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New Traffic Conflict Measure Based on a Potential Outcome Model

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Abstract: A key issue in the analysis of traffic accidents is to quantify the effectiveness of a given evasive action taken by a driver to avoid crashing. Since 1977, the widely accepted definition for this effectiveness measure, which is called traffic conflict, has been “the risk of a collision if the driver movement remains unchanged.” Although the definition is expressed counterfactually, the full power of counterfactual analysis was not utilized. In this paper, we propose a counterfactual measure of traffic conflict called Counterfactual Based Conflict (CBC). The CBC is interpreted as the probability that a driver avoided a crash actually by taking an evasive action in the counterfactual situation in which the crash would have occurred if he/she had not taken an evasive action and the crash would not have occurred if he/she had taken an evasive action. The CBC captures realistic aspects of the traffic situation, and lends itself to modern causal analysis. In addition, we provide some of identification conditions for the CBC. Furthermore, we formulate bounds on the CBC when the proposed identification conditions are violated. Finally, through an application of the CBC to the 100-Car Naturalistic Driving Study, we discuss the usefulness and limitations of the proposed measure.

Keywords: traffic conflict, crash-to-conflict ratio, counterfactual conditional, potential outcome model, structural causal model

1 Introduction

A traffic crash occurs when one vehicle collides with another vehicle or with a pedestrian, animal, road debris, or other stationary obstruction such as a tree or utility pole and often leads to severe consequences, including injury, death and property damage [1, 2, 3]. With the aim of preventing such consequences, a great deal of research effort has been expended in developing road safety theory (i. e., why crashes occur?), and identifying factors affecting road safety (i. e., what caused crashes?) [2, 4, 5].

It would be reasonable to observe crashes to conduct successful road safety research, if such observation were possible. However, there are two difficulties in conducting such research. First, crashes occur sporadically, and some of them might not be reported [6]. Second, waiting for such severe consequences in order to prevent crashes is questionable ethically [7]. To overcome these drawbacks, Perkins and Harris [8] developed a traffic conflict technique that analyzes observed situations in which drivers took an evasive action to avoid crashes.

Since the seminal work of Perkins and Harris [8], the concept of traffic conflicts has been refined and discussed by many researchers and practitioners in the field of road safety research [9, 10, 11]. In Oslo in 1977, a conceptual definition of traffic conflicts was proposed in the first workshop on traffic conflicts. The definition, which has been widely accepted by road safety researchers and practitioners since then, is as follows:

> “A traffic conflict is an observable situation in which two or more road users approach each other in space and time to such an extent that there is a risk of collision if their movements remain unchanged” [12].

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Regarding this definition, Davis et al. [13] pointed out that

> "An observed event qualifies as a conflict only if it satisfies a counterfactual test; if the movements had remained unchanged then a collision would probably have resulted,"

and introduced the idea of counterfactual conditionals on the basis of structural causal model [14] into traffic conflict technique [13, 15, 16].

In 1982, Hauer [17] pointed out the importance of crash-to-conflict ratio in the traffic conflict technique. Since then, over the past decades, the research on crash-to-conflict ratio has attracted interest in the field of traffic conflict technique. Recently, Davis et al. [13] proposed a novel type of crash-to-conflict ratio based on the level of initial conditions in traffic situations and clarified some of the properties. We refer to this crash-to-conflict ratio as "Davis's crash-to-conflict ratio." Different from the existing traffic conflict techniques, Davis's crash-to-conflict ratio is formulated based on the conceptual aspect of the potential outcomes in the form of the initial conditions. Although it is assumed that the information on the initial conditions are fully available in Davis et al. [13], considering that crashes occur as a result of complex interactions among drivers, their vehicles, and their road environments [7], it would be reasonable to consider that the initial conditions cannot be observed fully in actual field studies. Thus, it might be unrealistic to evaluate Davis's crash-to-conflict ratio under such an assumption. Solving this problem requires formulating a new traffic conflict measure.

This paper proposes a new traffic conflict measure, the Counterfactual Based Conflict (CBC). To overcome the drawback of existing traffic conflict techniques, the CBC is formulated based on the potential outcome model, and thus reflects the counterfactual statement. The CBC can be interpreted as the probability that a driver avoided a crash actually by taking an evasive action in the counterfactual situation in which the crash would have occurred if he/she had not taken an evasive action and the crash would not have occurred if he/she had taken an evasive action. In this sense, the CBC evaluates the effectiveness of a given evasive action to avoid crashing, that is, the rate at which drivers in the abovementioned counterfactual situation actually avoided crashes by taking an evasive action. In addition, we provide identification conditions for the CBC. Furthermore, we formulate the bounds on the CBC, that is, the ranges within which the true values of the CBC must lie, when the proposed identification conditions are violated. Finally, through an application of the CBC to the 100-Car Naturalistic Driving Study (hereafter, the 100-car study) [1], we discuss the usefulness and limitations of the proposed measures. The extension of the CBC to a case of multiple evasive actions is also discussed in Appendix.

2 Potential outcome model

In this section, we introduce the potential outcome model [14, 18, 19, 20], to propose our new traffic conflict measure.

Let X and Y be random variables indicating an evasive action of a driver and a collision-related outcome, respectively. For simplicity, regarding a dichotomous evasive action X, x_0 indicates that the driver does not take an evasive action, and x_1 indicates that the driver takes an evasive action ($x_0 < x_1$). In this paper, an evasive action is defined as any action of the driver that is aimed at avoiding a severe consequence. Thus, this can include actions that were both successful and unsuccessful, and both abrupt and non-abrupt in avoiding the severe consequence. In some observational studies, it may be difficult to directly judge whether a driver took an evasive action. However, in other cases, such evidence is available from the behavior of a vehicle, for example, the activation of the brake signal or the abrupt change of course. Although the discussion in the main part of this paper is based on a distinction between the presence and absence of an evasive action, in Appendix, we extend the discussion to a case of multiple evasive actions. It is noted that the decision as to whether we should focus on a dichotomous evasive action or multiple evasive actions is dependent on the problem setting of interest. For example, if (i) road safety researchers and practitioners wish to analyze several types of evasive actions or (ii) the assumption of "no multiple versions of treatment" (the definition is to be mentioned later) for a dichotomous evasive action is violated, we should focus on multiple evasive actions.

A collision-related outcome Y is the consequence of an evasive action taken in the event of interest, such as a crash or a near crash defined by physical quantities (e. g., the distance between vehicles and the time required for two vehicles to collide). Thus, Y can be a variable that takes a sampled value over time such as the time to collision (TTC), or a properly selected single value such as the minimum TTC or the time gap. The selection of the collision-related outcome depends on the problem setting of interest. Although it is important to develop a protocol to observe the collision-related outcome, we do not focus on this because the discussion in the paper does not depend on a specific accident occurrence process. For the discussion of the protocol to observe the collision-related outcome, for example, refer to Brown [21], Hayward [22], and Van der Horst and Hogema [23] for the TTC, and refer to Sun and Benekohal [24] for the time gap.

When $Y > y$ for a given y, a smaller value of Y represents a situation in which a driver is closer to a severe consequence of interest. In contrast, when $Y \leq y$, the driver is involved in the severe consequence of interest. For example, letting Y be a variable representing the TTC, if we are interested in whether the driver avoids crashing, then y would be zero. If we wish to clarify the driver's judgment of braking timing in a severe situation based on the TTC [25, 26], y would be taken as a threshold value that determines whether he/she perceives to be in the severe situation. Although this paper mainly considers that a smaller value of y for $Y > y$ represents a situation in which the driver was closer to a crash (but not occurred) and $Y \leq y$ represents a situation in which the crash occurred, the discussion of the paper is not limited to this problem setting.

$\mathrm{pr}(X = x) = \mathrm{pr}(x)$ and $\mathrm{pr}(Y = y|X = x) = \mathrm{pr}(y|x)$ indicate a marginal probability of $X = x$ and a conditional probability of $Y = y$ given $X = x$, respectively. In addition, $\mathrm{pr}(X = x, Y > y) = \mathrm{pr}(x, Y > y)$ and $\mathrm{pr}(Y > y)$ indicate a joint probability of $X = x$ and $Y > y$ and a marginal probability of $Y > y$, respectively. Similar notation is used for other probabilities.

Regarding the ith of the N drivers, for $X = x$ ($x \in \{x_0, x_1\}$), we use potential outcome $Y_x(i) = y$ to denote the counterfactual statement "Y would have a value y, had X been x for the ith driver." In the potential outcome model, we adopt the "stable unit treatment value assumption" (SUTVA), which combines the "no interference between units" assumption, the "no multiple versions of treatment" assumption, and the "consistency" assumption to evaluate causal quantities. The "no interference between units" assumption means that $Y_x(i)$ for the ith driver is not dependent on the set of evasive actions taken by other drivers [18, 27, 28]. When drivers in the study are treated as random samples from the population under consideration, it is reasonable to expect that the assumption would be satisfied [29], and $Y_x(i)$ can be treated as the random variables Y_x. In this paper, a set of random variables is assumed to be independent and identically distributed (i. i. d.), that is, each random sample is obtained independently from the same joint probability distribution as the others. Then, the probabilities of the potential outcomes can be defined as $\mathrm{pr}(Y_x = y) = \mathrm{pr}(y_x)$, which is called a causal risk of $X = x$ on $Y = y$. The other part of SUTVA concerns the "no multiple versions of treatment" assumption, stating that for each driver there is only a single version of each level of an evasive action. For example, Wu and Jovanis [30] studied an abrupt evasive action as an indication of a traffic conflict and concluded that the abruptness of an evasive action may not be practical for defining/detecting a conflict in some cases. In the framework of the potential outcome model, their finding implies that the assumption of "no multiple versions of treatment" for a dichotomous evasive action is violated and thus a potential outcome with multiple evasive actions should be used in some cases. Based on the "no multiple versions of treatment" assumption, we assume "consistency" [14, 18, 31, 32], and state that $Y_x(i)$ is observed if the ith driver has taken an evasive action $X = x$. The "consistency" assumption is formulated as

$$X = x \quad \Rightarrow \quad Y_x = Y.$$

Note that neither the "no multiple versions of treatment" nor the "consistency" assumption is required in the framework of structural causal models [14]. They are derived from the assumption of autonomous data-generating process. Davis et al. [13] addressed structural causal models that represent collision-related outcomes resulting from interactions between initial conditions and evasive actions. Here, our results can also be derived in the framework of structural causal models, because the potential outcome can be constructed from structural causal models [14].

When a randomized experiment is conducted, X is independent of (Y_{x_1}, Y_{x_0}). This condition is known as "exogeneity", or referred to as "full exchangeability" [33]. Under exogeneity, $\mathrm{pr}(y_x)$ is identifiable and is given by

$$\mathrm{pr}(y_x) = \mathrm{pr}(y|x).$$

Here, "identifiable" means that the causal quantities such as $\mathrm{pr}(y_x)$ can be estimated consistently from a joint distribution of observed variables. When a randomized experiment is difficult to conduct, $\mathrm{pr}(y_x)$ can still be identified according to conditional ignorability [18, 34], or graphically, the back door criterion [14]. If there exists a set W of covariates such that X is conditionally independent of Y_x given W for any x, and $\mathrm{pr}(x|w) > 0$ for any x and w, we say that treatment assignment is conditionally ignorable given W, or that W satisfies the conditional ignorability condition. $\mathrm{pr}(y_x)$ can then be estimated using W and is given by

$$\mathrm{pr}(y_x) = E_w\{\mathrm{pr}(y|x, W)\}.$$

Here, $E_w\{\mathrm{pr}(y|x, W)\}$ represents the expectation of $\mathrm{pr}(y|x, W)$ regarding W. In the framework of structural causal models, Pearl [14] provided various kinds of identification conditions for $\mathrm{pr}(y_x)$, in addition to the conditional ignorability condition.

3 Counterfactual Based Conflict (CBC)

3.1 Definitions and basic properties

Letting U be the set of discrete and continuous variables representing driver conditions that could affect X and Y, both observed and unobserved, Davis et al. [13] described the causal mechanism of a collision-related outcome Y in terms of the directed graph shown in Fig. 1. In Davis et al. [13], a condition U is limited to the running status of vehicles (e. g., speed, deceleration, headway, and driver reaction time). In contrast, in this paper, condition U is not so limited but covers any factor necessary to make Y_x a deterministic function of x for the driver. This includes aspects of the wider traffic situation such as the status of a road and a vehicle, and the characteristics of a driver such as gender, age, and health [35]. In the figure, the directed edge from X to Y indicates that X could have an effect on Y. In addition, the absence of a directed edge from Y to X indicates that Y cannot be a cause of X, and the directed path from U to Y through X indicates that some elements of U could have an effect on Y mediated by X. Fig. 1 also provides the graphical representation of the data generating process

$$Y = g_y(X, U, \epsilon_y), \quad X = g_x(U, \epsilon_x), U = g_u(\epsilon_u), \tag{1}$$

where ϵ_x, ϵ_y, and ϵ_u are independent random disturbances.

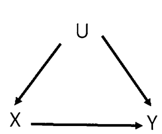

Figure 1: Simple graphical representation of the causal mechanism of a collision-related outcome.

In the situation shown in Fig. 1, irrespective of the complexity of U, the impact of U on Y cannot amount to more than a modification of the functional relationship between X and Y. Thus, for example, letting Y be a dichotomous variable based on a threshold y, there are exactly four functions regarding two dichotomous variables X and Y, and thus the value taken by U selects one of these four functions [14, 36]. Considering these observations, based on the level of an evasive action X, Davis et al. [13] divided driver conditions into the following four types:

$u_1 = \{u | Y_{x_1}(u) > y, Y_{x_0}(u) > y\}$ is the "safe condition" in which a driver would not have crashed regardless of the level of an evasive action. In this paper, we call a driver who satisfies this condition "a safe driver" for simplicity.

$u_2 = \{u | Y_{x_1}(u) > y, Y_{x_0}(u) \leq y\}$ is the "normal condition" in which a driver would not have crashed if he/she had taken an evasive action and would have crashed if he/she had not taken an evasive action. We call a driver who satisfies this condition "a normal driver."

$u_3 = \{u | Y_{x_1}(u) \leq y, Y_{x_0}(u) \leq y\}$ is the "doomed condition" in which a driver would have crashed regardless of the level of an evasive action. We call a driver who satisfies this condition "a doomed driver."

$u_4 = \{u | Y_{x_1}(u) \leq y, Y_{x_0}(u) > y\}$ is the "incompetent condition" in which a driver would have crashed if he/she had taken an evasive action and would not have crashed if he/she had not taken an evasive action. We call a driver who satisfies this condition "an incompetent driver."

Then, following Amundsen and Hyden [12] and Davis et al. [13], under the assumption of $\text{pr}(u_2) \neq 0$, we define the traffic conflict measure called the counterfactual based conflict (CBC) motivated by Davis's crash-to-conflict ratio, $\text{pr}(Y \leq y | u_j)$ $(j = 1, \ldots, 4)$ [13], as

$$\text{pr}(x_1, Y > y | u_2). \tag{2}$$

As seen from Eq. (2), the CBC is the probability that a normal driver took an evasive action and did not crash actually. Here, $\text{pr}(u_2)$ is also called the probability of necessity and sufficiency (PNS) [14, 37]. The PNS is the probability that a crash would respond to an evasive action both ways and therefore measures both the necessity and sufficiency of the evasive action to avoid crashing.

Davis et al. [13] pointed out two features of the definition of traffic conflicts:

> "First, the situation referred to appears to have three components: an initial condition, the actions of the road users, and a collision-related outcome. Second, an observed event qualifies as a conflict only if it satisfies a counterfactual test."

In Eq. (2), the initial condition is given by u_2 through the relationships between an evasive action X and a collision-related outcome Y. In addition, since Y_x is the mathematical expression of the counterfactual statement, both observed and counterfactual quantities are included in Eq. (2). Thus, these two features are reflected in the CBC.

According to Davis et al. [13], $u_1, \ldots, u_4$ are considered as conflict severities, and also referred to as "response types" in causal inference [14] and epidemiology [38]. Here, it is noted that these conflict severities are related to "principal stratification", which is a cross-classification of subjects defined by the joint potential values of the collision-related outcome under each of the evasive actions being compared [39, 40]. However, different from the principal stratification approach whose main purpose is to estimate the causal effects within principal strata, we are concerned with evaluating the effectiveness of an evasive action actually taken by a driver to avoid crashing.

For a dichotomous variable X, since $\text{pr}(x_0, Y > y | u_2) = 0$, Eq. (2) is transformed into

$$\text{pr}(x_1, Y > y | u_2) = \text{pr}(x_0, Y > y | u_2) + \text{pr}(x_1, Y > y | u_2) = \text{pr}(Y > y | u_2). \tag{3}$$

Here,

$$\text{pr}(Y \leq y | u_2) = 1 - \text{pr}(Y > y | u_2)$$

is the probability that a normal driver crashed actually. This is also interpreted as the special case of "the crash-to-conflict ratio for initiating events in set U_j" in Davis et al. [13]. However, as seen from Appendix A, 1–CBC is generally different from Davis's crash-to-conflict ratio in the case of multiple evasive actions. In addition, since $\text{pr}(x_1, Y \leq y | u_2) = 0$, Eq. (2) is transformed into

$$\text{pr}(x_1, Y > y | u_2) = \text{pr}(x_1, Y > y | u_2) + \text{pr}(x_1, Y \leq y | u_2) = \text{pr}(x_1 | u_2). \tag{4}$$

Eqs. (3) and (4) show that the probability that a normal driver took an evasive action actually is equivalent to the probability that he/she did not crash actually, as Davis et al. [13] pointed out.

Guttinger [41] stated,

> "For some, the conflict is an event that precedes an evasive action that can be either successful or not (collision). For others, it is the same as a near-miss situation after an evasive action. In this last view, a conflict cannot lead to a collision but is an event parallel with a collision."

Regarding Guttinger's statement, the CBC probabilistically evaluates the second interpretation in the sense that it takes an actual evasive action into account. Here, although the word "traffic conflict" implies Guttinger's first interpretation in some existing studies, this paper uses the word in the sense of Guttinger's second interpretation that includes both crash and near-miss situations. The term "near-miss" used in Guttinger's statement is considered to be the same as the concept of "near crash" in this paper.

As for Guttinger's first interpretation, letting Z be a set of covariates not affected by evasive action X, we can also propose a traffic conflict measure, $\mathrm{pr}(z|u_2)$. This measure is transformed into

$$\mathrm{pr}(z|u_2) = \frac{\mathrm{pr}(u_2|z)}{\mathrm{pr}(u_2)}\mathrm{pr}(z), \tag{5}$$

which includes the ratio between the PNS, $\mathrm{pr}(u_2)$ [14, 37] and the conditional PNS given Z, $\mathrm{pr}(u_2|z)$ [42]. We are not concerned with the evaluation problem of $\mathrm{pr}(z|u_2)$, because PNS has been discussed in detail by, for example, Kuroki and Cai [42], Cai and Kuroki [43], Pearl [14, 37] and Tian and Pearl [44].

It is worth emphasizing that the use of the CBC becomes significant when we wish to judge the effectiveness of an evasive action. For a crash that would have occurred if a driver had not taken an evasive action, the CBC evaluates the extent to which the crash was avoided by taking an evasive action actually. A higher value of the CBC suggests that an evasive action actually taken by a driver was more effective in avoiding the crash, represented by the given value y. In addition, the CBC is associated with the probability of the crash $\mathrm{pr}(Y < y)$ through Proposition 2 of Davis et al. [13]:

$$\begin{aligned}\mathrm{pr}(Y < y) &= \sum_{i=1}^{4} \mathrm{pr}(Y < y|u_i)\mathrm{pr}(u_i) \\ &= (1 - \mathrm{CBC})\mathrm{pr}(u_2) + \mathrm{pr}(u_3) + \mathrm{pr}(Y < y|u_4)\mathrm{pr}(u_4). \end{aligned} \tag{6}$$

Therefore, when the value of the CBC is higher, the probability of the crash may be lower. This implies that the use of the CBC with a continuous collision-related outcome enables us to evaluate the effectiveness of an evasive action to avoid a severe consequence at any level.

The CBC is defined based on normal drivers, but we can also focus on other conditions to formulate "CBC-like measures" if necessary. For example, regarding safe drivers, doomed drivers, and incompetent drivers, the "CBC-like measures" that we can consider are

$$\mathrm{pr}(x_1, Y > y|u_1), \tag{7}$$
$$\mathrm{pr}(x_1, Y > y|u_3), \tag{8}$$
$$\mathrm{pr}(x_1, Y > y|u_4), \tag{9}$$

respectively. In addition, if we observe some of traffic conditions Z such as the status of a road or a vehicle, or the characteristics of a driver, we can formulate conditional CBC given $Z = z$ as

$$\mathrm{pr}(x_1, Y > y|u_2, z), \tag{10}$$

and the conditional CBC-like measures as

$$\mathrm{pr}(x_1, Y > y|u_1, z), \tag{11}$$
$$\mathrm{pr}(x_1, Y > y|u_3, z), \tag{12}$$
$$\mathrm{pr}(x_1, Y > y|u_4, z). \tag{13}$$

Here, note that Eqs. (8), (9), (12) and (13) provide no information in the sense that they are zero. In this paper, we do not focus on Eqs. (7), (10) and (11) since the related results can be derived by the similar procedure to the CBC.

3.2 Identification conditions

Since the CBC is adapted from Davis et al. [13], when incompetent drivers do not exist in the population of interest, the CBC would be identifiable if we derive the information stated in Davis et al. [13]: (i) a structural model of crash events that supports counterfactual situation, (ii) understanding how evasive actions are achieved over a range of conflict severities, and (iii) knowledge of the relative frequencies of the different conflict severities. However, it may be difficult to derive such information in reality. In addition, the CBC is formulated on the basis of the joint distributions of potential outcomes Y_{x_1} and Y_{x_0}, that is, $\mathrm{pr}(u_2)$ and $\mathrm{pr}(x_1, u_2)$. These joint distributions cannot be expressed through $\mathrm{pr}(Y_{x_1} > y)$, $\mathrm{pr}(Y_{x_0} \le y)$, $\mathrm{pr}(x_1, Y > y)$, and $\mathrm{pr}(x_0, Y \le y)$, without further causal assumptions. This means that the CBC is not identifiable without further causal assumptions, even if both $\mathrm{pr}(x, y)$ and $\mathrm{pr}(y_x)$ ($x \in \{x_1, x_0\}$) are available. Thus, we discuss the identification problem of the CBC.

Proposition 1. *Assume that the whole population consists only of normal drivers for a given y (i. e.,* $\mathrm{pr}(u_2) = 1$ *for a given y). If* $\mathrm{pr}(x_1)$ *is available under the assumption, then the CBC is identifiable and is given by*

$$\mathrm{pr}(x_1, Y > y | u_2) = \mathrm{pr}(x_1) \tag{14}$$

for a given y. In addition, if $\mathrm{pr}(Y > y)$ *is available under the assumption, then the CBC is also identifiable and is given by*

$$\mathrm{pr}(x_1, Y > y | u_2) = \mathrm{pr}(Y > y) \tag{15}$$

for a given y.

The proof is straightforward from Eqs. (3) and (4).

Perkins and Harris [8] stated that

> "Over 20 objective criteria for traffic conflicts (or impending accident situations) have been defined as to specific accident patterns at intersections. Essentially, these traffic conflicts are defined by the occurrence of an evasive action, such as braking or weaving, which are forced on a driver by an impending accident situation or a traffic violation."

Eq. (14) implies that the concept of traffic conflicts proposed by Perkins and Harris [8] does not contradict the counterfactual statement given at the first workshop on traffic conflicts. In addition, Eq. (15) implies that the concept of traffic conflicts defined by collision-related outcomes such as the TTC is justified when the population under consideration consists only of normal drivers.

From Proposition 1, when both $\mathrm{pr}(x_1)$ and $\mathrm{pr}(Y > y)$ are available, we have

$$\mathrm{pr}(x_1) = \mathrm{pr}(Y > y).$$

This property is useful for supporting the hypothesis that the population under consideration includes various types of drivers through the use of statistical data. Consider the statistical hypothesis

$$H_0 : \mathrm{pr}(x_1) = \mathrm{pr}(Y > y) \quad \text{vs} \quad H_1 : \mathrm{pr}(x_1) \ne \mathrm{pr}(Y > y).$$

If the null hypothesis H_0 is rejected at the given significance level, we can judge at the level that various types of drivers exist in the population under consideration. Here, note that there can exist a situation in which the statistical null hypotheses $\mathrm{pr}(x_1) = \mathrm{pr}(Y > y)$ or $\mathrm{pr}(x_0) = \mathrm{pr}(Y > y)$ are not rejected at the same time. However, this situation does not imply that both $\mathrm{pr}(u_2) = 1$ and $\mathrm{pr}(u_4) = 1$ are reasonable. Actually, $\mathrm{pr}(u_2) = 1$

and $\mathrm{pr}(u_4) = 1$ cannot occur simultaneously. As seen from these considerations, the statistical hypothetical testing described above can be used to check the necessity condition of $\mathrm{pr}(u_2) = 1$, but not to check the sufficiency condition. Here, the equivalence between Eqs. (14) and (15) holds for the normal condition defined by a given specific value y. It is noted that a normal driver under a given specific value y may be classified as other type of driver for a different value $y'(\neq y)$.

Proposition 2. *Assume that* $X = x_1$ *is independent of* (Y_{x_1}, Y_{x_0})*. If* $\mathrm{pr}(x_1)$ *is available under the assumption, then the CBC is identifiable and is given by*

$$\mathrm{pr}(x_1, Y > y|u_2) = \mathrm{pr}(x_1). \tag{16}$$

The proof is straightforward from Eq. (4).

Proposition 3. *Assume that incompetent drivers do not exist in the population under consideration. If* $\mathrm{pr}(Y_{x_1} > y|x_1)$, $\mathrm{pr}(Y_{x_0} > y|x_1)$, $\mathrm{pr}(Y_{x_1} > y)$ *and* $\mathrm{pr}(Y_{x_0} > y)$ *are available under the assumption, then the CBC is identifiable and is given by*

$$\mathrm{pr}(x_1, Y > y|u_2) = \frac{\mathrm{pr}(Y_{x_1} > y|x_1) - \mathrm{pr}(Y_{x_0} > y|x_1)}{\mathrm{pr}(Y_{x_1} > y) - \mathrm{pr}(Y_{x_0} > y)}\mathrm{pr}(x_1). \tag{17}$$

The proof can be achieved as a special case of Proposition 8 in Appendix A.

We assume that $\mathrm{pr}(Y_x > y|x')$ and $\mathrm{pr}(Y_x > y)$ are available for x and x' $(x, x' \in \{x_0, x_1\})$ through the identification condition of causal risk or an expert's knowledge of road safety mechanisms. $\mathrm{pr}(Y_{x_1} > y|x_1) - \mathrm{pr}(Y_{x_0} > y|x_1)$ is the causal risk difference among drivers who took an evasive action actually. In addition, $\mathrm{pr}(Y_{x_1} > y) - \mathrm{pr}(Y_{x_0} > y)$ is the causal risk difference among drivers in the entire population [45]. Note that a collision-related outcome is used to formulate the CBC, that is, the CBC may be difficult to evaluate in the absence of data on the collision-related outcome. To address this, we suggest the use of proxy variables of the collision-related outcome. Such research designs have been discussed in the context of effect restoration [46, 47], and latent structure analysis [48, 49]. A further suggestion is the use of results from pilot studies or past studies in the main study. Such research designs can be found in the literature on the data fusion problem [50].

3.3 Bounds

As can be seen from the definition, the CBC is not identifiable based on the knowledge of observed data alone, even under the conditional ignorability condition. When the CBC is not identifiable, one possible solution is to derive the closed-form formulas of the bounds on the CBC under the milder conditions.

Proposition 4. *Assume that*

$$\mathrm{pr}(x_1|u_2) \geq \max\{\mathrm{pr}(x_1|u_1), \mathrm{pr}(x_1|u_3)\}, \tag{18}$$

$$\mathrm{pr}(u_3) \geq \mathrm{pr}(u_1) \tag{19}$$

hold. If both $\mathrm{pr}(x_1, Y > y)$ *and* $\mathrm{pr}(x_0, Y \leq y)$ *are available under the assumptions, then the bounds on the CBC are given by*

$$\frac{\mathrm{pr}(x_1, Y > y)}{\mathrm{pr}(x_1, Y > y) + \mathrm{pr}(x_0, Y \leq y)} \leq \mathrm{pr}(x_1, Y > y|u_2) \leq 1. \tag{20}$$

The proof can be achieved as a special case of Proposition 9 in Appendix A.

Intuitively, Eq. (18) implies that drivers who took an evasive action actually in the normal condition are more numerous than both those in the safe condition and those in the doomed condition. In addition, Eq. (19) shows that doomed drivers are more numerous than safe drivers in the population of interest. These assumptions would be reasonable when we focus on a traffic situation such as a potential rear-end collision in which a driver of the following vehicle adequately recognizes the risk of crashing. Note that Eq. (18) or Eq. (19) might not hold for specific values of y. In this case, the lower bound on the CBC is undefined. In this context, the assumption in Eq. (18) may be thought questionable when y is close to zero. However, Eq. (18) would still be reasonable when the main focus is on traffic safety research with a categorized collision-related outcome as, for example, in the 100-car study discussed in Section 4.

Proposition 5. *If both* $\mathrm{pr}(Y_x > y)$ *and* $\mathrm{pr}(x, Y > y)$ *are available for* $x \in \{x_1, x_0\}$, *then the bounds on the CBC are given by*

$$\begin{aligned} &\frac{\alpha_l(x_0,x_1)\times \mathrm{pr}(x_1, Y>y)}{\alpha_l(x_0,x_1)\times \mathrm{pr}(x_1, Y>y)+\beta_u(x_0,x_1)\times \mathrm{pr}(x_0, Y\le y)} \\ &\le \mathrm{pr}(x_1, Y>y|u_2) \\ &\le \frac{\alpha_u(x_0,x_1)\times \mathrm{pr}(x_1, Y>y)}{\alpha_u(x_0,x_1)\times \mathrm{pr}(x_1, Y>y)+\beta_l(x_0,x_1)\times \mathrm{pr}(x_0, Y\le y)}, \end{aligned} \tag{21}$$

where

$$\alpha_l(x_0,x_1) = \max\left\{0, \frac{\mathrm{pr}(Y>y)-\mathrm{pr}(Y_{x_0}>y)}{\mathrm{pr}(x_1, Y>y)}\right\}, \tag{22}$$

$$\alpha_u(x_0,x_1) = \min\left\{1, \frac{\mathrm{pr}(Y_{x_0}\le y)-\mathrm{pr}(x_0, Y\le y)}{\mathrm{pr}(x_1, Y>y)}\right\}, \tag{23}$$

$$\beta_l(x_0,x_1) = \max\left\{0, \frac{\mathrm{pr}(Y_{x_1}>y)-\mathrm{pr}(Y>y)}{\mathrm{pr}(x_0, Y\le y)}\right\}, \tag{24}$$

$$\beta_u(x_0,x_1) = \min\left\{1, \frac{\mathrm{pr}(Y_{x_1}>y)-\mathrm{pr}(x_1, Y>y)}{\mathrm{pr}(x_0, Y\le y)}\right\}, \tag{25}$$

respectively.

Proposition 5 can be proved as a special case of Proposition 10 in Appendix A.

4 Application to the 100-car study

4.1 Background

We apply the proposed traffic conflict measure to the data from the 100-car study, as shown in Table 1. The data was collected by the National Highway Traffic Safety Administration, Virginia Department of Transportation, and Virginia Tech Transportation Institute over a period of more than a year, from January 2003 to July 2004, from 100 drivers, to observe their behavior in various traffic situations that arise in daily life. The drivers were recruited through a newspaper advertisement. Thus, the 100-car study was an observational study.

According to Dingus et al. [1], first, events of interest that are candidate collision-related outcomes were selected based on the sensing values of lateral acceleration, longitudinal acceleration, forward TTC, rear TTC, yaw rate, and event button. The thresholds of these values were determined from a sensitivity analysis. Invalid events were then excluded by trained operators, whom Dingus et al. [1] referred to as “data reductionists.” They took account of the following: (a) video data on the driver’s view and behavior, and (b) time plots of the sensing values. They also assigned each valid event (i. e., collision-related outcome) to one of three levels: crash (y_0), near crash (y_1), and incident (y_2). The “crash” category was used if “there is any contact with an object, in which kinetic energy is measurably transferred or dissipated.” The “near crash” category was used if “there is any circumstance that requires a rapid maneuver (“rapid evasive action” in our paper) by the subject vehicle to avoid crashing.” Other events were categorized as an “incident.” Dingus et al. [1] defined a “rapid maneuver” as one involving “subject vehicle braking greater than 0.5g (g is the constant of gravitation) or steering input that results in a lateral acceleration greater than 0.4g to avoid crashing.” In this section, we assume that $y_0 < y_1 < y_2$ according to the severity. For details of the 100-car study, refer to Dingus et al. [1].

Table 1: Drivers of the following vehicles: dichotomous evasive action.

	Crash (y_0)	Near crash (y_1)	Incident (y_2)
No evasive action (x_0)	7	0	29
Evasive action (x_1)	8	380	5754

4.2 Analysis

Based on the data provided in Table 1, we present an analysis of rear-end collisions by the following vehicles. In Table 1, "evasive action (x_1)" indicates that a driver took an evasive action such as braking, turning the steering wheel, accelerating, or a combination of these, and "no evasive action (x_0)" indicates that the driver did not take any evasive action. In this situation, it would be reasonable to consider that the presence or absence of an evasive action taken by a driver of the following vehicle strongly affects the occurrence or non-occurrence of a crash. Then, the number of drivers in the safe, doomed and incompetent conditions can be assumed to be negligible, that is, $\mathrm{pr}(Y_{x_1} > y_0, Y_{x_0} = y_0) \simeq 1$. Making this assumption, from Proposition 1, we have

$$\mathrm{pr}(x_1) \simeq \mathrm{pr}(Y > y_0). \tag{26}$$

Note that y_0, y_1 and y_2 are the level taken by the collision-related outcome Y, but they do not mean the potential outcome Y_{x_0} or Y_{x_1}.

From Table 1, the unbiased estimates of $\mathrm{pr}(x_1)$ and $\mathrm{pr}(Y > y_0)$ are given by $\hat{\mathrm{pr}}(x_1) = 0.9942$ (standard error: 0.0010) and $\hat{\mathrm{pr}}(Y > y_0) = 0.9976$ (standard error: 0.0006), respectively, assuming that Table 1 was generated from a multinomial distribution. Thus, since there appears to be no strong reason for $\mathrm{pr}(x_1)$ and $\mathrm{pr}(Y > y_0)$ to be significantly different, it would be reasonable to consider that Table 1 was based on normal drivers for a given y_0. In this situation, the CBC $\mathrm{pr}(x_1, Y > y_0 | Y_{x_1} > y_0, Y_{x_0} = y_0)$ can be evaluated using $\hat{\mathrm{pr}}(x_1)$ and $\hat{\mathrm{pr}}(Y > y_0)$. In contrast, if we believe that Eq. (26) does not hold, then it is not appropriate to evaluate the CBC using $\hat{\mathrm{pr}}(x_1)$ or $\hat{\mathrm{pr}}(Y > y_0)$. In this situation, it would be useful to evaluate the bounds on the CBC. In the 100-car study, since the drivers of the following vehicles are considered to be able to recognize the crash risks adequately, it would be reasonable to assume that both Eqs. (18) and (19) hold for y_0. Then, according to Eq. (20), the lower bound on $\mathrm{pr}(x_1, Y > y_0 | Y_{x_1} > y_0, Y_{x_0} = y_0)$ is given by 0.9989 (standard error: 0.0004). Thus, the CBC takes a value within the range [0.9989, 1.0000], indicating that nearly all normal drivers took an evasive action and did not crash actually.

Here, regarding $\mathrm{pr}(x_1, Y > y_1 | Y_{x_1} > y_1, Y_{x_0} \leq y_1)$, since the unbiased estimate of $\mathrm{pr}(Y > y_1)$ is given by 0.9361 (standard error: 0.0031), one might think that $\mathrm{pr}(x_1)$ and $\mathrm{pr}(Y > y_1)$ are different. Therefore, it might be judged that $\mathrm{pr}(Y_{x_1} > y_1, Y_{x_0} \leq y_1) \simeq 1$ does not hold. Then, as in the case of $\mathrm{pr}(x_1, Y > y_0 | Y_{x_1} > y_0, Y_{x_0} = y_0)$, when we assume that both Eqs. (18) and (19) hold for y_1, the lower bound on the CBC is given by 0.9988 (standard error: 0.0005). Thus, $\mathrm{pr}(x_1, Y > y_1 | Y_{x_1} > y_1, Y_{x_0} \leq y_1)$ takes a value within the range [0.9988, 1.0000]. Consequently, it is considered that nearly all normal drivers took an evasive action and did not have a near crash actually.

5 Conclusions

In recent road safety studies, researchers and practitioners have proposed a range of traffic conflict techniques. However, most of these did not consider the counterfactual statement "there is a risk of collision if their movements remain unchanged." To address this, we have proposed a new traffic conflict measure, the CBC. In contrast with most existing traffic conflict techniques, the CBC is formulated based on the potential outcome model in causal inference; hence, reflecting the counterfactual statement. We also provided identification conditions for the CBC and formulated the bounds on the CBC under particular assumptions when the proposed identification conditions are violated. In addition, through an application to the 100-car study, we showed that the CBC is helpful in reliably evaluating the effectiveness of an evasive action.

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Appendix A. CBC with multiple evasive actions

The occurrence of a crash depends not only on whether or not a driver takes an evasive action but also on what type of evasive action is taken. To address this issue, we discuss the case in which a driver chooses one of a set of possible evasive actions $\{x_0, \ldots, x_p\}$. In this setting, under the assumption that $\mathrm{pr}(Y_{x_j} > y, Y_{x_k} \le y) \neq 0$, Eq. (2) can be extended as follows:

$$\mathrm{pr}(x_j, Y > y | Y_{x_j} > y, Y_{x_k} \le y) \tag{27}$$

$(j, k = 0, 1, \ldots, p; j \neq k)$. Eq. (27) represents the probability that for a driver who would not have crashed if he/she had taken the evasive action $X = x_j$ and would have crashed with the evasive action $X = x_k$, the driver took the evasive action $X = x_j$ and avoided crashing actually. In this paper, we consider both Eqs. (2) and (27) as CBCs because Eq. (27) is equivalent to Eq. (2) when $j = 1$ and $k = 0$.

A.1 Identification conditions and bounds

In this section, we provide the identification conditions and bounds for the CBC with multiple evasive actions. The proofs are given in the Appendix D.

Proposition 6. *Assume that* $\mathrm{pr}(Y_{x_j} > y, Y_{x_k} \le y) = 1$ *holds for a given* y. *If* $\mathrm{pr}(x_j)$ *is available under the assumption, then the CBC is identifiable and is given by*

$$\mathrm{pr}(x_j, Y > y | Y_{x_j} > y, Y_{x_k} \le y) = \mathrm{pr}(x_j) \tag{28}$$

for a given y. *If* $\mathrm{pr}(x_j, Y > y)$ *is available under the assumption, then the CBC is also identifiable and is given by*

$$\mathrm{pr}(x_j, Y > y | Y_{x_j} > y, Y_{x_k} \le y) = \mathrm{pr}(x_j, Y > y) \tag{29}$$

for a given y.

Proposition 7. *Assume that* $X = x_j$ *is independent of* (Y_{x_j}, Y_{x_k}). *If* $\mathrm{pr}(x_j)$ *is available under the assumption, then the CBC is identifiable and is given by*

$$\mathrm{pr}(x_j, Y > y | Y_{x_j} > y, Y_{x_k} \le y) = \mathrm{pr}(x_j). \tag{30}$$

Proposition 8. *Assume that* $\mathrm{pr}(Y_{x_j} \le y, Y_{x_k} > y) = 0$ *holds for a given* y. *If* $\mathrm{pr}(Y_{x_j} > y | x_j)$, $\mathrm{pr}(Y_{x_k} > y | x_j)$, $\mathrm{pr}(Y_{x_j} > y)$ *and* $\mathrm{pr}(Y_{x_k} > y)$ *are available under the assumption, then the CBC is identifiable and is given by*

$$\mathrm{pr}(x_j, Y > y | Y_{x_j} > y, Y_{x_k} \le y) = \frac{\mathrm{pr}(Y_{x_j} > y | x_j) - \mathrm{pr}(Y_{x_k} > y | x_j)}{\mathrm{pr}(Y_{x_j} > y) - \mathrm{pr}(Y_{x_k} > y)} \mathrm{pr}(x_j). \tag{31}$$

In addition, the following equations hold:

$$\left.\begin{array}{r} \mathrm{pr}(Y_{x_j} > y, Y_{x_k} > y) = \mathrm{pr}(Y_{x_k} > y) \\ \mathrm{pr}(Y_{x_j} > y, Y_{x_k} \leq y) = \mathrm{pr}(Y_{x_j} > y) - \mathrm{pr}(Y_{x_k} > y) \\ \mathrm{pr}(Y_{x_j} \leq y, Y_{x_k} \leq y) = \mathrm{pr}(Y_{x_j} \leq y) \end{array}\right\}. \tag{32}$$

Proposition 9. *Assume that*

$$\mathrm{pr}(x_j | Y_{x_j} > y, Y_{x_k} \leq y) \geq \max\{\mathrm{pr}(x_l | Y_{x_j} > y, Y_{x_k} \leq y), \mathrm{pr}(x_j | Y_{x_j} > y, Y_{x_k} > y)\} \tag{33}$$

for $l \neq j, k$,

$$\mathrm{pr}(x_k | Y_{x_j} \leq y, Y_{x_k} \leq y) \geq \mathrm{pr}(x_k | Y_{x_j} > y, Y_{x_k} \leq y), \tag{34}$$

and

$$\mathrm{pr}(Y_{x_j} \leq y, Y_{x_k} \leq y) \geq \mathrm{pr}(Y_{x_j} > y, Y_{x_k} > y). \tag{35}$$

If both $\mathrm{pr}(x_j, Y > y)$ *and* $\mathrm{pr}(x_k, Y \leq y)$ *are available under the assumption, then the bounds on the CBC are given by*

$$\frac{\mathrm{pr}(x_j, Y > y)}{p \times \mathrm{pr}(x_j, Y > y) + \mathrm{pr}(x_k, Y \leq y)} \leq \mathrm{pr}(x_j, Y > y | Y_{x_j} > y, Y_{x_k} \leq y) \leq 1, \tag{36}$$

where p stands for the number of evasive action types.

Proposition 10. *If both* $\mathrm{pr}(Y_x > y)$ *and* $\mathrm{pr}(x, Y > y)$ *are available, then the bounds on the CBC are given by*

$$\begin{aligned} &\frac{\alpha_l(x_k, x_j) \times \mathrm{pr}(x_j, Y > y)}{\alpha_l(x_k, x_j) \times \mathrm{pr}(x_j, Y > y) + \beta_u(x_k, x_j) \times \mathrm{pr}(x_k, Y \leq y) + \sum_{l \neq j,k} \mathrm{pr}(x_l)} \\ &\leq \mathrm{pr}(x_j, Y > y | Y_{x_j} > y, Y_{x_k} \leq y) \\ &\leq \frac{\alpha_u(x_k, x_j) \times \mathrm{pr}(x_j, Y > y)}{\alpha_u(x_k, x_j) \times \mathrm{pr}(x_j, Y > y) + \beta_l(x_k, x_j) \times \mathrm{pr}(x_k, Y \leq y)}. \end{aligned} \tag{37}$$

Note that the probabilities of causation $\mathrm{pr}(Y_{x_k} \leq y | x_j, Y > y)$ *and* $\mathrm{pr}(Y_{x_j} > y | x_k, Y \leq y)$ *lie within the ranges* $[\alpha_l(x_k, x_j), \alpha_u(x_k, x_j)]$ *and* $[\beta_l(x_k, x_j), \beta_u(x_k, x_j)]$*, respectively. Here, according to Tian and Pearl* [44], $\alpha_l(x_k, x_j)$, $\alpha_u(x_k, x_j)$, $\beta_l(x_k, x_j)$, *and* $\beta_u(x_k, x_j)$ *are derived by replacing* x_0 *and* x_1 *of Eqs.* (22)–(25) *with* x_k *and* x_j*, respectively.*

Appendix B. Application to the 100-car study (CBC with multiple evasive actions)

Here, we apply the results from the CBC with multiple evasive actions in Appendix A to the rear-end collision data from the 100-car study. According to Dingus et al. [1], we classified evasive actions into four types. “Ordinary evasive action (x_1)” indicates that a driver only applied the brake. “Aggressive evasive action (x_2)” indicates that a driver turned the steering wheel, with or without the ordinary evasive action. “Skilled evasive action (x_3)” indicates that a driver accelerated, with or without the aggressive evasive action. “No evasive action (x_0)” indicates that a driver did not take any evasive action. Here, we assume that $x_0 < x_1 < x_2 < x_3$. Using the same data aggregated in Table 1, we re-aggregated the data in Table 2 to reflect this classificatory scheme. Note that the total number of drivers shown in Table 2 is slightly different from that shown in Table 1, as we excluded 10 drivers whose evasive actions were unknown.

Table 2: Drivers of the following vehicles: multiple evasive actions.

	Crash (y_0)	Near crash (y_1)	Incident (y_2)
No evasive action (x_0)	7	0	29
Ordinary evasive action (x_1)	6	265	4930
Aggressive evasive action (x_2)	1	111	746
Skilled evasive action (x_3)	0	4	69

Table 3: Unbiased estimates and standard errors.

	$\hat{\mathrm{pr}}(x_j)$	$\hat{\mathrm{pr}}(x_j, Y > y_0)$	$\hat{\mathrm{pr}}(x_j, Y > y_1)$
No evasive action (x_0)	0.0058	0.0047	0.0047
	(0.0010)	(0.0009)	(0.0009)
Ordinary evasive action (x_1)	0.8432	0.8423	0.7993
	(0.0046)	(0.0046)	(0.0051)
Aggressive evasive action (x_2)	0.1391	0.1389	0.1209
	(0.0044)	(0.0044)	(0.0042)
Skilled evasive action (x_3)	0.0118	0.0118	0.0112
	(0.0014)	(0.0014)	(0.0013)

Based on Table 2, we address whether the following equation from Proposition 6 is reasonable to hold for x_j, x_k, and y_l ($j, k = 0, \ldots 3; j \neq k; l = 0, 1$).

$$\mathrm{pr}(x_j) \simeq \mathrm{pr}(x_j, Y > y_l) \tag{38}$$

Here, in Table 3, under the assumption that Table 2 is generated from a multinomial distribution, we provide some statistics derived from Table 2. In Table 3, the first and second rows show the unbiased estimates and the standard error of the corresponding probabilities, respectively.

Table 3 indicates that there appears to be no strong reason for $\mathrm{pr}(x_j)$ and $\mathrm{pr}(x_j, Y > y_0)$ to be significantly different from each other for $j = 0, 1, 2, 3$, but these results do not enable us to determine which of the four evasive actions are to be compared with $X = x_j$ to evaluate the CBCs. As stated in Section 3.2, it is not reasonable to consider that $\mathrm{pr}(Y_{x_j} > y_0, Y_{x_k} = y_0) = 1$ for any $j, k = 0, 1, 2, 3 (j \neq k)$, since they cannot hold simultaneously. From these observations, noting that $\hat{\mathrm{pr}}(x_0, Y > y_0)$ is smaller than the other probabilities, it might be possible to assume that $\mathrm{pr}(Y_{x_1} > y_0, Y_{x_2} > y_0, Y_{x_3} > y_0, Y_{x_0} = y_0) \simeq 1$. This assumption does not contradict the statistical results derived from Table 3. From this assumption, it would be reasonable to consider that Table 2 was based on drivers who would take $Y > y_0$ if they had taken any of the evasive actions and would take $Y = y_0$ if they had not taken any evasive action. This observation is consistent with the case of a dichotomous evasive action discussed in Section 4.2, and the CBCs, $\mathrm{pr}(x_j, Y > y_0 | Y_{x_j} > y_0, Y_{x_0} = y_0)$, can then be evaluated using $\hat{\mathrm{pr}}(x_j)$ and $\hat{\mathrm{pr}}(x_j, Y > y_0)$.

In contrast, since there appear to be significant differences between $\mathrm{pr}(x_1)$ and $\mathrm{pr}(x_1, Y > y_1)$ and between $\mathrm{pr}(x_2)$ and $\mathrm{pr}(x_2, Y > y_1)$, it would be difficult to judge that $\mathrm{pr}(Y_{x_1} > y_1, Y_{x_k} \leq y_1) = 1$ for $k = 0, 2, 3$ or that $\mathrm{pr}(Y_{x_2} > y_1, Y_{x_k} \leq y_1) = 1$ for $k = 0, 1, 3$. This observation does not contradict the case of a dichotomous evasive action discussed in Section 4.2. In addition, one might believe that the population of interest comprised various types of drivers and thus that Eq. (38) does not hold. Therefore, it is inappropriate to evaluate the CBCs, $\mathrm{pr}(x_j, Y > y_1 | Y_{x_j} > y_1, Y_{x_k} \leq y_1)$, through $\hat{\mathrm{pr}}(x_j)$ or $\hat{\mathrm{pr}}(x_j, Y > y_1)$ for $j = 1, 2; k = 0, 1, 2, 3; j \neq k$ even if the null hypothesis in Eq. (38) is not rejected. In this situation, it would be useful to evaluate the bounds on the CBCs. Here, if it would be reasonable to assume that Eqs. (33), (34), and (35) hold for y_0 and y_1, then the lower bounds on the CBCs are as given in Table 4. The lower triangular part of Table 4 provide the lower bounds on $\mathrm{pr}(x_j, Y > y_1 | Y_{x_j} > y_1, Y_{x_k} \leq y_1); j > k$, and the upper triangular part of Table 4 provides the lower bounds on $\mathrm{pr}(x_j, Y > y_0 | Y_{x_j} > y_0, Y_{x_k} = y_0); j > k$. In Table 4, the first and second rows provide the estimates and the standard error corresponding to the lower bounds, respectively.

Table 4: Lower bounds on the CBCs.

	No(x_0)	Ordinary(x_1)	Aggressive(x_2)	Skilled(x_3)
No(x_0)	–	0.3332	0.3324	0.3230
	–	(0.0001)	(0.0003)	(0.0040)
Ordinary(x_1)	0.3332	–	0.3326	0.3244
	(0.0001)	–	(0.0003)	(0.0037)
Aggressive(x_2)	0.3323	0.2973	–	0.3318
	(0.0004)	(0.0023)	–	(0.0015)
Skilled(x_3)	0.3224	0.1444	0.2163	–
	(0.0043)	(0.0111)	(0.0117)	–

The results in Table 4 indicate that the lower bounds on $\hat{\mathrm{pr}}(x_j, Y > y_0 | Y_{x_j} > y_0, Y_{x_k} = y_0)$ are greater than the lower bounds on $\hat{\mathrm{pr}}(x_j, Y > y_1 | Y_{x_j} > y_1, Y_{x_k} \leq y_1)$ for $j, k = 0, 1, 2, 3; j > k$. In addition, the lower bounds in Table 4 are smaller than those in the case of a dichotomous evasive action, perhaps reflecting the following observation. Intuitively, since the dichotomous evasive action case implies that a driver can choose from a wide range of evasive actions, including braking, turning the steering wheel, accelerating, or a combination of these, the probability of a crash avoidance or a near crash avoidance by taking an evasive action is relatively high. In contrast, when considering a crash or a near crash based on multiple evasive actions, a driver is allowed to take only a designated evasive action, that is, there is no other choice of evasive actions.

Appendix C. Numerical examples

In this appendix, we present comparisons of the CBC and $\mathrm{pr}(Y > y)$ using numerical examples. $\mathrm{pr}(Y > y)$ can be considered as a measure based on existing traffic conflict techniques such as the TTC and the time gap. The TTC and the time gap are empirically known to follow the Weibull distribution [24, 51, 52]. Thus, we assume that (Y_{x_1}, Y_{x_0}) follows the bivariate Weibull distribution for a given actual evasive action $X = x_m (m = 0, 1)$. Y_{x_n} represents a potential outcome associated with a counterfactual evasive action $X = x_n (n = 0, 1)$. The bivariate Weibull distribution of interest here has the following survival function [53, 54]:

$$\begin{aligned} &\mathrm{pr}(Y_{x_0} > y_0, Y_{x_1} > y_1 | X = x_m; \gamma_{m0}, \gamma_{m1}, \lambda_{m0}, \lambda_{m1}, \alpha_m) \\ &= \exp\left(-\left(\left(\frac{y_0}{\lambda_{m0}}\right)^{\frac{\gamma_{m0}}{\alpha_m}} + \left(\frac{y_1}{\lambda_{m1}}\right)^{\frac{\gamma_{m1}}{\alpha_m}}\right)^{\alpha_m}\right) \\ &y_n > 0, \gamma_{mn} > 0, \lambda_{mn} > 0, 0 < \alpha_m \leq 1; m, n = 0, 1. \end{aligned} \tag{39}$$

Here, γ_{mn}, λ_{mn}, and α_m are the shape, scale, and association parameters, respectively.

In accordance with previous empirical studies [24, 51], we assume that the collision-related outcomes follow the Weibull distribution with a shape parameter of approximately 1.0 ~ 2.5. With such a parameter setting, the marginal distribution of Eq. (39) has a unimodal right-skewed shape with a long right tail. In addition, regarding the scale parameters, a smaller value given $X = x_m$ can be considered to indicate that a driver would take evasive action $X = x_n$ as he/she would be close to a severe situation given $X = x_m$. In contrast, a larger value given $X = x_m$ can be considered to indicate that a driver would take evasive action $X = x_n$ even if he/she would be far from a severe situation given $X = x_m$. Furthermore, regarding the association parameter, if it takes a smaller value given $X = x_m$, potential outcomes Y_{x_0} and Y_{x_1} largely depend on each other given $X = x_m$. If it takes a larger value given $X = x_m$, the dependence between potential outcomes Y_{x_0} and Y_{x_1} is lower given $X = x_m$. This situation arises when a randomized experiment of X is conducted.

Here, although we compare the CBC and $\mathrm{pr}(Y > y)$ under an ideal situation in which γ_{mn}, λ_{mn}, and α_m are known, the true values of these parameters would be unknown in an actual road safety study. However, for example, when exogeneity holds in experiments such as the present simulation, Y_{x_m} are observed as the

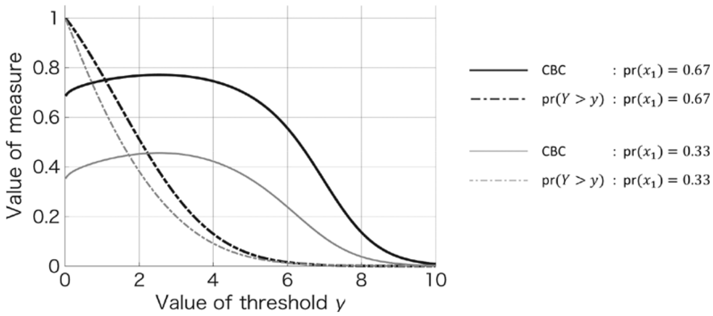

Figure 2: Comparison between the CBC and pr($Y > y$) (1).

values from the distribution of Y in the group of drivers who take evasive action $X = x_m$, $(m = 0, 1)$, under proper random sampling. In this case, both γ_{mn} and λ_{mn} can be estimated using, for example, a maximum likelihood method based on the conditional distribution of Y given $X = x_m$. However, since the values of $Y_{x_1}(i)$ and $Y_{x_0}(i)$ cannot be obtained simultaneously at the same time, the association parameter α_m, which remains unknown, would be evaluated using a sensitivity analysis.

First, based on observations from the empirical studies, letting

$$(\gamma_{00}, \gamma_{01}) = (1.10, 1.50), \quad (\gamma_{10}, \gamma_{11}) = (1.10, 2.00),$$
$$(\lambda_{00}, \lambda_{01}) = (1.50, 2.50), \quad (\lambda_{10}, \lambda_{11}) = (1.50, 3.00), \quad \alpha_0 = \alpha_1 = 0.10,$$

we show the difference between the CBC and pr($Y > y$) in Fig. 2. In the figure, the CBC shows a unimodal function shape, whereas pr($Y > y$) shows a simple decreasing function shape for y. Here, when the value of y is zero, the value of the CBC is equal to the value of pr(x_1). The CBC takes the maximum value 0.770 at $y = 2.55$ when pr(x_1) = 0.67, and 0.456 at $y = 2.55$ when pr(x_1) = 0.33. The values of the CBC depend strongly on the value of pr(x_1). These results indicate that the CBC is considered to reflect evasive actions of the drivers. In contrast, unlike the CBC, pr($Y > y$) may not reflect evasive actions of the drivers adequately, because the shapes of pr($Y > y$) are similar at pr(x_1) = 0.67 and 0.33.

Next, letting

$$(\gamma_{00}, \gamma_{01}) = (1.50, 1.70), \quad (\gamma_{10}, \gamma_{11}) = (1.50, 2.20),$$
$$(\lambda_{00}, \lambda_{01}) = (\lambda_{10}, \lambda_{11}) = (1.50, 2.50), \quad \alpha_0 = 0.90, \quad \alpha_1 = 0.10,$$

Fig. 3 shows the difference between the CBC and pr($Y > y$). In the figure, both the CBC and pr($Y > y$) are decreasing functions of y, but the slope of the CBC is less steep than that of pr($Y > y$). The values of the CBC depend more strongly on the value of pr(x_1) compared to pr($Y > y$). Thus, the CBC is considered to reflect evasive actions of the drivers, whereas pr($Y > y$) may not reflect these actions adequately.

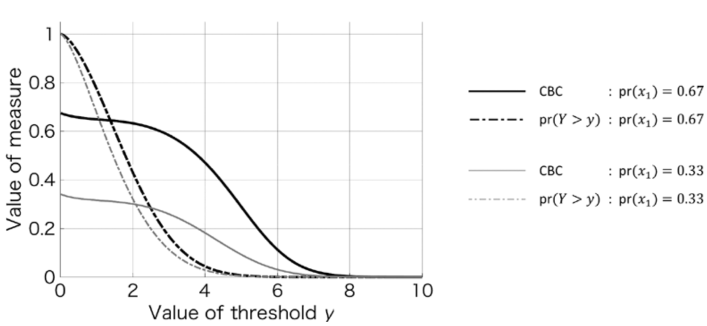

Figure 3: Comparison between the CBC and pr($Y > y$) (2).

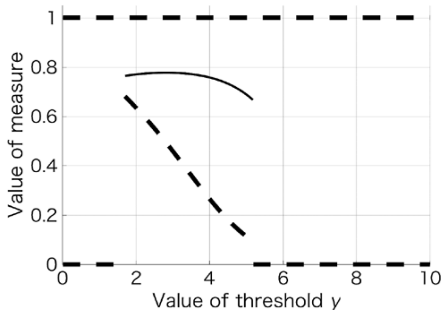

Figure 4: CBC and bounds (when $\mathrm{pr}(x_1) = 0.67$; the CBC is represented by solid lines, and the upper and lower bounds are represented by dotted lines).

Finally, Fig. 4, which was obtained using the same parameter settings that were used to generate Fig. 2, shows the function shapes of the CBC and Eq. (20). In the figure, the lower bound on the CBC is a decreasing function of y, and the lower bound is smaller than the CBC. Here, the lower bound is undefined when y is smaller than 1.690 and larger than 5.190 because the proposed assumptions in Proposition 4 do not hold.

Appendix D. Proofs

Proofs of Propositions 6 *and* 7. Note that

$$\begin{aligned}\mathrm{pr}(x_j, Y > y | Y_{x_j} > y, Y_{x_k} \le y) &= \mathrm{pr}(x_j, Y_{x_j} > y | Y_{x_j} > y, Y_{x_k} \le y)\\ &= \mathrm{pr}(x_j | Y_{x_j} > y, Y_{x_k} \le y).\end{aligned} \tag{40}$$

Thus, when $\mathrm{pr}(Y_{x_j} > y, Y_{x_k} \le y) = 1$ holds for a given y, we obtain Eqs. (28) and (29) immediately. In addition, when $X = x_j$ is independent of (Y_{x_j}, Y_{x_k}), we obtain Eq. (30).

Proof of Proposition 8. When $\mathrm{pr}(Y_{x_j} \le y, Y_{x_k} > y) = 0$ for any y holds, we have

$$\begin{aligned}\mathrm{pr}(x_j, Y > y | Y_{x_j} > y, Y_{x_k} \le y) &= \frac{\mathrm{pr}(x_j, Y > y, Y_{x_k} \le y)}{\mathrm{pr}(Y_{x_j} > y, Y_{x_k} \le y)}\\ &= \frac{\mathrm{pr}(x_j, Y > y) - \mathrm{pr}(x_j, Y_{x_k} > y) + \mathrm{pr}(x_j, Y \le y, Y_{x_k} > y)}{\mathrm{pr}(Y_{x_j} > y) - \mathrm{pr}(Y_{x_k} > y) + \mathrm{pr}(Y_{x_j} \le y, Y_{x_k} > y)}\\ &= \frac{\mathrm{pr}(x_j, Y > y) - \mathrm{pr}(x_j, Y_{x_k} > y)}{\mathrm{pr}(Y_{x_j} > y) - \mathrm{pr}(Y_{x_k} > y)} = \frac{\mathrm{pr}(Y_{x_j} > y | x_j) - \mathrm{pr}(Y_{x_k} > y | x_j)}{\mathrm{pr}(Y_{x_j} > y) - \mathrm{pr}(Y_{x_k} > y)}\mathrm{pr}(x_j).\end{aligned}$$

Proof of Proposition 9. First, we show that the following equation can be obtained:

$$\mathrm{pr}(Y_{x_k} \le y | x_j, Y > y) \ge \mathrm{pr}(Y_{x_j} > y | x_k, Y \le y). \tag{41}$$

To do this, from Eqs. (33) and (34), we derive

$$\begin{aligned}&\mathrm{pr}(x_j | Y_{x_j} > y, Y_{x_k} \le y)\mathrm{pr}(x_k, Y \le y) - \mathrm{pr}(x_k | Y_{x_j} > y, Y_{x_k} \le y)\mathrm{pr}(x_j, Y > y)\\ &= \mathrm{pr}(x_j | Y_{x_j} > y, Y_{x_k} \le y)\{\mathrm{pr}(x_k, Y \le y, Yx_j \le y) + \mathrm{pr}(x_k, Y \le y, Yx_j > y)\}\\ &\quad - \mathrm{pr}(x_k | Y_{x_j} > y, Y_{x_k} \le y)\{\mathrm{pr}(x_j, Y > y, Yx_k > y) + \mathrm{pr}(x_j, Y > y, Yx_k \le y)\}\\ &= \mathrm{pr}(x_j | Y_{x_j} > y, Y_{x_k} \le y)\mathrm{pr}(x_k | Y \le y, Y_{x_j} \le y)\mathrm{pr}(Y_{x_j} \le y, Y_{x_k} \le y)\\ &\quad - \mathrm{pr}(x_k | Y_{x_j} > y, Y_{x_k} \le y)\mathrm{pr}(x_j | Y > y, Y_{x_k} > y)\mathrm{pr}(Y_{x_j} > y, Y_{x_k} > y)\\ &\ge \mathrm{pr}(x_j | Y_{x_j} > y, Y_{x_k} \le y)\mathrm{pr}(x_k | Y_{x_j} > y, Y_{x_k} \le y)\mathrm{pr}(Y_{x_j} \le y, Y_{x_k} \le y)\end{aligned}$$

$$
\begin{aligned}
&- \mathrm{pr}(x_k|Y_{x_j} > y, Y_{x_k} \le y)\mathrm{pr}(x_j|Y_{x_j} > y, Y_{x_k} \le y)\mathrm{pr}(Y_{x_j} > y, Y_{x_k} > y) \\
&= \mathrm{pr}(x_j|Y_{x_j} > y, Y_{x_k} \le y)\mathrm{pr}(x_k|Y_{x_j} > y, Y_{x_k} \le y) \\
&\quad \times (\mathrm{pr}(Y_{x_j} \le y, Y_{x_k} \le y) - \mathrm{pr}(Y_{x_j} > y, Y_{x_k} > y)).
\end{aligned}
$$

Then, from Eq (35), we derive

$$\mathrm{pr}(x_j|Y_{x_j} > y, Y_{x_k} \le y)\mathrm{pr}(x_k, Y \le y) - \mathrm{pr}(x_k|Y_{x_j} > y, Y_{x_k} \le y)\mathrm{pr}(x_j, Y > y) \ge 0.$$

Thus, we have

$$\mathrm{pr}(x_j, Y_{x_j} > y, Y_{x_k} \le y)\mathrm{pr}(x_k, Y \le y) \ge \mathrm{pr}(x_k, Y_{x_j} > y, Y_{x_k} \le y)\mathrm{pr}(x_j, Y > y),$$

which leads to Eq. (41).

Second, we show that the following equation can be obtained:

$$
\begin{aligned}
\mathrm{pr}(Y_{x_j} > y, Y_{x_k} \le y) \le p \times \mathrm{pr}(Y_{x_k} \le y|x_j, Y > y)\mathrm{pr}(x_j, Y > y)& \\
+ \mathrm{pr}(Y_{x_k} \le y|x_j, Y > y)\mathrm{pr}(x_k, Y \le y).& \qquad (42)
\end{aligned}
$$

To do this, we derive

$$
\begin{aligned}
\mathrm{pr}(Y_{x_j} > y, Y_{x_k} \le y) &= \sum_{l=0}^{p} \mathrm{pr}(x_l, Y_{x_j} > y, Y_{x_k} \le y) \\
&= \sum_{l \ne j,k} \mathrm{pr}(x_l, Y_{x_j} > y, Y_{x_k} \le y) + \mathrm{pr}(x_j, Y_{x_j} > y, Y_{x_k} \le y) \\
&\quad + \mathrm{pr}(x_k, Y_{x_j} > y, Y_{x_k} \le y) \\
&= \sum_{l \ne j,k} \mathrm{pr}(x_l, Y_{x_j} > y, Y_{x_k} \le y) + \mathrm{pr}(Y_{x_k} \le y|x_j, Y > y)\mathrm{pr}(x_j, Y > y) \\
&\quad + \mathrm{pr}(Y_{x_j} > y|x_k, Y \le y)\mathrm{pr}(x_k, Y \le y). \qquad (43)
\end{aligned}
$$

From Eq. (33), the first term of Eq. (43) can be transformed into

$$
\begin{aligned}
\sum_{l \ne j,k} \mathrm{pr}(x_l, Y_{x_j} > y, Y_{x_k} \le y) &= \sum_{l \ne j,k} \mathrm{pr}(x_l|Y_{x_j} > y, Y_{x_k} \le y)\mathrm{pr}(Y_{x_j} > y, Y_{x_k} \le y) \\
&\le \sum_{l \ne j,k} \mathrm{pr}(x_j|Y_{x_j} > y, Y_{x_k} \le y)\mathrm{pr}(Y_{x_j} > y, Y_{x_k} \le y) \\
&= (p-1)\mathrm{pr}(x_j, Y_{x_j} > y, Y_{x_k} \le y) = (p-1)\mathrm{pr}(Y_{x_k} \le y|x_j, Y > y)\mathrm{pr}(x_j, Y > y).
\end{aligned}
$$

And from Eq. (41), the third term of Eq. (43) can be transformed into

$$\mathrm{pr}(Y_{x_j} > y|x_k, Y \le y)\mathrm{pr}(x_k, Y \le y) \le \mathrm{pr}(Y_{x_k} \le y|x_j, Y > y)\mathrm{pr}(x_k, Y \le y).$$

By combining these equations, we derive Eq. (42). Since the CBC with multiple evasive actions can be transformed into

$$
\begin{aligned}
\mathrm{pr}(x_j, Y > y|Y_{x_j} > y, Y_{x_k} \le y) &= \frac{\mathrm{pr}(x_j, Y > y, Y_{x_k} \le y)}{\mathrm{pr}(Y_{x_j} > y, Y_{x_k} \le y)} \\
&\ge \frac{\mathrm{pr}(Y_{x_k} \le y|x_j, Y > y)\mathrm{pr}(x_j, Y > y)}{\left(\begin{array}{r} p \times \mathrm{pr}(Y_{x_k} \le y|x_j, Y > y)\mathrm{pr}(x_j, Y > y) \\ + \mathrm{pr}(Y_{x_k} \le y|x_j, Y > y)\mathrm{pr}(x_k, Y \le y) \end{array} \right)} \\
&= \frac{\mathrm{pr}(x_j, Y > y)}{p \times \mathrm{pr}(x_j, Y > y) + \mathrm{pr}(x_k, Y \le y)}
\end{aligned}
$$

from Eq. (42), we have Eq. (36).

Proof of Proposition 10. From Eq. (43), the CBC can be transformed as follows:

$$\begin{aligned}&\mathrm{pr}(x_j, Y > y \mid Y_{x_j} > y, Y_{x_k} \le y)\\&= \frac{\mathrm{pr}(Y_{x_k} \le y \mid x_j, Y > y)\mathrm{pr}(x_j, Y > y)}{\left(\begin{array}{l}\mathrm{pr}(Y_{x_k} \le y \mid x_j, Y > y)\mathrm{pr}(x_j, Y > y)\\ \quad + \mathrm{pr}(Y_{x_j} > y \mid x_k, Y \le y)\mathrm{pr}(x_k, Y \le y)\\ \quad + \sum_{l \neq j,k} \mathrm{pr}(Y_{x_j} > y, Y_{x_k} \le y \mid x_l)\mathrm{pr}(x_l)\end{array}\right)}.\end{aligned}$$

This leads to Eq. (37) by considering bounds on probabilities of causation (i. e., $\mathrm{pr}(Y_{x_k} \le y \mid x_j, Y > y)$ and $\mathrm{pr}(Y_{x_j} > y \mid x_k, Y \le y)$).

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