 0.7 \cap 0.17], \\ 0.6 \cup 0.16\rangle \end{matrix} \right\},$$

$$b_1 \oplus b_2 = \left\{ \begin{matrix} s_{4+5}, [0.4 + 0.7 - (0.4)(0.7), 0.6 + 0.9 - (0.6)(0.9), 0.8 + 0.11 - (0.8)(0.11), \\ 0.10 + 0.13 - (0.10)(0.13)], \langle[0.5 + 0.15 - (0.5)(0.15), 0.7 + 0.17 - (0.7)(0.17)], \\ (0.6)(0.16)\rangle = s_9, [0.82, 0.96, 0.822, 0.217], \langle[0.575, 0.751], 0.096\rangle \end{matrix} \right\},$$

$$b_1 \otimes b_2 = \left\{ \begin{matrix} s_{4\times 5}, [(0.4)(0.7), (0.6)(0.9), (0.8)(0.11), (0.10)(0.13)], [(0.5)(0.15), (0.7)(0.17)], \\ 0.6 + 0.16 - (0.6)(0.16)\rangle = s_{20}, [0.28, 0.54, 0.088, 0.013], \langle [0.075, 0.119], 0.664\rangle \end{matrix} \right\},$$

$$\lambda = 0.25, 0.25, 0.25, 0.25$$

$$\lambda b = \left\{ \begin{matrix} s_{2\times 0.25}, [1-(1-0.1)^{0.25}, 1-(1-0.2)^{0.25}, 1-(1-0.3)^{0.25}, 1-(1-0.4)^{0.25}], \\ \langle [1-(1-0.6)^{0.25}, 1-(1-0.8)^{0.25}], (0.7)^{0.25}\rangle = s_5, [0.0259, 0.0542, 0.0853, 0.1198], \\ \langle [0.2047, 0.3312], 0.916\rangle \end{matrix} \right\},$$

$$b^{\lambda} = \left\{ \begin{matrix} s_{2^{0.25}}, [(0.1)^{0.25}, (0.2)^{0.25}, (0.3)^{0.25}, (0.4)^{0.25}], \langle [(0.6)^{0.25}, (0.8)^{0.25}], 1-(1-0.7)^{0.25}\rangle = \\ s_{1.1892}, [0.5623, 0.6687, 0.7400, 0.7952], \langle [0.8801, 0.9457], 0.2599\rangle \end{matrix} \right\}.$$

Definition 8: Let $b_i = \left\{ \begin{matrix} s_{\theta_i}, [r_i, s_i, t_i, u_i], \\ \langle [\omega^-_{h_i}, \omega^+_{h_i}], \eta_{h_i}\rangle \end{matrix} \right\}$ $(i = 1, 2, \ldots, n)$ be a collection of TrLCFNs and let $\Psi = (\Psi_1, \Psi_2, \ldots, \Psi_n)^T$ be the weight vector of TrLCFNs h_i $(i = 1, 2, \ldots, n)$, where $\Psi_i \in [0, 1]$ $\sum_{i=1}^{n} \Psi_i = 1$. The trapezoidal linguistic cubic fuzzy weighted geometric operator is a mapping $h^n \to h$ such that $TrLCFWG(h_1, h_2, \ldots, h_n) = \bigotimes_{i=1}^{n} (h_i^{\Psi_i})$. If $\Psi = (\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})^T$, then the TrLCFWG operator reduces to the trapezoidal linguistic cubic fuzzy averaging operator: $TrLCFG(b_1, b_2, \ldots, b_n) = \bigotimes_{i=1}^{n} (b_i^{\frac{1}{n}})$.

$$\left\{ \begin{matrix} \text{TrLCFWG}(b_1, b_2, \ldots, b_n) = \\ \end{matrix} \begin{matrix} s_{\prod_{i=1}^{n} \theta(\alpha_i)^{\Psi_i}}, \left[\begin{matrix} \prod_{i=1}^{n} (r_i)^{\Psi_i} [\prod_{i=1}^{n} (s_i)^{\Psi_i}, \\ [\prod_{i=1}^{n} (t_i)^{\Psi_i}, \prod_{i=1}^{n} (u_i)^{\Psi_i} \end{matrix} \right], \\ \left\langle \left[\prod_{i=1}^{n} (\omega_i^-)^{\Psi_i}, \prod_{i=1}^{n} (\omega_i^+)^{\Psi_i} \right], \right. \\ \left. 1 - \prod_{i=1}^{n} (1-\eta_i)^{\Psi_i} \right\rangle \end{matrix} \right\}.$$

Definition 9: Let $b_1 = \left\{ \begin{matrix} s_{\theta_1}, [r_1, s_1, t_1, u_1]; \\ \langle [\omega^-_{b_1}, \omega^+_{b_1}], \eta_{b_1}\rangle \end{matrix} \right\}$ and $b_2 = \left\{ \begin{matrix} s_{\theta_2}, [r_2, s_2, t_2, u_2]; \\ \langle [\omega^-_{b_2}, \omega^+_{b_2}], \eta_{b_2}\rangle \end{matrix} \right\}$ be two TrLCFNs. The Hamming distance between b_1 and b_2 is defined as follows:

$$\underset{H}{d}(\widetilde{b}_1, \widetilde{b}_2) = \left\{ \begin{matrix} \frac{1}{12}[\mid s_{\theta_1-\theta_2} \mid, \langle [\mid r_1 - r_2 \mid + \mid s_1 - s_2 \mid + \mid t_1 - t_2 \mid + \\ \mid u_1 - u_2 \mid] + \max[\left\{ \mid \omega^-_{b_1} - \omega^-_{b_2} \mid, \mid \omega^+_{b_1} - \omega^+_{b_2} \mid \right\}, \mid \eta_{b_1} - \eta_{b_2} \mid]] \end{matrix} \right\}$$

the TrLCFN $b_1 = \left\{ \begin{matrix} s_{\theta_1}, [r_1, s_1, \\ t_1, u_1]; \\ \langle [\omega_{b_1^-}, \omega_{b_1^+}], \\ \eta_{b_1}\rangle \end{matrix} \right\}$ and $b_2 = \left\{ \begin{matrix} s_{\theta_2}, [r_2, s_2, \\ t_2, u_2]; \\ \langle [\omega_{b_2^-}, \omega_{b_2^+}], \\ \eta_{b_2}]\rangle \end{matrix} \right\}$ reduces to a TrLCF $b_1 = \left\{ \begin{matrix} s_{\theta_1}, [r_1, s_1, \\ t_1, u_1]; \\ \langle [1, 1], \\ 0\rangle \end{matrix} \right\}$ and $b_2 = \left\{ \begin{matrix} s_{\theta_2}, [r_2, s_2, \\ t_2, u_2]; \\ \langle [1, 1], \\ 0\rangle \end{matrix} \right\}$. If $\omega_{b_1^-} = 1, \omega_{b_2^-} = 1$ and $\omega_{b_1^+} = 1, \omega_{b_2^+} = 1$, if $\eta_{b_1} = 0$ and $\eta_{b_2} = 0$.

Example 3: Let $b_1 = \begin{Bmatrix} s_6, [0.2, 0.3, \\ 0.4, 0.5]; \\ \langle [0.11, 0.15], \\ 0.13 \rangle \end{Bmatrix}$ and $b_2 = \begin{Bmatrix} s_3, [0.4, \\ 0.6, 0.8, 0.10]; \\ \langle [0.7, 0.9], \\ 0.8 \rangle \end{Bmatrix}$ be two TrLCFNs. The Hamming distance between b_1 and b_2 is defined as follows:

$$\underset{H}{d}(\widetilde{b}_1, \widetilde{b}_2) = \left\{ \begin{array}{c} \frac{1}{12}[\mid s_{6-3} \mid, \langle [\mid 0.2 - 0.4 \mid + \mid 0.3 - 0.6 \mid + \mid 0.4 - 0.8 \mid \\ + \mid 0.5 - 0.10 \mid] + \max[\begin{Bmatrix} \mid 0.11 - 0.7 \mid, \\ \mid 0.15 - 0.9 \mid \end{Bmatrix}, \mid 0.13 - 0.8 \mid]] \\ \frac{1}{12}[\mid s_3 \mid, \langle [\mid 0.2 \mid + \mid -0.3 \mid + \mid -0.4 \mid + \mid 0.4 \mid] + \\ \max[\begin{Bmatrix} \mid -0.59 \mid, \\ \mid -0.75 \mid \end{Bmatrix}, \mid -0.67 \mid]] = s_{0.5125} \end{array} \right\}.$$

3 Trapezoidal Linguistic Cubic Fuzzy TOPSIS Method

The trapezoidal linguistic cubic fuzzy TOPSIS is a decision making technique. It is a goal based approach for finding the alternative that is closest to the ideal solution.

In this section, we apply a trapezoidal cubic fuzzy set to a linguistic TOPSIS method. We define a new extension of the trapezoidal linguistic cubic fuzzy TOPSIS method by using a trapezoidal cubic fuzzy set.

Step 1: Suppose that a trapezoidal linguistic cubic fuzzy TOPSIS method decision-making problem under multiple attributes has m students and n decision attributes. The framework of the trapezoidal linguistic cubic decision matric can be exhibit as follows:

$$\beta = \left\{ \begin{array}{c} s_{\theta_{11}}[r_{11}, s_{11}, t_{11}, u_{11}]\langle [B^-_{11}, B^+_{11}], \eta_{11} \rangle \\ s_{\theta_{12}}[r_{12}, s_{12}, t_{12}, u_{12}]\langle [B^-_{12}, B^+_{12}], \eta^S_{12} \rangle \ldots\ldots \\ s_{\theta_{n1}}[r_{n1}, s_{n1}, t_{n1}, u_{n1}]\langle [B^-_{n1}, B^+_{n1}], \eta_{1n} \rangle \\ s_{\theta_{12}}[r_{21}, s_{21}, t_{21}, u_{21}]\langle [B^-_{21}, B^+_{21}], \eta_{21} \rangle \\ s_{\theta_{22}}[r_{22}, s_{22}, t_{22}, u_{22}]\langle [B^-_{22}, B^+_{22}], \eta_{22} \rangle \ldots\ldots \\ s_{\theta_{n2}}[r_{n2}, s_{n2}, t_{n2}, u_{n2}]\langle [B^-_{n2}, B^+_{n2}], \eta_{2n} \rangle \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ s_{\theta_{m1}}[r_{m1}, s_{m1}, t_{m1}, u_{m1}]\langle [B^-_{m1}, B^+_{m1}], \eta_{m1} \rangle \\ s_{\theta_{m2}}[r_{m2}, s_{m2}, t_{m2}, u_{m2}]\langle [B^-_{m2}, B^+_{m2}], \eta_{m2} \rangle \ldots\ldots \\ s_{\theta_{mn}}[r_{mn}, s_{mn}, t_{mn}, u_{mn}]\langle [B^-_{mn}, B^+_{mn}], \eta_{mn} \rangle \end{array} \right\} \tag{1}$$

Step 2: Construct a normalized trapezoidal linguistic cubic fuzzy TOPSIS decision matrix $R = [\beta_{ij}]$. The normalized value β_{ij} is calculated as:

$$\beta = \left\{ \begin{array}{c} \frac{s_\theta}{\sqrt{\sum_{i=1}^{n} (s_{\theta_{ij}})^2}}, \left[\frac{r}{\sqrt{\sum_{i=1}^{n} (r_{ij})^2}}, \frac{s}{\sqrt{\sum_{i=1}^{n} (s_{ij})^2}}, \frac{t}{\sqrt{\sum_{i=1}^{n} (t_{ij})^2}}, \frac{u}{\sqrt{\sum_{i=1}^{n} (u_{ij})^2}} \right], \\ \left\langle \left[\frac{B^-}{\sqrt{\sum_{i=1}^{n} (B^-_{ij})^2}}, \frac{B^+}{\sqrt{\sum_{i=1}^{n} (B^+_{ij})^2}} \right], \frac{\eta}{\sqrt{\sum_{i=1}^{n} (\eta)^2}} \right\rangle \end{array} \right\}. \tag{2}$$

Step 3: Construct the weighted normalized trapezoidal linguistic cubic fuzzy TOPSIS decision matrix by multiplying the normalized trapezoidal linguistic cubic decision matrix by its associated weights. The weight vector $W = (w_1, w_2, \ldots w_n)$ collected of the isolated weights w_j $(j = 1, 2, 3, \ldots n)$ for each attribute C_j satisfying $\sum_{j=1}^{n} W_j = 1$. The weighted normalized value is deliberate by Equation (3)

$$B_j = v_{ij} w_j \tag{3}$$

where $0 \leq B_j \leq 1\, i = 1, 2, 3\ldots., m$ and $j = 1, 2, 3\ldots, n$.

Step 4: Determine the positive ideal solution (α^+) and negative ideal solution (α^-). The trapezoidal linguistic cubic fuzzy TOPSIS method positive-ideal solution (TrLCFPIS, α^+) and the trapezoidal linguistic cubic fuzzy negative-ideal solution (TrLCFNIS, α^-) are shown as

$$\begin{aligned} \alpha_i^+ &= \left\{ \begin{matrix} [s_{\theta_1}(r_1, s_1, t_1, u_1), s_{\theta_2}(r_2, s_2, t_2, u_2) \\ \ldots\ldots s_{\theta_n}(r_n, s_n, t_n, u_n)]\{(B_1^+, \eta_1)(B_2^+, \eta_2)\ldots\ldots\ldots(B_n^+, \eta_n) \end{matrix} \right\} \\ &= \max_i s_{\theta_{ij}}, \max_i(r_{ij}, s_{ij}, t_{ij}, u_{ij})\{max_i(B_{ij}^+)\}\{min_i(\eta_i)\}, \end{aligned} \tag{4}$$

$$\begin{aligned} \alpha_i^- &= \left\{ \begin{matrix} [s_{\theta_1}(r_1, s_1, t_1, u_1), s_{\theta_2}(r_2, s_2, t_2, u_2) \\ \ldots\ldots s_{\theta_n}(r_n, s_n, t_n, u_n)], \{(B_1^-, \eta_1)(B_2^-, \eta_2)\ldots\ldots\ldots(B_n^-, \eta_n) \end{matrix} \right\} \\ &= \min_i s_{\theta_{ij}}, \min_i(r_{ij}, s_{ij}, t_{ij}, u_{ij}), \{\min_i(B_1)\}\{\max_i(\eta_i)\}. \end{aligned} \tag{5}$$

Step 5: Calculate the separation measures, using the n-dimensional Euclidean distance. The separation of each candidate from the trapezoidal linguistic cubic positive ideal solution $q_i^* \langle [B^-, B^+], \eta \rangle$ is given as

$$q_i^+ \langle [B^-, B^+], \eta \rangle = \left\langle \frac{1}{12n} \left(\begin{matrix} \left| s_{\theta_{ij} - \theta_j} \right|, |r_{ij} - r_j| + |s_{ij} - s_j| + |t_{ij} - t_j| \\ + |u_{ij} - u_j|, \max\{ \left| B_{ij} - B_j^- \right|, \left| B_{ij} - B_j^+ \right|, \\ |\eta_{ij} - \eta_j| \} \end{matrix} \right) \right\rangle. \tag{6}$$

The separation of each candidate from the trapezoidal linguistic cubic negative ideal solution $q_i^- \langle [B^-, B^+], \eta \rangle$ is given as

$$q_i^- \langle [B^-, B^+], \eta \rangle = \left\langle \frac{1}{12n} \left(\begin{matrix} \left| s_{\theta_{ij} - \theta_j} \right|, |r_{ij} - r_j| + |s_{ij} - s_j| + |t_{ij} - t_j| \\ + |u_{ij} - u_j|, \max\{ \left| B_{ij} - B_j^- \right|, \left| B_{ij} - B_j^+ \right|, \\ |\eta_{ij} - \eta_j| \} \end{matrix} \right) \right\rangle. \tag{7}$$

Step 6: Calculate the relative closeness to the ideal solution. This progression comprehends the relative closeness to an ideal solution by the equation

$$Z_i = \frac{q_i^- \langle [B^-, B^+], \eta \rangle}{q_i^- \langle [B^-, B^+], \eta \rangle + q_i^+ \langle [B^-, B^+], \eta \rangle}. \tag{8}$$

3.1 Decision Support Algorithm

In this subsection, we present the MAGDM approach based on an extended trapezoidal cubic linguistic fuzzy TOPSIS method using TCFN. In the MAGDM, decision makers should firstly determine the preferences of alternatives on the criteria. Based on these preference values, decision makers can select the best alternative. Because the information available for decision makers is vague and imprecise under uncertain environments, the decision makers cannot easily use an exact value to estimate their preferences.

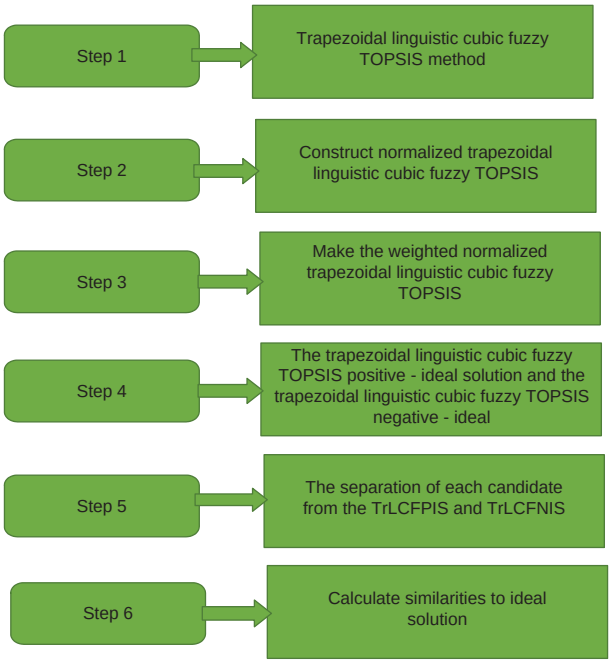

Figure 1: Flow Chart of the Extended Trapezoidal Linguistic Cubic Fuzzy TOPSIS Method.

Referring to Figure 1, the decision support algorithm will be as follows:

1. Suppose that a trapezoidal linguistic cubic fuzzy TOPSIS method decision-making problem under multiple attributes has m students and n decision attributes. After all the TrLCFN calculation provided by decision-makers of Eq. (1).
2. Determining normalized value r_{ij} by using Eq. (2)
3. The weight vector $W = (w_1, w_2, \ldots w_n)$ collected of the isolated weights $w_j (j = 1, 2, 3, \ldots n)$ for each attribute C_j satisfying $\sum_{j=1}^{n} W_j = 1$. The weighted normalized value is calculate of criteria using Eq. (3)
4. Identifying the trapezoidal linguistic cubic fuzzy TOPSIS method positive-ideal solution (TrLCFPIS, α^+) and the trapezoidal linguistic cubic fuzzy negative-ideal solution (TrLCFNIS, α^-) by using Eqs. (4)–(5)
5. Calculate distance from the trapezoidal linguistic cubic positive ideal solution $q_i^+ \langle [B^-, B^+], \eta \rangle$ and trapezoidal linguistic cubic negative ideal solution $q_i^- \langle [B^-, B^+], \eta \rangle$ separation measures by using Eqs. (6)–(7)
6. For each alternative the relative closeness to the ideal solution Z_i is computed by Eq. (8).
7. Ranking the alternatives according to the relative closeness coefficient.

3.2 Numerical Example

In this subsection we present a numerical example to illustrate the proposed approach. Suppose there is an investment company, which wants to invest a sum of money in the best choice (alternative). There is a board with three possible choices (alternatives) to invest the money: B_1 is a car company, B_2 is a food company, B_3 is a computer company as shown in Table 1. The investment company must take a decision according to the

Table 1: Linguistic Variables of Rating of Alternatives by Decision Maker.

	C_1	C_2	C_3
B_1	$\left\{ \begin{matrix} s_4, [0.1, 0.2, 0.3, 0.4], \\ \langle [0.10, 0.12], 0.11 \rangle \end{matrix} \right\}$	$\left\{ \begin{matrix} s_2, [0.2, 0.3, 0.4, 0.5], \\ \langle [0.7, 0.9], 0.8 \rangle \end{matrix} \right\}$	$\left\{ \begin{matrix} s_3, [0.5, 0.6, 0.7, 0.8] \\ \langle [0.1, 0.3], 0.2 \rangle \end{matrix} \right\}$
B_2	$\left\{ \begin{matrix} s_3, [0.5, 0.6, 0.7, 0.8], \\ \langle [0.1, 0.3], 0.2 \rangle \end{matrix} \right\}$	$\left\{ \begin{matrix} s_4, [0.1, 0.2, 0.3, 0.4], \\ \langle [0.10, 0.12], 0.11 \rangle \end{matrix} \right\}$	$\left\{ \begin{matrix} s_2, [0.2, 0.3, 0.4, 0.5], \\ \langle [0.7, 0.9], 0.8 \rangle \end{matrix} \right\}$
B_3	$\left\{ \begin{matrix} s_2, [0.2, 0.3, 0.4, 0.5], \\ \langle [0.7, 0.9], 0.8 \rangle \end{matrix} \right\}$	$\left\{ \begin{matrix} s_3, [0.5, 0.6, 0.7, 0.8], \\ \langle [0.1, 0.3], 0.2 \rangle \end{matrix} \right\}$	$\left\{ \begin{matrix} s_4, [0.1, 0.2, 0.3, 0.4], \\ \langle [0.10, 0.12], 0.11 \rangle \end{matrix} \right\}$

Table 2: Linguistic Variables of Ratings of Alternatives by Decision Maker.

	C_1	C_2	C_3
B_1	$\left\{\begin{matrix} s_6, [0.1, 0.3, 0.5, 0.7], \\ \langle [0.11, 0.13], 0.10\rangle \end{matrix}\right\}$	$\left\{\begin{matrix} s_4, [0.2, 0.4, 0.6, 0.8], \\ \langle [0.13, 0.15], 0.14\rangle \end{matrix}\right\}$	$\left\{\begin{matrix} s_3, [0.4, 0.8, 0.10, 0.12], \\ \langle [0.1, 0.3], 0.2\rangle \end{matrix}\right\}$
B_2	$\left\{\begin{matrix} s_3, [0.4, 0.8, 0.10, 0.12], \\ \langle [0.1, 0.3], 0.2\rangle \end{matrix}\right\}$	$\left\{\begin{matrix} s_6, [0.1, 0.3, 0.5, 0.7], \\ \langle [0.11, 0.13], 0.10\rangle \end{matrix}\right\}$	$\left\{\begin{matrix} s_4, [0.2, 0.4, 0.6, 0.8], \\ \langle [0.13, 0.15], 0.14\rangle \end{matrix}\right\}$
B_3	$\left\{\begin{matrix} s_4, [0.2, 0.4, 0.6, 0.8], \\ \langle [0.13, 0.15], 0.14\rangle \end{matrix}\right\}$	$\left\{\begin{matrix} s_3, [0.4, 0.8, 0.10, 0.12], \\ \langle [0.1, 0.3], 0.2\rangle \end{matrix}\right\}$	$\left\{\begin{matrix} s_6, [0.1, 0.3, 0.5, 0.7], \\ \langle [0.11, 0.13], 0.10\rangle \end{matrix}\right\}$

following three attributes: C_1 is the risk analysis, C_2 is the growth analysis, C_3 is the social-political impact analysis as shown in Table 2. The environmental impact refers to the impact on the companies' environment and the processes used in making the product, such as the management methods and work environment. The risk involves more than one risk factor, including product risk and development environment risk. The growth prospects include increased profitability and returns. The social political impact refers to the governments and local residents support for company. The three criteria are correlated with each other in the assessment process. The evaluation values r_{ij} $(i = 1, 2, 3)$, $(j = 1, 2, 3)$ should be in the form of a trapezoidal linguistic cubic fuzzy TOPSIS method which are provided by two decision-makers based on their knowledge and experience. In the case where two decision-makers give the same value, then it is counted repeatedly, and r_{ij} is the set of evaluation values for two decision-makers. Three faculty candidates (alternatives) B_i $(i = 1, 2, 3)$ are to be evaluated using the term set $S = \{s_0 =$ extremely poor; $s_1 =$ very poor; $s_2 =$ poor; $s_3 =$ slightly poor; $s_4 =$ fair; $s_5 =$ slightly good; $s_6 =$ good; $s_7 =$ very good; $s_8 =$ extremely good$\}$. The three possible alternatives B_i $(i = 1, 2, 3)$ are to be evaluated using the trapezoidal linguistic cubic fuzzy TOPSIS method information of two decision makers as presented in the following matrix $R = [r_{ij}]_{3\times 3}$ $(k = 1, 2, 3)$

$$v_1 = 0.5, v_2 = 0.3, v_3 = 0.2$$

Step 1: In this step, we aggregated the TrLCF-decision matrix D_1, D_2 based on the opinions of the experts after weights values for the experts are obtained, the evaluating values provided by different experts can be aggregated based on the TrLCFWG operator as: The aggregated TrLCF-decision matrix can be defined as follows Table 3:

Step 2: Construct a normalized trapezoidal linguistic cubic fuzzy TOPSIS decision matrix $R = [\beta_{ij}]$. The normalized value β_{ij} is calculated as (Table 4):

Step 3 : Make the weighted normalized trapezoidal linguistic cubic fuzzy TOPSIS decision matrix by multiplying the normalized trapezoidal linguistic cubic decision matrix by its associated weights. $w_1 = 0.3721, w_2 = 0.3057, w_3 = 0.3221$ (Table 5).

Table 3: The Aggregated TrLCF-Decision Matrix.

	C_1	C_2	C_3
B_1	$\left\{\begin{matrix} s_{4.8989}, [0.1, \\ 0.2449, 0.3872, \\ 0.5291], \langle [0.1048, \\ 0.1248], 0.1051\rangle \end{matrix}\right\}$	$\left\{\begin{matrix} s_{1.8660}, [0.3807, \\ 0.5293, 0.6517, \\ 0.7596], \langle [0.4872, \\ 0.5484], 0.4102\rangle \end{matrix}\right\}$	$\left\{\begin{matrix} s_{1.5518}, [0.7247, \\ 0.8634, 0.5875, \\ 0.6258], \langle [0.2511, \\ 0.6178], 0.5253\rangle \end{matrix}\right\}$
B_2	$\left\{\begin{matrix} s_{3.0000}, [0.4472, \\ 0.6928, 0.2645, \\ 0.3098], \langle [0.1, \\ 0.3], 0.2\rangle \end{matrix}\right\}$	$\left\{\begin{matrix} s_{2.5945}, [0.2511, \\ 0.4299, 0.5661, \\ 0.6825], \langle [0.2584, \\ 0.2871], 0.0644\rangle \end{matrix}\right\}$	$\left\{\begin{matrix} s_{1.5157}, [0.5253, \\ 0.6543, 0.7516, \\ 0.8325], \langle [0.6191, \\ 0.6699], 0.4746\rangle \end{matrix}\right\}$
B_3	$\left\{\begin{matrix} s_{4.0000}, [0.2, \\ 0.3464, 0.4898, \\ 0.6324], \langle [0.3016, \\ 0.3674], 0.5852\rangle \end{matrix}\right\}$	$\left\{\begin{matrix} s_{1.9331}, [0.6171, \\ 0.8023, 0.4503, \\ 0.4951], \langle [0.2511, \\ 0.4855], 0.3807\rangle \end{matrix}\right\}$	$\left\{\begin{matrix} s_{1.8881}, [0.3981, \\ 0.5696, 0.6842, \\ 0.7752], \langle [0.4057, \\ 0.4351], 0.0434\rangle \end{matrix}\right\}$

Table 4: The Normalized Trapezoidal Linguistic Cubic Fuzzy TOPSIS Decision Matrix.

C_1	C_2	C_3
$\left\{\begin{array}{c} s_{0.8246}, [0.0168, \\ 0.0412, 0.0651, \\ 0.0891], \langle [0.0176, \\ 0.02100], 0.0176\rangle \end{array}\right\}$	$\left\{\begin{array}{c} s_{0.3141}, [0.0641, \\ 0.0891, 0.1097, \\ 0.1278], \langle [0.0821, \\ 0.0923], 0.0691\rangle \end{array}\right\}$	$\left\{\begin{array}{c} s_{0.2612}, [0.1219, \\ 0.1453, 0.0989, \\ 0.1053], \langle [0.0422, \\ 0.1041], 0.0884\rangle \end{array}\right\}$
$\left\{\begin{array}{c} s_{0.6235}, [0.0929, \\ 0.1439, 0.0549, \\ 0.0643], \langle [0.0207, \\ 0.0623], 0.0415\rangle \end{array}\right\}$	$\left\{\begin{array}{c} s_{0.5392}, [0.0521, \\ 0.0893, 0.1176, \\ 0.1418], \langle [0.0537, \\ 0.0596], 0.0133\rangle \end{array}\right\}$	$\left\{\begin{array}{c} s_{0.3151}, [0.1091, \\ 0.1359, 0.1562, \\ 0.1731], \langle [0.1286, \\ 0.1392], 0.0986\rangle \end{array}\right\}$
$\left\{\begin{array}{c} s_{0.7941}, [0.0397, \\ 0.0687, 0.0972, \\ 0.1255], \langle [0.0598, \\ 0.0729], 0.1161\rangle \end{array}\right\}$	$\left\{\begin{array}{c} s_{0.3837}, [0.1225, \\ 0.1592, 0.0893, \\ 0.0982], \langle [0.0498, \\ 0.0963], 0.0755\rangle \end{array}\right\}$	$\left\{\begin{array}{c} s_{0.3748}, [0.0781, \\ 0.1131, 0.1358, \\ 0.1538], \langle [0.0805, \\ 0.0863], 0.0086\rangle \end{array}\right\}$

Table 5: The Weighted Normalized Trapezoidal Linguistic Cubic Fuzzy TOPSIS Decision Matrix.

C_1	C_2	C_3
$\left\{\begin{array}{c} s_{0.3068}, [0.0062, \\ 0.0153, 0.0242, \\ 0.0331], \langle [0.0065, \\ 0.0781], 0.0065\rangle \end{array}\right\}$	$\left\{\begin{array}{c} s_{0.0961}, [0.2096, \\ 0.2914, 0.3588, \\ 0.4181], \langle [0.2685, \\ 0.3019], 0.2261\rangle \end{array}\right\}$	$\left\{\begin{array}{c} s_{0.0841}, [0.3784, \\ 0.4511, 0.3071, \\ 0.3269], \langle [0.1311, \\ 0.3231], 0.2744\rangle \end{array}\right\}$
$\left\{\begin{array}{c} s_{0.2320}, [0.0345, \\ 0.0535, 0.0204, \\ 0.0239], \langle [0.0077, \\ 0.0231], 0.0154\rangle \end{array}\right\}$	$\left\{\begin{array}{c} s_{1.7638}, [0.1704, \\ 0.2921, 0.3846, \\ 0.4638], \langle [0.1756, \\ 0.1949], 0.0435\rangle \end{array}\right\}$	$\left\{\begin{array}{c} s_{0.1014}, [0.3387, \\ 0.4219, 0.4849, \\ 0.5374], \langle [0.3992, \\ 0.4321], 0.3061\rangle \end{array}\right\}$
$\left\{\begin{array}{c} s_{0.2954}, [0.0147, \\ 0.0255, 0.0361, \\ 0.0466], [0.0223, \\ 0.0271], 0.0432\rangle \end{array}\right\}$	$\left\{\begin{array}{c} s_{0.1172}, [0.4007, \\ 0.5207, 0.2921, \\ 0.3212], \langle [0.1629, \\ 0.3151], 0.2469\rangle \end{array}\right\}$	$\left\{\begin{array}{c} s_{0.1207}, [0.0251, \\ 0.0364, 0.0437, \\ 0.0495], \langle [0.2499, \\ 0.2679], 0.0027\rangle \end{array}\right\}$

Step 4: Calculate the positive ideal solution (α^*) and negative ideal solution (α^-). The trapezoidal linguistic cubic fuzzy TOPSIS method positive-ideal solution (TrLCFTPIS, α^*) and the trapezoidal linguistic cubic fuzzy TOPSIS method negative-ideal solution (TrLCFTNIS, α^-) is shown as follows:

$$\alpha_i^+ = \left\{\begin{array}{c} s_{0.3068}, [0.3784, \\ 0.4511, 0.3588, \\ 0.4181], \langle [0.2685, \\ 0.3231], 0.0065\rangle \end{array}\right\}, \left\{\begin{array}{c} s_{1.7638}, [0.3387, \\ 0.4219, 0.4849, \\ 0.5374], \langle [0.3992, \\ 0.4321], 0.0154\rangle \end{array}\right\}, \left\{\begin{array}{c} s_{0.2954}, [0.4007, \\ 0.5207, 0.2921, \\ 0.3212], \langle [0.2499, \\ 0.3151], 0.0027\rangle \end{array}\right\}.$$

$$\alpha_i^- = \left\{\begin{array}{c} s_{0.0841}, [0.0062, \\ 0.0153, 0.0242, \\ 0.0331], \langle [0.0065, \\ 0.0781], 0.2744\rangle \end{array}\right\}, \left\{\begin{array}{c} s_{0.1014}, [0.0345, \\ 0.0535, 0.0204, \\ 0.0239], \langle [0.0077, \\ 0.0231], 0.3061\rangle \end{array}\right\}, \left\{\begin{array}{c} s_{0.1172}, [0.0147, \\ 0.0255, 0.0361, \\ 0.0466], \langle [0.0223, \\ 0.0271], 0.2469\rangle \end{array}\right\}.$$

Step 5: Estimated separation measures, using the n-dimensional Euclidean distance. The separation of each candidate from the TrLCPIS $q_i^* \langle [B^-, B^+], \eta\rangle$ is given as $q_1^+ \langle [B^-, B^+], \eta\rangle = s_{1.3466}$, $q_2^+ \langle [B^-, B^+], \eta\rangle = s_{11.8025}$, $q_3^+ \langle [B^-, B^+], \eta\rangle = s_{1.3153}$. The separation of each candidate from the TrLCNIS $q_i^- \langle [B^-, B^+], \eta\rangle$ is given as the separation of each candidate from the TrLCNIS $q_i^- \langle [B^-, B^+], \eta\rangle$ is given as $q_1^- \langle [B^-, B^+], \eta\rangle = s_{0.7912}$, $q_2^- \langle [B^-, B^+], \eta\rangle = s_{8.1092}$, $q_3^- \langle [B^-, B^+], \eta\rangle = s_{0.3365}$.

Step 6: Calculate the similarities to the ideal solution. This progression comprehends the similitudes to an ideal solution by Eqs. $Z_1 = s_{0.3701}$, $Z_2 = s_{0.6807}$, $Z_3 = s_{0.2037}$.

4 Comparison Analyses

In order to verify the validity and effectiveness of the proposed approach, a comparative study is conducted using the methods of trapezoidal IFN [37], intuitionistic linguistic fuzzy number (ILFN) [22] and triangular cubic fuzzy number [11], which are special cases of TrLCTrFNs, to the same illustrative example.

4.1 A Comparison Analysis with the Existing MCDM Method Intuitionistic Trapezoidal Fuzzy Number

The intuitionistic trapezoidal fuzzy number can be considered as a special case of trapezoidal linguistic cubic fuzzy numbers (TrLCFNs) when there is only a four element in membership and a non-membership degree. For comparison, the intuitionistic trapezoidal fuzzy number (ITrFNs) can be transformed to ITrFNs by calculating the average value of the membership and non-membership degrees. After transformation, the ITrFNs information is given in Table 6

Then, we utilize the proposed procedure to get the most desirable alternative(s).

Step 1: Utilize the decision information given in the intuitionistic trapezoidal fuzzy decision matrix R_k, and the ITFWAA operator to derive the individual overall preference intuitionistic trapezoidal fuzzy values r_i^k of the alternative A_i and (whose weighting vector is $(0.25, 0.35, 0.40)^T$ as shown in Table 7

Step 2: Utilize the ITFHA operator to derive the collective overall preference intuitionistic trapezoidal fuzzy values r_i $(1, 2, \ldots, m)$ of the alternative A_i (Let $\omega = (0.30, 0.30, 0.40)^T$) as shown in Table 8.

Step 3: Calculate the score value

$Z_1 = 0.1561, Z_2 = 0.1700, Z_3 = 0.1975.$

Step 4: The ranking of all alternatives $Z_3 > Z_2 > Z_1$ and Z_3 is the best selection.

Obviously, the ranking derived from the method proposed by Wei [37], is different from the result of the proposed method. TrLCFNs are more flexible than ITrFNs because they consider the situations where decision-makers would like to use several possible values to express the membership and non-membership degrees.

Table 6: Intuitionistic Trapezoidal Fuzzy Decision Matrix.

	C_1	C_2	C_3
B_1	$\left\{\begin{matrix}[0.1, 0.2, 0.3, 0.4], \\ \langle[0.10, 0.12]\rangle\end{matrix}\right\}$	$\left\{\begin{matrix}[0.2, 0.3, 0.4, 0.5], \\ \langle[0.7, 0.9]\rangle\end{matrix}\right\}$	$\left\{\begin{matrix}[0.5, 0.6, 0.7, 0.8], \\ \langle[0.1, 0.3]\rangle\end{matrix}\right\}$
B_2	$\left\{\begin{matrix}[0.5, 0.6, 0.7, 0.8], \\ \langle[0.1, 0.3]\rangle\end{matrix}\right\}$	$\left\{\begin{matrix}[0.1, 0.2, 0.3, 0.4], \\ \langle[0.10, 0.12]\rangle\end{matrix}\right\}$	$\left\{\begin{matrix}[0.2, 0.3, 0.4, 0.5], \\ \langle[0.7, 0.9]\rangle\end{matrix}\right\}$
B_3	$\left\{\begin{matrix}[0.2, 0.3, 0.4, 0.5], \\ \langle[0.7, 0.9]\rangle\end{matrix}\right\}$	$\left\{\begin{matrix}[0.5, 0.6, 0.7, 0.8], \\ \langle[0.1, 0.3]\rangle\end{matrix}\right\}$	$\left\{\begin{matrix}[0.1, 0.2, 0.3, 0.4], \\ \langle[0.10, 0.12]\rangle\end{matrix}\right\}$

Table 7: Preference Intuitionistic Trapezoidal Fuzzy Values.

B_1	{[0.2, 0.275, 0.35, 0.425], [0.2978, 0.4242]}
B_2	{[0.28, 0.385, 0.49, 0.595], [0.3905, 0.3011]}
B_3	{[0.32, 0.44, 0.56, 0.68], [0.4321, 0.2536]}

Table 8: Utilize the ITFHA Operator.

B_1	{[0.06, 0.0825, 0.105, 0.1275], [0.1006, 0.7731]}
B_2	{[0.084, 0.1155, 0.147, 0.1785], [0.1381, 0.6976]}
B_3	{[0.128, 0.176, 0.224, 0.272], [0.2025, 0.5776]}

4.2 A comparison analysis with the existing MCDM method intuitionistic linguistic fuzzy numbers

ILFNs can be considered as a special case of TrLCFNs when decision makers only consider membership degrees in the evaluation. For comparison, the TrLCFNs can be transformed to ILFNs by retaining only the linguistic number, membership degrees, and non-membership degrees. After transformation, the ILFNs information is given in Table 9

Step 1: Calculate the comprehensive evaluation values r_i^k (suppose $\lambda = 1$) as shown in Table 10

Step 2: Calculate the degree of similarity $s(r_i^k, x_i)$ as shown in Table 11

Step 3: Calculate the score function $Z_1 = 0.4463, Z_2 = 0.6694, Z_3 = 1.2529$

Step 4: Rank all the alternatives. According to the ranking of score function $S(z_i)$, the ranking is $Z_3 > Z_2 > Z_1$.

The ranking of all alternatives $Z_3 > Z_2 > Z_1$ and Z_3 is the best selection. Obviously, the ranking derived from the method proposed by Liu [22], is different from the result of the proposed method. The main reasons are that an ILFN only consider the linguistic number, membership degrees of an element and non-membership degrees, which may result in information linguistic number are not equal.

4.3 A Comparison Analysis with the Existing MCDM Method Triangular Cubic Fuzzy Number

A triangular cubic fuzzy number can be considered as a special case of TrLCFNs when decision makers only consider membership degrees in an evaluation and non-membership degrees [9]. For comparison, the TrLCFNs can be transformed to the triangular cubic fuzzy number by retaining only the interval-valued TFN and TFN. After transformation, the triangular cubic fuzzy number information is given in Table 12

Step 1: Calculate the triangular cubic fuzzy hybrid aggregation (TCFHA) operator and $\omega = (0.2, 0.5, 0.3)$ as shown in Table 13

Step 3: Calculate of the score value

$Z_1 = 0.0167, Z_2 = 0.2064, Z_3 = 0.0667.$

Table 9: Intuitionistic Linguistic Fuzzy Decision Matrix.

	C_1	C_2	C_3
B_1	$\langle s_6, [0.11, 0.13]\rangle$	$\langle s_4, [0.13, 0.15]\rangle$	$\langle s_3, [0.1, 0.3]\rangle$
B_2	$\langle s_3, [0.1, 0.3]\rangle$	$\langle s_6, [0.11, 0.13]\rangle$	$\langle s_4, [0.13, 0.15]\rangle$
B_3	$\langle s_4, [0.13, 0.15]\rangle$	$\langle s_3, [0.1, 0.3]\rangle$	$\langle s_6, [0.11, 0.13]\rangle$

Table 10: Comprehensive Evaluation Values.

B_1	$\langle s_{2.5}, [0.0863, 0.2765]\rangle$
B_2	$\langle s_{3.25}, [0.0864, 0.2765]\rangle$
B_3	$\langle s_{6.5}, [0.1652, 0.0764]\rangle$

Table 11: Degree of Similarity.

B_1	$\langle s_{1.5}, [0.9137, 0.7235]\rangle$
B_2	$\langle s_{2.25}, [0.9137, 0.7235]\rangle$
B_3	$\langle s_{5.5}, [0.8348, 0.9236]\rangle$

Table 12: Triangular Cubic Fuzzy Decision Matrix.

	C_1	C_2	C_3
B_1	$\left\{\begin{array}{c}[0.1, 0.2, 0.3], \\ \langle[0.10, 0.12], 0.11\rangle\end{array}\right\}$	$\left\{\begin{array}{c}[0.2, 0.3, 0.4], \\ \langle[0.7, 0.9], 0.8\rangle\end{array}\right\}$	$\left\{\begin{array}{c}[0.5, 0.6, 0.7], \\ \langle[0.1, 0.3], 0.2\rangle\end{array}\right\}$
B_2	$\left\{\begin{array}{c}[0.5, 0.6, 0.7], \\ \langle[0.1, 0.3], 0.2\rangle\end{array}\right\}$	$\left\{\begin{array}{c}[0.1, 0.2, 0.3], \\ \langle[0.10, 0.12], 0.11\rangle\end{array}\right\}$	$\left\{\begin{array}{c}[0.2, 0.3, 0.4], \\ \langle[0.7, 0.9], 0.8\rangle\end{array}\right\}$
B_3	$\left\{\begin{array}{c}[0.2, 0.3, 0.4], \\ \langle[0.7, 0.9], 0.8\rangle\end{array}\right\}$	$\left\{\begin{array}{c}[0.5, 0.6, 0.7], \\ \langle[0.1, 0.3], 0.2\rangle\end{array}\right\}$	$\left\{\begin{array}{c}[0.1, 0.2, 0.3], \\ \langle[0.12, 0.14], 0.13\rangle\end{array}\right\}$

Table 13: Triangular Cubic Fuzzy Hybrid Aggregation Operator.

B_1	$[0.16, 0.22, 0.28], \langle[0.2464, 0.4273], 0.4457\rangle$
B_2	$[0.4, 0.55, 0.7], \langle[0.5071, 0.7518], 0.1326\rangle$
B_3	$[0.24, 0.33, 0.42], \langle[0.3502, 0.5695], 0.3129\rangle$

Step 4: The ranking is $Z_2 > Z_3 > Z_1$. The ranking of all alternatives $Z_2 > Z_3 > Z_1$ and Z_2 is the best selection. Obviously, the ranking derived from the method proposed by Fahmi et al. [11], is different from the result of the proposed method. The main reasons are that a triangular cubic fuzzy number only consider the interval-valued TFN and TFN, which may result in information are equal.

The ranking values of those discussion are given in Table 14 and shown in Figure 2

The following advantages of our proposal can be summarized on the basis of the given comparison analyses. TrLCFNs are very suitable for illustrating uncertain or fuzzy information in MCDM problems because the membership and non-membership degrees can be two sets of several possible values, which cannot be achieved by ITrFNs, ILFNs and triangular cubic fuzzy number. On the bases of basic operations, aggregation operators and comparison method of trapezoidal linguistic cubic fuzzy number can be also used to process intuitionistic trapezoidal fuzzy numbers, ILFN and triangular cubic fuzzy number after slight adjustments, because TrLCNs can be considered as the generalized form of ITrFNs, ILFNs and triangular cubic fuzzy number. The defined operations of TrLCNs give us more accurate than the existing operators.

Table 14: The Ranking Values of the Discussed Methods.

Method	Ranking
Trapezoidal linguistic cubic fuzzy TOPSIS method	$Z_2 > Z_1 > Z_3$
Intuitionistic trapezoidal fuzzy number [37]	$Z_3 > Z_2 > Z_1$
Intuitionistic linguistic fuzzy number [22]	$Z_3 > Z_2 > Z_1$
Triangular cubic fuzzy number [11]	$Z_2 > Z_3 > Z_1$

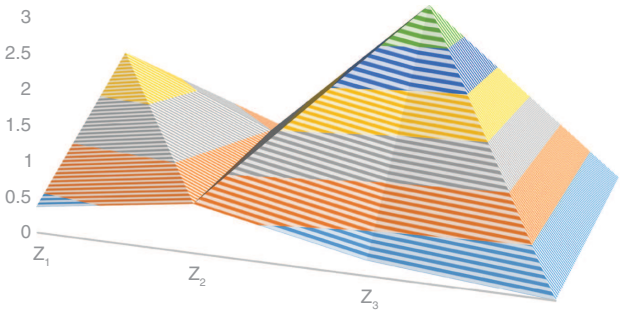

Figure 2: Ranking Value.

4.4 Discussion

Compared with other methods, the advantages of the trapezoidal linguistic cubic fuzzy TOPSIS method are shown as follows:

(1) Comparing with the intuitionistic trapezoidal fuzzy number by Wei [37], they are only the special cases of the proposed operator in this paper. The intuitionistic trapezoidal fuzzy number proposed by Wei [37] is based on the membership and non-membership, algebraic operations, and those in this paper is based on a trapezoidal linguistic cubic fuzzy TOPSOS method.

(2) The existing decision-making methods based on prospect theory in the literature only express the preferences of alternatives on criteria with crisp values, fuzzy numbers, and linguistic variables. However, due to the complexity of the socio-economic environment, there may be hesitation about preferences in decision-making. Recently, prospect theories under LIFNs and trapezoidal intuitionistic fuzzy information have been developed, such as [4] and [19], which also consider the hesitation about preferences in decision-making. However, LIFS and TrIF can only express the extent to which a criterion to a fuzzy concept "Excellence" or "Good" and they only use discrete domains. The TrLCFNs method is the extending of LIFS which extend discrete sets to continuous sets. Compared with LIFS and TrIF, TrLCFNs used in our proposed method, by introducing two trapezoidal intuitionistic fuzzy numbers as a reference, can describe and character the fuzziness of the objective world meticulously and accurately, it also allows criteria to use different dimensions. Thus, compared with the previous decision-making methods, the proposed method can express more abundant and flexible information, thus have a stronger expression ability to deal with the uncertain information.

(3) The existing triangular cubic fuzzy number based on an interval-valued TFN and TFN assume that decision-makers are totally rational, but in most practical decision-making problems, decision makers have bounded rationality under uncertainty. Thus, these methods can deal with this complex situation. Compared with the triangular cubic fuzzy number based on the interval-valued TFN and TFN, the proposed method fully considers the decision makers' bounded rationality for decision-making and incorporates the decision maker's risk psychological factors into decision-making, which is more reasonable and thus more applicable to practical problems.

5 Conclusion

In this paper, we initiated the concept of TrLCFNs and defined some operational laws. We initiated the concept of the trapezoidal linguistic cubic fuzzy TOPSIS method. The concept of the trapezoidal linguistic cubic fuzzy number is the generalization of cubic number, trapezoidal linguistic intuitionistic fuzzy numbers, trapezoidal linguistic cubic fuzzy numbers, and interval-valued trapezoidal fuzzy numbers. As investigated in the introduction that trapezoidal linguistic cubic fuzzy numbers fuzzy numbers become cubic fuzzy number when removing the linguistic numbers form it, trapezoidal linguistic cubic fuzzy number become trapezoidal ILFN if we take only fuzzy number instead of interval in the membership degree, trapezoidal linguistic cubic fuzzy numbers become interval-valued linguistic fuzzy number if we remove non-membership degree from it and trapezoidal linguistic cubic fuzzy numbers becomes linguistic intuitionistic fuzzy numbers if we remove the non-membership degree and take fuzzy number instead of interval in the membership degree. The trapezoidal linguistic cubic fuzzy information is more abundant and flexible than cubic sets, TrILFSs, LIFS, IVLIFSs. We propose a new decision method to solve the MCDM problems. We assumed that the ratings of alternatives on the given attributes are expressed using trapezoidal linguistic cubic fuzzy TOPSIS method and the weight of attributes is completely unknown. The relative distance of the alternatives is then obtained. It satisfies the closest to the trapezoidal linguistic cubic fuzzy TOPSIS positive ideal solution and the farthest from the trapezoidal linguistic cubic fuzzy TOPSIS negative ideal solution simultaneously, and finally, the alternatives are ranked based on the main trapezoidal linguistic cubic fuzzy operations. Finally, we compared the proposed method to the existing methods, which shows the trapezoidal linguistic cubic fuzzy TOPSIS method is more flexible to deal uncertainties and fuzziness. In fact, this method is very simple

and flexible. Hence, it is expected that what is proposed in this study may have more potential management applications, that is, the group decision-making problems, because the experts usually come from different specialty fields and have different backgrounds and levels of knowledge. These operators can be applied to many other fields, such as information fusion, economics, sports, computer science, and pattern recognition, which may be a possible topic for future research. In the future, we shall apply the distance measure of the cubic intuitionistic fuzzy set to other domains, such as pattern recognition, clustering analysis, algebraic structure, and medical diagnosis. Also, in the future, we shall apply the distance measure of the cubic triple linguistic fuzzy set to the other fields, such as kidney stage 1, 2, 3, 4, 5, liver and stomach patients. A future study can be extended to cover interactions and inner or outer dependencies among criteria or alternatives with the trapezoidal cubic uncertain fuzzy ANP to verify the findings of the present study.

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