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NO-SHOW PARADOX IN SLOVAK PARTY-LIST PROPORTIONAL SYSTEM

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Abstract: The phenomenon of the paradoxes of the largest remainders methods has been studied by numerous authors. Nevertheless, the examples presented in their studies do not deal with the case where a party's possible additional votes can directly lead to a loss in the party's number of representatives. This paradox, which can be called the no-show apportionment paradox, has not previously been mentioned in the literature. It is based on the assumption that a voter's favourite party may lose a seat if the voter votes honestly but get more seats if he or she abstains. The article gives simple examples illustrating the no-show paradox in a party-list proportional representation electoral system using the quota method of apportionment. In these situations it would be better for the party if some of their voters abstained, otherwise it would lose one or more seats.

Key words: no-show paradox; negative vote weight; party-list proportional representation; apportionment paradoxes; voting paradox.

Introduction

The voter generally assumes that a preferred party will be more likely to gain seats if the voter votes for it and, of course, less likely if that voter abstains. The voter operates under the assumption that it is impossible to do any harm if he or she votes for a specific party. P. C. Fisburn and S. J. Brams (1983) exposed a phenomenon known as the "no-show paradox", showing that in some situations it was rational for a voter to withhold his vote to achieve a better electoral outcome for the preferred alternative.

It is well known that the largest remainders methods are non-monotone. The largest remainders methods can cause voting paradoxes known as the population paradox, Alabama paradox or new state paradox (see the seminal work on apportionment by Balinski & Young 2001). However, in the apportionment literature relating to the non-monotonicity problem of the largest remainders methods, there is no mention of the possibility that a voter can directly hurt his or her favourite political party depending on how he or she votes. In the article it will be shown that the Slovak party-list proportional representation system is not immune to the no-show paradox.

Under normal circumstances, we would expect a voting system to satisfy the straightforward criterion that giving a vote to any candidate will not make the candidate lose.

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It is well-known (see e.g. Doron & Kronick, 1977; Brams & Fishburn, 1984 or Nurmi, 1999, 2004), however, that many two-round voting systems do not satisfy this condition. The no-show paradox has been studied by numerous authors, e.g. H. Moulin (1988), H. Nurmi (1999), D. Lepelley and V. Merlin (2001), and J. Pérez (2001). The examples presented in their studies use Condorcet voting methods (see Moulin, 1988) or scoring run-off methods (see Nurmi, 1999; Saari, 1994). The no-show paradox arises when one or more voters are better off not voting than casting a sincere ballot. That is why the paradox is also known as the abstention paradox. The strong version of the no-show paradox occurs when the voter's most favoured political party wins more seats if he or she does not participate in the election. If he or she does participate and vote sincerely for the favoured party, the party may lose one or more seats. H. Nurmi (1999, p. 53) emphasizes that "it is not possible that the candidate ranked first by the abstainers would be elected when they abstain and not be elected when they vote". The no-show paradox is based on hypothetical reasoning, the same is true about the additional support paradoxes. The no-show paradox (using Felsenthal's terminology) is a conditional paradox, "where changing one relevant datum while holding constant all other relevant data leads to a 'surprising' and arguably undesirable outcome" (Felsenthal, 2010, p. 3)[1].

In the next section it will be shown that it is not only two-round voting systems that are vulnerable to the additional support, or monotonicity, paradox; party-list proportional representation systems (such as that in Slovakia) that use quota methods (the largest remainders methods) to allocate mandates to the parties are vulnerable to these paradoxes too.

No-show apportionment paradox

The effect of negative voting weight may lead to a situation where a vote can have the opposite effect to what the voter intended. Under certain circumstances it can be seen to be beneficial for a voter not to cast his or her vote for the preferred party or, from the point of view of the political party, the party might gain an extra seat by not receiving so many votes. It is possible to conclude that it would be better for that political party to give away some of its votes and thereby gain a seat. The no-show paradox can also be illustrated by an example in which one voter can change the voting results in a paradoxical way.

The allocation method used in Slovakia is defined in section 68 of Act No. 180/2014, Coll. on the conditions for the execution of voting rights and on amendments and supplements to certain laws as follows:

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[1] The no-show paradox should not be confused with the paradox of non-voting. The paradox of non-voting is generally (see e.g. Owen & Grofman, 1984) explained as follows: "...in a general election, in which many citizens vote, the probability that a single voter can affect the outcome is so small that in general, citizens have no rational reason for voting. However, if all citizens accept this reasoning, then none will vote, and so each vote has a large probability of affecting the outcome. Hence all should vote after all" (Owen & Grofman, 1984, p. 311). On the other hand, as pointed out by H. Nurmi, "[t]he no-show paradox belongs to a family of related counterintuitive features in some voting systems which have in common the possibility that in certain preference profiles the act of doing what one is supposed to do in the election, *viz.* to indicate one's opinion with regard to the candidates, undermines the reason for the act. In the no-show paradox by not voting one gets a result that is preferred to the one ensuing from one's voting *ceteris paribus*" (Nurmi, 1999, p. 55).

(1) The sum of valid votes cast for the proceeding political parties or coalitions shall be divided by the number 151 (the number of seats plus one). The result of this division rounded off to a whole number is the republic electoral number.
(2) The sum of valid votes obtained by a political party or coalition shall be divided by the republic electoral number; a political party or coalition shall be allocated a number of seats equal to the number of times the republic electoral number divides into the sum of valid votes which it obtained.
(3) In the event that there is allocated one seat more than should have been, the surplus seat shall be deducted from the political party or coalition which has the smallest remainder from the division. If remainders are equal, the seat shall be deducted from the political party or coalition which obtained the fewer number of votes. If the numbers of votes are equal, the deduction shall be decided by drawing lots.
(4) In the event that not all seats are allocated or that a political party or coalition should be allocated more seats than it has candidates, then the Central Electoral Commission shall allocate such seats among other political parties or coalitions in order of their remainder from the division beginning with the highest. If remainders are equal, the seat shall be allocated to the political party or coalition which obtained the larger number of votes. If the numbers of votes are the equal, the allocation shall be decided by drawing lots. [2]

The quota (the number of votes required for a seat) is calculated using the formula Q = [V/(S+1)+0.5].[3] The number of votes for each qualifying party is then divided by the electoral quota. The result for each party usually consists of an integer part plus a fractional remainder. In the first part of the allocation process each party receives the number of seats indicated by the integer. This process almost always leaves some seats unallocated. Unallocated seats are distributed according to the largest remainder methods, i.e. the parties are ranked on the basis of the fractional remainders, and the first unallocated seat goes to the party with the highest remainder. If there are any more unallocated seats, the second one goes to the party with the second highest remainder and so on. If the remainders of two or more parties are the same, the unallocated seat goes to the party with more votes. If two or more parties obtained the same number of votes, the deadlock is decided by a lot. In other words, the allocation of seats is generally performed in two rounds. In the first round, each party obtains one seat for each whole number produced—i.e. each party *i* receives $x_i = [v_i/Q]$. After the first round of seat allocation, if there are any unallocated seats, the remainders of the parties are compared and the parties with the largest remainders are allocated the remaining seats. The remainder of party *i* is calculated by the formula $r_i = v_i - x_i Q$.

In the examples presented in Tables 1 and 2, the quota is calculated using the formula Q = [V/(S+1)+0.5]. In Table 1, party A gets the majority of seats (76). Adding one more vote to

[2] The same method is also used to allocate seats in the Slovak elections to the European Union Parliament (there are 13 Slovak MEPs in the European Union Parliament, so the quota is calculated by taking the total number of valid votes polled via qualifying lists and dividing it by 14) (see Section 94, Act No. 180/2014 Coll).

[3] In the article the following notation is used: *Q* means quota, *V* represents the total number of votes, *S* represents the total number of seats, q_i is the quota of party *i*, v_i is the number of votes obtained by party *i*, s_i is the number of seats assigned to party *i*, x_i is the number of seats assigned to party *i* in first round of allocation and r_i represents the remainder of the party *i*. Square brackets [] denote the floor function, which rounds the real number down to the next integer.

party A causes party A to lose its overall majority (Table 2)[4]. One vote for party A thus has a negative impact on the voting results.

Table 1

I	v_i	$q_i = v_i/Q$	$[q_i]$	Remainder (r_i)	Additional seat (ads_i)	$\sum s_i$
A	683 622	76.0002	76	0.0002		76
B	269 844	29.9993	29	0.9993		29
C	162 000	18.0100	18	0.0100		18
D	89 945	9.9994	9	0.9994	1	10
E	80 950	8.9994	8	0.9994	1	9
F	71 959	7.9998	7	0.9998	1	8
Total	1 358 320	Q = 8 995	147		3	150

Table 2

i	v_i	$q_i = v_i/Q$	$[q_i]$	Remainder (r_i)	Additional seat (ads_i)	$\sum s_i$
A	683 623	75.9918	75	0.9918		75
B	269 844	29.9959	29	0.9959	1	30
C	162 000	18.0080	18	0.0080		18
D	89 945	9.9983	9	0.9983	1	10
E	80 950	8.9984	8	0.9984	1	9
F	71 959	7.9989	7	0.9989	1	8
Total	1 358 321	Q = 8 996	146		4	150

Balinski and Young point out that

> if one party increases its vote total relative to another in two successive elections it is inconceivable that the first should lose seats to the second…. This is particularly important in PR systems since otherwise it would be possible for a party to misrepresent part of its vote by giving it to a rival party and thereby gain seats (Balinski & Young, 2001, p. 89).

In the paradox described above, however, party A would still lose one seat, even when one (additional) vote is given to any other party.

The no-show paradox as discussed by P. C. Fishburn and S. J. Brams (1983), H. Nurmi (1999), and D. Lepelley and V. Merlin (2001) hinges on the existence of a third alternative. In the following examples, it will be shown that the no-show paradox can also be observed in a party-list proportional representation system where there are two parties. This can be done using a formula such as Q = [V/S], Q = [V/S+0.5], Q = [V/(S+1)+0.5] or Q = [V/(S+1)+1]. For example, if party A receives 100 200 votes and party B 500 votes, the quota is Q =

[4] In Table 1: Q = [V/(S+1)+0.5] = [1 358 320/151+0.5] = 8 995. In Table 2: Q = [V/(S+1)+0.5]= [1 358 321/151+0.5] = 8 996.

[V/S] = [100 700/400] = [251.75] = 251. Party A gets 399 seats and party B one seat. In the new scenario, party A receives 7 000 more votes (107 200 votes). The quota is Q = [V/S] = [107 700/400] = [269.25] = 269. Paradoxically, in the new scenario, party A gets mores votes but fewer seats (cf. Tables 3 and 4).

Table 3

i	v_i	$q_i = v_i/Q$	$[q_i]$	Remainder (r_i)	Additional seat	Σs_i
A	100 200	399.203	399	0.203	0	399
B	500	1.992	1	0.992	0	1
	100 700	Q = 251	400		0	400

Table 4

i	v_i	$q_i = v_i/Q$	$[q_i]$	Remainder (r_i)	Additional seat	Σs_i
A	107 200	398.513	398	0.513	0	398
B	500	1.858	1	0.858	1	2
	107 700	Q = 269	399		1	400

The no-show paradox can also be illustrated using similar examples for other quota formulae. For example, if we use quota Q = [V/S+0.5], the no-show paradox occurs when party A receives 101 189 votes and party B receives 210 votes. The quota is calculated using the formula Q = [V/S+0.5] = [101 399/400+0.5] = [253.9975] = 253. Party A gets all the seats. The no-show paradox can be seen in the new scenario—party A receives 65 000 more votes. In that scenario the quota changes: Q = [V/S+0.5] = [166 399/400+0.5] = [416.4975] = 416, and party A gets only 399 seats and party B one seat (cf. Tables 5 and 6).

Table 5

i	v_i	$q_i = v_i/Q$	$[q_i]$	Remainder (r_i)	Additional seat (ads_i)	Σs_i
A	101 189	399.956	399	0.956	1	400
B	210	0.830	0	0.830	0	0
	101 399	Q = 253	399		1	400

Table 6

i	v_i	$q_i = v_i/Q$	$[q_i]$	Remainder (r_i)	Additional seat (ads_i)	Σs_i
A	166 189	399.492	399	0.492	0	399
B	210	0.504	0	0.504	1	1
	166 399	Q = 416	399		1	400

Conclusions

Although not everyone agrees that non-monotonicity is a huge defect in electoral systems (see e. g. Austen-Smith & Banks, 1991 or Tideman, 2006), it can be concluded (at least from a moral point of view) that there is no place for largest remainders methods in party-list proportional representation systems. The electoral law should always protect the voters' decisions, and the impact of their votes should always be positive. In this context H. Nurmi points out that

> vulnerability to the no-show paradox is a serious drawback in a voting system. After all, any reasonable voter would expect that by voting he is contributing to the possibility that his favourite wins. The realization that the very act of communicating his true preferences by voting makes the outcome worse from his point of view than it would have been had he decided not to vote all, may be demoralizing (Nurmi, 1999, p. 53).

Similar conclusions were drawn by the Federal Constitutional Court of Germany (2 BvC 1/07. 2 BvC 7/07 – published 3 July 2008) in a judgment stating

> the effect of negative voting weight violates the principles of the equality and directness of elections. The provision is therefore unconstitutional insofar as the effect of negative voting weight is facilitated thereby ... The equal contribution towards success requires the contribution of each vote towards success to be equal, whatever party it was cast for. This also means that it must be able to have a positive effect for the party for which it was cast. An electoral system which is designed or at least in typical constellations permits an increase in the number of votes to lead to a loss of mandates or the electoral proposal of a party to achieve more total mandates if it itself attracts fewer votes, ... , leads to arbitrary results and to a situation in which democratic competition for the approval of the electorate appears to be paradoxical (Press release no. 68/2008 of 3 July 2008)[5].

The electoral system should prevent even the slightest possibility of the occurrence of the no-show paradox. More support should always help a party. Abstention should not be a rational choice for a voter in a party-list proportional representation system. Methods of highest averages (divisor methods) are monotonic, so they are also immune to voting paradoxes. From that point of view, there is no rational reason for using largest remainders methods in party-list proportional representation.

References

Austen-Smith, D., & Banks, J. (1991). Monotonicity in electoral systems. *American Political Science Review, 85*(2), 531-537.

Balinski, M. L., & Young, H. P. (2001). *Fair representation. Meeting the ideal of one man, one vote*. Washington, D. C.: Brookings.

Brams, S. J., & Fishburn, P. C. (1984). Some logical defects of the single transferable vote. In A. Lijphart & B. Grofman (Eds.), *Choosing an electoral system: Issues and alternatives* (pp. 147-151). New York: Praeger.

[5] For further information on the problem of negative vote weight in Germany see Pukelsheim (2014).

Doron, G., & Kronick, R. (1977). Single transferable vote: An example of a perverse social choice function. *American Journal of Political Science, 21*(2), 303-311.

Felsenthal, D. S. (2010). *Review of paradoxes afflicting various voting procedures where one out of m candidates (m ≥ 2) must be elected*. Paper presented at the Leverhulme Trust sponsored 2010 Voting Power in Practice workshop held at Chateau du Baffy, Normandy, France, 30 July – 2 August, 2010.

Fishburn, P. C., & Brams, S. J. (1983). Paradoxes of preferential voting. *Mathematics Magazine, 56*(4), 207-214.

Lepelley, D., & Merlin, V. (2001). Scoring run-off paradoxes for variable electorates. *Economic Theory, 17*(1), 53-80.

Moulin, H. (1988). Condorcet's principle implies the no show paradox. *Journal of Economic Theory, 45*(1), 53- 64.

Nurmi, H. (1987). *Comparing voting systems*. Dordrecht: D. Reidel.

Nurmi, H. (1999). *Voting paradoxes and how to deal with them*. Berlin – Heidelberg – New York: Springer.

Nurmi, H. (2004). Monotonicity and its cognates in the theory of choice. *Public Choice, 121*(1-2), 25-49.

Owen, G., & Grofman, B. (1984). To vote or not to vote: The paradox of nonvoting. *Public Choice, 42*, 311-325.

Pérez, J. (2001). The strong no show paradoxes are a common flaw in Condorcet voting correspondences. *Social Choice and Welfare, 18*(3), 601- 616.

Pukelsheim, F. (2014). *Proportional representation. Apportionment methods and their applications*. Dordrecht: Springer.

Saari, D. G. (1994). *Geometry of voting*. Berlin – Heidelberg – New York: Springer.

Tideman, N. (2006). *Collective decisions and voting: The potential for public choice*. Aldershot: Ashgate.

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