

Appendix 2

Consider settings with one exposure (A), k mediators (M_1, M_2, \dots, M_k), baseline confounder (V), and k sets of mediator-outcome confounder (L_1, L_2, \dots, L_k). Since the number of PSEs is 2^k and there is no close form for all PSEs, here we use $k = 3$ as example to illustrate the methods.

When $k = 3$, the number of PSE is $2^3 = 8$, which are listed as follows:

$$PSE_{A \rightarrow Y} = r\Phi(a_1, a_0, a_0, a_0, a_0, a_0, a_0, a_0) - r\Phi(a_0, a_0, a_0, a_0, a_0, a_0, a_0, a_0)$$

$$PSE_{A \rightarrow M_1 \rightarrow Y} = r\Phi(a_1, a_1, a_0, a_0, a_0, a_0, a_0, a_0) - r\Phi(a_1, a_0, a_0, a_0, a_0, a_0, a_0, a_0)$$

$$PSE_{A \rightarrow M_2 \rightarrow Y} = r\Phi(a_1, a_1, a_1, a_0, a_0, a_0, a_0, a_0) - r\Phi(a_1, a_1, a_0, a_0, a_0, a_0, a_0, a_0)$$

$$PSE_{A \rightarrow M_1 \rightarrow M_2 \rightarrow Y} = r\Phi(a_1, a_1, a_1, a_1, a_0, a_0, a_0, a_0) - r\Phi(a_1, a_1, a_1, a_0, a_0, a_0, a_0, a_0)$$

$$PSE_{A \rightarrow M_3 \rightarrow Y} = r\Phi(a_1, a_1, a_1, a_1, a_1, a_0, a_0, a_0) - r\Phi(a_1, a_1, a_1, a_1, a_0, a_0, a_0, a_0)$$

$$PSE_{A \rightarrow M_1 \rightarrow M_3 \rightarrow Y} = r\Phi(a_1, a_1, a_1, a_1, a_1, a_1, a_0, a_0) - r\Phi(a_1, a_1, a_1, a_1, a_1, a_0, a_0, a_0)$$

$$PSE_{A \rightarrow M_2 \rightarrow M_3 \rightarrow Y} = r\Phi(a_1, a_1, a_1, a_1, a_1, a_1, a_1, a_0) - r\Phi(a_1, a_1, a_1, a_1, a_1, a_1, a_0, a_0)$$

$$PSE_{A \rightarrow M_1 \rightarrow M_2 \rightarrow M_3 \rightarrow Y} = r\Phi(a_1, a_1, a_1, a_1, a_1, a_1, a_1, a_1) - r\Phi(a_1, a_1, a_1, a_1, a_1, a_1, a_1, a_0)$$

$r\Phi(a^{(1)}, a^{(2)}, a^{(3)}, a^{(4)}, a^{(5)}, a^{(6)}, a^{(7)}, a^{(8)})$ is defined as $E[Y(a^{(1)}, G_1(a^{(2)}), G_2(a^{(3)}), G_1(a^{(4)}))]$

$$G_3(a^{(5)}, G_1(a^{(6)}, G_2(a^{(7)}, G_1(a^{(8)}))))]. \text{ Using the similar procedure in Appendix 1, } r\Phi = \\ \sum_{m_3} [\sum_{m_2} \{ \sum_{m_1} E[Y(a^{(1)}, m_1, m_2, m_3)] \times \Pr(M_1(a^{(2)}) = m_1) \} \times \{ \sum_{m'_1} \Pr[M_2(a^{(3)}, m'_1) = \\ m_2] \times \Pr(M_1(a^{(4)}) = m'_1) \} \times \sum_{m''_1, m'''_1, m'_2} \Pr(M_3(a^{(5)}, m''_1, m'_2) = m_3) \times \Pr(M_1(a^{(6)}) = \\ m''_1) \times \Pr(M_2(a^{(7)}, m'''_1) = m'_2) \times \Pr(M_1(a^{(8)}) = m'''_1)]$$

Under no unmeasured confounding assumptions, each component of above expression can be identified by the g-formul as follows.

$$E[Y(a, m_1, m_2, m_3)] =$$

$$\sum_{l_1, l_2, l_3} E[Y|a, l_1, m_1, l_2, m_2, l_3, m_3] \Pr(l_1|a) \Pr(l_2|a, l_1, m_1) \Pr(l_3|a, l_1, m_1, l_2, m_2)$$

$$\Pr(M_1(a)=m_1) = \Pr(m_1|a)$$

$$\Pr(M_2(a, m_1)=m_2) = \sum_{l_1} \Pr(m_2|a, l_1, m_1) \Pr(l_1|a)$$

$$\Pr(M_3(a, m_1, m_2)=m_3) = \sum_{l_1, l_2} \Pr(m_3|a, l_1, m_1, l_2, m_2) \Pr(l_1|a) \Pr(l_2 = l_2|a, l_1, m_1)$$

We can extend the above formula to any k by the similar procedure.