SUPPLEMENTARY MATERIALS for

Bayesian estimation and prediction for network meta-analysis with contrast-based approach

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e-Appendix A: Supplementary results of the simulation studies in Section 5

In Section 5, we provided simulation results concerning an estimation of log-OR on A vs. C. We here present the other results for the simulation studies concerning estimations for log-OR on A vs. B, A vs. D, A vs. E, and A vs. F (e-Tables 1–8) and for the heterogeneity standard deviation τ. For estimating the log-ORs, the overall trends were similar to those of log-OR on A vs. C. Also, for the estimation of τ, the REML estimation generally underestimated the heterogeneity, which is a well-known property of the ordinary ML and REML estimations for meta-analysis [1-3]. In addition, the Bayes estimators based on the uniform priors for τ^2 and τ, along with the Jeffreys prior, overestimated the actual heterogeneity. The relatively overcoverage trends of the corresponding CrIs and PIs would reflect these properties. In addition, the Bayes estimator based on the proper inverse gamma prior underestimated actual statistical heterogeneity; however, the overestimation biases were generally moderate compared with the REML estimation.

e-Table 1. Results of the simulation studies (B vs. A): Bias, standard error (SE) and root-mean-squared error (RMSE).

					Proposed Bayesian analyses											
		REML estimation			Uniform improper prior for τ^2			Uniform improper prior for τ			Jeffreys prior			Proper inverse gamma prior		
N	τ	Bias	SE	RMSE	Bias	SE	RMSE	Bias	SE	RMSE	Bias	SE	RMSE	Bias	SE	RMSE
22	0.10	−0.001	0.069	0.069	−0.002	0.069	0.069	−0.001	0.069	0.069	−0.002	0.070	0.070	−0.001	0.069	0.069
22	0.20	0.005	0.108	0.108	0.005	0.108	0.108	0.005	0.108	0.108	0.004	0.108	0.108	0.005	0.108	0.108
22	0.30	0.009	0.191	0.192	0.008	0.192	0.192	0.008	0.191	0.192	0.007	0.192	0.192	0.008	0.191	0.192
22	0.40	0.006	0.348	0.348	0.005	0.349	0.348	0.006	0.349	0.349	0.004	0.349	0.349	0.006	0.348	0.348
44	0.10	0.002	0.048	0.048	0.002	0.048	0.048	0.002	0.048	0.048	0.002	0.048	0.048	0.002	0.048	0.048
44	0.20	0.006	0.074	0.074	0.006	0.074	0.074	0.006	0.074	0.074	0.006	0.074	0.074	0.006	0.074	0.074
44	0.30	0.009	0.131	0.131	0.009	0.131	0.131	0.009	0.131	0.131	0.008	0.131	0.131	0.009	0.131	0.131
44	0.40	0.001	0.256	0.256	0.000	0.257	0.256	0.000	0.256	0.256	0.000	0.257	0.257	0.001	0.257	0.257

e-Table 2. Results of the simulation studies (D vs. A): Bias, standard error (SE) and root-mean-squared error (RMSE).

					Proposed Bayesian analyses											
		REML estimation			Uniform improper prior for τ^2			Uniform improper prior for τ			Jeffreys prior			Proper inverse gamma prior		
N	τ	Bias	SE	RMSE	Bias	SE	RMSE	Bias	SE	RMSE	Bias	SE	RMSE	Bias	SE	RMSE
22	0.10	0.000	0.067	0.067	0.000	0.067	0.067	0.000	0.067	0.067	0.000	0.067	0.067	0.000	0.067	0.067
22	0.20	0.006	0.105	0.105	0.005	0.105	0.105	0.006	0.105	0.105	0.005	0.105	0.105	0.006	0.105	0.105
22	0.30	0.017	0.185	0.186	0.016	0.185	0.186	0.017	0.185	0.186	0.016	0.185	0.186	0.017	0.185	0.186
22	0.40	0.015	0.353	0.354	0.014	0.354	0.354	0.015	0.354	0.354	0.013	0.354	0.354	0.015	0.353	0.354
44	0.10	0.003	0.046	0.046	0.003	0.046	0.046	0.003	0.046	0.046	0.003	0.046	0.046	0.003	0.046	0.046
44	0.20	0.004	0.076	0.076	0.004	0.076	0.076	0.004	0.076	0.076	0.004	0.076	0.076	0.005	0.076	0.076
44	0.30	0.005	0.130	0.130	0.005	0.130	0.130	0.005	0.130	0.130	0.005	0.130	0.130	0.005	0.130	0.130
44	0.40	0.005	0.262	0.262	0.005	0.262	0.262	0.005	0.262	0.262	0.004	0.263	0.263	0.006	0.262	0.262

e-Table 3. Results of the simulation studies (E vs. A): Bias, standard error (SE) and root-mean-squared error (RMSE).

					Proposed Bayesian analyses											
		REML estimation			Uniform improper prior for τ^2			Uniform improper prior for τ			Jeffreys prior			Proper inverse gamma prior		
N	τ	Bias	SE	RMSE	Bias	SE	RMSE	Bias	SE	RMSE	Bias	SE	RMSE	Bias	SE	RMSE
22	0.10	0.000	0.066	0.066	−0.001	0.066	0.066	0.000	0.066	0.066	-0.001	0.066	0.066	0.000	0.066	0.066
22	0.20	0.008	0.104	0.104	0.007	0.104	0.104	0.008	0.104	0.104	0.007	0.104	0.104	0.008	0.104	0.104
22	0.30	0.016	0.184	0.184	0.015	0.184	0.184	0.016	0.184	0.184	0.014	0.184	0.184	0.016	0.184	0.184
22	0.40	0.016	0.349	0.349	0.015	0.349	0.349	0.016	0.349	0.349	0.014	0.349	0.349	0.016	0.349	0.349
44	0.10	0.003	0.047	0.047	0.003	0.047	0.047	0.003	0.047	0.047	0.003	0.047	0.047	0.003	0.047	0.047
44	0.20	0.006	0.074	0.074	0.005	0.074	0.074	0.005	0.074	0.074	0.005	0.074	0.074	0.006	0.074	0.074
44	0.30	0.012	0.132	0.132	0.012	0.132	0.132	0.012	0.132	0.132	0.011	0.132	0.132	0.012	0.132	0.132
44	0.40	0.002	0.258	0.258	0.001	0.258	0.258	0.002	0.258	0.258	0.001	0.259	0.258	0.002	0.258	0.258

e-Table 4. Results of the simulation studies (F vs. A): Bias, standard error (SE) and root-mean-squared error (RMSE).

					Proposed Bayesian analyses											
		REML estimation			Uniform improper prior for τ^2			Uniform improper prior for τ			Jeffreys prior			Proper inverse gamma prior		
N	τ	Bias	SE	RMSE	Bias	SE	RMSE	Bias	SE	RMSE	Bias	SE	RMSE	Bias	SE	RMSE
22	0.10	0.000	0.067	0.067	0.000	0.067	0.067	0.000	0.067	0.067	−0.001	0.068	0.067	0.000	0.067	0.067
22	0.20	0.005	0.100	0.100	0.005	0.100	0.100	0.005	0.100	0.100	0.004	0.100	0.100	0.006	0.100	0.100
22	0.30	0.012	0.176	0.177	0.011	0.176	0.177	0.011	0.176	0.177	0.010	0.177	0.177	0.012	0.176	0.177
22	0.40	0.013	0.344	0.344	0.012	0.344	0.345	0.013	0.344	0.344	0.011	0.345	0.345	0.013	0.344	0.344
44	0.10	0.002	0.047	0.047	0.002	0.047	0.047	0.002	0.047	0.047	0.002	0.047	0.047	0.002	0.047	0.047
44	0.20	0.007	0.070	0.070	0.007	0.070	0.070	0.007	0.070	0.070	0.006	0.070	0.070	0.007	0.070	0.070
44	0.30	0.012	0.125	0.126	0.011	0.125	0.126	0.011	0.125	0.126	0.011	0.126	0.126	0.012	0.126	0.126
44	0.40	0.007	0.237	0.237	0.006	0.237	0.237	0.006	0.237	0.237	0.006	0.237	0.237	0.007	0.237	0.237

e-Table 5. Results of the simulation studies (B vs. A): Coverage probability (CP) and expected length (EL) of 95% CrI and PI.

						Proposed Bayesian analyses															
		REML estimation				Uniform improper prior for τ^2				Uniform improper prior for τ				Jeffreys prior				Proper inverse gamma prior			
		95%CI		95%PI		95%CrI		95%PI		95%CrI		95%PI		95%CrI		95%PI		95%CrI		95%PI	
N	τ	CP	EL	CP	EL	CP	EL	CP	EL	CP	EL	CP	EL	CP	EL	CP	EL	CP	EL	CP	EL
22	0.10	0.939	0.265	0.911	0.483	0.966	0.309	0.976	0.598	0.957	0.290	0.946	0.539	0.974	0.334	0.988	0.669	0.950	0.278	0.935	0.504
22	0.20	0.931	0.407	0.940	0.922	0.958	0.462	0.954	1.019	0.951	0.442	0.948	0.968	0.969	0.495	0.974	1.111	0.941	0.425	0.936	0.920
22	0.30	0.932	0.726	0.941	1.824	0.962	0.818	0.960	1.945	0.952	0.790	0.957	1.878	0.975	0.873	0.973	2.089	0.944	0.766	0.953	1.813
22	0.40	0.946	1.375	0.949	3.583	0.964	1.544	0.952	3.777	0.960	1.497	0.951	3.662	0.972	1.646	0.967	4.040	0.955	1.453	0.944	3.552
44	0.10	0.935	0.187	0.923	0.432	0.954	0.199	0.947	0.478	0.943	0.193	0.934	0.453	0.963	0.206	0.965	0.504	0.941	0.190	0.932	0.437
44	0.20	0.944	0.289	0.942	0.850	0.957	0.304	0.950	0.884	0.956	0.299	0.952	0.864	0.963	0.312	0.971	0.914	0.949	0.294	0.955	0.847
44	0.30	0.947	0.512	0.947	1.679	0.959	0.538	0.949	1.718	0.956	0.530	0.947	1.693	0.964	0.551	0.954	1.767	0.952	0.524	0.945	1.671
44	0.40	0.938	0.981	0.949	3.344	0.951	1.029	0.961	3.404	0.946	1.016	0.950	3.360	0.957	1.055	0.967	3.492	0.945	1.003	0.957	3.319

e-Table 6. Results of the simulation studies (D vs. A): Coverage probability (CP) and expected length (EL) of 95% CrI and PI.

						Proposed Bayesian analyses															
		REML estimation				Uniform improper prior for τ^2				Uniform improper prior for τ				Jeffreys prior				Proper inverse gamma prior			
		95%CI		95%PI		95%CrI		95%PI		95%CrI		95%PI		95%CrI		95%PI		95%CrI		95%PI	
N	τ	CP	EL	CP	EL	CP	EL	CP	EL	CP	EL	CP	EL	CP	EL	CP	EL	CP	EL	CP	EL
22	0.10	0.932	0.259	0.907	0.479	0.966	0.304	0.968	0.596	0.956	0.284	0.955	0.536	0.976	0.328	0.984	0.666	0.947	0.273	0.943	0.501
22	0.20	0.929	0.401	0.940	0.920	0.958	0.456	0.964	1.017	0.949	0.437	0.953	0.965	0.972	0.490	0.974	1.108	0.940	0.420	0.935	0.918
22	0.30	0.938	0.721	0.956	1.821	0.968	0.812	0.966	1.943	0.959	0.786	0.952	1.876	0.976	0.867	0.974	2.085	0.954	0.761	0.946	1.811
22	0.40	0.930	1.368	0.948	3.580	0.953	1.536	0.958	3.780	0.953	1.490	0.961	3.663	0.968	1.637	0.967	4.040	0.943	1.446	0.952	3.551
44	0.10	0.941	0.183	0.923	0.430	0.956	0.195	0.950	0.477	0.951	0.189	0.944	0.451	0.960	0.201	0.963	0.502	0.946	0.185	0.930	0.435
44	0.20	0.934	0.285	0.935	0.848	0.945	0.300	0.955	0.883	0.941	0.295	0.951	0.863	0.948	0.309	0.963	0.912	0.937	0.290	0.945	0.846
44	0.30	0.944	0.508	0.949	1.677	0.954	0.534	0.962	1.718	0.951	0.526	0.945	1.692	0.958	0.546	0.966	1.765	0.946	0.520	0.949	1.668
44	0.40	0.937	0.976	0.948	3.342	0.944	1.023	0.958	3.403	0.944	1.010	0.947	3.360	0.948	1.048	0.962	3.490	0.941	0.999	0.949	3.319

e-Table 7. Results of the simulation studies (E vs. A): Coverage probability (CP) and expected length (EL) of 95% CrI and PI.

						Proposed Bayesian analyses															
		REML estimation				Uniform improper prior for τ^2				Uniform improper prior for τ				Jeffreys prior				Proper inverse gamma prior			
		95%CI		95%PI		95%CrI		95%PI		95%CrI		95%PI		95%CrI		95%PI		95%CrI		95%PI	
N	τ	CP	EL	CP	EL	CP	EL	CP	EL	CP	EL	CP	EL	CP	EL	CP	EL	CP	EL	CP	EL
22	0.10	0.937	0.261	0.913	0.480	0.977	0.305	0.972	0.596	0.965	0.286	0.951	0.537	0.983	0.329	0.987	0.667	0.954	0.274	0.936	0.502
22	0.20	0.938	0.403	0.939	0.920	0.966	0.457	0.963	1.018	0.960	0.438	0.944	0.965	0.974	0.491	0.976	1.109	0.952	0.422	0.936	0.919
22	0.30	0.936	0.720	0.945	1.821	0.963	0.811	0.962	1.943	0.955	0.784	0.957	1.875	0.973	0.867	0.971	2.086	0.952	0.760	0.951	1.810
22	0.40	0.935	1.366	0.948	3.579	0.957	1.532	0.965	3.774	0.952	1.487	0.958	3.658	0.968	1.634	0.976	4.037	0.940	1.443	0.944	3.550
44	0.10	0.945	0.184	0.930	0.431	0.957	0.196	0.958	0.477	0.949	0.190	0.942	0.452	0.961	0.203	0.959	0.503	0.950	0.187	0.942	0.436
44	0.20	0.934	0.285	0.946	0.848	0.948	0.301	0.957	0.882	0.943	0.296	0.940	0.863	0.953	0.309	0.961	0.914	0.943	0.291	0.938	0.847
44	0.30	0.937	0.508	0.940	1.677	0.950	0.533	0.952	1.718	0.944	0.526	0.945	1.692	0.956	0.546	0.958	1.765	0.944	0.519	0.948	1.668
44	0.40	0.941	0.974	0.940	3.342	0.951	1.021	0.951	3.405	0.948	1.008	0.953	3.361	0.953	1.048	0.956	3.490	0.945	0.997	0.941	3.319

e-Table 8. Results of the simulation studies (F vs. A): Coverage probability (CP) and expected length (EL) of 95% CrI and PI.

						Proposed Bayesian analyses															
		REML estimation				Uniform improper prior for τ^2				Uniform improper prior for τ				Jeffreys prior				Proper inverse gamma prior			
		95%CI		95%PI		95%CrI		95%PI		95%CrI		95%PI		95%CrI		95%PI		95%CrI		95%PI	
N	τ	CP	EL	CP	EL	CP	EL	CP	EL	CP	EL	CP	EL	CP	EL	CP	EL	CP	EL	CP	EL
22	0.10	0.925	0.255	0.903	0.478	0.963	0.296	0.968	0.591	0.947	0.278	0.952	0.533	0.974	0.320	0.989	0.663	0.944	0.267	0.937	0.498
22	0.20	0.940	0.388	0.948	0.917	0.962	0.441	0.959	1.010	0.957	0.422	0.950	0.957	0.970	0.473	0.973	1.101	0.949	0.406	0.944	0.912
22	0.30	0.936	0.692	0.944	1.815	0.964	0.778	0.963	1.929	0.956	0.753	0.955	1.861	0.972	0.832	0.973	2.072	0.949	0.730	0.952	1.798
22	0.40	0.932	1.310	0.950	3.568	0.953	1.470	0.960	3.750	0.949	1.427	0.947	3.634	0.966	1.569	0.967	4.011	0.944	1.385	0.943	3.523
44	0.10	0.941	0.179	0.913	0.429	0.955	0.190	0.953	0.475	0.949	0.185	0.936	0.449	0.961	0.196	0.966	0.501	0.946	0.181	0.932	0.433
44	0.20	0.946	0.276	0.949	0.846	0.958	0.290	0.955	0.879	0.955	0.285	0.947	0.860	0.964	0.298	0.958	0.911	0.952	0.281	0.941	0.842
44	0.30	0.943	0.488	0.954	1.674	0.955	0.513	0.954	1.710	0.951	0.505	0.955	1.685	0.958	0.526	0.960	1.759	0.946	0.499	0.945	1.662
44	0.40	0.947	0.934	0.959	3.336	0.957	0.980	0.956	3.392	0.953	0.967	0.955	3.348	0.957	1.005	0.958	3.478	0.951	0.957	0.954	3.305

e-Table 9. Results of the simulation studies (τ estimator): Bias, standard error (SE) and root mean squared error (RMSE).

					Proposed Bayesian analyses											
		REML estimation			Uniform improper prior for τ^2			Uniform improper prior for τ			Jeffreys prior			Proper inverse gamma prior		
N	τ	Bias	SE	RMSE	Bias	SE	RMSE	Bias	SE	RMSE	Bias	SE	RMSE	Bias	SE	RMSE
22	0.10	−0.009	0.040	0.041	0.019	0.034	0.038	0.004	0.034	0.035	0.035	0.036	0.050	−0.004	0.031	0.031
22	0.20	−0.006	0.044	0.045	0.022	0.047	0.052	0.010	0.047	0.048	0.042	0.049	0.064	−0.001	0.047	0.047
22	0.30	−0.006	0.071	0.071	0.037	0.077	0.086	0.022	0.076	0.079	0.069	0.082	0.107	0.008	0.074	0.075
22	0.40	−0.019	0.129	0.130	0.059	0.141	0.153	0.033	0.137	0.141	0.116	0.150	0.189	0.009	0.134	0.134
44	0.10	−0.005	0.026	0.027	0.005	0.025	0.025	−0.001	0.026	0.026	0.012	0.025	0.027	−0.005	0.025	0.025
44	0.20	−0.003	0.030	0.030	0.008	0.030	0.031	0.003	0.030	0.031	0.015	0.031	0.034	−0.001	0.030	0.030
44	0.30	−0.006	0.047	0.047	0.012	0.048	0.050	0.006	0.048	0.048	0.023	0.049	0.055	0.000	0.048	0.048
44	0.40	−0.012	0.084	0.085	0.021	0.088	0.090	0.010	0.087	0.087	0.041	0.090	0.099	0.000	0.086	0.086

e-Appendix B: Supplementary results of the applications in Section 6

For comparative purpose, we also performed arm-based analyses of the real network meta-analysis datasets in Section 6. We used a representative Bayesian network meta-analysis tool, `gemtc` [4] in R. We adopted the `gemtc` default setting, which assumes a proper noninformative prior; please refer to the help manual for `gemtc` [4]. The results are presented in e-Tables 10 and 11. (The functions to calculate PIs are currently not available in `gemtc`; thus, we only provided the estimates and CrIs.) In general, the posterior summaries were comparable to the contrast-based Bayesian analyses with noninformative priors. We also compared the computational times of `gemtc` and `BANMA`, which we developed for implementing the proposed methods. Computations by `gemtc` were relatively fast because of the difference in the designs of the computational modules (they are not comparable). However, the computational time of our proposed methods was nonproblematic for practical uses and sufficiently short.

e-Table 10. Results of network meta-analysis for the antihypertensive drug example using the arm-based approach by R `gemtc` package (default setting) †.

	Odds-ratio estimate and 95% CrI
Diuretic vs.	
ACE inhibitor	0.659 (0.548, 0.779)
ARB	0.610 (0.481, 0.758)
β-Blocker	0.927 (0.770, 1.103)
CCB	0.780 (0.655, 0.923)
Placebo	0.741 (0.611, 0.880)
Between-studies SD (95% CrI)	0.134 (0.058, 0.238)

† ACE: angiotensin-converting enzyme; ARB: angiotensin-receptor blocker; CCB: calcium-channel blocker.

e-Table 11. Results of network meta-analysis for the anticoagulant example using the arm-based approach by R `gemtc` package (default setting) †.

	Odds-ratio estimate and 95% CrI
Placebo vs.	
DOAC	2.286 (0.815, 6.508)
LMWH (low dose)	1.057 (0.445, 2.272)
LMWH (intermediate dose)	1.358 (0.694, 2.670)
UFH (low dose)	1.447 (0.685, 3.892)
UFH (intermediate dose)	2.626 (0.988, 6.234)
Pentasaccharide	0.980 (0.030, 40.49)
No treatment	0.670 (0.227, 2.120)
Between-studies SD (95%CrI)	0.430 (0.027, 1.023)

† DOAC: direct oral anticoagulant; LMWH: low-molecular-weight heparin; UFH: unfractionated heparin.

e-Table 12. Computational times of the Bayesian analyses for the two real data examples (by a general desktop computer with an Intel Core i9-12900T CPU) †.

	Antihypertensive drug example	Anticoagulant example
Arm-based approach using `gemtc` (default setting)	10.765 sec	13.093 sec
Contrast-based approach using `BANMA`		
Uniform improper prior for τ^2	1.044 min	1.533 min
Uniform improper prior for τ	1.022 min	1.585 min
Jeffreys prior	1.663 min	2.805 min
Proper inverse gamma prior	1.030 min	1.660 min

† The arm-based approach used 20,000 posterior samples after a 5,000 burn-in period and did not involve prediction interval calculations. The contrast-based methods also used 20,000 posterior samples.

References

1. Noma H, Hamura Y, Gosho M, Furukawa TA. Kenward-Roger-type corrections for inference methods of network meta-analysis and meta-regression. ISM Research Memorandum No 1222. 2023.
2. Noma H, Nagashima K, Maruo K, Gosho M, Furukawa TA. Bartlett-type corrections and bootstrap adjustments of likelihood-based inference methods for network meta-analysis. *Stat Med*. 2018;37:1178-1190.
3. Veroniki AA, Jackson D, Bender R, et al. Methods to calculate uncertainty in the estimated overall effect size from a random-effects meta-analysis. *Res Synth Methods*. 2019;10:23-43.
4. gemtc: Network meta-analysis using Bayesian Methods. R package version 1.0-1. 2021. https://cran.r-project.org/web/packages/gemtc/