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Fuzzy Adaptive Genetic Algorithm for Improving the Solution of Industrial Optimization Problems

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Abstract: In the industrial and manufacturing fields, many problems require tuning of the parameters of complex models by means of exploitation of empirical data. In some cases, the use of analytical methods for the determination of such parameters is not applicable; thus, heuristic methods are employed. One of the main disadvantages of these approaches is the risk of converging to "suboptimal" solutions. In this article, the use of a novel type of genetic algorithm is proposed to overcome this drawback. This approach exploits a fuzzy inference system that controls the search strategies of genetic algorithm on the basis of the real-time status of the optimization process. In this article, this method is tested on classical optimization problems and on three industrial applications that put into evidence the improvement of the capability of avoiding the local minima and the acceleration of the search process.

Keywords: Optimization, genetic algorithms, fuzzy inference system, adaptive genetic algorithms, industrial problems.

2010 Mathematics Subject Classification: 68T37.

1 Introduction

One of the most frequent problems within the industrial and manufacturing fields is the tuning of the parameters of existing empirical or theoretical models for enhancing the performance of a process or the quality of a product on the basis of experimental data. In these cases, the task is driven by the idea of improving the available knowledge (theoretical or empirical) on a general phenomenon – usually formalized as a model – by adapting such general model to the peculiarities of a particular plant or process that can be extracted from the collected data.

Available data are normally used for tuning a subset of the internal parameters of the model. This latter operation can be performed either manually, if the effect of the modification of each parameter on the behavior of the model is known and the number of parameters to tune is limited, or by means of analytical techniques that exploit the model and the available data within an optimization problem that can be expressed as finding the global minimum x^* of an arbitrary objective function

$$f:\Gamma \to \mathbb{R}, \tag{1}$$

where the compact set $\Gamma \subset \mathbb{R}^D$ is a D-dimensional parallelepiped and the minimization problem corresponds to finding the point x^* so that

$$f(x^*) \le f(x),\ \forall x \in \Gamma. \tag{2}$$

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Unfortunately, analytical techniques are not always applicable, for instance in the presence of objective functions that are non-linear and non-differentiable; moreover, their performance is strongly affected by the presence of noise and outliers in the data used for the tuning. These situations are very frequent in the industrial and manufacturing fields, and often prevent the use of traditional analytical techniques. For this reason, in the last decades, more sophisticated methods based on probabilistic approaches, artificial intelligence, and heuristics have been employed with satisfactory results on a multitude of practical real-world problems. For instance, artificial neural networks (ANNs) have been successfully used by Cabrera and Elbuluk [2] for tuning the main parameters of the control of an induction motor of a tool designed for assembly purposes in the manufacturing industry. Chen and Huang [3] employed ANNs for tuning the parameters of a proportional–integral–derivative controller that faces the inherent time-varying non-linearity and complexity of a chemical process. Further, the use of evolutionary algorithms (EAs), such as simulated annealing, genetic algorithms (GAs), and particle swarm optimization, is very popular in this field. For example, Colla et al. [4] used GAs for the tuning of a model for the estimation of the mechanical properties of steel.

When analytical techniques are not applicable, the advantages of EAs are manifold: free derivative characteristics, simple preparation of the optimization model, and parallel nature. However, one crucial issue for the success of EAs is the capability of avoiding the premature convergence to suboptimal regions that leads to the selection of local minima instead of the global one. This problem is related to the characteristics of the model to tune and, by consequence, of the surface of the objective function: because of a high number of variables whose interaction is complex and highly non-linear, the quality of the data used for the tuning usually generates a complex surface objective function, characterized by a high number of local minima that increase the risk of converging to suboptimal solutions. Unfortunately, these situations are often encountered in the manufacturing field and limit the effective performance (or even the usability) of EA-based approaches.

Successful EAs should efficiently explore the objective function domain, taking into account computational constraints and actively avoiding the potentially “dangerous” situations. In the study by Vannucci and Colla [26], a novel adaptive GA approach expressly designed for avoiding local minima and based on the real-time adjustment of search strategies is proposed and tested on literature problems. In this article, the capabilities of this novel GA are assessed on real-world problems related to industrial applications in order to put into evidence its practical advantages.

The article is organized as follows: in Section 2, an introduction on GAs is provided focusing on the role of recombination rates and on the most advanced strategies for their adjustment. In Section 3, the proposed method is described by putting into evidence its elements of novelty. Sections 4 and 5 are devoted to the tests pursued in order to assess the efficiency of the adaptive GA. Section 4 includes tests performed on classical optimization problems, while in Section 5 the advantages achieved by the use of the adaptive GAs on several industrial applications involving optimization tasks affected by the local minima problem are depicted. Some final conclusions are drawn in Section 6 together with the future perspective of the proposed method.

2 Background on GAs

GAs, first proposed by Holland [14], is the most popular evolutionary method and is often used to solve a wide range of optimization problems. The GA population, if convergence is not reached, evolves through generations. At each generation, the new population is formed by a rate $r_s \in [0; 1]$ of survivors from the previous generation and a rate of r_c of individuals created by mating existing individuals by means of the crossover operator. Chromosomes are selected for mating and survival on the basis of an arbitrary fitness function that assesses the goodness of each candidate solution (the higher the fitness, the more probable the selection). Finally, the mutation operator is applied to a subset of the individuals with uniform probability $r_m \in [0; 1]$.

GAs efficiently explore the most promising regions of the objective function as the consequence of the combination of two divergent research lines: the exploration of the search space and the exploitation of the knowledge gained during the exploration. The selection and mating of survivors contribute to the exploitation phase as fittest individuals are awarded and a more accurate search in the most promising zones of the problem domain is promoted. On the other hand, the mutation operator directs the search toward unexplored regions of the search space.

One of the main drawbacks of traditional GAs is their tendency of premature convergence if certain adverse conditions – such as the presence of a high number of local minima within a complex objective function surface – hold. This issue arises when the genes of some high-rated individuals quickly attain to dominate the population, constraining it to converge toward a local optimum. In this case, the genetic operators cannot produce any more descendants better than the parents, as proven by Fogel [9]. One of the most efficient ways to avoid premature convergence is to preserve both the population diversity and the fittest individuals during the evolution.

In this context, such purpose can be achieved by a suitable tuning of the recombination rates r_c and r_m that, at each generation of the GA, determine the composition of the new population by determining the rate of chromosomes obtained by the mating of parent chromosomes and the subsequent rate of mutation of the so-formed population, respectively.

The influence of mutation and crossover rates on the GA convergence speed and on the quality of the achieved solution has been demonstrated since the first work concerning this technique [10]. For this reason, many researches have been focused on their convenient tuning with the twofold aim of reducing the convergence time and avoiding local minima [8, 12].

2.1 Influence and Control of Recombination Rates in GAs

In a GA framework based on a population of candidate chromosomes whose cardinality is N, the population P_{t+1} at an arbitrary generation $t+1$ is formed by $(1-r_c)\cdot N$ individuals directly taken from P_t. The remaining $r_c \cdot N$ chromosomes are generated by the mating of individuals taken from population P_t through the crossover operator. In both cases, the survivors and the parents are selected in a probabilistic manner according to their fitness: the higher the fitness, the higher the selection probability. In addition, the $r_m \cdot N$ values of these individuals are selected and undergo the mutation operation that slightly modifies them.

In this framework, r_c and r_m directly manage the formation of new populations and the interaction between exploitation and exploration. In terms of exploitation, the rate of survivors $(1-r_c)$ preserves within the GA population all the characteristics of the best individuals, while r_c promotes the exchange and mix of these characteristics belonging to two distinct solutions and that can potentially lead to a better solution [10]. Exploration is influenced by r_m that determines the rate of chromosomes, which, after the modification of a limited number of genes, move the search to other promising regions of the domain. Mutation can often avoid the undesirable convergence of the algorithm toward a local optimum rather than a global one. Despite their fundamental role within the search process, the optimal values of r_c and r_m are very controversial and seem to strongly depend on the peculiarity of the faced problems. The various approaches for the determination or control of these rates are summarized in Table 1, where the main ideas are reported together with significant contributions to each family of approaches.

3 Fuzzy Adaptive GAs

The fuzzy adaptive recombination strategy (FARS) GA proposed by Vannucci and Colla [26] was aimed at the improvement of GA efficiency and particularly at the avoidance of suboptimal solutions by suitably controlling some key GA parameters in order to successfully combine the exploration and exploitation phases. In particular, FARS operates on the adaptive update of the crossover (r_c) and mutation (r_m) operator rates that influence the evolution of the candidate population and the way the search space is explored in order to accelerate the attainment of the optimal solution and to avoid the stoppage in a local optimum through a synergism of mutation and crossover.

Table 1: Summary of Approaches and Relevant Methods for the Control of Recombination Rates within GAs.

Approach	Relevant methods
Static: r_c and r_m constant through generations and often empirically determined	Typical values: $r_c \in$ ([0.5–0.95]), $r_m \in$ ([0.001–0.05]) [16, 18]
Deterministic: r_c and r_m vary according to aforethought criteria	Variation according to the generation number [1, 8, 13, 21]
Adaptive: r_c and r_m updated in response to some feedbacks on the actual status of the search	– r_m tuned on the basis of stochastic learning related to the best individual fitness [24] – r_m directly related to the fitness of the best individual of the current generation: the higher its fitness, the lower r_m [1] – r_c and r_m jointly modified expressly to avoid local minima. If a stall situation is detected by analyzing the fitness of all the individuals of the current population, r_c and r_m are increased [22]
Fuzzy adaptive GAs: r_c and r_m tuned by means of a FIS that interprets the status of the search and accordingly modifies the GA parameters. The FIS rules base reproduces adaptive strategies whose formalization would be difficult otherwise [11]	– A FIS based on rules provided by experts involving current generation fitness figures (highest, variance) manages r_c and r_m [19] – Recombination rates controlled by a FIS including rules provided by experts and involving population diversity figures [17] – A meta-GA at a higher level is used for the learning of an optimal rules set for the control of r_c and r_m

The adjustment of r_m and r_c is performed on the basis of some indicators of the status of search process calculated by FARS, such as the trend of the fitness of the candidate solutions and the temporal phase of the search. The fitness trend indicator is used to evaluate the eventual stall of the algorithm in a local minimum while the phase is taken into account, as different values of r_m and r_c are suitable for different phases of the search (see Ref. [13]): at the beginning of the search, for instance, higher values of r_c will be used in order to exploit the desirable characteristics of fittest solutions, while in the ending phases r_m could be increased for solution fine tuning and avoidance of local optima.

The set of employed (crisp) indicators – once fuzzified – is fed to two fuzzy inference systems (FISs) that, by means of a set of `IF-THEN` rules, put into practice varying strategies based on r_m and r_c modifications aimed at GA efficiency improvement.

The use of a FIS allows the design of complex strategies that take into account the interactions among all the considered variables, as well as the incorporation and exploitation of intuitive considerations and strategies driven by the experience.

The main elements of the novelty of FARS with respect to existing fuzzy adaptive GAs lie in the characteristics of the performance indicators taken into account and fed to the FIS by FARS (i.e. the involvement of the "phases" concept, the computation of indicators related to the performance of the 25% fittest individuals) combined to the simultaneous variation of r_m and r_c.

The rules managing the crossover rate adaptation have the objective of keeping r_c constant at the beginning of the optimization in order to allow the successful mating of fittest individuals. Should the trend of the average fittest population be positive (which is not desirable in the context of a minimization problem), r_c is lowered in order to limit the mating of low-fitted chromosomes. In the subsequent phase, the search process is expected to be stabilized; thus, r_c can be raised together with r_m to promote a fine tuning of the final solution. An exemplar rule taken from those included in the FIS controlling r_m is as follows:

$$\texttt{IF PHASE is initial and TREND is negative} \Rightarrow \delta_m \texttt{ is constant,}$$

which is used when, in the initial phase of the search process, the GA is fruitfully converging to a set of solutions with lower fitness: in this case, r_c is kept constant in order to continue the exploitation of this favorable situation.

The main purpose of the FIS in charge of r_m update is the avoidance of local minima. The basic idea within the design of the r_m control is to detect as early as possible the occurrence of stall situations and to update r_m in order to promote the exploration of new and possibly very different candidate solutions. The detection of situations where the search process is stuck in a potential local minimum is performed through the use of the FIS that basically takes into consideration the trend of population fitness (i.e. if it is rather constant, the risk of a stall is high) and the search phase. The output of the FIS in these cases will lead to an improvement of r_m. In the opposite situation, when the search is converging toward solutions with a better fitness, r_m will be kept constant or even decreased in order to exploit the good characteristics of reached solutions and evolve new and better ones. In the initial phase, the behavior of this FIS is analogous to that of the other one. In fact, until the search is fruitful (i.e. the trend is decreasing), r_m does not vary; otherwise, it is increased.

A fully detailed description of the method for the calculation of the search status indicator, the fuzzification process, and the FIS rules can be found in Ref. [26].

4 Testing on Classical Problems

In this section, in order to preliminarily assess the FARS approach, its performance has been analyzed on two classical literature problems and compared to those achieved by GAs adopting a wide set of constant recombination rates. In more detail, the following values for r_c and r_m have been used for generating 45 (r_c, r_m) combinations that have been evaluated in the tests:

$$r_c: 0.5, 0.6, 0.7, 0.8, 0.9.$$

$$r_m: 0.001, 0.002, 0.005, 0.01, 0.02, 0.05, 0.1, 0.2, 0.5.$$

In the constant approach, all the couples generated by the candidate r_c and r_m values have been assessed and, for each problem in this work, the best performing one in terms of number of convergences firstly (main criterion) and convergence generation secondly has been selected. The best (r_c, r_m) couple and related results are reported in each table summarizing the achievements of methods.

4.1 Function Minimization

Within the classical optimization problems, the function minimization is a typical comparison field. From the wide ranges of function, three known minimization problems, selected from De Jong's [5] set, have been considered: these functions have been selected in order to put into evidence different aspects of the improvements achieved by means of FARS.

Sphere function. A 20-dimensional hypersphere [see Eq. (3)] is considered for an elemental optimization problem, which is characterized by a smooth gradient, the absence of local minima, and the global optimum point in the null value $\vec{x}=\vec{0}$.

$$f(\vec{x})=\sum_{i=1}^{N} x_i^2. \tag{3}$$

The domain of the search space for the sphere problem is the hyper-cube $x_i \in [-5.12; 5.12] \cdot \forall i$.

Rastrigin function. The Rastrigin function is defined as follows:

$$f(x)=10N+\sum_{i=1}^{N}(x_i^2-10\times\cos(2\pi x_i)). \tag{4}$$

Within this work, $N=10$ and the search domain is defined by the hypercube determined as for the sphere function $x_i \in [-5.12; 5.12]\ \forall i$. The global optimum of the Rastrigin function is located in $\vec{x}=\vec{0}$. A two-dimensional representation of the Rastrigin function is shown in Figure 1. Due to the large number of local minima,

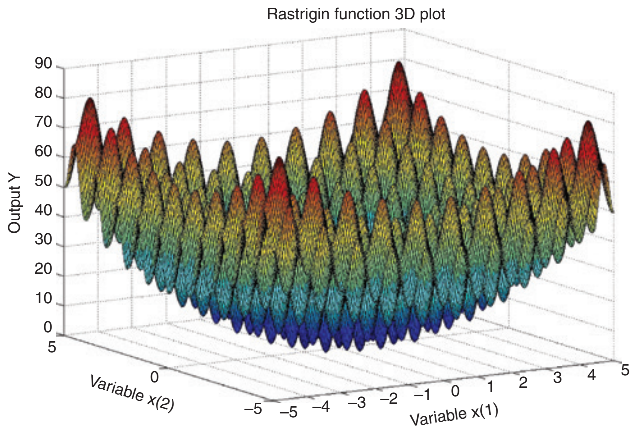

Figure 1: Two-Dimensional Rastrigin Function.

the Rastrigin function is a complex optimization problem for which the algorithms often converge to non-optimal solutions [23].

Schwefel function. The Schwefel function is another typical benchmark for optimization algorithms because of the large number of local minima. The function is defined as follows:

$$f(x)=\sum_{i=1}^{N}-x_i\cdot\sin\left(\sqrt{|x_i|}\right),\tag{5}$$

whose global minimum is located in $\vec{x}$ so that $\forall i\ x_i=420.9687$. In this function, global and local minima are potentially distant and optimization algorithms, including GA, often converge to a non-optimal solution. Tests have been performed on the Schwefel function with $N=2$ in the search space $x_i\in[-500; 500]$, for $i=1, 2$.

The comparison between classical GA (with constant recombination rates) and FARS method has been performed starting from the same configuration. In particular, the settings of the algorithms (the same used in Ref. [23]) are defined as follows:
- Each individual is an array of real elements, where each chromosome represents one dimension.
- The number of population individuals has been set equal to 30.
- The initial population is created with candidate solutions uniformly distributed in the search space.
- Two parent solutions are chosen for the crossover method, and their crossover is computed as the gene-wise average of the two individuals.
- The mutation operator, applied to preserve the population diversity, replaces the value of a randomly chosen gene x_i of an existing individual into a mutated $\hat{x}_i$ one by adding a random value $\mu\in[-M_\mu; M_\mu]$: $\hat{x}_i=x_i+\mu$, with M_μ the maximum allowed gene mutation. In particular, in the tests, M_μ is set equal to 1.
- The GA algorithm stops when 500 generations have been completed or if fitness $(x)<0.001$.

For each optimization problem and approach, 100 tests have been performed. The results have been compared to those achieved by GA with constant recombination rates (all the mentioned combinations) according to three performance indexes: the percentage of convergence to global optimum within the maximum number of generations, the average number of generations to reach the convergence, and the average fitness of the solutions within the maximum number of generations.

The results summarized in Table 2 show that in the case of the sphere function minimization, the simplicity of the problem leads to obtain 100% convergence for both recombination approaches, with an increased convergence speed achieved by FARS and a lower number of generations required with respect to the constant recombination method, with an improvement of 26% to reach the target fitness.

Figure 2 shows an example of the evaluation of fitness function and recombination rates in function of the number of generations, during the minimization of the sphere function. In more detail, in the first

Table 2: Results Obtained on the Function Minimization Problems: The Best (r_c, r_m) Combination for the Constant Approach is Reported.

	Sphere		Rastrigin		Schwefel	
	Const r_m=0.6 r_m=0.1	FARS	Const r_c=0.6 r_c=0.2	FARS	Const r_c=0.5 r_c=0.05	FARS
Conv. %	100	100	7	84	90	100
Conv. gen.	244	182	421	389	109	37
Average sol.	2.4E – 5	1.2E – 5	0.13	8E – 4	4.7E – 4	8E – 5

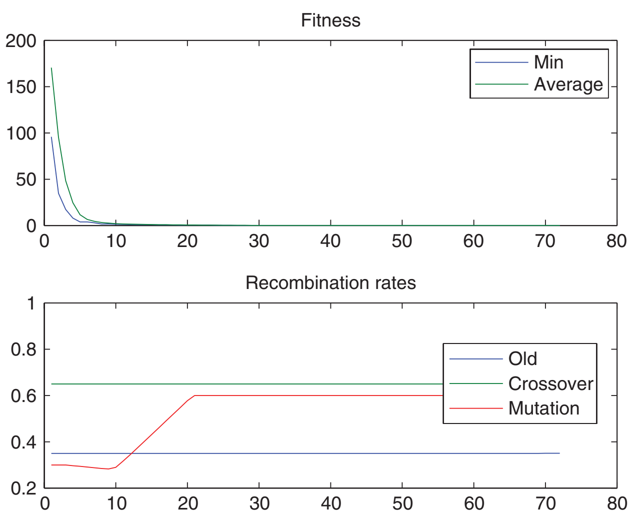

Figure 2: Sphere Function Minimization: Trend of GA Average Population and Best Individual Fitness (Upper Figure) and Recombination Rates (Lower Figure).

generations while the fitness function decreases, the mutation rate r_m decreases in order to allow the search process to focus on the deep exploration of the promising zone of the search space. In successive phases, when the optimization seems to stall on a rather explored area, r_m is increased in order to explore new zones of the search space to avoid convergence to a local minimum.

As far as the minimization of Rastrigin function is concerned, the results show the difficulty of reaching the global optimum by a constant recombination method, due to the large number of local minima and probably coupled to the low mutation rate, which makes it more difficult to escape from a local minimum once reached. In more detail, the constant approach converges in 7% of the tests compared to the 84% success obtained by FARS. Furthermore, FARS allows improving the average solution of three orders of magnitude compared to the constant method.

Figure 3 shows an example of the evaluation of fitness function and recombination rates in function of the number of generations, during the minimization of the Rastrigin function. As the case of sphere function, in the first generations, both fitness and the mutation rate r_m decrease. The FARS method leads to adaptation and increases the mutation parameter while the optimization process is trapped in local minimum (see the zones with constant fitness); otherwise, when the fitness decreases, the algorithm focuses on the exploitation of the acquired knowledge in order to fine tune the solution of the optimization problem. At the end of the search process, the crossover parameter r_c is adapted and decreased in order to keep into account the stability of the fitness function.

Finally, the results obtained on minimization of the Schwefel function highlight once again the limit of the classical recombination approach facing a function with a large number of local minima. In particular,

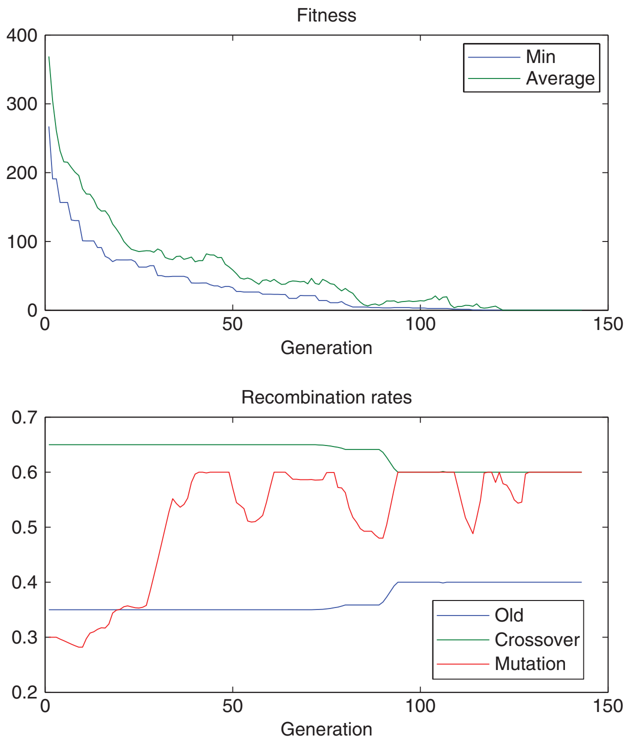

Figure 3: Rastrigin Function Minimization: Trend of GA Average Population and Best Individual Fitness (Upper Figure) and Recombination Rate (Lower Figure).

the constant method does not always ensure the convergence to global minimum, with a 90% success with respect to the 100% achieved by FARS. In more detail, FARS converges faster than the constant recombination method and improves the accuracy of the solution. The reasons for the difficulties encountered by classical GA are attributable to the characteristics of the Schwefel function, in particular to the large distance between local and global minima.

For the sake of synthesis, the results obtained by FARS prove the validity of the approach used with respect to the classical recombination method. FARS allows obtaining more accurate results with decreased computation time. In more detail, the three minimization problems put into evidence three possible issues to be faced in problems of this type: in the sphere problem, the main objective is related to the convergence speed in the absence of local minima. In the Rastrigin function, the main target is represented by a large number of local minima with respect to the volume of search space. Finally, in the Schwefel function, the main issue is the large distance between local and global minima. These three issues have been faced by FARS with good behavior compared to the classical recombination approach.

4.2 The Counting-Ones Problem

A typical benchmark for testing novel optimization algorithms is the "counting-ones" problem. In more detail, the idea is to count the number of 1-valued bits in an array of bits whose length is l. The problem corresponds to the maximization of the following binary function:

$$f(a_1, \dots a_l) = \sum_{i=1}^{l} a_i. \tag{6}$$

In this work – without affecting the validity of the obtained results – the analogous problem of the minimization of this function has been considered. This problem is rather challenging for algorithms such as GA. In particular, the larger the dimension l, the more difficult the problem is due to the fact that standard recombination methods, in tasks where chromosomes are binary coded, have difficulty in converging to global optimum in a short time and often convergence is not ensured within the prefixed maximum number of generations.

A set of 100 tests for the assessment of FARS performance with respect to standard GA employing constant recombination rates has been performed. The configuration of the main parameters of the algorithms is defined as follows:

- All tests share the same fitness function, defined in Eq. (6).
- The search space dimension l and chromosome length are set to 300.
- Two parent solutions are chosen for the crossover method, and their child chromosomes are computed by randomly picking a bit from each parent.
- The mutation operator mutates a maximum of 2 bits of the selected chromosome.
- The GA stops when 300 generations have been completed or the function evaluation reach the maximum fitness l.

Results summarized in Table 3 highlight the great difference in terms of performance between the classical recombination methods and FARS: the former converges only in 7% of the tests while the convergence rate of FARS is 50%. It is also worth noting that the average solution reached by FARS (0.74 bits wrongly set) is far better with respect to standard GA (3.01 bits wrongly set). In the case of the constant approach, the best (r_c, r_m) resulting from the tests is characterized by a rather low mutation rate with respect to other test problems.

5 Exploitation of FARS on Industrial Problems

In this section, the successful use of FARS on three different optimization problems from the industrial field is shown. In the following case studies, the performances achieved by means of FARS are compared to those obtained through the use of standard methods such as GA in a two-fold manner: on the one hand, in terms of the goodness of the solution in order to put into evidence the capability of FARS of avoiding local minima and reaching a solution with higher fitness, and on the other hand in terms of the time required for the convergence of the optimization method in order to highlight the capability of FARS of exploring the search space with higher efficiency.

5.1 Estimation of Final Carbon Content at the Endpoint in a Converter

This application of the FARS approach concerns the basic oxygen furnace (BOF) plant within a steelmaking industry. The aim of the BOF within the steel production process is the reduction of carbon (C) content within a C-rich liquid steel by means of oxygen blowing. The blowing operation is performed through a sublance that

Table 3: Results on the Counting-Ones Problem in Terms of Convergences to the Global Optimum and Generation of Convergence: The Best (r_c, r_m) Combination for the Constant Approach is Reported.

	Constant r_c=0.7 r_m=0.002	**FARS**
Convergences %	7	50
Conv. generation	319	273
Average solution	3.01	0.74

blows the oxygen directly into the BOF. In addition, the sublance measures the temperature and carbon content throughout the process until 2–3 min before the end of the blowing due to technical reasons. Due to this early stopping of sensor measurement, the actual C content at the end of the blowing is unknown (the temperature is assumed to remain constant), although it is important for the management of the next phases of the manufacturing. For these reasons, the C content at the endpoint in the BOF is usually estimated by means of Eq. (7):

$$[\mathrm{C}] = \frac{P_c}{[\mathrm{O}]} \cdot 10^{-\left(\frac{A}{T}+B\right)}, \tag{7}$$

where [C] and [O] are the carbon and oxygen (available) content expressed in wt.%, respectively; T is the temperature expressed in Kelvin degrees; P_c is a model parameter related to pressure, and its theoretical value must vary in the range [1, 1.5] atm; and A and B are two further parameters whose theoretical values, according to Turkdogan [25], must lie in the ranges $A \in [1700, 2500]$ and $B \in [1.5, 1.9]$, respectively.

In this application, Eq. (7) is used to set up a model for the estimation of the final carbon content of an existing BOF plant. The model has been tuned by optimizing Eq. (7) on the basis of an experimental dataset related to >400 heats and including the measured C, O contents as well as the temperatures within the BOF at the endpoint. The main aim of the tuning is finding the optimal values of the parameters P_c, A, B in order to minimize the discrepancy between actual and estimated values of [C] expressed in terms of standardized root mean square error (SRMSE) depicted in Eq. (8), where M is the number of samples used for the test and $\hat{y}_i$ and y_i are the estimated and measured C content, respectively.

$$\epsilon = \sqrt{\frac{1}{M}\sum_{i=1}^{M}[\hat{y}_i - y_i]^2}. \tag{8}$$

The straightforward solution of exploiting Eq. (7) and building an overdimensioned linear system of equations (one for each input-output data sample) to be solved through pseudoinversion does not lead to an acceptable solution. Thus, in a first attempt, GAs have been employed for the model optimization, taking into account the allowed variability ranges for the parameters to tune. Several runs of GAs have been performed, and for each run a different result was achieved. This behavior may signal the presence of a number of local minima that are reached, in turn, by the different runs of the standard GA. In order to try to overcome this drawback, the FARS approach has been employed by using the same configuration (in terms of cardinality of candidate populations, number of generations, operators, etc.) as for the standard GA. A set of 100 runs for both the standard GAs and FARS has been performed for the optimization of the model of Eq. (7). The best performances achieved by each approach are reported in Table 4 together with those of the model employing the nominal values of the parameters according to Turkdogan [25]. In the case of standard GAs employing the constant recombination rate couples listed in Section 4, the best combination is reported, together with its performance, in the table.

Table 4 puts into evidence the parameter values, the SRMSE-based performance, and the time required for computation corresponding to the tested approaches. In the row related to constant recombination rates, the best-performing couple and its achievements are reported.

In addition, in Table 5, the variation of the model parameters and standard deviation of the performance throughout the 100 tests performed are reported.

Table 4: Summary of Best Achievements of the Optimized Models for the Estimation of Carbon Content at the Endpoint in a Converter.

Model	P_c	A	B	Time (s)	ϵ
Nominal	1	1895	1.6	–	1.7E – 3
GA (r_c = 0.5, r_m = 0.2)	1.41	2228	1.57	16	1.54E – 4
FARS	1.44	2148	1.52	16	1.25E – 4

Table 5: Variation Ranges of Optimized Model Parameters and Performance Standard Deviation for GAs and FARS within the 100 Performed Optimization Runs.

Model	P_c	A	B	Std (ϵ)
GA ($r_c=0.5$, $r_m=0.2$)	1.37–1.49	1936–2412	1.52–1.72	0.07E – 4
FARS	1.39–1.47	2001–2410	1.60–1.79	0.03E – 4

The results reported in Table 4 put into evidence the improvement related to the use of GA-based approaches with respect to the model employing literature parameter values. Further, the approach based on FARS obtains a lower error than the model optimized with standard GAs: FARS in this case is able to avoid the local minima reached by the different runs of GA. In terms of performance, the error reduction attributable to FARS is about 20% while the computational time is the same. The results reported in Table 5 show a higher stability of FARS with respect to standard GAs both in terms of the achieved solution (the parameter variability range is smaller) and performance.

5.2 Mean Flow Stress (MFS) Model Optimization

A second application where FARS has been employed is related to the estimation of MFS during the hot rolling of steel. A correct estimation of this measure is fundamental for the efficiency of the rolling process and influences the final quality of the product. In the literature, it is possible to find a number of works that focus on MFS prediction: most of them try to put into correlation the steel chemical composition and some key process parameters such as temperature, strains, and strain rates during the rolling to the MFS. The model proposed by Siciliano et al. [20], shown in Eq. (9), estimates MFS in kgf/mm^2 and involves the content of some chemical elements (niobium and molybdenum, in wt.%), the temperature T (in Kelvin), the applied strain ϵ, and the strain rate ϵ_s together with the $\alpha=0.21$ and $\beta=0.13$ model parameters whose values have been empirically determined.

$$\mathrm{MFS}_{Sic}=e^{\frac{2704+3345[Nb]+220[Mn]}{T}}\cdot\epsilon^{\alpha}\cdot\epsilon_s^{\beta} \tag{9}$$

GAs have been successfully exploited by Dimatteo et al. [6] for the optimization of Eq. (9) by tuning the parameters α and β according to a dataset collected during standard industrial operations on an industrial rolling mill and minimizing the percent difference between the predicted and actual MFS. FARS has been subsequently applied in order to try to improve the performance of the model.

This problem has been used to compare the performance of the different recombination approaches. As there is no *a priori* information on the actual global minimum of the function to be minimized, only the fitness of the solution achieved by the different methods has been considered as a performance indicator. Table 6 reports the average values and standard deviations over 100 tests of this latter indicator and shows evidence that FARS is able to find a better solution in terms of accuracy with respect to alternative approaches that include Eq. (9) employing nominal parameter values, the one tuned by means of standard

Table 6: Performance of Literature, Standard GA, and FARS Tuned Siciliano Models [Eq. (9)] in Terms of Percent Discrepancy between Actual and Predicted MFS.

Model	Average % Error	Std % Error	Time (s)
Nominal	12.9	–	–
GA [6]	9.1	1.3	28
GA ($r_c=0.8$, $r_m=0.1$)	9.0	0.6	22
FARS	8.8	0.6	19

GA by Dimatteo et al. [6], and the best performing one among constant GAs adopting the recombination rate couples described in this work. In terms of stability of performance, FARS is comparable to the best configuration of GAs and more stable than the GA-optimized model proposed by Dimatteo et al. [6].

As shown by the results reported in Table 6, the FARS approach achieves better results with respect to standard GA not only in terms of average percent error [reduction in this case is 4% with respect to GA with ($r_c = 0.6$, $r_m = 0.2$) and 3% with respect to the best constant recombination rate] but also in terms of the time necessary for reaching the convergence (reduction in this case is 33% and 14%, respectively).

5.3 Optimization of Metallurgical Phase Transformation Temperature Models

Dimatteo et al. [7] developed a simplified finite element model (FEM) for the prediction of the mechanical characteristics of steel bars for concrete reinforcement. The FEM simulates the cooling of the bar for the determination of the internal microstructure of the bar. One of the main tasks of the simulator is the estimation of the critical temperatures that determine the steel phase transformations into the bainitic and martensitic microstructures. In more detail, the temperatures of interest are the ones at which the bainitic and martensitic transformations starts, indicated as *Bs* and *Ms*, respectively.

These temperatures are usually estimated by means of simple linear literature models that involve the steel chemical composition as reported by Kirkaldy and Venugopalan [15]. Dimatteo et al. [7] tested several of these linear models on an experimental dataset formed by about 200 samples that include the steel bar chemical compositions as well as the *Ms* and *Bs* obtained by means of laboratory tests performed on purpose. According to the results of these tests, the best-performing models for *Bs* and *Ms* estimation are those of Eqs. (10) and (11), respectively:

$$Bs_{\mathrm{Lit}} = 718 - 425 \cdot [C] - 42.5 \cdot [\mathrm{Mn}], \tag{10}$$

$$Ms_{\mathrm{Lit}} = 561 - 474 \cdot [\mathrm{C}] - 33 \cdot [\mathrm{Mn}] - 17 \cdot [\mathrm{Cr}] - 17 \cdot [\mathrm{Ni}] - 21 \cdot [\mathrm{Mo}], \tag{11}$$

where [C], [Mn], [Cr], [Ni], and [Mo] are the concentrations in wt.% of the chemical elements and *Bs* and *Ms* temperatures are expressed in K.

In order to improve the accuracy of the simulator, two additional models for *Bs* and *Ms* estimation have been developed and tested on the same experimental dataset. The models, depicted in Eqs. (12) and (13), respectively, employ more variables concerning chemical composition and are non-linear. Although in this case, *Bs* and *Ms* models share the same structure, the values of internal parameters (including the constant values k_{Bs} and k_{Ms}) are different:

$$Bs_{\mathrm{New}} = k_{\mathrm{Bs}} + \alpha_1 \cdot [\mathrm{Cu}] + \alpha_2 \cdot [\mathrm{C}]^2 + \alpha_3 \cdot [\mathrm{C}] \cdot [\mathrm{Mn}] + \alpha_4 \cdot [\mathrm{C}] + \alpha_5 \cdot [\mathrm{SI}]^2 \cdot [\mathrm{Mn}] + \alpha_6 \cdot e^{-([\mathrm{Mn}]+[\mathrm{Cr}])}, \tag{12}$$

$$Ms_{\mathrm{New}} = k_{\mathrm{Ms}} + \beta_1 \cdot [\mathrm{Cu}] + \beta_2 \cdot [\mathrm{C}]^2 + \beta_3 \cdot [\mathrm{C}] \cdot [\mathrm{Mn}] + \beta_4 \cdot [\mathrm{C}] + \beta_5 \cdot [\mathrm{SI}]^2 \cdot [\mathrm{Mn}] + \beta_6 \cdot e^{-([\mathrm{Mn}]+[\mathrm{Cr}])}. \tag{13}$$

A set of 100 optimization runs by means of standard GAs and FARS has been performed in order to find the optimal values for the parameters α_i, β_i, k_{Bs}, and k_{Ms}. The performances achieved by these approaches have been compared to those obtained by the literature models of Eqs. (10) and (11) and by a slightly upgraded version of such models obtained by calculating the model parameters through multiple linear regression, minimizing the discrepancy between measured and estimated temperatures (literature optimized models). As far as the standard GA is concerned, all the recombination rate couples considered in this work have been assessed and the performances of the best ones (one for *Ms* and the other for *Bs*) are reported. The results of the comparison are summarized in Table 7 in terms of the mean absolute error in the temperature estimation (in K) and the standard deviation of this error over the 100 performed tests. The optimal values of the parameters cannot be reported for confidentiality reasons.

The results summarized in Table 7 put into evidence several aspects of this problem. On the one hand, the benefit gained from the use of optimized models is evidenced by the performance improvement of the

Table 7: Comparison among the Different Approaches Tested for the *Bs* and *Ms* Estimation: Time Is Expressed in Seconds, Error in K; The Optimal Recombination Rates for Eq. (12) GA are ($r_c = 0.5$, $r_m = 0.05$) and for Eq. (13) GA are ($r_c = 0.7$, $r_m = 0.2$).

Model	*Bs*			*Ms*		
	Err.	Std	Time	Err.	Std	Time
Literature	18.5	–	–	9.4	–	–
Lit. opt.	11.4	–	1	8.5	–	1
Eq. (12) GA	9.8	1.4	16	–	–	
Eq. (13) GA	–	–	–	6.2	0.3	14
FARS	6.4	0.2	12	6.0	0.04	12

optimized linear literature model with respect to the one exploiting nominal parameter values. On the other hand, it draws attention to the additional improvement due to the use of a non-linear model (that cannot be optimized through multiple regression). In this latter case, the use of FARS instead of plain GAs for the optimization task leads to a much more accurate model, likely due to the better exploration of the search space that this approach can perform. FARS outperforms GAs also in terms of stability of performance, as the standard deviation of the error is sensibly reduced by using this approach.

6 Conclusions

In this paper, the use of a novel adaptive GA within industrial optimization tasks is presented. The method, referred to as FARS, has the twofold aim of speeding up the optimization process and, mainly, preventing the optimization from reaching a local minimum instead of a global one, as it often occurs when facing complex problems involving the exploitation of experimental data for model tuning. FARS is based on the control of GA recombination rates by means of the extraction of features describing the status of optimization. The extracted features are fed to a FIS that controls the GA mutation and crossover rates in order to fulfill its aims through a set of rules implementing suitable strategies.

Within this work, the proposed approach has been tested both on classical optimization tasks affected by the problem of local minima achievement and especially on three real-world applications from the steelmaking industry in order to assess FARS capabilities when facing real practical problems and datasets. The results achieved by FARS on classical problems demonstrated its efficiency. Moreover, the use of FARS on industrial problems improved the performances of the models optimized with respect to GAs employing a variety of constant recombination rate couples and standard methods, proving its capability of efficiently exploring the objective function surfaces and avoiding local minima even in the presence of industrial datasets and problems. Further, tests showed a reduction of the computational time required for the optimization that is likely due to the improvement of the exploration strategy derived by the use of FARS.

In the future, FARS will be used for controlling more GA parameters, such as the cardinality of the candidate population through GA generations and the types of crossover and mutation operators to employ according to the status of the optimization. Further, more literature and industrial problems will be utilized to assess the method performance.

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