Online Appendix: Quantile Difference in Differences with Time-varying Qualification in Panel Data

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This is online appendix for "Quantile Difference in Differences with Time-varying Qualification in Panel Data." Appendix A provides estimation and inference for the parameters of interest explained in Section 3. Appendix B provides proofs for our main theorems. Appendix C describes identification and inference for the QTIM, i.e., the estimates for *in-movers*. Appendix D explains background, data, and additional results for our application. Appendix E checks the robustness of our application results using another distributional DID method by Athey and Imbens (2006). Appendix F provides untreated moving effects in our application.

Appendix A. Estimation and Inference

The section proposes estimators for the parameters of interest and establishes a functional central limit theorem for the QTIS estimator over a compact subset strictly included in the unit interval and noted $\mathcal{T}$[1]. We also establish the results on consistency and asymptotic distribution of the estimator.

For all $\tau \in \mathcal{T}$, we estimate:

$$\widehat{QTIS}(\tau) = \hat{F}^{-1}_{Y_3^1|Q_2=1,Q_3=1}(\tau) - \hat{F}^{-1}_{Y_3^0|Q_2=1,Q_3=1}(\tau).$$

For $i, j \in \{0, 1\}$, letting $\alpha^k_{ij} = 1\{Q_{k2} = i, Q_{k3} = j\}$ and $n_{ij} = \sum_{k=1}^{n} \alpha^k_{ij}$, the estimator of

[1] The same argument can be applied to the QTIM.

$F_{Y_s|ij}(y)$ is given by

$$\hat{F}_{Y_s|ij}(y) = \frac{1}{n_{ij}} \sum_{k=1}^{n} 1\{Y_{ks}\}\alpha_{ij}^{k}.$$

The first term in $Q\widehat{TI}S(\tau)$ is estimated by inverting the empirical distributions of observed outcomes for *in-stayers*:

$$\hat{F}^{-1}_{Y_3^1|Q_2=1,Q_3=1}(\tau) = \inf\{y : \hat{F}_{Y_3^1|Q_2=1,Q_3=1}(y) \geq \tau\},$$

The estimator of the counterfactual quantiles in the second term is derived from the identification results in Theorem 1:

$$\hat{F}^{-1}_{Y_3^0|Q_2=1,Q_3=1}(\tau) = \inf\{y : \hat{F}_{Y_3^0|Q_2=1,Q_3=1}(y) \geq \tau\},$$

where

$$\hat{F}_{Y_3^0|Q_2=1,Q_3=1}(y) = \tfrac{1}{n_{11}} \textstyle\sum_{i=1}^{n_{11}} 1\{\hat{F}^{-1}_{\Delta Y_3^0|1,1}(\hat{F}_{\Delta Y_2|1,1}(\Delta Y_2)) \leq y - \hat{F}^{-1}_{Y_2|1,1}(\hat{F}_{Y_1|1,1}(Y_1))\}.$$

The final challenge remaining is the estimation of $\hat{F}^{-1}_{\Delta Y_3^0|1,1}(\cdot)$. Using the identification result in Theorem 1, we estimate it as:

$$\hat{F}^{-1}_{\Delta Y_3^0|1,1}(\delta) = \frac{1}{n}\sum_{i=1}^{n} \frac{(1-Q_{i2})(1-Q_{i3})}{\hat{p}_{11}} \frac{\hat{p}_{11}(X)}{\hat{p}_{00}(X)} 1\{\Delta Y_{i3} \leq \delta\} \Big/ \frac{1}{n}\sum_{i=1}^{n} \frac{(1-Q_{i2})(1-Q_{i3})}{\hat{p}_{11}} \frac{\hat{p}_{11}(X)}{\hat{p}_{00}(X)},$$

The denominator ensures that the estimator lies between 0 and 1. We also need to estimate the propensity scores, $p_{00}(X)$ and $p_{11}(X)$, in the first step.

Next, under standard assumptions in the literature (Hirano, Imbens, and Ridder 2003), we show that the estimator of the $QTIS$ converges at the parametric rate of $\sqrt{n}$, even when the propensity scores are estimated nonparametrically. Consider the following assumptions.

Assumption 5.1

$E[1\{\Delta Y_3^0 \leq y\}|0,0,X]$ is continuously differentiable for all $x \in Supp(X)$.

Assumption 5.2

(i) $Supp(X) = \prod_{j=1}^{r}[x_{lj}, x_{uj}]$ is a Cartesian product of compact intervals with r the dimension of X.

(ii) $f_X(\cdot)$ the density of X is bounded away from 0 on $Supp(X)$.

Assumption 5.3

(i) $p_{00}(x)$ and $p_{11}(x)$ are continuously differentiable of order $s \geq 7r$.

(ii) $p_{00}(x)$ and $p_{11}(x)$ are bounded away from 0 and 1.

Assumption 5.4

$p_{00}(x)$ and $p_{11}(x)$ are estimated nonparametrically by series logit where the power series are of order $K = n^{\nu}$ with $\frac{1}{4(s/r-1)} < \nu < \frac{1}{9}$.

Assumption 5.5

The observed data $(Y_{i3}, Y_{i2}, Y_{i1}; X_i, Q_{i3}, Q_{i2})$ are independently and identically distributed.

First, we show preliminary results on the weak convergence of some empirical distributions, which allows us to use the functional delta method. Additional notations are used below: $\hat{G}_{Y_s|ij}(y) = \sqrt{n}(\hat{F}_{Y_s|ij}(y) - F_{Y_s|ij}(y))$, $\hat{G}_{Y_s^d|ij}(y) = \sqrt{n}(\hat{F}_{Y_s^d|ij}(y) - F_{Y_s^d|ij}(y))$, $\hat{G}_{\Delta Y_v|ij}(\delta) = \sqrt{n}(\hat{F}_{\Delta Y_v|ij}(\delta) - F_{\Delta Y_v|ij}(\delta))$, $\hat{G}_{\Delta Y_v^d|ij}(\delta) = \sqrt{n}(\hat{F}_{\Delta Y_v^d|ij}(\delta) - F_{\Delta Y_v^d|ij}(\delta))$ for $s \in \{1, 2, 3\}$, $v \in \{3, 2\}$ and $i, j, d \in \{0, 1\}$. We also assume that n_{ij} goes to ∞ as the sample size n becomes ∞ and $\sqrt{n/n_{ij}}$ converges to $\sqrt{1/p_{ij}}$.

Proposition 2 *Define* $\tilde{Y}_{i3} = F^{-1}_{\Delta Y_3^0|1,1}(F_{\Delta Y_2|1,1}(\Delta Y_{i2})) + F^{-1}_{Y_2|1,1}(F_{Y_1|1,1}(Y_{i1}))$. *Denote* $\tilde{F}_{Y_3^0|1,1}(y)$ *the estimator for the counterfactual distribution based on observations* $\{\tilde{Y}_{i3}\}$. *Let* $\tilde{G}_{Y_3^0|1,1}(y) = \sqrt{n}(\tilde{F}_{Y_3^0|1,1}(y) - F_{Y_3^0|1,1}(y))$. *Under Assumptions 1-4, and 5.1-5.5:*

$$(\hat{G}_{\Delta Y_3^0|1,1}, \hat{G}_{\Delta Y_2|1,1}, \tilde{G}_{Y_3^0|1,1}, \hat{G}_{Y_3|1,1}, \hat{G}_{Y_2|1,1}, \hat{G}_{Y_1|1,1}) \rightarrow (\mathbb{W}_1^1, \mathbb{W}_2^1, \mathbb{V}_0^1, \mathbb{V}_1^1, \mathbb{W}_3^1, \mathbb{W}_4^1)$$

in the space $\mathcal{S} = l^{\infty}(\Delta\mathcal{Y}_{3|11}(0)) \times l^{\infty}(\Delta\mathcal{Y}_{2|11}) \times l^{\infty}(\mathcal{Y}_{3|11}(0)) \times l^{\infty}(\mathcal{Y}_{3|11}) \times l^{\infty}(\mathcal{Y}_{2|11}) \times l^{\infty}(\mathcal{Y}_{1|11})$ *where* $l^{\infty}(\Omega)$ *is the space of all uniformly bounded functions on the set* Ω, *associated with the supremum norm* $\|\cdot\|_{\infty}$ *and* $(\mathbb{W}_1^1, \mathbb{W}_2^1, \mathbb{V}_0^1, \mathbb{V}_1^1, \mathbb{W}_3^1, \mathbb{W}_4^1)$ *is a Gaussian process with mean*

0 and covariance $V^1(g,g') = E[\psi^1(g)\psi^1(g')]$ *for* $(g,g') \in \mathcal{S}\times\mathcal{S}$*, with*

$$\psi^1(g) = \begin{pmatrix} \frac{E[1\{\Delta Y_3 \le g_1\}|X,0,0]}{p_{11}p_{00}(X)}(p_{00}(X)Q_2^3 - p_{11}(X)D_2^3) + \frac{D_2^3}{p_{11}}\frac{p_{11}(X)}{p_{00}(X)}1\{\Delta Y_3 \le g_1\} - F_{\Delta Y_3^0|1,1}(g_1) \\ \frac{(1-Q_2)(1-Q_3)}{p_{11}}1\{\Delta Y_2 \le g_2\} - F_{\Delta Y_2|1,1}(g_2) \\ \frac{(1-Q_2)(1-Q_3)}{p_{11}}1\{\tilde{Y}_3 \le g_3\} - F_{\tilde{Y}_3|1,1}(g_3) \\ \frac{(1-Q_2)(1-Q_3)}{p_{11}}1\{Y_3 \le g_4\} - F_{Y_3|1,1}(g_4) \\ \frac{(1-Q_2)(1-Q_3)}{p_{11}}1\{Y_2 \le g_5\} - F_{Y_2|1,1}(g_5) \\ \frac{(1-Q_2)(1-Q_3)}{p_{11}}1\{Y_1 \le g_6\} - F_{Y_1|1,1}(g_6) \end{pmatrix}$$

where $Q_2^3 = Q_2Q_3$ *and* $D_2^3 = (1-Q_2)(1-Q_3)$*.*

The next proposition provides the joint limiting distribution of observed *in-stayers* outcomes and counterfactual untreated potential outcomes for *in-stayers*.

Proposition 3 *Let* $\hat{G}_0^1(y) = \sqrt{n}(\hat{F}_{Y_3^0|1,1}(y) - F_{Y_3^0|1,1}(y))$ *and let* $\hat{G}_1^1(y) = \sqrt{n}(\hat{F}_{Y_3^1|1,1}(y) - F_{Y_3^1|1,1}(y))$*. Under Assumptions 1-4, and 5.1-5.5:*

$$(\hat{G}_0^1, \hat{G}_1^1) \to (\mathbb{G}_0^1, \mathbb{G}_1^1)$$

where $\mathbb{G}_0^1$ *and* $\mathbb{G}_1^1$ *are tight Gaussian processes with mean 0 and almost surely uniformly continuous paths on the space* $\mathcal{Y}_{3|11}(0) \times \mathcal{Y}_{3|11}(1)$ *given by*

$$\mathbb{G}_1^1 = \mathbb{V}_1^1$$

and

$$\begin{aligned} \mathbb{G}_0^1 \;=\; & \mathbb{V}_0^1 + \int \Bigg\{ \mathbb{W}_1^1 \circ F^{-1}_{Y_2|1,1} \circ F_{Y_1|1,1}(v) - F_{\Delta Y_3^0|1,1}\left(y - \frac{\mathbb{W}_2^1 - \mathbb{W}_2^1 \circ F^{-1}_{Y_2|1,1} \circ F_{Y_2|1,1}(v)}{f_{Y_2|1,1} \circ F^{-1}_{Y_2|1,1} \circ F_{Y_2|1,1}(v)} \right) \\ & - \mathbb{W}_2^1 \circ F_{\Delta Y_3^0|1,1}(y - F^{-1}_{Y_2|1,1} \circ F_{Y_1|1,1}(v)) \Bigg\} K(y,v) \mathrm{d}F_{Y_1|1,1} \end{aligned}$$

where

$$K(y,v) = \frac{f_{\Delta Y_2|Y_1,1,1}(F^{-1}_{\Delta Y_2|1,1} \circ F_{\Delta Y_3^0|1,1}(y - F^{-1}_{Y_2|1,1} \circ F_{Y_1|1,1}(v)))}{f_{\Delta Y_2|1,1} \circ F^{-1}_{\Delta Y_2|1,1} \circ F_{\Delta Y_3^0|1,1}(y - F^{-1}_{Y_2|1,1} \circ F_{Y_1|1,1}(v))}.$$

The next theorem establishes asymptotic properties of the QTIS estimator.

Theorem 2 *Suppose* $F_{Y_3^0|1,1}$ *admits a positive continuous density* $f_{Y_3^0|1,1}$ *on an interval* $[a,b]$ *containing an* ϵ*-enlargement of the set* $\{F^{-1}_{Y_3^0|1,1}(\tau) : \tau \in \mathcal{T}\}$ *in* $\mathcal{Y}_{3|11}(0)$ *with* $\mathcal{T} \subset (0,1)$*. Under Assumptions 1-4, and 5.1-5.5:*

$$\sqrt{n}(\widehat{QTIS}(\tau) - QTIS(\tau)) \rightarrow \mathbb{G}_1^{1+}(\tau) - \mathbb{G}_0^{1+}(\tau),$$

where $(\mathbb{G}_1^{1+}(\tau), \mathbb{G}_0^{1+}(\tau))$ *is a stochastic process in the metric space* $(l^\infty(\mathcal{T}))^2$ *such that*

$$\mathbb{G}_0^{1+}(\tau) = \frac{\mathbb{G}_0^1(F^{-1}_{Y_3^0|1,1}(\tau))}{f_{Y_3^0|1,1}(F^{-1}_{Y_3^0|1,1}(\tau))} \quad \textit{and} \quad \mathbb{G}_1^{1+}(\tau) = \frac{\mathbb{G}_1^1(F^{-1}_{Y_3^1|1,1}(\tau))}{f_{Y_3^1|1,1}(F^{-1}_{Y_3^1|1,1}(\tau))}$$

Because our estimators do not have standard asymptotic distributions, we achieve inference using a nonparametric bootstrap. Here, we present the algorithm procedure only for $\widehat{QTIS}(\tau)$, while the same technique can be used to perform inference on $\widehat{QTIM}(\tau)$.

Algorithm

Let B the number of bootstrap iterations. For $b = 1, ..., B$,

1. Draw a sample of size n with replacement from the original data

2. Compute $\widehat{QTIS}^b(\tau) = \hat{F}^{-1b}_{Y_3^1|Q_2=1,Q_3=1}(\tau) - \hat{F}^{-1b}_{Y_3^0|Q_2=1,Q_3=1}(\tau)$, where

$$\hat{F}^b_{Y_3^0|1,1}(y) = \frac{1}{n_{11}^b} \sum_i 1\{\hat{F}^{-1b}_{\Delta Y_3^0|1,1}(\hat{F}^b_{\Delta Y_2|1,1}(\Delta Y_{i2})) \leq y - \hat{F}^{-1b}_{Y_2|1,1}(\hat{F}^b_{Y_1|1,1}(Y_{i1}))\}.$$

3. Let $I^b = \sup_\tau |\widehat{QTIS}^b(\tau) - \widehat{QTIS}(\tau)|$, a $(1-\alpha)$ confidence band is given by

$$\widehat{QTIS}(\tau) - c^B_{1-\alpha}/\sqrt{n} \leq QTIS(\tau) \leq \widehat{QTIS}(\tau) + c^B_{1-\alpha}/\sqrt{n}$$

   where $c^B_{1-\alpha}$ is the $(1-\alpha)$ quantile of $\{I^b\}_{b=1}^B$

The next proposition establishes the validity of the nonparametric bootstrap for our inference procedure.

Proposition 4 *Under Assumptions 1-4, and 5.1-5.5:*

$$\sqrt{n}(Q\widehat{TIS}^*(\tau) - Q\widehat{TIS}(\tau)) \to_* \mathbb{G}_1^{1+}(\tau) - \mathbb{G}_0^{1+}(\tau)$$

where $\mathbb{G}_1^{1+}(\tau)$ *and* $\mathbb{G}_0^{1+}(\tau)$ *are defined above, and* $\to_*$ *indicates weak convergence in probability under the bootstrap law.*

Appendix B. Proofs

B.1 Identification

B.1.1 Proof of Theorem 1

Proof. We use the following result in the proof. Given two random variables U and V, their joint density in terms of the copula pdf is given by,

$$f(u, v) = c(F_U(u), F_V(v))f_U(u)f_V(v). \tag{B1}$$

The copula pdf in term of their joint density is

$$c(a, b) = f(F_U^{-1}(a), F_V^{-1}(b))\frac{1}{f_U(F_U^{-1}(a))}\frac{1}{f_V(F_V^{-1}(b))}. \tag{B2}$$

For a given copula C, let c the its associated pdf. Let c_3 be the copula pdf associated to $C_{\Delta Y_3^0, Y_2^0|Q_2=1,Q_3=1}$ and c_2 the copula cdf associated to $C_{\Delta Y_2^0, Y_1^0|Q_2=1,Q_3=1}$. Let $f_3 = f_{\Delta Y_3^0, Y_2^0|Q_2=1,Q_3=1}$ be the joint pdf of ΔY_3^0 and Y_2^0 conditional on $Q_2 = 1$ and $Q_3 = 1$ so that $f_2 = f_{\Delta Y_2^0, Y_1^0|Q_2=1,Q_3=1}$. Finally, define $\Delta\mathcal{Y} = Supp(\Delta Y_3^0|Q_2 = 1, Q_3 = 1)$ and $\mathcal{Y} = Supp(Y_2|Q_2 = 1, Q_3 = 1)$. We have the following equalities,

$$
\begin{aligned}
Pr(Y_3^0 \leq y|1,1) &= Pr(\Delta Y_3^0 + Y_2^0 \leq y|Q_2 = 1, Q_3 = 1) \\
&= E[1\{\Delta Y_3^0 \leq y - Y_2^0\}|Q_2 = 1, Q_3 = 1] \\
&= \int_{\mathcal{Y}} \int_{\Delta\mathcal{Y}} 1\{\delta \leq y - y'\} f_3(\delta, y') \mathrm{d}\delta \mathrm{d}y' \\
&= \int_{\mathcal{Y}} \int_{\Delta\mathcal{Y}} 1\{\delta \leq y - y'\} c_3(F_{\Delta Y_3^0|1,1}(\delta), F_{Y_2^0|1,1}(y')) f_{\Delta Y_3^0|1,1}(\delta) f_{Y_2^0|1,1}(y') \mathrm{d}\delta \mathrm{d}y' \qquad \text{(B3)} \\
&= \int_{\mathcal{Y}} \int_{\Delta\mathcal{Y}} 1\{\delta \leq y - y'\} c_2(F_{\Delta Y_3^0|1,1}(\delta), F_{Y_2^0|1,1}(y')) f_{\Delta Y_3^0|1,1}(\delta) f_{Y_2^0|1,1}(y') \mathrm{d}\delta \mathrm{d}y' \qquad \text{(B4)}
\end{aligned}
$$

Using equation (B2), we know that

$$
\begin{aligned}
c_2(F_{\Delta Y_3^0|1,1}(\delta), F_{Y_2^0|1,1}(y')) &= f_2(F^{-1}_{\Delta Y_3^0|1,1}(F_{\Delta Y_3^0|1,1}(\delta)), F^{-1}_{Y_1^0|1,1}(F_{Y_2^0|1,1}(y'))) \\
&\times \frac{1}{f_{\Delta Y_2^0|1,1}(F^{-1}_{\Delta Y_2^0|1,1}(F_{\Delta Y_3^0|1,1}(\delta)))} \frac{1}{f_{Y_1^0|1,1}(F^{-1}_{Y_1^0|1,1}(F_{Y_2^0|1,1}(y')))} \qquad \text{(B5)}
\end{aligned}
$$

Equality (B3) follows from equation (B1) and equality (B4) follows from Assumption 2. Let us make the following change of variables, $u = F^{-1}_{\Delta Y_2^0|1,1}(F_{\Delta Y_3^0|1,1}(\delta))$ and $v = F^{-1}_{Y_1^0|1,1}(F_{Y_2^0|1,1}(y'))$. We have,

1. $\delta = F^{-1}_{\Delta Y_3^0|1,1}(F_{\Delta Y_2^0|1,1}(u))$
2. $y' = F^{-1}_{Y_2^0|1,1}(F_{Y_1^0|1,1}(v))$
3. $\mathrm{d}\delta = \frac{f_{\Delta Y_2^0|1,1}(u)}{f_{\Delta Y_3^0|1,1}(F^{-1}_{\Delta Y_3^0|1,1}(F_{\Delta Y_2^0|1,1}(u)))} \mathrm{d}u$
4. $\mathrm{d}y' = \frac{f_{Y_1^0|1,1}}{f_{Y_2^0|1,1}(F^{-1}_{Y_2^0|1,1}(F_{Y_1^0|1,1}(v)))} \mathrm{d}v$

We replace (B4) and equalities 1-4 in equation (B3), and we obtain

$$
\begin{aligned}
Pr(Y_3^0 \leq y|11) &= \int_{\mathcal{Y}_2}\int_{\Delta\mathcal{Y}_2} 1\{F^{-1}_{\Delta Y_3^0|11}(F_{\Delta Y_2^0|11}(u)) \leq y - F^{-1}_{Y_2^0|11}(F_{Y_1^0|11}(v))\} f_2(u,v)\mathrm{d}u\mathrm{d}v \\
&= E[1\{F^{-1}_{\Delta Y_3^0|11}(F_{\Delta Y_2^0|11}(u)) \leq y - F^{-1}_{Y_2^0|11}(F_{Y_1^0|11}(v))\}|Q_2 = 1, Q_3 = 1] \quad \text{(B6)}
\end{aligned}
$$

Where equality (B6) follows from the definition of expectation.

$$
\begin{aligned}
Pr(\Delta Y_3^0 \leq \delta | Q_2 = 1, Q_3 = 1) &= \frac{Pr(\Delta Y_3^0 \leq \delta, Q_2 = 1, Q_3 = 1)}{p_{11}} \\
&= E\left[\frac{Pr(\Delta Y_3^0 \leq \delta, Q_2 = 1, Q_3 = 1)|X}{p_{11}}\right] \\
&= E\left[\frac{p_{11}(X)}{p_{11}} Pr(\Delta Y_3^0 \leq \delta | Q_2 = 1, Q_3 = 1), X)\right] \\
&= E\left[\frac{p_{11}(X)}{p_{11}} Pr(\Delta Y_3^0 \leq \delta | Q_2 = 0, Q_3 = 0), X)\right] \quad \text{(B7)} \\
&= E\left[\frac{p_{11}(X)}{p_{11}} E\left[(1 - Q_2)(1 - Q_3)1\{\Delta Y_3 \leq \delta\} | Q_2 = 0, Q_3 = 0, X\right]\right] \quad \text{(B8)} \\
&= E\left[\frac{p_{11}(X)}{p_{11}p_{00}(X)} E\left[(1 - Q_2)(1 - Q_3)1\{\Delta Y_3 \leq \delta\}|X\right]\right] \quad \text{(B9)} \\
&= E\left[\frac{(1 - Q_2)(1 - Q_3)}{p_{00}(X)} \frac{p_{11}(X)}{p_{11}} 1\{\Delta Y_3 \leq \delta\}\right] \quad \text{(B10)}
\end{aligned}
$$

Where the first three equalities hold by definition of conditional probability and conditional expectation. Equality (B7) holds by Assumption 1. Equality (B8) is true since $Q_2 = Q_3 = 0$ implies $\Delta Y_3^0 = \Delta Y_3$ and $(1 - Q_2)(1 - Q_3) = 1$. Equality (B9) holds because

$$
E[(1-Q_2)(1-Q_3)1\{\Delta Y_3 \leq \delta\}|X] = p_{00}(X) E\left[(1 - Q_2)(1 - Q_3)1\{\Delta Y_3 \leq \delta\} | Q_2 = 0, Q_3 = 0, X\right].
$$

Finally, equality (B10) results from the Law of Iterated Expectations. ■

B.1.2 Proof of Example 1

Proof. First, notice that for in-stayers and out-stayers, $Q_2 = Q_3$ so that for both subgroups, $\Delta Y_3^0 = q(U_3, X) - q(U_2, X)$.

$$\begin{aligned} P(\Delta Y_3^0 \leq \delta | X = x, Q_2 = 1, Q_3 = 1) &= \int \mathbb{1}\{q(u,x) - q(\tilde{u},x) \leq \delta\} dF_{U_3,U_2|X,Q_2=1,Q_3=1}(u,\tilde{u}|x) \\ &= \int \mathbb{1}\{q(u,x) - q(\tilde{u},x) \leq \delta\} dF_{U_3,U_2|X,Q_2=0,Q_3=0}(u,\tilde{u}|x) \\ &= P(\Delta Y_3^0 \leq \delta | X = x, Q_2 =, Q_3 = 0) \end{aligned}$$

This proves that Assumption 1 is satisfied. The second equality holds because $(U_3, U_2|X, Q_2 = 0, Q_3 = 0)$ and $(U_3, U_2|X, Q_2 = 1, Q_3 = 1)$ have the same distribution.

We now show that Assumption 2 is also satisfied. We have:

$$\begin{aligned} &P(\Delta Y_3^0 \leq \delta, Y_2^0 \leq y | Q_2 = 1, Q_3 = 1) \\ &\quad = P(q(U_3, X) - q(U_2, X) \leq \delta, q(U_2, X) + \eta + \beta_q \leq y | Q_2 = 1, Q_3 = 1) \\ &\quad = P(q(U_2, X) - q(U_1, X) \leq \delta, q(U_1, X) + \eta + \beta_q \leq y | Q_2 = 1, Q_3 = 1) \\ &\quad = P(\Delta Y_2^0 \leq \delta, Y_1^0 \leq y | Q_2 = 1, Q_3 = 1), \end{aligned}$$

where the second equality holds because $(U_3, U_2, X, \eta | Q_2 = 1, Q_3 = 1)$ and $(U_2, U_1, X, \eta | Q_2 = 1, Q_3 = 1)$ have the same distribution.

■

B.1.3 Proof of Proposition 1

Proof. Straightforward, identical approach to proof of Theorem 1. ■

B.2 Inference

B.2.1 Proof of Proposition 2

Let us introduce the notations $F = (F_1, F_2, F_2, F_4)$, where $F_1 = F_{\Delta Y_3^0|11}$, $F_2 = F_{\Delta Y_2^0|11}$, $F_3 = F_{Y_2^0|11}$ and $F_4 = F_{Y_1^0|11}$. Define

$$\phi_n(F) = \frac{1}{n_{11}} \sum_{i=1}^{n} 1\{F_1^{-1}(F_2(\Delta Y_{i2})) \leq y - F_3^{-1}(F_4(Y_{i1}))\}\delta_{11}^i$$

and

$$\phi(F) = E[1\{F_1^{-1}(F_2(\Delta Y_{i2})) \leq y - F_3^{-1}(F_4(Y_{i1}))\}|Q_2 = 1, Q_3 = 1]$$

The first step of the proof is to establish Hadamard differentiability of $\phi(F)$ and this follows from Lemma B.6. in Callaway and Li (2017). Defining $v_n(F) =\sqrt{n}(\phi_n(F) - \phi(F))$, we know by Lemma B.7. in Callaway and Li (2017) that

$$\sup_y |v_n(\hat{F})(y) - v_n(F)(y)| \rightarrow_p 0.$$

Now let $W_3 = (Q_2, Q_3, X, \Delta Y_3)$ and define

$$h(W_3, \delta) = \frac{E[1\{\Delta Y_3 \leq g_1\}|X, 0, 0]}{p_{11}p_{00}(X)}(p_{00}(X)Q_2^3 - p_{11}(X)D_2^3) + \frac{D_2^3}{p_{11}}\frac{p_{11}(X)}{p_{00}(X)}1\{\Delta Y_3 \leq \delta\}.$$

Using similar argument as Lemma B.9 in Callaway and Li (2017) we know that the class of functions $\mathcal{K} = \{h(W_3, \delta)|\delta \in \Delta\mathcal{Y}_3\}$ is a Donsker class. Define $p(\cdot) = (p_{11}(\cdot), p_{00}(\cdot))$ and let $F_{\Delta Y_3^0|1,1}(\delta, p(\cdot)) = E\left[\frac{(1-Q_2)(1-Q_3)}{p_{00}(X)}\frac{p_{11}(X)}{p_{11}}1\{\Delta Y_3 \leq \delta\}\right]$ be the identified distribution of the change in untreated potential outcomes for the *in-stayers* for scores $p(\cdot)$. Then, the pathwise derivative $\Gamma(\delta, p)(\hat{p} - p)$ exists and is given by

$$\Gamma(\delta, p)(\hat{p} - p) = E\left[\frac{(1 - Q_2)(1 - Q_3)}{p_{11}}\frac{1\{\Delta Y_3 \leq \delta\}}{p_{00}^2(X)}(p_{00}(X)\hat{p}_{11}(X) - p_{11}(X)\hat{p}_{00}(X))\right],$$

which is the limit as $h \to 0$ of $\frac{F_{\Delta Y_3^0|1,1}(\delta, p+h(\hat{p}-p)) - F_{\Delta Y_3^0|1,1}(\delta,p)}{h}$. Next, under assumptions in Proposition 2, we established the following result:

$$\sqrt{n}|F_{\Delta Y_3^0|1,1}(\delta,\hat{p}) - F_{\Delta Y_3^0|1,1}(\delta,p) - \Gamma(\delta,p)(\hat{p}-p)|_\infty = o_p(1)$$

Proof. Similar proof to Lemma B.11. in Callaway and Li (2017). ■

Also, under assumptions in Proposition 2, we know that (see Lemma B.12. in Callaway and Li, 2017):

$$\sup_\delta |\sqrt{n}(\hat{F}_{\Delta Y_3^0|1,1}(\delta,\hat{p}) - F_{\Delta Y_3^0|1,1}(\delta,p)) - \sqrt{n}(\frac{1}{n}\sum_{i=1}^{n} h(W_{i3},\delta) - F_{\Delta Y_3^0|1,1}(\delta,p))| = o_p(1)$$

Finally we combine all the previous results to show Proposition 2 as follows,

$$\sqrt{n}(\hat{F}_{Y_3^0}(y) - F_{Y_3^0}(y)) = \sqrt{n}(\phi_n(F) - \phi(F)) + \phi_F^{'}\sqrt{n}(\hat{F} - F) + o_p(1)$$

Where the right side of the equality results from an application of the functional central limit theorem.

B.2.2 Proof of Proposition 3

Proof. Application of the functional delta method to result in Proposition 2. ■

B.2.3 Proof of Theorem 2

Proof. The result is guaranteed by the Hadamard differentiability of the quantile map (Van der Vaart and Wellner 1996, Lemma 3.9.23(ii)) and application of the functional delta method to result in Proposition 3. ■

B.2.4 Proof of Proposition 4

Proof. The result follows from identical reasoning as in Lemma B.13. and Lemma B.14. in Callaway and Li (2017). ■

Appendix C. Identification and Inference for in-movers

Now we turn to the identification of the quantile treatment effect for the *in-movers* (QTIM). Before stating the identification result in the next theorem, let us first provide the assumptions under which that result is valid.

Assumption 6 (Distributional Difference in Differences)
For all $x \in Supp(X)$, for all δ,

$$F_{\Delta Y_3^0|Q_2=0,Q_3=1,X=x}(\delta) = F_{\Delta Y_3^0|Q_2=1,Q_3=0,X=x}(\delta).$$

This assumption postulates that conditional on covariates X, the distribution of the change in untreated potential outcomes for the *in-movers* (treatment group) is identical to the distribution of the change in untreated potential outcomes for the *out-movers* (control group).

Assumption 7 (Copula Stability Assumption)

$$C_{\Delta Y_3^0,Y_2^0|Q_2=0,Q_3=1}(\cdot,\cdot) = C_{\Delta Y_2^0,Y_1^0|Q_2=0,Q_3=1}(\cdot,\cdot)$$

This assumption says that the dependence between the marginal distributions, $F_{\Delta Y_3^0|0,1}$ and $F_{Y_2^0|0,1}$ is identical to the dependence between the distribution $F_{\Delta Y_2^0|0,1}$ and $F_{Y_1^0|0,1}$.

Assumption 8 (Continuity)
$\Delta\mathcal{Y}_{3|10}(0)$, $\Delta\mathcal{Y}_{2|01}(0)$, $\mathcal{Y}_{2|01}(0)$, and $\mathcal{Y}_{1|01}(0)$ are compact and each of the associated random variables are continuously distributed on their support with densities that are bounded above and away from 0.

Assumption 9 (Overlap)
$p_{01} > 0$ and $p_{10}(x) > 0$ for all $x \in Supp(X)$

The first part of this assumption states that there is some positive probability that individuals are *in-movers*. The second part postulates that for an individual with any value of covariates x, there is a positive probability that she belongs to the group of *out-movers*.

Proposition 5 *Under Assumptions 6-9,*

$$F_{Y_3^0|0,1}(y) = E[1\{F^{-1}_{\Delta Y_3^0|1,0}(F_{\Delta Y_2|0,1}(\Delta Y_2)) \leq y - F^{-1}_{Y_2|0,1}(F_{Y_1|0,1}(Y_1))\}|Q_2 = 0, Q_3 = 1],$$

where

$$F_{\Delta Y_3^0|1,0}(\delta) = E\left[\frac{Q_2(1-Q_3)}{p_{10}(X)}\frac{p_{01}(X)}{p_{01}}1\{\Delta Y_3 \leq \delta\}\right], \tag{C1}$$

and

$$QTIM(\tau) = F^{-1}_{Y_3^1|0,1}(\tau) - F^{-1}_{Y_3^0|0,1}(\tau).$$

The proof of Proposition 5 is similar to Theorem 1's proof and is therefore relegated to Online Appendix B1.

Next we provide similar results for the *in-movers*.

Assumption 2.2 (Conditional Copula Stability Assumption)

$$C_{\Delta Y_3^0, Y_2^0|Q_2=0,Q_3=1,X=x}(\cdot,\cdot) = C_{\Delta Y_2^0, Y_1^0|Q_2=0,Q_3=1,X=x}(\cdot,\cdot)$$

Lemma 1 *Assume that, for all $x \in Supp(X)$, the random variables $\Delta Y_3^0|1,0$, $\Delta Y_2^0|0,1$, $Y_2^0|0,1$, and $Y_1^0|0,1$ are continuously distributed conditional on x. Under Assumptions 6, 2.2, 8 and 9,*

$$F_{Y_3^0|0,1,x}(y) = E[1\{F^{-1}_{\Delta Y_3^0|0,1,x}(F_{\Delta Y_2|0,1,x}(\Delta Y_2|x)) \leq y - F^{-1}_{Y_2|0,1,x}(F_{Y_1|0,1,x}(Y_1|x))\}|0,1,x],$$

and

$$QTIM(\tau, x) = F^{-1}_{Y_3^1|0,1,x}(\tau|x) - F^{-1}_{Y_3^0|0,1,x}(\tau|x),$$

which is identified, with

$$F_{Y_3^0|0,1}(y) = \int_{Supp(X)} F_{Y_3^0|0,1,x}(y)\mathrm{d}F(x|0,1) \quad \text{and} \quad QTIS(\tau) = F^{-1}_{Y_3^1|0,1}(\tau) - F^{-1}_{Y_3^0|0,1}(\tau).$$

Appendix D. Background, Data, and Additional Results for Application

D.1 Background

To be eligible for SNAP benefits, a household needs to clear three means tests: a gross income test, a net income test, and a resource test. The first two tests require a household to have a monthly income (gross income test) and a monthly income minus several deductions (net income test) below different threshold levels depending on the household size. The resource test requires a household not to have countable resources, such as cash money in bank accounts, higher than a maximum level[2]. Once a household clears these tests and is accepted by a state agency to receive SNAP benefits, it can receive benefits for a specific period of time, called the certification period (e.g., three months), after which it has to obtain a re-certification to continue receiving benefits.

These standards of income tests are adjusted once a year in October based on the change in the cost of living. The allotment amounts are also adjusted each year. The American Recovery and Reinvestment Act (ARRA), implemented in April 2009, expanded the budget for SNAP in response to the Great Recession. The main change in the ARRA was to increase monthly SNAP benefits for the next 5 years[3]. The maximum benefit level was increased by 13.6%, and as a result, an average 4-person household received an extra $80 in benefits each month. Because households received, on average, $260 a month before the policy change, this change was significant. We use this exogenous change in SNAP benefits in our analysis.

[2]The maximum level is $2,250 for a normal household and $3,500 for a household with an elderly or disabled member. There are also restrictions on vehicle ownership. The resource test excludes assets such as a home, lot, and benefits from some other government programs. However, most states eliminated the resource test for most participants as of 2018.

[3]Aside from the change in SNAP benefits, it temporarily allowed Able-Body-Adults-Without-Dependents (ABAWDs) to receive SNAP benefits beyond 3 months in 3 years even without working.

D.2 Data

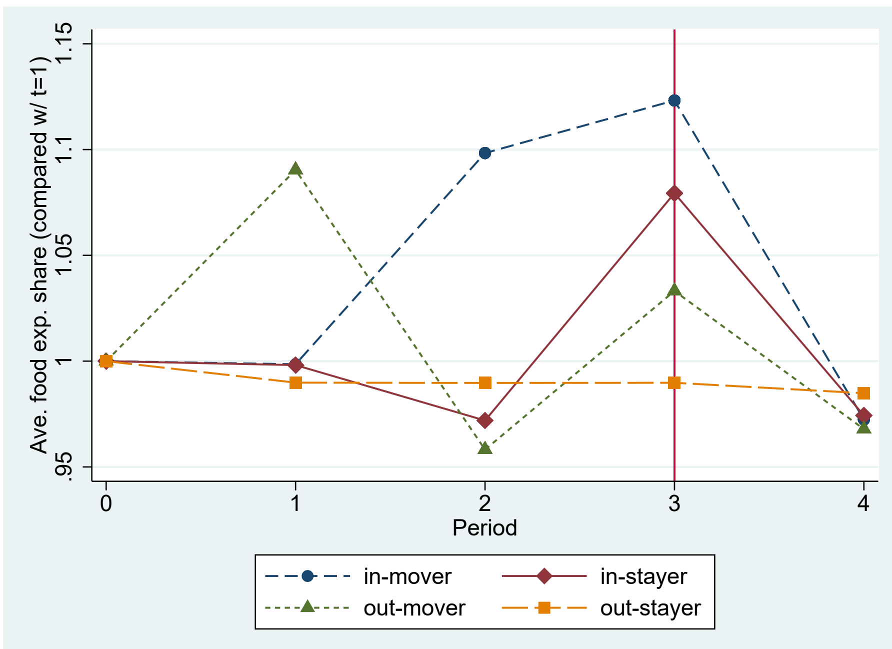
New panels in the Consumer Expenditure Quarterly Interview Survey (CEX) are initiated every month of the year. The CEX follows specific addresses, not specific households, across quarters. Therefore, if a household moves into a survey participant's address, the new household is interviewed from then on to the end of the panel. To mitigate this problem, we drop a household if it reports an unreasonable change in the age of the reference person or the number of household members. Furthermore, We drop households (1) without two pre-treatment periods due to our methodological requirement, and (2) with more than 150% of the average total expenditure of the treatment group.

A caveat on the qualification variable Q_{it} in the data is that the CEX has a variable concerning whether a household received SNAP benefits in the previous 12 months, not in the interviewed quarter. Therefore, I treat a household receiving SNAP benefits in the previous 12 months as a SNAP participant in the quarter. In addition, information on SNAP participation in the second and third quarters is imputed from information in the first quarter. Therefore, a household is defined as *in-movers* (or *out-movers*) if it participates in SNAP in the first quarter, but not in the fourth quarter (or not participating in SNAP in the first quarter, but participating in the program in the final quarter).

Finally, we choose our final sample in the following way. We used the coarsened exact matching method in order to obtain a better control group (Iacus, King, and Porro 2011). The method balances observable variables between treatment and control groups by coarsening specific demographic variables into categories and then matching households. The observable variables include the family size, the average age of head of household, the race of the reference person, marital status, income before taxes, and working status.

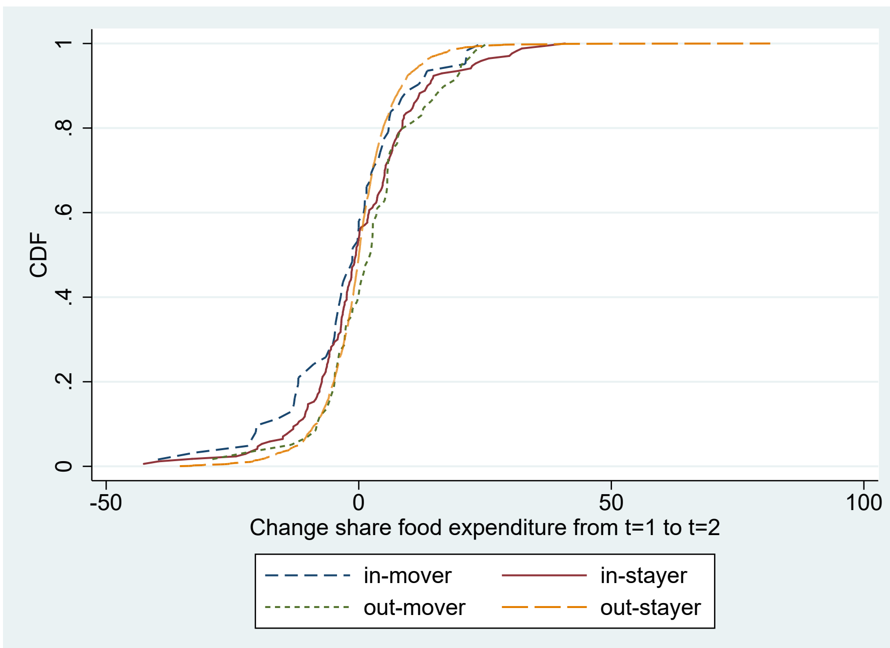
There is a data-driven reason why we choose *out-stayers* as our control group. Figures D1 and D2 show the average and the distribution of the change in the food-at-home expenditure share during the pre-treatment periods. Figure D1 shows that the trend of the average outcome variable for *in-stayers* is the closest to the trend for *out-stayers*. The distributions of these groups are also close to each other, as shown in Figure D2. Table D1 shows the result of the Kolmogorov-Smirnov tests on the equality of conditional distributions (from panels 2 to 7 of Table D1) as well as unconditional distributions (in panel 1 of Table D1). The distributions are conditioned with respect to marital status, race (i.e., white or not), and the discretized version of covariates such as education, total expenditure, family size, and age, respectively. In almost all cases, we cannot reject the null hypothesis of equality of both conditional distributions. These results support that the *out-stayers* are the best control group.

Figure D1: Trend in the average food expenditure share at home



Note: The figure shows the average food expenditure share at home for each group of households, normalizing that in $t = 1$. Treatment, i.e., an increase in SNAP benefits is in $t = 3$. The dotted line is a trend for *out-movers*, the dashed line is that for *in-movers*, the long-dashed line is that for *out-stayers*, and the solid line is that for *in-stayers*.

Figure D2: CDF of the change in food expenditure share at home between $t = 1$ and $t = 2$



Note: The figure shows a cumulative distribution function of a change in food expenditure share at home for each group. The dotted line is a trend for *out-movers*, the dashed line is that for *in-movers*, the long-dashed line is that for *out-stayers*, and the solid line is that for *in-stayers*.

Table D1: Kolmogorov-Smirnov tests on the equality of distributions

	High/Large/Yes			Low/Small/No		
	(1)	(2)	(3)	(4)	(5)	(6)
	Difference	p-value	# sample	Difference	p-value	# sample
Unconditional						
In-stayer vs out-stayer	0.138	0.005	2,347			
In-stayer vs in-mover	0.110	0.645	232	-	-	-
In-stayer vs out-mover	0.146	0.307	229			
Conditional on each variable						
Conditional on education						
In-stayer vs out-stayer	0.207	0.517	740	0.121	0.035	1,607
Conditional on total expenditure						
In-stayer vs out-stayer	0.143	0.477	1,216	0.151	0.009	1,131
Conditional on family size						
In-stayer vs out-stayer	0.156	0.042	758	0.159	0.043	1,589
Conditional on age						
In-stayer vs out-stayer	0.118	0.353	1,131	0.170	0.008	1,216
Conditional on marital status						
In-stayer vs out-stayer	0.124	0.606	1,054	0.184	0.001	1,293
Conditional on white						
In-stayer vs out-stayer	0.121	0.071	1,914	0.205	0.053	433

Note: The top panel shows the Kolmogorov-Smirnov (KS) test for the equality of unconditional distributions, for *in-stayers* vs. *out-stayers*, *in-stayers* vs. *in-movers*, and *in-stayers* vs. *out-movers* respectively. Panels 2 to 7 are the KS tests conditional on each variable. The first three columns of them show the KS test for the sub-group of a high (or large, or dummy variable being 1) conditioning variable. The last three columns show the KS test for sub-group of a low (or small or dummy variable being 0) conditioning variable. For example, the first column of panel 2 shows the difference in the distribution of food expenditure share between *in-stayers* and *out-movers* for sub-groups with a high education level. Similarly, the fourth column of panel 2 shows the difference in the distribution of food expenditure share between *in-stayers* and *out-movers* for sub-groups with a low education level.

The summary statistics of our final sample are reported in Table D2. There is substantial under-reporting in SNAP participation status in the CEX. The SNAP participation rate is 7.2%, while the rates calculated from other administrative data are 12.8%, 13.5%, and 14.5% in 2007, 2008, and 2009, respectively. This under-reporting implies that our control groups include some SNAP participants, leading to a potential underestimation of the effect of increases in SNAP benefits on food expenditure shares.

Table D2: Summary Statistics on households in the first period

Mean	**Treatment Group**		**Control Group**	**Difference**
	Infra-marginal	**Extra-marginal**		
Food exp. share at home	23.14	19.16	14.16	-7.43
Food exp. at home	448.91	452.32	579.13	-128.90
Food exp. away from home	70.36	109.49	280.38	-194.83
Total expenditure	2146.56	2799.18	4973.71	-2573.78
Family size	2.80	3.09	2.21	0.70
Head ages	50.55	45.02	52.62	-4.22
Urban	0.92	0.94	0.92	0.004
White	0.70	0.71	0.82	-0.12
Black	0.27	0.27	0.12	0.15
Married	0.23	0.23	0.46	-0.23
Working	0.38	0.56	0.68	-0.23
Male head	0.21	0.27	0.46	-0.23
Households	104	66	2,177	

Note: The table shows summary statistics for infra-marginal households in the treatment group (column 2), those for extra-marginal households in the treatment group (column 3), those in the control group (column 4), and the difference in averages between the treatment and control groups. Households in the treatment group are divided into two groups depending on the relationship between food expenditure at home and SNAP allotments.

D.3 Comparison with Existing Studies

While many studies in the literature focus on the ATT, to our knowledge, Valizadeh and Smith (2020) are the only studies that analyze the heterogeneous impact of additional SNAP benefits on food expenditure. One of their findings (Figure A2 in the Online Appendix of their 2019 paper) is that the positive treatment effect (i.e., an increase in food expenditure share due to the additional SNAP benefit) is larger for higher quantiles of the distribution of food expenditure for the inframarginal households. A similar result can be replicated with their method using our data in Table D3.

While the positive relationship between the treatment effects and the percentile in the distribution is the same as our results, the magnitude of their estimates seems to be larger in each percentile than ours. This distinction can be due to differences in identification approaches. Valizadeh and Smith (2020) use a non-additive fixed-effect quantile regression, and thus their identification comes from a variation within a household over time. Therefore, it could be vulnerable to measurement errors and time-varying unobserved shocks that are correlated with both treatment and food expenditure during policy changes.

The ARRA of 2009 was a package of several policies, such as unemployment benefits and temporary welfare payments (TANF and WIC). Therefore, relying on within-variation might contaminate their estimates. On the other hand, our identification comes from a comparison of SNAP households with those having similar characteristics in the control group. Because the control group also has similar household characteristics as the treatment group, unobserved time-varying shocks could be canceled out.

Beatty and Tuttle (2015) use a linear DID approach for the same exogenous policy changes as ours. Their ATT is 0.72 in their main specification (Table 4 in their paper), which is different from the ATT obtained from our methodology, 2.27. One reason could be that they focus on inframarginal SNAP recipients, i.e., households whose food expenditure is larger than SNAP benefits, while we include both inframarginal and extramarginal SNAP recipients in our sample. Once we focus on inframarginal SNAP recipients as our treatment group, the ATT becomes 1.93, as reported in Table D4. Another reason for the difference in s could be our methodological requirement. Our methodology requires, at least, three-period panel data with two pre-treatment periods. Once we regress a linear DID without requiring two periods before treatment, the ATT declines to 1.47.

Table D3: Quantile treatment effects using Powell's (2020) method

	With covariates	
Expenditure:	Share food at home	
Quantile	Estimate	s.e.
10	2.21	(0.77)
20	1.28	(0.65)
30	0.95	(0.74)
40	2.68	(0.83)
50	1.25	(0.69)
60	5.99	(1.51)
70	5.21	(0.95)
80	7.72	(3.41)
90	5.85	(1.03)

Note: The table shows the quantile treatment effects for each quantile when they are estimated by a non-additive fixed-effect quantile regression approach in Powell (2020). STATA code *qregpd* is available in Powell (2020).

D.4 Additional Results

Figure D3 and Table D4 show the results when focusing only on inframarginal SNAP households. They provide a positive relationship between the increment of the food expenditure share and a percentile in the food expenditure share distribution even within the sample of inframarginal participants.

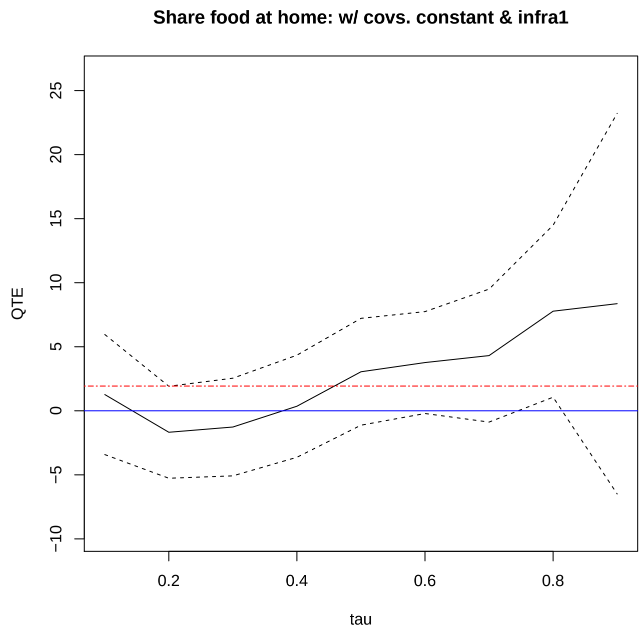
We also estimate the impact of additional SNAP benefits on the food-away-from-home expenditure share, including eating at restaurants and ordering fast food. Because SNAP benefits can be used only for food prepared at home, we do not expect SNAP participants to increase this share. Figures D4 and D5 show the results. These figures do not provide any noticeable patterns and therefore imply that additional SNAP benefits are not fungible (not the same as extra cash).

Table D4: Quantile treatment effects for inframarginal households

	With covariates		Without covariates	
Expenditure:	Share food at home		Share food at home	
Quantile	Estimate	s.e.	Estimate	s.e
10	1.27	(2.39)	0.73	(2.29)
20	-1.68	(1.83)	-1.70	(1.75)
30	-1.27	(1.95)	-2.42	(1.73)
40	0.36	(2.03)	-0.30	(1.77)
50	3.05	(2.13)	2.09	(2.08)
60	3.77	(2.03)	3.88	(1.88)
70	4.31	(2.65)	4.37	(2.87)
80	7.78	(3.43)	7.70	(3.73)
90	8.36	(7.58)	6.28	(7.51)
ATE	1.93	(1.42)	1.69	(1.39)

Note: The table shows the QTIS and their standard errors from the 10th to 90th percentiles when we focus our treatment group on inframarginal SNAP households. The first two columns show the results when using the share of food expenditure consumed at home as the outcome variable and controlling for covariates. Columns 3 and 4 show those when using the share of food expenditure at home as the outcome variable without covariates. The standard errors in parentheses are calculated by the bootstrap with 300 iterations.

Figure D3: QTT on food expenditure share at home (inframarginal): ATT 1.93 [1.42]

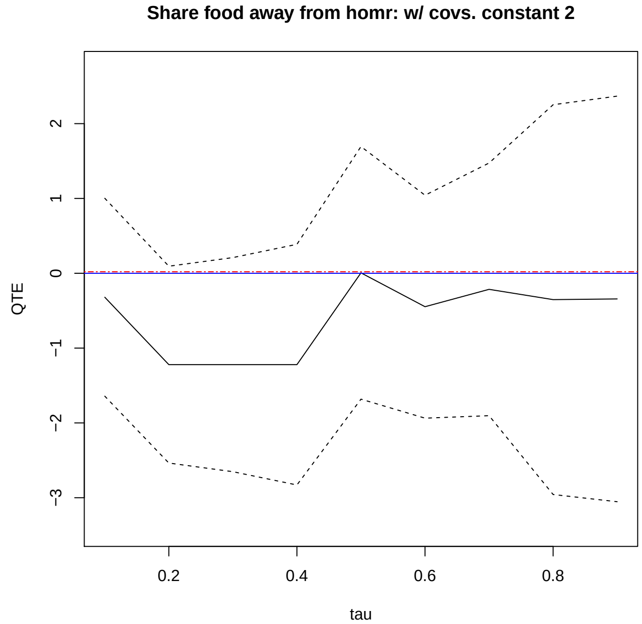


Note: The figure provides estimates of the QTIS with covariates on the effect of an increase in SNAP benefits on food expenditure level at home, for the group of infra-marginal households. The solid line denotes the estimate for each quantile measured on the horizontal axis. The black dotted line is a 95% confidence interval calculated by the bootstrap with 300 iterations. The red dashed horizontal line is the average treatment effect.

D.5 Checking the Empirical Validity of Assumptions

We have already discussed the empirical validity of the first assumption using the Kolmogorov-Smirnov tests in Section D1. The second assumption, the CSA, requires that the dependence between (i) the distribution of the change in the untreated potential outcomes for the treated (i.e., $\Delta Y_{03}|Q_3 = 1$) and (ii) the distribution of the initial untreated outcome for the treated group (i.e., $Y_{02}|Q_3 = 1$) remains stable over time. The validity of this assumption could be especially concerning because our analysis includes the periods during the Great Recession. If SNAP households with large food-at-home expenditure shares in the initial period have larger increases in the share before the Great Recession, but the relationship does not hold

Figure D4: QTT on food expenditure share away from home:
ATT 0.02 [0.56]

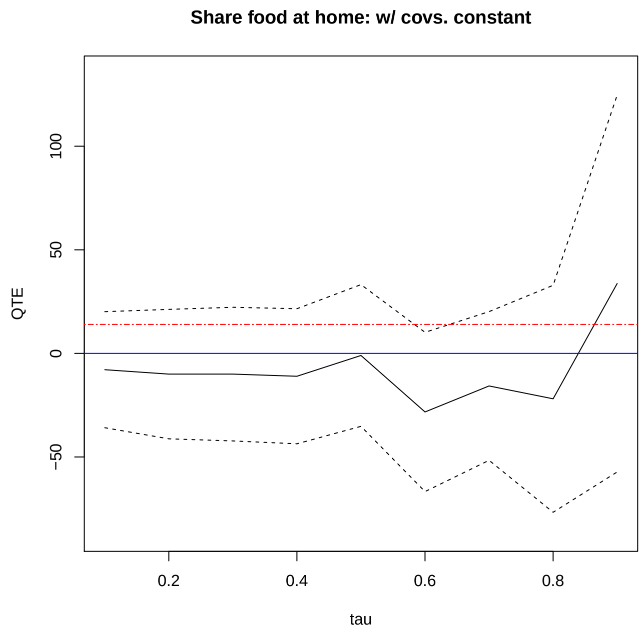


Note: The figure provides estimates of the QTIS with covariates on the effect of an increase in SNAP benefits on food expenditure share away from home. The solid line denotes the estimate for each quantile measured on the horizontal axis. The black dotted line is a 95% confidence interval calculated by the bootstrap with 300 iterations. The red dashed horizontal line is the average treatment effect.

during the Recession, then the assumption is violated.

The validity of the CSA is assessed by the test of Kendall's Tau (a standard dependence measure that depends only on the copula), as suggested by Callaway and Li (2019). Kendall's Tau for the change in the food expenditure share at home and the initial level of the food expenditure share for treated households must be constant over time during the pre-treatment period. Because each household is recorded in the sample for at most four periods in our data and most households have two pre-treatment periods, we calculate Kendall's Tau for the change in the food expenditure share at home from the 1st to 2nd periods and the expenditure share in the 1st period for the treated households. The stability of Kendall's Tau over the pre-treatment is further checked by using another sample of house-

Figure D5: QTT on food expenditure away from home:
ATT 13.98 [10.90]

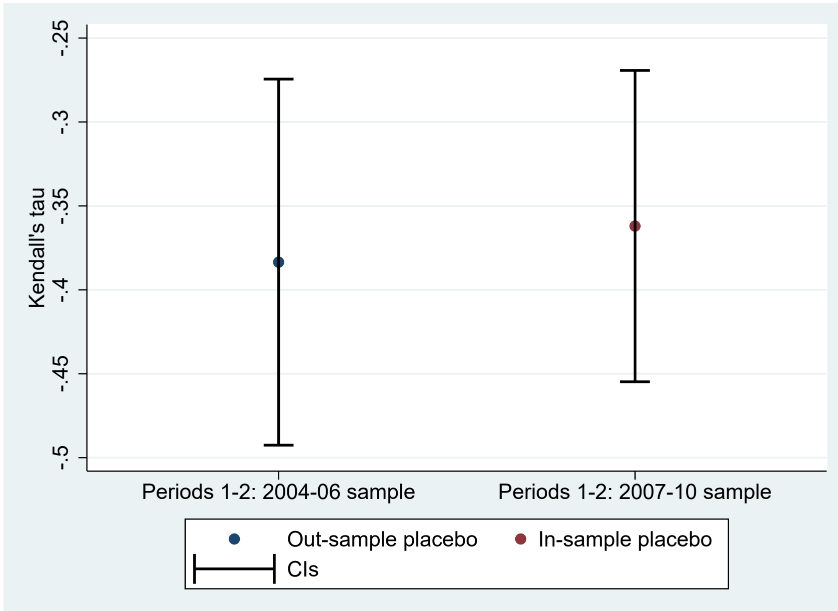


Note: The figure provides estimates of the QTIS with covariates on the effect of an increase in SNAP benefits on food expenditure level away from home. The solid line denotes the estimate for each quantile measured on the horizontal axis. The black dotted line is a 95% confidence interval calculated by the bootstrap with 300 iterations. The red dashed horizontal line is the average treatment effect.

holds from the 1st to 2nd periods from 2004 to 2006 (rather than 2007 to 2010, which is used in our main analysis). As illustrated in Figure D6, Kendall's Tau varies little over time and is approximately -0.3. Figure D7 shows the same Kendall's Tau for the placebo sample of households but also calculates the same statistics from the 2nd to 3rd periods (the second plot in the figure) and from the 3rd to 4th periods (the third plot in the figure). Again, the measure changes slightly over the periods, thus indirectly supporting the CSA even without conditioning on covariates.

We can also check the validity of the DDID Assumption by performing a placebo test while assuming the other assumptions hold. Ideally, we want to test the assumption by estimating the QTIS during the pre-treatment periods if the panel is long enough. However,

Figure D6: Kendall's Tau

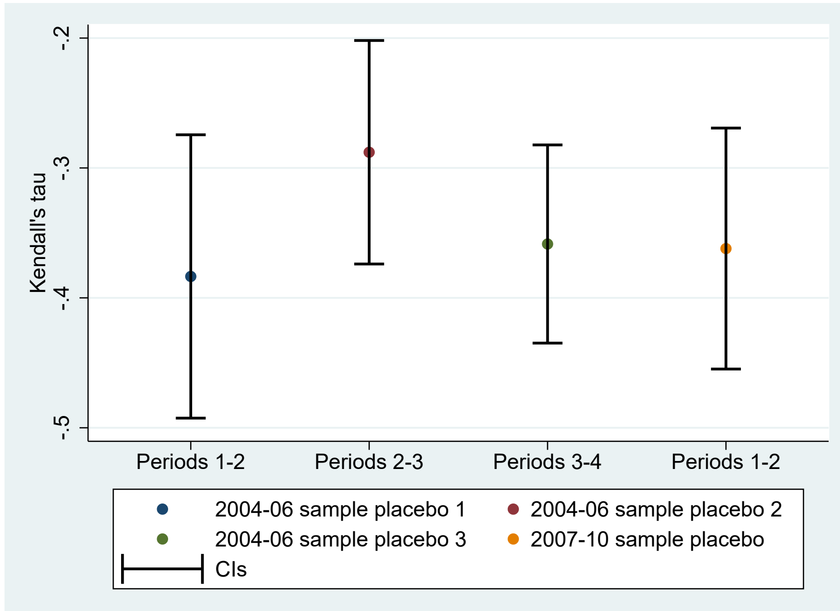


Note: The figure shows Kendall's tau for the pre-treatment period. The left Kendall's tau (denoted a circle) is calculated for the dependence between the distribution of the share of food-expenditure-at-home in the first period and that for a change in share from the first to second periods, using a sample of households in the CEX during 2004-06. The right Kendall's tau (denoted a circle) is calculated for the same dependence using the same sample of households as the main analysis, but during the pre-treatment period. A range of a vertical line corresponds to a 95% confidence interval.

each household in the CEX is recorded, at most, only for 4 quarters and the QTIS requires two periods before treatment, and thus the placebo experiment cannot be implemented using our main sample. Therefore, we construct another sample of households using the 2004-2006 CEX and implement the same quantile estimation. SNAP recipients face a marginal adjustment of SNAP benefits in the 4th quarter of each year and the changes in the benefits are therefore used as a quasi-placebo experiment (the increase in the maximum SNAP benefits was at most 2.4% in October 2006, while that in April 2009 was 13.6%). Figures D8 and D9 show the estimates of the QTIS with the placebo sample of households. While the lower quantiles have statistically significant negative effects, most of the QTISs are statistically insignificant, supporting the fact that all four assumptions potentially hold jointly.

The final assumption to check is the overlap assumption. It requires that (i) there is a

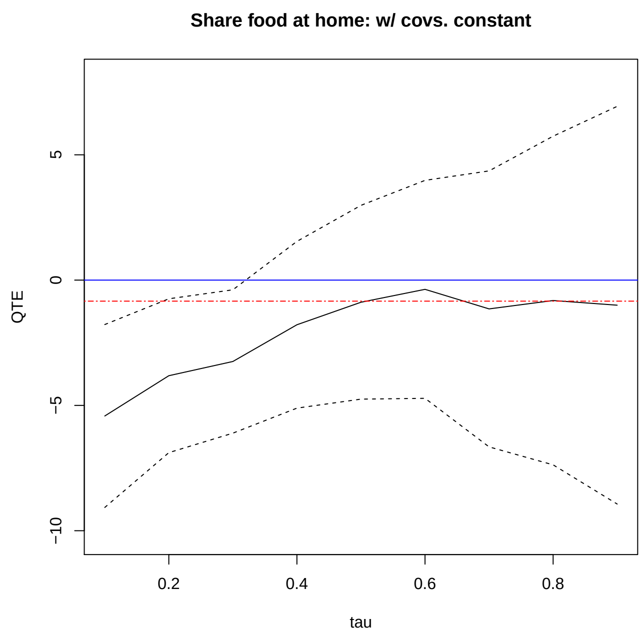
Figure D7: Kendall's Tau for different quarters



Note: The figure shows Kendall's tau for the pre-treatment period. The very left and right statistics show the same as those in Figure 11. The second left Kendall's tau (denoted a circle) is calculated for the dependence between the distribution of the share of food-expenditure-share-at-home in the second period and that for a change in share from the second to third periods during 2004-06. The second right Kendall's tau (denoted a circle) is calculated for the same dependence over third and fourth periods during 2004-06. A range of a vertical line corresponds to a 95% confidence interval.

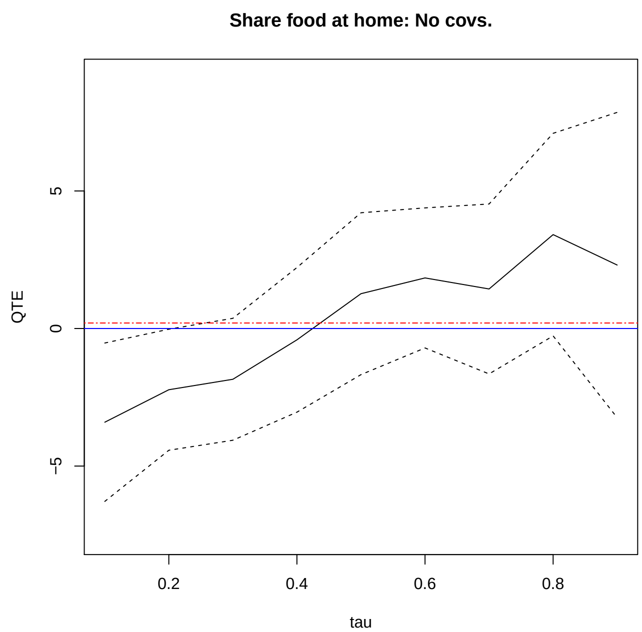
positive probability for a household to be *in-stayers* and (ii) there is a positive probability for a household with any covariates x to be *out-stayers*. While the former condition is true in our sample, the latter is indirectly tested by checking the existence of *out-stayers* for a range of covariates in our data. The results are shown in Tables D5 and D6. For most of the combinations of covariates, there are positive *out-stayers*, supporting the second part of the overlap assumption.

Figure D8: QTT on the share of food expenditure at home (Placebo): ATT -1.65 [s.e. 2.18]



Note: The figure provides a placebo estimate of the QTIS with covariates on the effect of an increase in SNAP benefits on food expenditure share at home. The sample period used here is from 2004-06, rather than 2007-10 when there were actual large increases in SNAP benefits. The solid line denotes the estimate for each quantile measured on the horizontal axis. The black dotted line is a 95% confidence interval calculated by the bootstrap with 300 iterations. The red dashed horizontal line is the average treatment effect.

Figure D9: QTT on the share of food expenditure at home (Placebo, no covariates): ATT 0.82 [s.e. 0.90]



Note: The figure provides a placebo estimate of the QTIS without covariates on the effect of an increase in SNAP benefits on food expenditure share at home. The sample period used here is from 2004-06, rather than 2007-10 when there were actual large increases in SNAP benefits. The solid line denotes the estimate for each quantile measured on the horizontal axis. The black dotted line is a 95% confidence interval calculated by the bootstrap with 300 iterations. The red dashed horizontal line is the average treatment effect.

Table D5: Number of observations in each group

			Family Size																	
			1		2		3		4		5		6		7		8		9	
Family income			L	H	L	H	L	H	L	H	L	H	L	H	L	H	L	H	L	H
Not work	Not urban	out-stayer	1,106	252	442	474	71	73	24	26	6	6	1	3	0	4	0	0	0	0
		in-stayer	45	3	21	6	15	5	3	5	0	0	0	0	0	0	0	0	0	0
	Urban	out-stayer	76	12	58	47	6	2	0	0	0	0	0	0	0	0	0	0	0	0
		in-stayer	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Work	Not urban	out-stayer	890	719	517	1,075	254	579	202	602	70	266	32	75	10	28	0	11	0	3
		in-stayer	12	0	32	3	37	6	1	11	7	5	6	6	3	1	0	0	0	0
	Urban	out-stayer	53	34	52	66	32	86	15	56	0	19	8	8	0	0	0	0	3	1
		in-stayer	3	1	3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Note: The table shows the number of observations in our final sample for each subgroup. The odd columns, denoted "L", are a group of households with low income. The even columns, denoted "H", are a group of households with high income. The first two columns report the number of observations for single households, the next two columns are that for two-person households, the fifth and sixth columns are that for three-person households, and so on. Each row divides the entire sample by working status, living area (urban or not urban), and their qualification status over time. Therefore, the fifth row shows, for example, the number of observations in non-urban living and SNAP-disqualified households whose heads work.

Table D6: Number of observations in each group 2

Family income			Marriage status: No L	No H	Yes L	Yes H
Not white	Male	out-stayer	268	59	196	110
		in-stayer	12	0	10	0
	Female	out-stayer	451	140	187	49
		in-stayer	55	2	20	0
White	Male	out-stayer	1,130	376	1,362	416
		in-stayer	29	0	20	0
	Female	out-stayer	1,165	470	1,299	335
		in-stayer	53	8	32	4

Note: The table shows the number of observations in our final sample for each subgroup defined differently from the last table. The odd columns, denoted "L", are a group of households with low income. The even columns, denoted "H", are a group of households with high income. The first two columns report the number of observations for households without married members and the next two columns are that for households with married members. Each row divides the entire sample by race (white nor not), male or female, and their qualification status over time. Therefore, the fifth row shows, for example, the number of observations in SNAP-disqualified households with white male heads.

Appendix E. Robustness Check

We check the robustness of our method by using an alternative approach by Athey and Imbens (2006)[4]. Their identifying assumptions are:

Assumption 10.1: In the absence of treatment, the outcome is $Y_t^0 = h(U, t)$, where U is the unobserved characteristics.

Assumption 10.2: The production function $h(u, t)$ is strictly increasing in u

Assumption 10.3: The distribution of u is independent of time conditional on the treatment: $U \perp t | Q$

Assumption 10.4: $Supp(U|Q = 1) \subseteq Supp(U|Q = 0)$

Under Assumptions 10.1-10.4, the counterfactual distribution is identified and the treatment effect on the treated for τ-th quantile is:

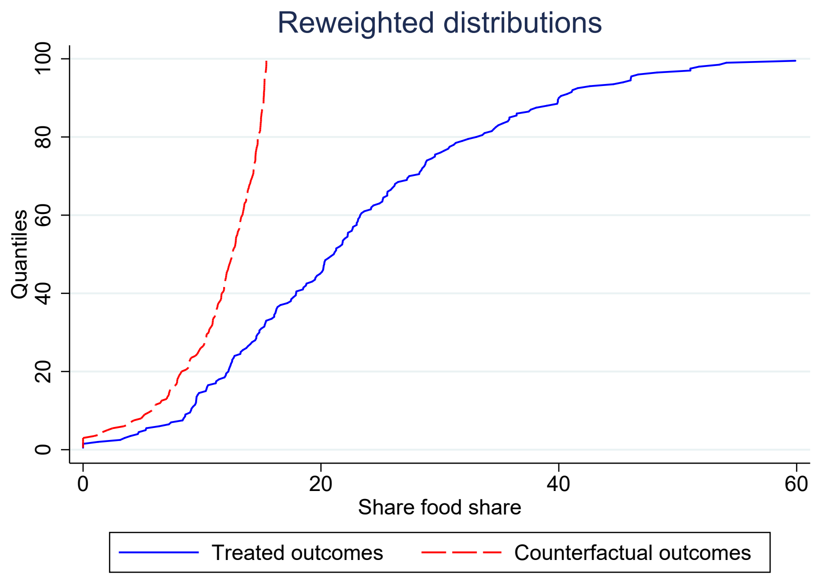
$$\Delta(\tau) = \mathbb{E}_x[F^{-1}_{Y|Q_t=1,T=t,X}(\tau)] - \underbrace{\mathbb{E}_x[F_{Y|Q_t=1,T=2,X}(F_{Y|Q_t=0,T=2,X}(F_{Y|Q_t=0,T=t,X}(\tau)))]}_{\text{Counterfactual distribution}}.$$

Using this identification strategy with the propensity score re-weighting, Figure E1 shows the cumulative distribution function and the counterfactual cumulative distribution function for the treated. Figure E2 shows the treatment effect on the treated, which is constructed by the difference between the cumulative distribution function and the counterfactual cumulative distribution function. The 95% confidence intervals are derived using the bootstrap with 300 iterations.

Figures E3 and E4 show the results when using the log food expenditure at home as the

[4] We use a STATA code by Garlick (2017).

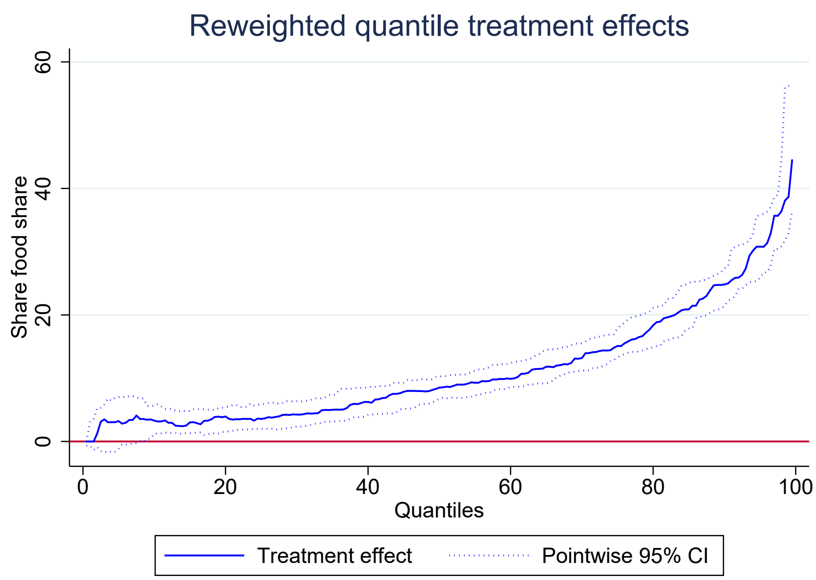
Figure E1: Distributions in changes-in-changes: food expenditure share



Note: The figure provides the share of food expenditure at home in each quantile for treated and counterfactual constructed using the change-in-changes method by Athey and Imbens (2006). The red dashed line is for the counterfactual and the solid blue line is for the treated group.

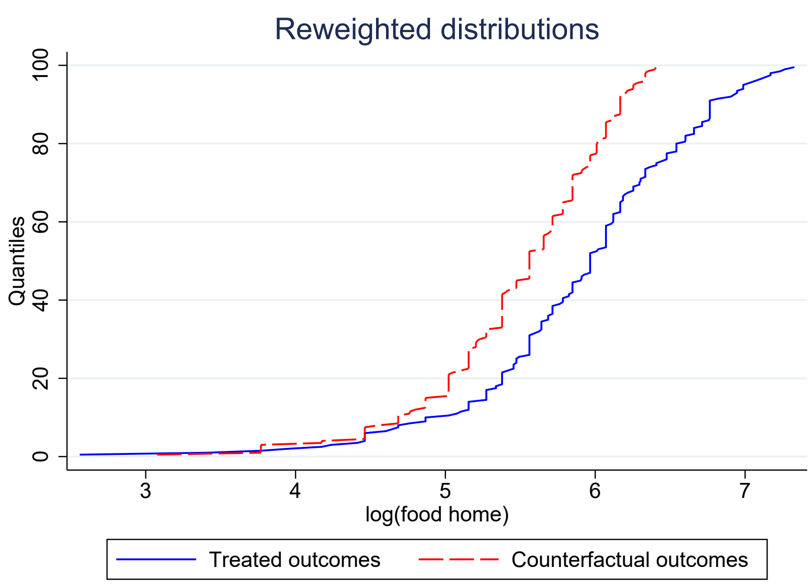
outcome variable.

Figure E2: Treatment effect in changes-in-changes: food expenditure share



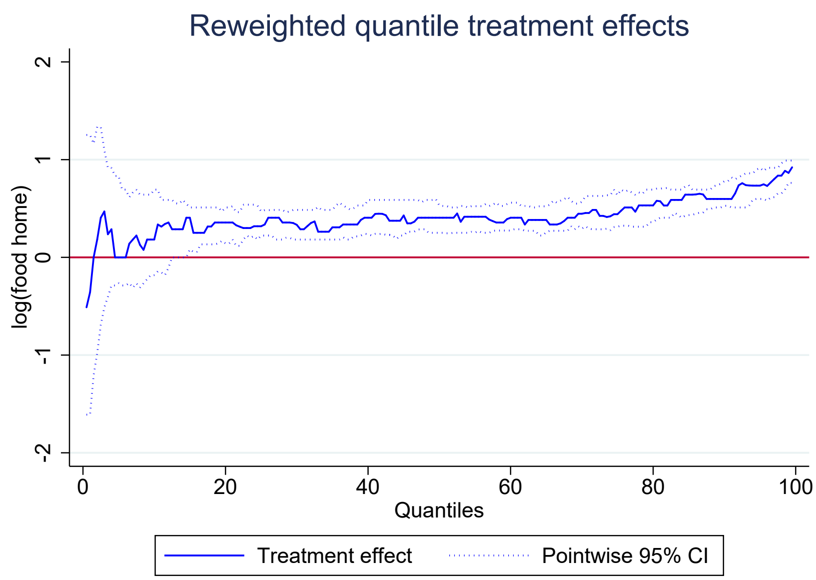
Note: The figure provides estimates of the change-in-changes approach by Athey and Imbens (2006) on the effect of an increase in SNAP benefits on food expenditure share at home. The solid line denotes the estimate for each quantile measured on the horizontal axis. The black dotted line is a 95% confidence interval calculated by the bootstrap with 300 iterations. The red dashed horizontal line is the average treatment effect.

Figure E3: Distributions in changes-in-changes: log food expenditure



Note: The figure provides the value of log food expenditure at home in each quantile for treated and counterfactual constructed using the change-in-changes method by Athey and Imbens (2006). The red dashed line is for the counterfactual and the solid blue line is for the treated group.

Figure E4: Treatment effect in changes-in-changes: log food expenditure



Note: The figure provides estimates of the change-in-changes approach by Athey and Imbens (2006) on the effect of an increase in SNAP benefits on log food expenditure at home. The solid line denotes the estimate for each quantile measured on the horizontal axis. The black dotted line is a 95% confidence interval calculated by the bootstrap with 300 iterations. The red dashed horizontal line is the average treatment effect.

Appendix F. Untreated Moving Effects

This section provides a way to quantify the potential bias due to untreated moving effects in the context of a linear DID model, following Lee and Kim (2014). Consider the following linear model:

$$Y_t^0 = \beta_t + \beta_q Q_t + \beta_w' W_t + V_t, \qquad \text{and} \qquad Y_t^1 = \beta_d Q_2 + \beta_m(1 - Q_2) + Y_t^0,$$

where $W_t = (C', X_t')'$ is composed of time-invariant and time-variant covariates, and $V_t = \delta + u_t$ is an error term of time-invariant and time-variant elements. The average treatment effect in this specification is different between the *in-movers* and *in-stayers*. Let $t = 3$ be a period with an increase in SNAP benefits, and $t = 2$ is the period before. Therefore, the treatment variable is $D_t = 1\{t = 3\} \times Q_t$ (i.e., the increase in SNAP benefits happens for households in the treatment group at period 0). Combining these equations gives

$$Y_t = \{\beta_d Q_2 + \beta_m(1 - Q_2)\}D_t + \beta_t + \beta_q Q_t + \beta_w' W_t + V_t,$$

Finally, by taking the difference between $t = 3$ and $t = 2$, the regression equation becomes

$$\Delta Y_3 = \Delta\beta_3 + \beta_d Q_2 Q_3 + \beta_m(1 - Q_2)Q_3 + \beta_q \Delta Q_3 + \beta_w' \Delta W_3 + \Delta u_3, \tag{F1}$$

ΔY_3 is the difference in the outcome variable Y_t between periods $t = 3$ and $t = 2$. β_d is the treatment effect on *in-stayers* (i.e., units with $Q_2 = 1$ and $Q_3 = 1$), β_m is the treatment effect on *in-movers* (i.e., units with $Q_2 = 0$ and $Q_3 = 1$), and β_q is the untreated moving effects (i.e., units with $Q_2 = 0$ and $Q_3 = 1$ or $Q_2 = 1$ and $Q_3 = 0$).

Estimation results are reported in Table F1. There are three things to note here. First, the average treatment effect on *in-stayers* is positive and statistically significant for the food-at-home expenditure share, but not significant for the food-away-from-home expenditure share. These are consistent with the results of the linear DID regression presented above. Second, the average treatment effect on *in-movers*, captured by β_m, is statistically insignificant but positive for all regressions. Third, the untreated moving effects are negative and statistically insignificant. The negative signs are plausible because the sample periods are mainly during the Great Recession. These results show different effects of the increases in SNAP benefits between *in-stayers* and *in-movers*, and therefore suggest the importance of separating them.

Table F1: Linear DID: untreated moving effects

	(1)	(2)	(3)	(4)
VARIABLES	Δ(Share food home)	Δ(Share food away)	Δ(Food home)	Δ(Food away)
β_d	3.053***	0.336	69.49**	36.36
	(0.595)	(0.424)	(28.35)	(24.43)
β_m	0.558	1.405	44.35	43.79
	(2.542)	(1.811)	(121.1)	(104.4)
β_q	-1.169	-0.398	-25.30	-13.09
	(1.799)	(1.282)	(85.73)	(73.89)
Observations	2,467	2,467	2,467	2,467
R-squared	0.117	0.009	0.051	0.037

Note: The table shows the estimation results from equation (F1). The first column shows the results using a change in food expenditure share at home as the outcome variable, and the second column shows the results using a change in food expenditure share away from home as the outcome variable. The third column shows the results when using a change in food expenditure at home as the outcome variable, and the final column shows the same results but using a change in food expenditure away from home as the outcome variable. Standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

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