MATHEMATICS AND THE HISTORY OF RELIGION*

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The aim of the paper is to describe some common patterns in the development of religion and of mathematics. We consider religion to be the place, where a culture establishes its contact with the transcendent reality. Nevertheless the objects of mathematics such as numbers or geometrical figures also transcend the physical world. This means that mathematics is also based on transcendence of physical reality. We try to show that this common basis, namely the transcendence of the physical world, gives rise to some common patterns in the development of religion and of mathematics. In our opinion, religion creates the basic means and ways of transcendence, which are used in the whole culture, including mathematics. So the common patterns in the development of religion and mathematics are not accidental. They belong to the very nature of these subjects.

It may seem unusual to connect mathematics and the history of religion. If somebody wants to study the cultural anchorage of mathematics, s/he usually chooses the relationship of mathematics to music, or to visual arts and analyses the parallels between a musical composition and the structure of a mathematical theory, or between the presentation of space in painting and in mathematics. Religion is mostly not paralleled with mathematics because a principal difference is evident at first sight and this complicates similar parallels. Religion is based on faith, which can rarely be justified by the believer who is often not able even to explain it. By contrast, mathematics is knowledge justified on a rational basis, which is strictly proved by deduction, the faith being thus irrelevant to its acceptance.

In spite of these differences, there are certain points of contact between religion and mathematics. One of them is the Pythagorean School, which stands at the beginnings of what is currently named mathematics. Pythagoras is the author of the term mathematics itself (it originally meant knowledge, teaching, science, art) with

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the meaning as used till today, namely as a denotation of arithmetic, geometry, and other exact disciplines together. From the point of view of mathematics the Pythagoreans represent the beginning of this discipline. Pythagoras was the one to discover the idea of a proof. We shall speak more about proofs later, now we only say that mathematicians regard Pythagoras as the founder of their science. On the other hand, Pythagoras is also a great figure from the perspective of the development of religion. The character of the Pythagorean School was that of a religious sect with strict moral regulations, believing in reincarnation and dealing with esoteric disciplines. Pythagoreanism has its origin in the Orphic cult, and therefore to understand the Pythagoreans, it is necessary to follow the religious practices of his adherents (see e.g. Cornford 1912, pp. 194-214, Russell 1946, pp. 48-56 or Gorman 1979). Pythagoreanism is of course not the only point of contact between mathematics and the sacred. There is another teaching, where we can find relations between mathematics and religion, namely Kabbalah. Kabbalah is a Jewish doctrine seeking the numbers hidden in the sacred Torah trying to uncover the concealed message of the Scripture by various mathematical calculations. Similarly, in some sacred literature of the ancient Indians we can find different kinds of mathematical knowledge. But we do not have to go to the remote Orient. The builders of Gothic cathedrals possessed in all probability an extensive amount of knowledge, some of which had a mathematical nature.

These examples open a possible route of investigating the relationship between mathematics and religion. This might be denoted as **hidden mathematics**, the mathematics that we find in sacred texts or sacral buildings. It is the mathematical knowledge concealed in the Torah, the Veda and the Koran or in the pyramids, temples and cathedrals. Priests in particular cultures had an extensive amount of knowledge, which we today regard as mathematical knowledge and which they concealed in sacred texts or sacral buildings. Uncovering the hidden mathematics is a very interesting and intellectually exciting undertaking. This is, however, not the route we want set out on. We are not much attracted by this hidden mathematics, because although we can learn some interesting things about particular religions, we do not learn much about mathematics. The knowledge hidden in sacred texts or coded in sacral buildings is very simple and is mostly of only marginal importance from the point of view of mathematics itself. In no case can we expect that a differential equation or integrals, which are the simplest objects of higher mathematics, might be coded in some sacred texts. If we want to explore the relationship between the development of mathematics and the history of religion, we want the part of mathematics which creates a parallel to religion, to be significant also from the point of view of mathematics itself. Therefore we have to leave the study of the hidden mathematics.

The second route, which opens up for the investigation of the relationship between mathematics and religion, is the exploration of the process of **mathematical creation**. We approach here the very centre of mathematics, the milestones in its history. The parallel between mathematics and religion is in this case based on the

fact that many renowned mathematicians describe the process of mathematical creation as a contact with a mystery, a contact with something that transcends humanity. The character of mathematical creation is that of transcendence of the known, and this transcendence links mathematics with religion. Thus the second route which opens up in the exploration of the relationship between mathematics and religion consists in a search for religious aspects of mathematics itself. That means the point is not to uncover mathematical aspects of sacred texts or buildings but the contrary: it is a search for the religious aspects of mathematical creation. The point is to analyse such moments, when prominent mathematicians face the radical crossing of the boundaries of knowledge and when they face the mystery directly. Many of them, for example Albert Einstein, describe this moment as a moment of contact with the higher harmony of the universe and often speak about this experience directly in the religious terminology (see Einstein 1931, pp. 5-8).

In his lecture *"On some tendencies in the development of mathematics"*, Igor Shafarevitch, the outstanding Russian mathematician, pointed to a similar aspect in connection with the discovery of non-Euclidean geometry: *"After Lobachevski and Bolyai laid the foundation of non-Euclidean geometry independently of one another, it became known that two other men, Gauss and Schweikart, also working independently, had both come to the same results ten years before. One is overwhelmed by a curious feeling when one sees the same designs as if drawn by a single hand in the work done by four scientists quite independently of one another."* (Davis and Hersh 1981, p. 53) Shafarevitch does not speak about any marginal current of mathematics. He describes one of the central events in the development of mathematics in the nineteenth century. He thus shows that in the very centre of the main stream of mathematics we face the phenomena which, according to him, point to the higher aim and deeper mission of mathematics. Shafarevitch explicitly calls this deeper sense of mathematics *"a supreme religious goal ...of the spiritual activity of mankind"*.

The second route is, from our viewpoint more interesting than the first route, and yet we shall not follow it. This is because of the fact that the area of mathematical creativity is not accessible to scientific exploration. We would be dependent on the testimonies of scientists, who are not able to describe their experiences and to support them with arguments. Therefore we would just have to rely on them. What they tell us is important and interesting, but it can hardly become the subject of a serious scientific analysis.

We are looking for a sort of the middle road which would, in contrast to the investigation of hidden mathematics, lean on the mathematics of the main current, and, in contrast to the analysis of evanescent moments of creation, it would be based on the analysis of their results. This middle route as is common with middle ways, is not so exciting as the two extremes between which it wanders. Its object of study is substantially more prosaic than the secret knowledge coded in sacred books and the analysis of mathematical works is, on the other hand, much less exciting than the investigation of the personal testimonies or confessions of great cre-

ators. The basic advantage offered by the middle route is that we can get a little further by following it, than by following the two extremes. Sooner or later, the analysis of hidden mathematics will certainly get involved in a network of contradictory interpretations, whereas, after some time, the exploration of the statements of great mathematicians will have to face the problem of trustworthiness. By its keeping to mathematical texts of the main stream, the middle road always has their standard interpretation at hand, which is verified by the mathematical community and, as such, is consistent and reliable with high probability. We are sure that we do not need either sacred texts or mysterious moments of creation in order to reveal connections between the development of mathematics and the history of religion. What we want to show is that the contact between mathematics and religion takes place in the main stream of mathematics and on a daily base.

1. RUPTURES IN THE HISTORY OF MATHEMATICS AND RELIGION

We have dealt with the systematic analysis of ruptures in the history of mathematics at greater length (see Kvasz 1998, Kvasz 1999a, Kvasz 1999b, and Kvasz 2000). We have found that particular ruptures have their parallels in various areas of culture. For instance, the discovery of non-Euclidean geometry is closely associated with the philosophy of German romanticism (see Scholz 1982 and Lewis 1977). Just as the romantic philosophers emphasized the role of subject in creating the picture of the world, mathematicians based the possibility of the creation of new non-Euclidean geometry on this creative force. When a subject is not a mere receptor of reality, it is not merely an empty notice board, on which the pictures of reality are pressed but the other way round – it is an active co-creator of these pictures, then it can also create a new geometry. A new, idealistic conception of the subject is thus concealed in the background of the discovery of the new geometry.

Different ruptures in the history of mathematics appear to spring from different depths of culture (see Kvasz 1999). Some, for example the discovery of non-Euclidean geometry, are linked with the parallel rebuilding of the conceptual apparatus used for the interpretation of reality. The roots of this rupture touch philosophy. Other ruptures go even deeper. It is not enough to re-build the conceptual apparatus but the overall view of the world has to be changed. **The history of mathematics can thus serve as a seismograph for precise recording of the ruptures in culture.** Some changes in the history of mathematics accompany parallel changes in conceptual structures. Others accompany changes in the disclosedness of the world. In our classification of ruptures in the history of mathematics there are, however, two ruptures we fail to reduce either to a change in philosophy or to a change in art. These are the Pythagorean discovery of the proof and the Galilean discovery of the principle of inertia. We think that in these cases we are concerned with ruptures in the development of mathematics that intervene in the still deeper layers of culture and their parallel are changes in religion. The analysis of ruptures in the history of

the exact sciences and efforts to anchor the ruptures within a wider cultural context shows two ruptures intervening as deeply as the layer of culture designated as religion. Now we shall describe these ruptures in more detail.

1.1. The Pythagorean rupture and the birth of proof

Pythagorean rupture is associated with the discovery of proof. Pythagoras was probably the first to realize that proof can help us to reach absolutely certain knowledge. The discovery of proof represents one of the deepest ruptures in the history of western thought. To be able to understand better the character of this rupture, we shall give one typical mathematical text on each side of this rupture.

The first is the example of the Moscow papyrus, which is a collection of mathematical tasks from ancient Egypt: *"Find the volume of a cylindrical granary of diameter 9 and height 10. Take away 1/9 of 9, namely 1, the remainder is 8. Multiply 8 times 8, it makes 64. Multiply 64 times 10, it makes 640 cubed cubits. Add 1/2 of it to it, it makes 960, its contents is khar. Take 1/20 of 960, namely 48. 4800 hekat of grain will go into it. "* (Fauvel and Gray 1987, p. 17)

As the second text, we take a theorem from Euclid's Elements: *"If in a triangle two angles be equal to one another, the sides which subtend the equal angles will also be equal to one another. Let ABC be a triangle having the angle ABC equal to the angle ACB, I say that the side AB is also equal to the side AC. For, if AB is unequal to AC, one of them is greater. Let AB be greater, and from AB the greater let DB be cut off equal to AC the less, let DC be joined. Then since DB is equal to AC, and BC is common, the two sides DB, BC are equal to the two sides AC, CB respectively, and the angle DBC is equal to the angle ACB. Therefore the base DC is equal to the base AB, and the triangle DBC will be equal to the triangle ACB, the less to the greater: which is absurd. Therefore AB is not unequal to AC, it is therefore equal to it."* (Euclid 1925, p. 255)

These two texts are entirely different in their character. The task from the papyrus reminds us of a cook's recipe rather than a mathematical text. It is an instruction what one should do to obtain the result. It is thereby not clear at all whether this procedure leads to a correct result, and, if so, why. We think that both aspects of Egyptian mathematics, both the character of the concrete instruction what to do and the non-clarity of why it is actually functioning, if at all, strongly remind of **ceremonies**. (Under ceremony we understand a physical activity with a particular symbolic content through which the performer of the ceremony turns to the transcendental.) The action of the Egyptian mathematician is, in our opinion, of ceremonial character. The contact with the mathematical truth is not given by arguments, but it is mediated through a set of concrete "manual" operations.

Euclidean proof is of a principally different character. In contrast to the Egyptian mathematician, Euclid does not want us to do something particular. He does not require any activity through which we would achieve something. He turns to us straightaway and wants us to look into something, to understand something. He

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wants to persuade us that the theorem is true. The proof has the character of argumentation, it is a direct talk with an imaginary partner. It reminds us of a **prayer**. (Prayer is understood as a verbal communication with the transcendental, through which the performer contacts directly, without any mediation the transcendental.) The proof reminds us of a prayer because, like prayer, proof is also verbal and nonmanipulative. In addition, proof is, similarly as prayer, based on direct, immediate communication.

We shall try to explain the transition from Egyptian to Greek mathematics as a transition from ceremony to prayer. The question why we use religious terms to express this difference between instruction and persuasion will be answered later. Now we only want to grasp in a preliminary way the similarities between what the Egyptian mathematician did when calculating the volume of the pyramid and ceremonies. Similarly, we want to fix a certain affinity between proof and prayer. Naturally, every analogy is a question rather than explanation; therefore we shall have to return to this point and to elucidate the basis of the preliminary analogy in detail. But first we give a brief account of the second rupture mentioned above.

1.2. The Galilean rupture and the birth of the experimental method

As an illustration of the second rupture we will use the different descriptions of planetary motion. If we wanted to compare the most typical pre-Galilean and post-Galilean planetary system, we would probably reach for the works of Claudius Ptolemy and Pierre Simon Laplace. It is not appropriate for our aims to select these typical examples because they are very remote and thus separated by a number of other changes – and not only the one we want to illustrate. Consequently, the reader would hardly know which aspects of the difference between the Ptolemaic and the Laplacean theory are a consequence of the Galilean rupture and which belong to other changes. Therefore we prefer to use the theories of pre-Galilean and post-Galilean science, which are as close as possible in time in order to minimize the danger of interference from any additional rupture. We decided to take the theories of planetary motion of Kepler and Newton.

Kepler was the successor to Tycho de Brahe in his post as astronomer at the court of Rudolph II. He inherited from his predecessor tables containing the then most precise observations of the position of planets. As a Platonist, he believed that behind the intricate set of numbers there were simple regularities. He set to work and began to plot the positions of Mars point by point. The observations were done from the Earth and therefore it was necessary to subtract from the apparent motion of Mars the part caused by the motion of the Earth and to plot the motion of Mars with respect to the Sun. He thought at first that he would obtain a nice circle, but gradually, the deviations from the circular orbit were more and more evident. Kepler came to the conclusion that the shape of the path is not circular but elliptical.

This discovery by Kepler is undoubtedly significant and Newton appreciated it appropriately. He realized, however, that Kepler was very lucky. De Brahe's tables

were quite precise to differentiate circle from ellipse but, at the same time, they were not too precise and thus the effect of Jupiter and of other planets on the orbit of Mars was not manifested there. This is why Kepler could argue with a clear conscience that Mars moves along the ellipse. In fact, the path of Mars does not have the shape of an ellipse. It has no shape at all. The more precise would be its plotting, the more complicated the effects would be shown, and the greater would be the deviations from the ellipse but also from any shape given in advance. There are no such regularities in nature, which give certain invariable forms or proportions. The path of Mars does not follow any particular prescribed pattern determining its shape. It is not the shape of the trajectory that is regular, but the way in which the trajectory is generated. Under certain simplifying assumptions the trajectory will have the shape of ellipse. But this is only an approximation to the reality and not the reality itself. It is not the invariable forms that should be sought behind phenomena, but the dynamic laws.

What is the difference between these two views? It consists in the fact that the order, which was sought in the planetary system by Kepler, prescribes a characteristic pattern or nature of behaviour to each individual planet, and the system is just a juxtaposition of such individual patterns. Such a way of interpretation of the world has its analogy in **polytheism**, which understands the world as a place where different, often contradictory tendencies represented by particular deities meet. On the other hand, Newton's interpretation of the planetary system is based on a universally valid law, to which all phenomena are subjected, from the stars to the depths of the Earth. This law does not describe the form of the trajectory, but the way of its creation. This reminds us of **monotheism**, where the world is understood as subjected to a universal Law. As Prigogine and Stengers say, Newton was compared in the eighteenth century to Moses because, just as the ten commandments were given by God to Moses, Newton was given the laws of mechanics (see Prigogine and Stengers 1984, part I, Chapter 1.1.). That means that in our view **the transition from Kepler's theory of celestial harmony to the Newtonian theory of motion of celestial bodies is paralleled by the transition from polytheism to monotheism.**

Thus parallels were found in the development of religion for both ruptures in the development of the exact sciences, for which no sufficient justification had been found in the development of either philosophy or art. We are facing the task, of interpreting in a systematic way these preliminary analogies and disclose what is behind them. We shall try to find common foundations for the history of religion and the history of mathematics.

2. RELIGION AS A PLACE OF TRANSCENDENCE IN CULTURE

In order to give a systematic account of the above-mentioned analogies, we shall have to shed more light on our understanding of religion in the first place. We shall try to interpret religion as the place where culture touches the transcendental. In

this view, there are two important aspects of religion. It is the **type of transcending activity** (ceremony, prayer, or meditation), through which culture realizes its contact with the transcendental and the **character of the transcendental object** (value, ideal) to which the religious activity turns.

In addition to the establishment of contact with the transcendental, each religion fulfils a number of other social and cultural functions; what follows, is thus the interpretation of only one aspect of religion, although, in our opinion, the central aspect. To avoid any misunderstanding, we shall not speak of religions but only of the **forms of transcendence**. Instead of monotheism or polytheism, we shall thus speak about the monotheistic form of transcendence and the polytheistic form of transcendence. There is another complication that might be a source of misinterpretations: the point is that the existing religions represent very complex systems using simultaneously different forms of transcendence. Thus monotheistic religions often preserve polytheistic forms of transcendence bound to angels, and saints as well as many ceremonies stemming from even older forms of transcendence. It is therefore important to underline that under the form of transcendence of certain religion (e.g. monotheism, or polytheism) we do not understand all possible activities, through which the particular religion establishes contact with the transcendental. We prefer to introduce the concept of the form of transcendence in a purely theoretical way in the following table:

	VALUE	IDEAL
MEDITATION		RHP
PRAYER	PFT	MFT
CEREMONY	TFT	

The table expresses for each of the four basic forms of transcendence the character of the activity through which religion contacts the transcendental as well as the character of the transcendental. Therefore the particular forms of the transcendence are defined as the combination of the particular type of activities and the particular type of the transcendental. In the case of the **Totemistic Form of Transcendence (TFT)** the contact with the transcendental is established through ceremonies, the transcendental having the character of a value. The transition to the **Polytheistic Form of Transcendence (PFT)** consists in the fact that the transcendental is associated with a mind and therefore one can turn to it through prayers.

The table also indicates that the transition from the totemistic form of transcendence to the polytheistic form of transcendence is smooth. On the other hand in the later stages of the totemistic form of transcendence spirits start to emerge, to whom

the members of the community turn with various pleas, with the structure very close to a prayer. On the other hand, in polytheism there are various ceremonies, which are remnants of the totemistic form of transcendence. They are only re-interpreted and adapted to the needs of the new form of transcendence. From the perspective of our interpretation all liturgical ceremonies represent activities belonging to the totemistic form of transcendence.

Transition from the polytheistic form of transcendence to the **Monotheistic Form of Transcendence (MFT)** consists in the change of the character of the transcendental object, to which religion turns. An ideal becomes the content of transcendence instead of a value. For a further form of transcendence, which follows monotheism and occurs in Christianity, Buddhism, or Taoism, we propose to create an independent category, namely **Religion of the Highest Principle (RHP).** Its birth consists in the transition from prayer to meditation as the transcending activity.

If we accept this interpretation of religion, the parallel between the development of mathematics and religion, described in the previous chapter, will become clearer. Mathematics is an activity within which transcendence of reality takes place. Numbers or geometrical bodies are not objects, which can be met commonly. Their creation means the transcendence of reality. If we accept the hypothesis that religion is the place in culture where the method of contact with transcendence is constituted, then, evidently, mathematics merely selects from the possibilities created by religion.

The parallel between Egyptian mathematics and the ceremonies, which has been preliminarily described above, chiefly consists in the analogy of the way of transcendence. The character of the object of transcendence is the same for both cases, it is the static value. The task of a mathematician is to find the unknown quantity, it's particular value. This value transcends the actual knowledge of the mathematician. The process of solving the problem is an activity, the purpose of which it is to find this particular value, that is the value, which transcends the known. So the character of the transcendental object as well as the type of the transcending activity of the Egyptian mathematician belong to the totemistic form of transcendence. Such a calculation resembles ceremony in the sense that it is a set of concrete tasks, which have to be done to obtain the required result. Similarly the parallel between Greek mathematics and the polytheistic form of transcendence lies in the analogy of the form of transcendence. Euclid's argumentation in the proof of the validity of a theorem does not represent any concrete manipulations. The proof is based on the dynamic value, on the continuous balance of the particular arguments, just as the polytheistic form of transcendence is built on continuous actions of individual gods.

Thus we discovered the ground, on which our preliminary analogies between the development of mathematics and religion rested. This ground is the common form of transcendence. Both religion and mathematics go beyond the world of phenomena and refer to something that transcends the physical world surrounding us. There are only a few forms of transcendence (so far we have shown four of them). There-

fore their typology creates a foundation for certain parallels between the development of mathematics and the development of religion. The parallels described above cease to be preliminary analogies based on some external resemblance and they can be subjected to deeper investigation. A possibility emerges for a theory of transcendence, which would explore such parallels in a systematic manner.

2.1. Parallels in the development of ethics

The most interesting aspect of our approach to culture based on the study of the forms of transcendence is the fact that our scheme can also be used for the analysis of the development of ethics. This possibility is opened up thanks to the fact that good as the basic ethical category also does not represent anything actually existing in the physical world but it transcends this world. We obtain four types of ethical systems that are parallel to the four forms of transcendence. The first stage in the development of ethics, parallel to the totemistic form of transcendence can be denoted by the term **role ethics**. If a boy passes through a ceremony of initiation and returns to his village as a man, some forms of behaviour become inadmissible to him. His behaviour is thus based on the imitation of a certain pattern, in the following of prescriptions (similar to the mathematical recipes of the Egyptian mathematician). In an analogous way if a child starts to cry because it is tired, it will be enough to mention that Winettou, Robin Hood or any other of its current heroes would not behave like that. The child is willing to mobilize its strengths to behave like this hero would. In role ethics it is exactly prescribed, what should be done, how a person should behave. Heroes are not discussed they are simply followed. The role ethics is thus based on the static value, the role is exactly given and does not change in the process of action.

The second stage in the development of ethics, parallel to the polytheistic form of transcendence is the so-called **relational ethics**. Here morality or immorality of an act depends on how this act will affect other people. One should not hurt others, not because Winettou did not hurt the weak, but because it causes other people pain. Here the ethical value is not fixed to a certain pattern of behaviour, but is a result of a complex network of social relations. If somebody offended me, I can take revenge on him/her and if my revenge is proportional to the offence, it is not regarded as immoral. Naturally, each society has a slightly different network of human relations and thus also a different measure of such moral evaluation. The moral value is not associated with the act as such. Relational ethics is based on the dynamic value. The moral value of an act constantly changes depending on social circumstances. The same act can be moral under certain circumstances and immoral under others. The individual has to conform his/her actions to the situation in which s/he acts.

The third stage in the development of ethics, parallel to the monotheistic form of transcendence is the so-called **oriented ethics**. This ethics draws its norms from the relation to a certain ideal. The working ethics of Protestantism, which is the ba-

sis of modern capitalist society, also belongs here together with many other cases. From the perspective of this ethics, idleness is immoral. The immorality of idleness does not consist in the fact that we would hurt somebody. Quite the opposite, a man with a lax relation to work usually has more time to spend with his family and thus he is much better from the point of view of relational ethics, than a businessman working 14 hours a day. Oriented ethics does not condemn idleness on the basis of its societal consequences but because it distracts us from the basic direction, from the basic orientation, which is in this particular case to maximize effectiveness and prosperity. Different commitments and resolutions concerning regimen, self-education, and self-improvement, which a teenager sets for him, manifest this type of ethics. Well-being or self-improvement is not the only possibility for the particular ideal. It can equally be charity, help to others, but also nation, race, or classless society. Thus we include the ethos of communist and fascist movements in the category of oriented ethics.

The fourth and last type of ethics parallel to the form of the transcendence of the religion of the highest principle might be called **internal ethics**, that is ethics, which lead us to responsibility to our inner voice. From the point of view of this ethics many activities in the field of work or charity, which seem highly positive from the external point of view, can be manifestations of escape from the self, and thus immoral from the point of view of internal ethics. A businessman or a committed activist in ecological or political movement whose evaluation is highly positive from the perspective of the oriented ethics, can, in their thriving and beneficial activities, run away from their deeper self, and thus betray themselves. Internal ethics binds us to accept our inner voice and be led by it.

No true ethical system is based on only one of these principles, similarly as no true religion is based on only one form of transcendence. In ethics, as in religion, there are combinations in which all four principles meet. In spite of this, one cannot deny that such a typology of ethical principles is possible and that it sheds light on the parallel, which has been revealed between the history of mathematics and the history of religion. The reason for the existence of this parallel between the development of ethics and religion is similar to that in the case of mathematics. Ethics, like mathematics, is based on the transcendence of reality and it thus only selects from the forms of transcendence as supplied by religion.

2.2. The history of Christianity and the forms of transcendence
To elucidate the relation between the forms of transcendence and the really existing religions, we shall try to give a brief outline of the development of Christianity from the point of view of the forms of transcendence. Primarily, Christianity belonged to the religions of the highest principle, that is its object was the ideal as a dynamic quality. It was originally constituted and codified in this shape.

Nations like the Celts, Germans or Slavs, who converted to Christianity, were at the level of polytheism in their religious development. An interesting situation

arose, when elements typical of the polytheistic form of transcendence were gradually built into Christianity. It is primarily the cult of the saints and the saints as patrons of certain societies and activities. In this perspective, the Reformation, with its rejection of the cult of the saints as well as with its emphasis on the moral dimension of Christianity, means a transition to the monotheistic form of transcendence. Christianity finally renovates the level at which it was born, in the ecumenical movement, namely the form of the transcendence of the religion of the highest principle based on the dynamic ideal.
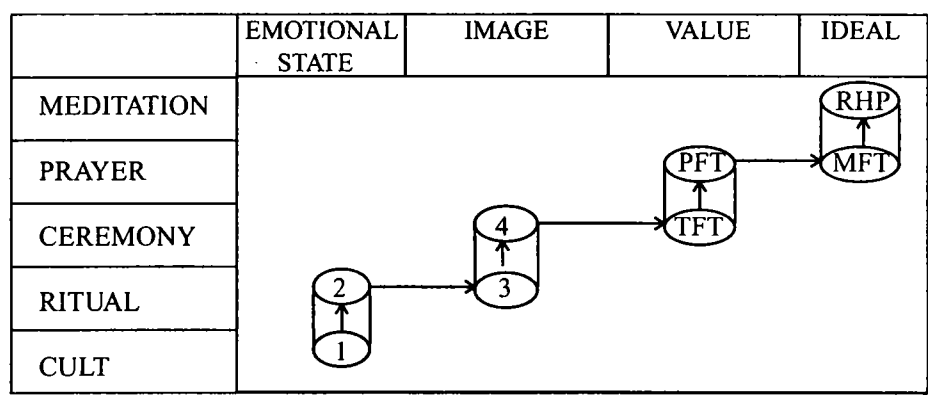
Naturally, this does not mean that there were no Christian thinkers, dignitaries or whole communities in various periods that based Christianity in its original and, at the same time, highest form of transcendence. Our point is the interesting development within Christianity, which is another illustration of the regularities of the development of the forms of transcendence described in our essay. Nations who accepted Christianity, were not able to accept its content. Instead of it, they introduced many aspects parallel to polytheistic and monotheistic forms of transcendence into Christianity and thus, within the Christian Church, they went through the sequence of forms of transcendence, we have described. If this is true, it is another argument that the analogy between the development of religion and the history of mathematics is not accidental, but a manifestation of the regularities of the development of the human mind.

3. RELIGION AS A PLACE OF THE CONSTITUTION OF THE MIND

Now we shall try to express the basic thesis of this essay: **religion is a gate through which the first mental objects of the particular type (i.e. first static values, first dynamic values, ...) enter the mind**. This means that we regard religion as the place where the human mind was constituted. A feeling of transcendence is only how we subjectively experience the process of this self-constitution. It is the particular mental object of the new type, which is transcendental to the mind. As this new object is gradually domesticated in the mind and becomes an ordinary part of its spiritual life, the feeling of transcendence, with which it was originally connected, diminishes, to make room for a new form of transcendence which will bring an object of transcendence of a new type. This means that it is not a coincidence that the history of mathematics and the development of ethics show a parallel with the history of religion. These are not some accidental or exceptional parallels. Quite the opposite, it was precisely religion with its forms of transcendence through which the objects of our mental life were constituted and gradually stabilized in human minds in the line of totemism, polytheism, monotheism, and the religion of the highest principle. At first a static value was born in the mind. It was incorporated in the totemistic form of transcendence. Then, thanks to the polytheistic form of transcendence the static value was turned into a dynamic value system. Then a static ideal emerged in the monotheistic form of transcendence. Finally the

static ideal was turned into the dynamic system of ideals within the form of the transcendence of the religion of the highest principle. Thus the history of mathematics or the development of ethics is naturally periodized by this constitutional process of the human mind. This periodization explains the analogy, which stands at the beginning of our essay.

If we accept the basic thesis of this essay namely **that it is religion through which the mental life of humans was constituted and this process of constitution is subjectively experienced as transcendence,** there emerges a significant consequence. Our spiritual life contains, in addition to values and ideals, images and emotional states. If we take seriously the constitutional role of religion for the mind, there is no other way than to postulate an additional four forms of transcendence, through which emotional states and images have been constituted in the mind. We come to the following scheme:

	EMOTIONAL STATE	IMAGE	VALUE	IDEAL
MEDITATION				RHP
PRAYER			PFT	MFT
CEREMONY		4	TFT	
RITUAL	2	3		
CULT	1			

We do not restrict the concept of **image** only to visual images although this is the most common type of image. Under image we mean rather all that we are able to single out and isolate in our imagination. Thus there are motor images, tactile images, acoustic images, smelling images, etc. Everybody can imagine more or less clearly the smell of violets, a friend's voice, motion of one's hand or stomach-ache. An image is thus everything we are able to separate from ourselves, place mentally in front of us, and analyse. In contrast, an **emotional state** is something, from which we are not distanced. It is something principally unreflected, unidentified, we find ourselves inside of it.

The concepts emotional state, image, value, and ideal denote four basic types of transcendental objects, which can be the content of a religion. Each of these four types exists in two kinds, first in a static kind, and then in a dynamic kind. The notions cult, ritual, ceremony, prayer, and meditation represent the basic ways of contact with the transcendental. Since these concepts are commonly used in the de-

scription of the religious practices and in order to avoid any misunderstanding, we shall try to define their meaning as we use them. Our use is mostly a narrowing of the common meaning of these terms consisting in the fact that **the particular activity is always defined merely in relation to the corresponding object**. That means that, for example, in contrast to its common use, prayer is not regarded as a sort of ceremony because the ceremony is bound to the dynamic image or the static value, whereas prayer is bound to the dynamic value or the static ideal. One can even say that ceremony allowed transition from the dynamic image to the static value, and prayer enables transition from the dynamic value to the static ideal.

It should be underlined that the first four forms of transcendence that we introduce (1, 2, 3, 4 in the table) have not been documented and only their residues in higher types of religion can be explored. This circumstance is not so negative as might seem at first sight, because, fortunately, religion is one of the most conservative components of human culture and thus many of its elements live long even after losing their function. Thus the cult of the sun that we meet in many polytheistic religions could be a remnant of the original form of transcendence of the type 1, with which actually all began. Similarly, various manifestations of fetishism could be the residues of the forms of transcendence, where the transcendental had the form of a static image.

If these hypothetical forms of transcendence could be at least indirectly documented by empirical material, it would mean the confirmation of the central thesis of our essay. According to this thesis all basic objects of human mind, that is emotional states, images, values and ideals were originally constituted within religion, being gradually interiorized, detached from religious activities, becoming, finally, autonomous components of our inner world. After interiorization and separation of the particular objects from religion, a space inside religion opens up for the emergence of the objects of a new type, which will become the objects of transcendence of a new religion. The four forms of transcendence – 1, 2, 3, 4 represent the earliest forms of both religion and the spiritual life of humans. Their exact characteristics are beyond the possibilities of current science; we shall therefore try to give a hypothetical outline of how they could have functioned.

Form of transcendence 1 represents in principle the separation of humans from nature, when humans became able to induce and stabilize a certain emotional state, which was independent of the actual state induced by their environment. Transition from form 1 to **form of transcendence 2** is, as all transitions from the static to the dynamic stage, continuous and consists in a gradual transformation of the cult into a ritual. The ritual, probably having the form of a ritual dance, makes it possible to modulate the emotional state by varying the rhythm and intensity from states of despair up to ecstasy. Different sequences of emotional states are experienced repeatedly, they become stabilized, and thanks to them humans become able to emotionally differentiate their environment because they have a wider spectrum of emotional states available in their minds.

We think that a crucial discovery takes place, consisting in the transition from form 2 to **form of transcendence 3**, the discovery of fire. The discovery of fire consists in the separation of the image of fire from the emotional state of animal fear invoked by fire in all wild animals, and thus with all probability also in our ancestors. This separation of the image of fire from the emotional state of fear, normally induced, was enabled by the fact that thanks to the ritual, humans acquired the ability to induce and maintain an autonomous emotional state and not to succumb to the pressure of the emotional state induced by the environment. The original function of ritual was to stabilize the emotional state. Gradually however, the ritual started to be used not only to stabilize the emotional state, but also to open up the possibilities of transcendence of a new type with a focus on images.

The transition from form 3 to **form of transcendence 4** consists in a gradual change of the ritual into ceremony. The ceremony differs from the ritual by its focus on something symbolic. A ceremony uses the fact, that in stage 3 the ritual induced static images, and the ceremony starts with a dynamization of these images. It turns from one image to another (in a similar way as a ritual was able to turn from one emotional state to another by the change of the rhythm and intensity of the dance). In this way sequences of images were introduced, and especially a very important kind of such sequences, named as work. We usually characterize work through the aim which it serves, that is by the value it brings. This way of grasping activities is, however, relatively late. Work was originally understood as a sequence of motor images, as a dynamic motor shape, similar to when a piano player has been practising a composition and his hands play it themselves. The activity is fixed as a sequence of motions. After each motion, another motion is induced automatically. It is just the ability to follow such a thread that characterizes the ability to use an image as a dynamic mental quality. This dynamic image was the original content of a ceremony. The ceremony brought something new in comparison with the ritual: the new whole made up of single motions. Thus the mind participates in the ceremony not only through executing its parts, but also by controlling its course.

The transition from form of transcendence 4 to the totemic form of transcendence, which is the first form of religion both ethnographically and historically documented, consists again in separation, now in the separation of the activities from their goal. As the ceremony stabilizes the sequence of operations that form the particular complex activity, by frequent repetition, the result of the activity begins to be associated at the very beginning of the activity itself, or even before it is started. In this way the result is mentally separated from the sequence of operations and becomes a value. This enables people to conform the activity to this value, that is to begin to act purposefully. The birth of the purposeful action is an important step forward in the process of human emancipation. This shift is manifested by the birth of the magic force with which the ceremony is associated.

We do not think it useful to describe the further development of religion after reaching the first empirically supported stage. There is enough literature available

and we also think that there are experts whose views would be much more relevant. We do not know whether it will be possible to document our four hypothetical forms of transcendence by ethnographic and historical material. It will not influence the relation between the development of mathematics and the history of religion since mathematics as a solution of certain problems is bound to the (at least static) value and mathematics is thus only possible from the form of transcendence of primal religious systems. Therefore there is no earlier stage of mathematics in the interpretation of which some of the four hypothetical types of religion might play any role.

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