

Supplementary Material

Estimating The Area Under A Receiver Operating
Characteristic Curve Using Partially Ordered Sets

Additional Simulation: RE Results under DPS models

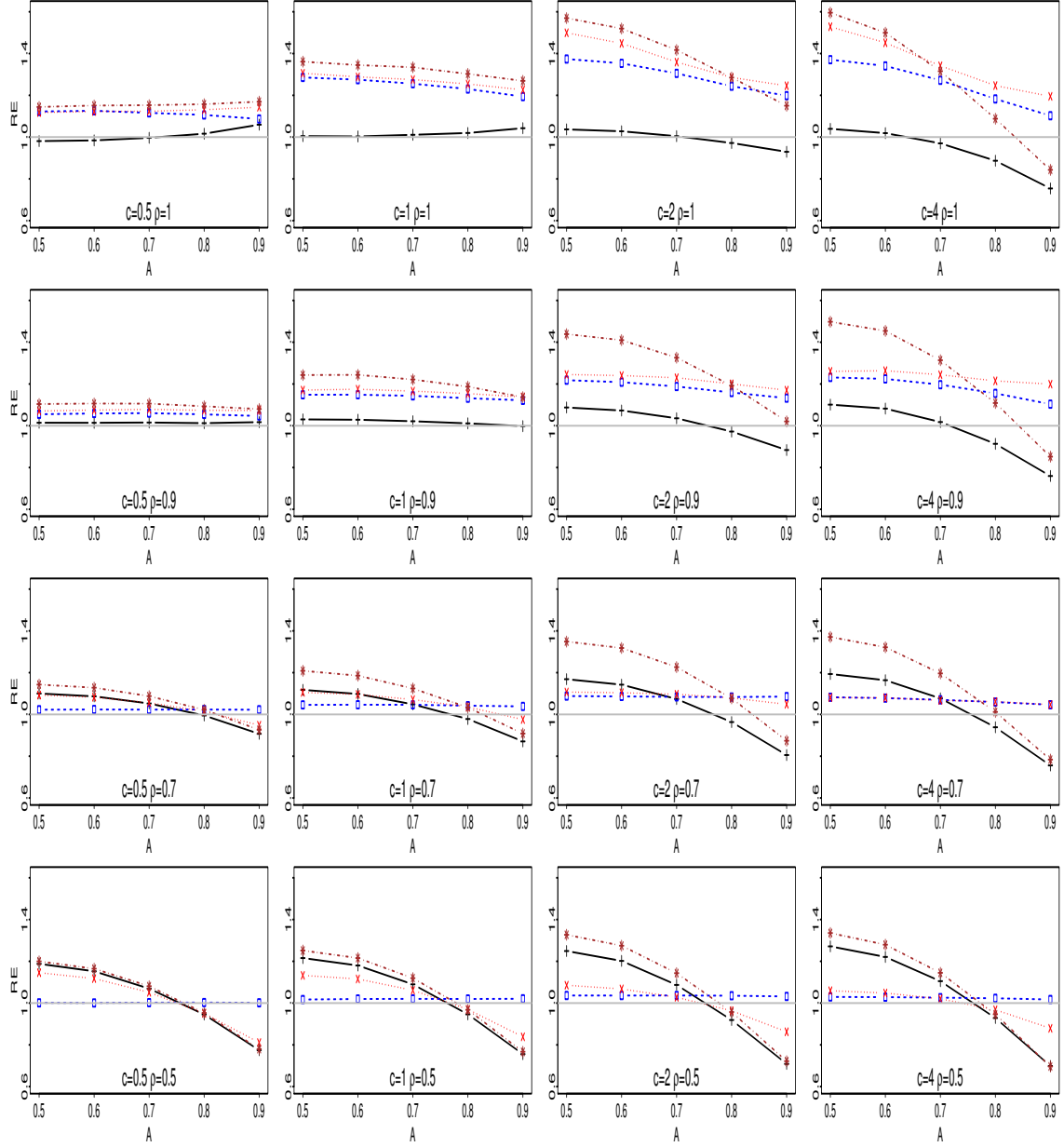


Figure S1: The empirical relative efficiency of $\hat{A}_{iso}$ (represented by “ $\square$ ” connected using dashed lines), $\hat{A}_{ml}$ (represented by “+” connected using solid lines), $\hat{A}_{t.ml}$ (represented by “ $\times$ ” connected using dotted lines) and $\hat{A}_{s.ml}$ (represented by “*” connected using dot-dashed lines) to $\hat{A}_{st}$ as a function of A when the subpopulation distributions are normal, under DPS models for $c \in \{0.5, 1, 2, 4\}$, $\rho \in \{0.5, 0.7, 0.9, 1\}$, $(n, m) = (15, 5)$.

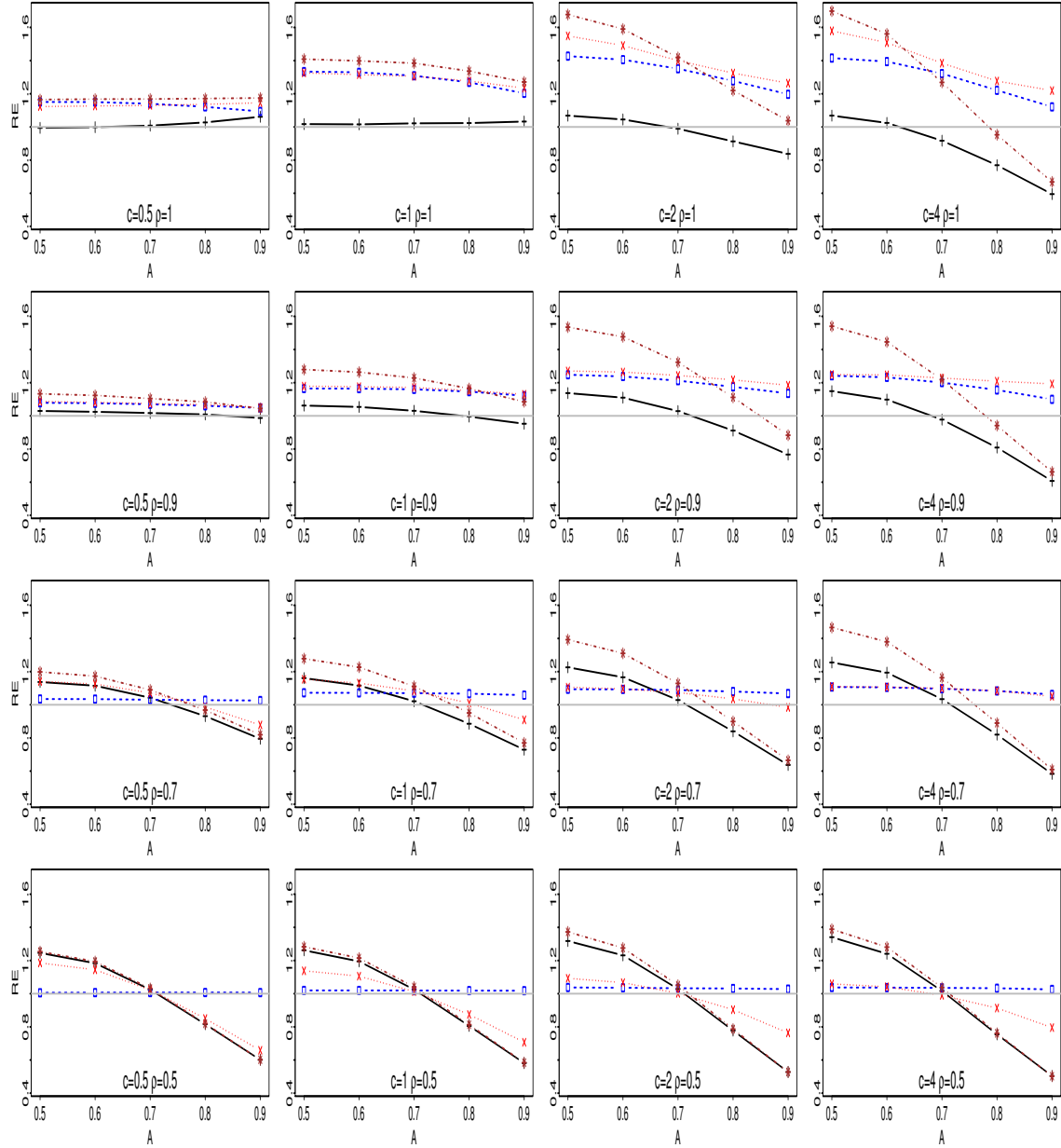


Figure S2: The empirical relative efficiency of $\hat{A}_{iso}$ (represented by “ $\square$ ” connected using dashed lines), $\hat{A}_{ml}$ (represented by “+” connected using solid lines), $\hat{A}_{t.ml}$ (represented by “ $\times$ ” connected using dotted lines) and $\hat{A}_{s.ml}$ (represented by “*” connected using dot-dashed lines) to $\hat{A}_{st}$ as a function of A when the subpopulation distributions are normal, under DPS models for $c \in \{0.5, 1, 2, 4\}$, $\rho \in \{0.5, 0.7, 0.9, 1\}$, $(n, m) = (30, 5)$.

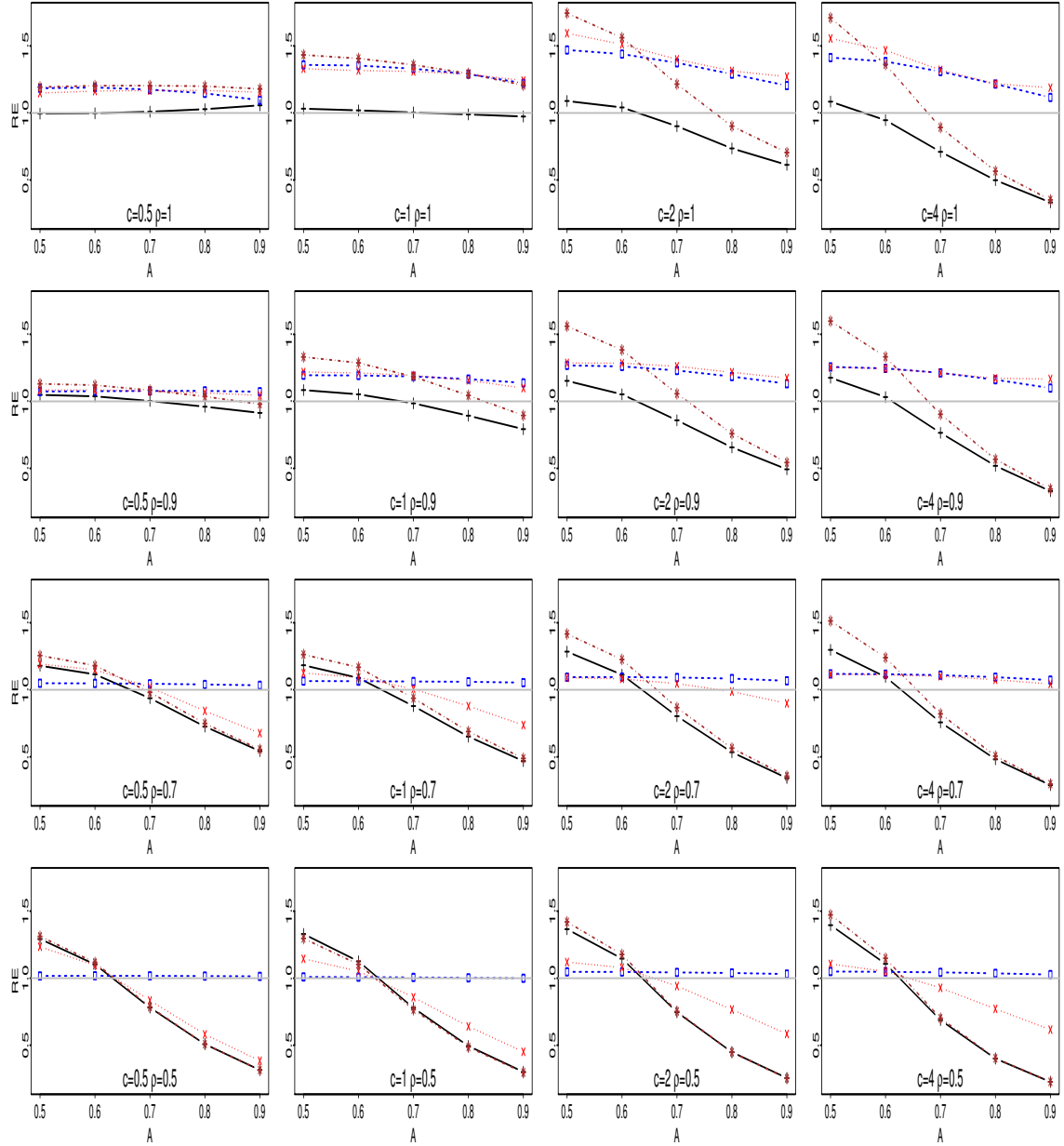


Figure S3: The empirical relative efficiency of $\hat{A}_{iso}$ (represented by “ $\square$ ” connected using dashed lines), $\hat{A}_{ml}$ (represented by “+” connected using solid lines), $\hat{A}_{t.ml}$ (represented by “ $\times$ ” connected using dotted lines) and $\hat{A}_{s.ml}$ (represented by “*” connected using dot-dashed lines) to $\hat{A}_{st}$ as function of A when the subpopulation distributions are normal, under DPS models for $c \in \{0.5, 1, 2, 4\}$, $\rho \in \{0.5, 0.7, 0.9, 1\}$, $(n, m) = (90, 5)$.

Additional Simulation: RE Results under TIC models

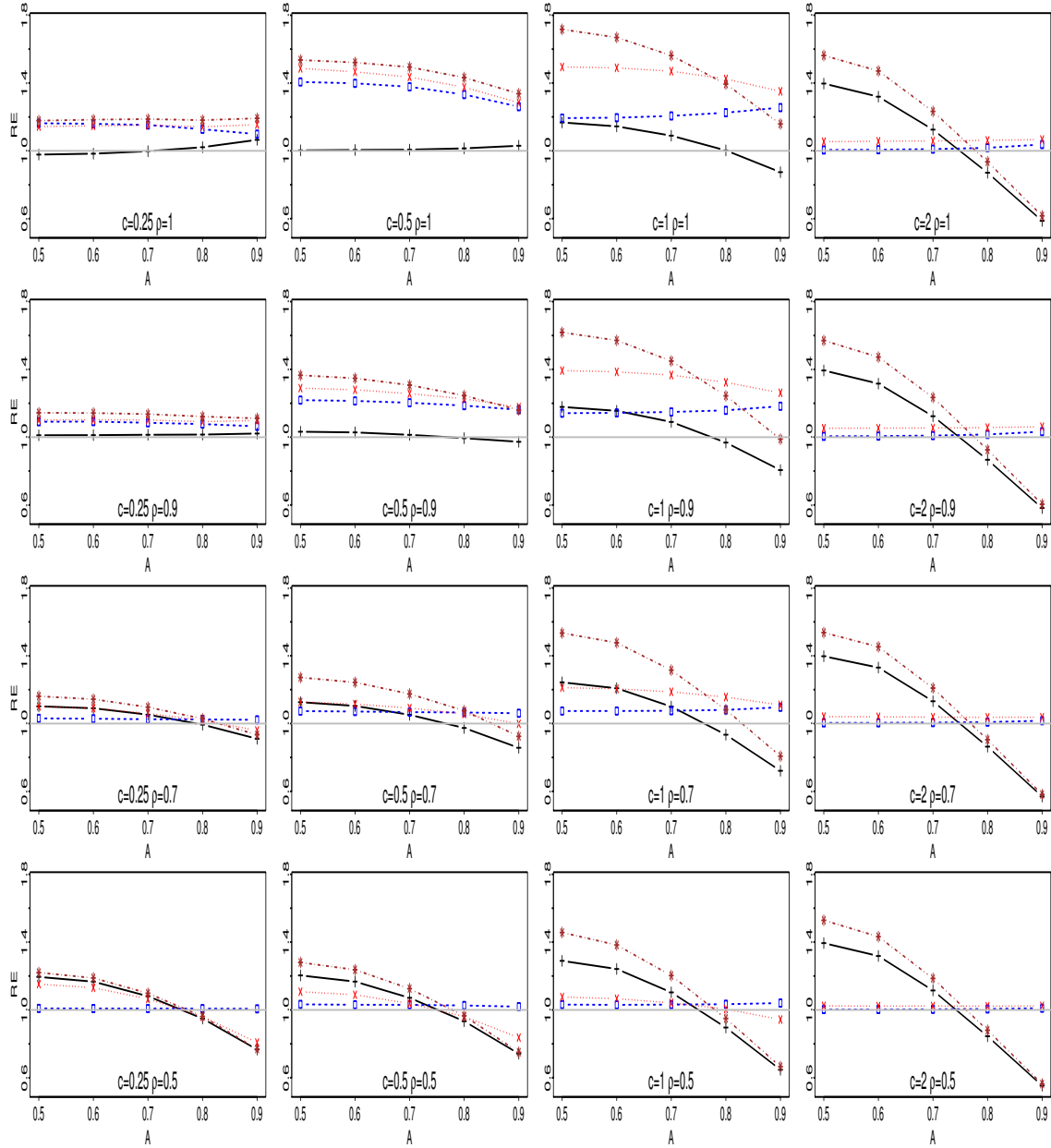


Figure S4: The empirical relative efficiency of $\hat{A}_{iso}$ (represented by “ $\square$ ” connected using dashed lines), $\hat{A}_{ml}$ (represented by “+” connected using solid lines), $\hat{A}_{t.ml}$ (represented by “ $\times$ ” connected using dotted lines) and $\hat{A}_{s.ml}$ (represented by “*” connected using dot-dashed lines) to $\hat{A}_{st}$ as function of A when the subpopulation distributions are normal, under TIC models for $c \in \{0.5, 1, 2, 4\}$, $\rho \in \{0.5, 0.7, 0.9, 1\}$, $(n, m) = (15, 5)$.

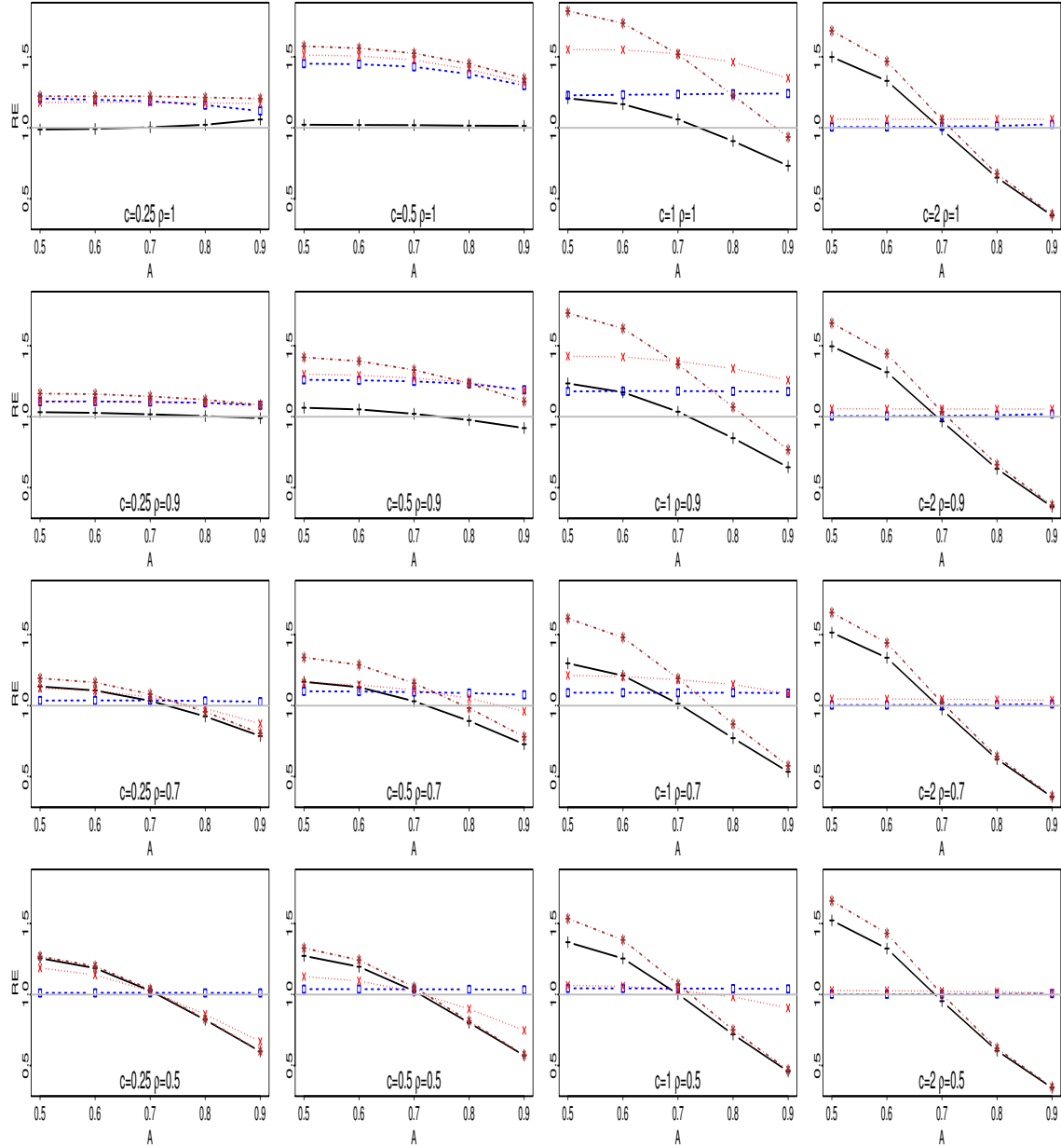


Figure S5: The empirical relative efficiency of $\hat{A}_{iso}$ (represented by “ $\square$ ” connected using dashed lines), $\hat{A}_{ml}$ (represented by “+” connected using solid lines), $\hat{A}_{t.ml}$ (represented by “ $\times$ ” connected using dotted lines) and $\hat{A}_{s.ml}$ (represented by “*” connected using dot-dashed lines) to $\hat{A}_{st}$ as function of A when the subpopulation distributions are normal, under TIC models for $c \in \{0.5, 1, 2, 4\}$, $\rho \in \{0.5, 0.7, 0.9, 1\}$, $(n, m) = (30, 5)$.

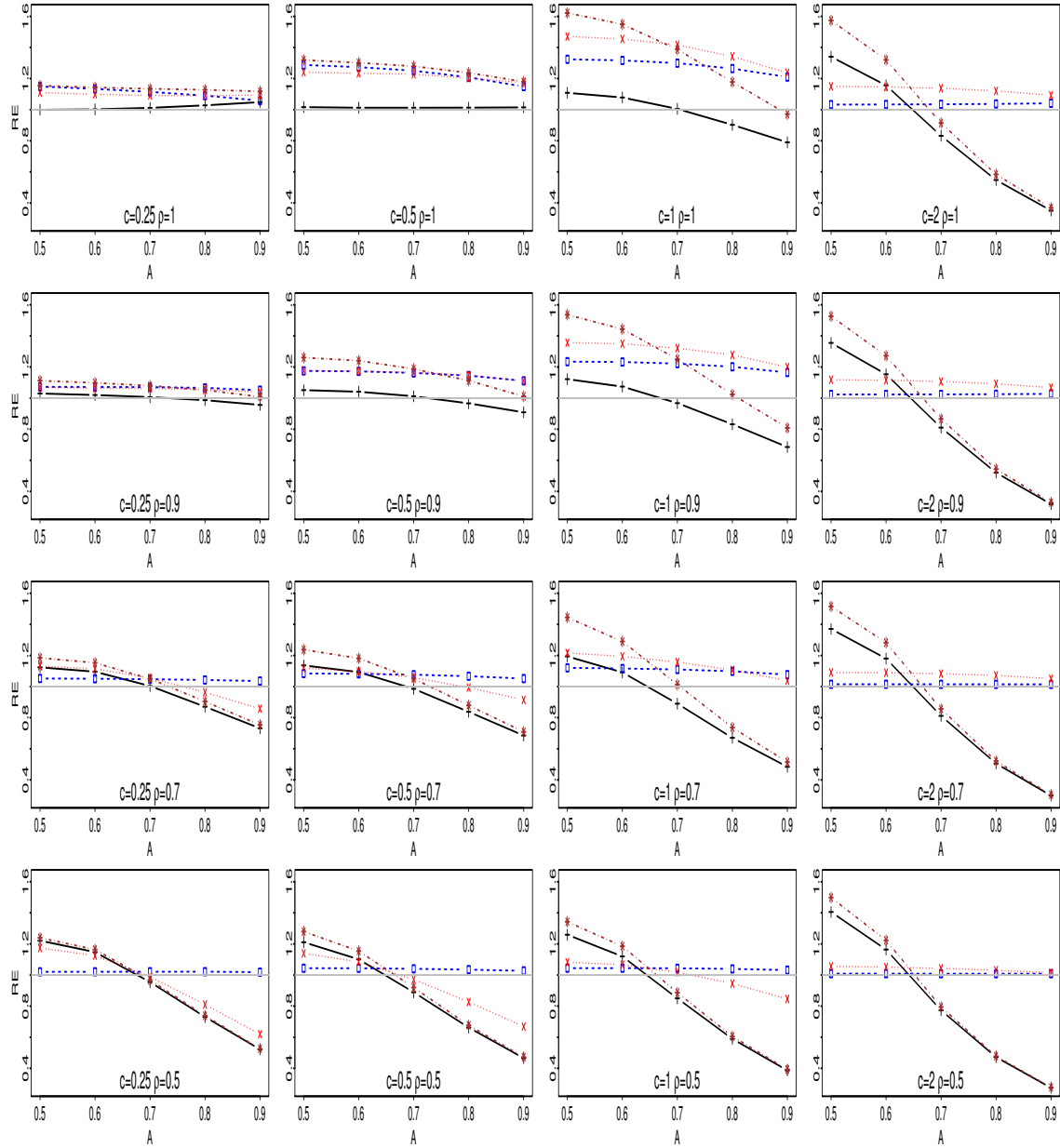


Figure S6: The empirical relative efficiency of $\hat{A}_{iso}$ (represented by “ $\square$ ” connected using dashed lines), $\hat{A}_{ml}$ (represented by “+” connected using solid lines), $\hat{A}_{t.ml}$ (represented by “ $\times$ ” connected using dotted lines) and $\hat{A}_{s.ml}$ (represented by “*” connected using dot-dashed lines) to $\hat{A}_{st}$ as function of A when the subpopulation distributions are normal, under TIC models for $c \in \{0.5, 1, 2, 4\}$, $\rho \in \{0.5, 0.7, 0.9, 1\}$, $(n, m) = (90, 5)$.

Additional Simulation: Bias of Different AUC estimators

Table S1: The empirical bias of different AUC estimators under DPS models for normal subpopulation distributions with $(n, m) = (15, 5)$ and perfect ranking ($\rho = 1$).

c	A	$\hat{A}_{st}$	$\hat{A}_{sp}$	$\hat{A}_{iso}$	$\hat{A}_{ml}$	$\hat{A}_{t.ml}$	$\hat{A}_{s.ml}$
0.5	0.5	0.00228	0.00210	0.00209	0.00127	0.00137	0.00129
	0.6	0.00145	0.00148	0.00149	0.00112	0.00154	0.00117
	0.7	0.00224	0.00227	0.00233	0.00232	0.00304	0.00232
	0.8	0.00240	0.00255	0.00258	0.00280	0.00356	0.00282
	0.9	0.00167	0.00188	0.00191	0.00154	0.00227	0.00164
1	0.5	0.00033	-0.00012	-0.00008	-0.00074	-0.00169	-0.00118
	0.6	0.00107	0.00109	0.00114	-0.00017	-0.00015	-0.00056
	0.7	0.00145	0.00174	0.00179	0.00004	0.00084	-0.00039
	0.8	0.00294	0.00357	0.00360	0.00134	0.00269	0.00100
	0.9	0.00255	0.00327	0.00329	0.00113	0.00221	0.00111
2	0.5	0.00244	0.00217	0.00211	0.00144	0.00104	0.00163
	0.6	0.00232	0.00233	0.00233	-0.00206	0.00281	-0.00217
	0.7	0.00339	0.00372	0.00376	-0.00414	0.00531	-0.00435
	0.8	0.00292	0.00341	0.00351	-0.00633	0.00523	-0.00642
	0.9	0.00244	0.00310	0.00318	-0.00542	0.00380	-0.00543
4	0.5	0.00431	0.00322	0.00321	0.00346	0.00164	0.00206
	0.6	0.00442	0.00340	0.00353	-0.00127	0.00488	-0.00309
	0.7	0.00544	0.00457	0.00480	-0.00474	0.00813	-0.00662
	0.8	0.00447	0.00374	0.00403	-0.00826	0.00786	-0.01028
	0.9	0.00309	0.00274	0.00301	-0.00876	0.00521	-0.01054

Table S2: The empirical bias of different AUC estimators under DPS models for normal subpopulation distributions with $(n, m) = (15, 5)$ and imperfect ranking ($\rho = 0.5$)

c	A	$\hat{A}_{st}$	$\hat{A}_{sp}$	$\hat{A}_{iso}$	$\hat{A}_{ml}$	$\hat{A}_{t,ml}$	$\hat{A}_{s,ml}$
0.5	0.5	0.00709	0.00786	0.00791	0.00566	0.00729	0.00705
	0.6	0.00677	0.00759	0.00762	-0.00083	0.00182	0.00031
	0.7	0.00596	0.00666	0.00660	-0.00707	-0.00355	-0.00617
	0.8	0.00513	0.00557	0.00557	-0.01092	-0.00693	-0.01028
	0.9	0.00291	0.00323	0.00325	-0.01145	-0.00803	-0.01110
1	0.5	0.00354	0.00318	0.00345	0.00236	0.00280	0.00238
	0.6	0.00401	0.00361	0.00395	-0.00352	-0.00078	-0.00377
	0.7	0.00417	0.00416	0.00433	-0.00933	-0.00451	-0.00969
	0.8	0.00435	0.00432	0.00448	-0.01228	-0.00680	-0.01310
	0.9	0.00322	0.00328	0.00339	-0.01186	-0.00668	-0.01266
2	0.5	-0.00398	-0.00384	-0.00384	-0.00510	-0.00433	-0.00468
	0.6	-0.00504	-0.00478	-0.00469	-0.01333	-0.00808	-0.01337
	0.7	-0.00457	-0.00422	-0.00413	-0.01949	-0.00968	-0.01977
	0.8	-0.00307	-0.00268	-0.00255	-0.02273	-0.00994	-0.02303
	0.9	-0.00339	-0.00305	-0.00285	-0.02178	-0.00958	-0.02204
4	0.5	0.00042	-0.00012	-0.00029	-0.00059	-0.00318	-0.00245
	0.6	0.00055	0.00009	0.00007	-0.00855	-0.00570	-0.01109
	0.7	0.00045	-0.00008	-0.00005	-0.01547	-0.00773	-0.01874
	0.8	0.00052	0.00007	0.00018	-0.01936	-0.00832	-0.02306
	0.9	0.00139	0.00115	0.00132	-0.01729	-0.00637	-0.02037