KUHN'S STRUCTURE OF SCIENTIFIC REVOLUTIONS – AND HOW TO CONTINUE*

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Kuhn's book *The Structure of Scientific Revolutions* gave rise to a vivid and lasting controversy among the historians and philosophers of science. One of the reasons for this controversy was the use of rather vague and non-specific concepts, such as paradigm and scientific revolution. The aim of this paper is to offer a way to make Kuhn's concepts more precise and thus the discussions more productive. The author suggests a classification of scientific revolutions into three different kinds, which makes it possible to describe the specific and characteristic structure of scientific revolution for each of the three kinds separately. In the author's opinion, Kuhn's concept of scientific revolution is vague because it is a superposition of three different concepts.

In modern society, science has taken the place which philosophy had in Ancient Greece, religion in the culture of the Middle Ages and art in the Renaissance. Science has become the prototype of legitimacy. If somebody wants to show that his or her views are legitimate, s/he does not have to write a philosophical tract or to prove that they are in accord with the Scriptures. The sign of original inspiration is not necessary either. It is sufficient to show that the views are scientific. The philosophy of science was therefore not only the philosophical reflection of the background and the achievements of science. The philosophy of science became something much more: it became an institution which was able to determine a certain discipline to be scientific and so give its exponents the hallmark of honesty or, the other way round, to declare it non-scientific and give its supporters the stigma of dishonesty. Such a denial afflicted alchemy, astrology, telepathy, which probably deserved it but also psychoanalysis or Darwinism, the scientific level of which is indeed not a simple question. The central position in philosophy of science was taken by physics, which was presented as a model for other disciplines. Therefore psychology, sociology or linguistics began to copy the methods of physics and

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more or less successfully started to obey its norms. Whether it helped them still remains an open question.

Contemporary science is gradually losing its central position in culture. It is too early to decide whether that which perceives itself as postmodernism is a mere transition to a new homogeneous culture, in which a new form of knowledge will take over the central position after philosophy, faith, art, and science or whether postmodernism itself is a new epoch with no privileged kind of knowledge in its centre. Our times are, however, appropriate for looking back at science which is withdrawing from its positions in order to better understand its history. We have the advantage of the ethos of science surviving within the scientific community; but this ethos is not so self-evident any more as a result of changes in contemporary culture and society. This opens up the possibility of keeping a critical distance in preserving the understanding for implicit aspects of knowledge.

One of the most important critical analyses of science in the second half of the twentieth century was undoubtedly the book of Thomas Kuhn *The Structure of Scientific Revolutions* (Kuhn 1962). The number of the copies of the book sold and its place in the citation indexes of the last thirty years serve as evidence. The virtue of the book was that it destroyed the cumulativistic image of science in a decisive manner. The author's aim "to draft an entirely different conception of science that can emerge from the historical record of scientific research" was undoubtedly achieved. Kuhn's conception is today part of the standard courses on the philosophy of science. However, more than 30 years have passed from the appearance of Kuhn's book and we think it is time to take a step forward and to try to better define and develop Kuhn's conception. Where Kuhn outlines a global vision of scientific revolution, a finer typology can be developed today. Where Kuhn puts forward a unique universal scenario, according to which the revolution proceeds, a spectrum of alternative scenarios for individual types of revolutions can be submitted.

1. Some motives for making Kuhn's conception of the development of science more precise

A question may arise why we need to make Kuhn's theory more precise. The level of precision of the argumentation used by Kuhn was sufficient to show what he wanted to show – the inadequacy of the positivist cumulativistic image of science. It can therefore seem that Kuhn's precision of argumentation is sufficient for the purpose of the philosophy of science and an attempt to increase the precision raises the suspicion of purely academic efforts at preciseness. However, this is not the case. Kuhn's conception has been widely criticized as too general and ambiguous. The critics found in *The Structure of Scientific Revolutions* 22 different meanings of the term paradigm. Due to this criticism, Kuhn replaced the notion of paradigm by the concept of disciplinary matrix (Kuhn 1974). That means that some philosophers felt a necessity to make Kuhn's conception more precise and Kuhn himself accepted this criticism. In our opinion, the introduction of the term disciplinary matrix is not a solution to the problem. It provides a more precise explication of the concept of paradigm which will extend and enrich the term by a number of additional aspects, but this extension will merely lead to an explicit articulation of its implicit generality and ambiguity. The concept of disciplinary matrix is no more specific than the original concept of paradigm.

In our opinion, excessive generality and ambiguity of the concept of paradigm is not due to the insufficient explication but rather due to the fact that Kuhn included into the concept of scientific revolution several processes of different character. This fact will not be changed by introducing the term of disciplinary matrix in any way. This is why we think that only after dividing the concept of scientific revolution into several types will a possibility arise of defining a specific type of paradigm for every type of revolution and thus to do away with the generality and ambiguity of Kuhn's notion. Kuhn's concept of paradigm is general and ambiguous for a simple reason – it is a superposition of several concepts. We think that it is possible to differentiate three types of revolutions – idealization, re-presentation, and objectivization (see Kvasz 1998a) and thus there are three different types of paradigms. This means that Kuhn's 22 different meanings of the term paradigm will be divided into three groups of approximately seven. Seven is still quite a lot but it is much less than 22. It is a variation which could probably be interpreted as various aspects of one concept.

More precise description of the mechanisms of the development of science can help in current discussions in the philosophy of science. Many classical discussions (e.g. between Kuhn and Lakatos) contained a disturbing element consisting in the fact that the participants analysed different types of scientific revolution. Since the concept of scientific revolution was not internally specified, each of them presented the results of his analysis as if they would concern the development of science in general and so transferred the knowledge valid for a certain type of change in science to the development of science as such. Therefore, various misunderstandings emerged. Discussion on incommensurability is a typical discussion of this type. The thesis on incommensurability between the old and the new paradigms is entirely valid in the case of idealization, partially in the case of re-presentation but it is invalid in the case of objectivization. Since Kuhn's primary examples were representations whereas Lakatos analysed changes of substantially lower order, they evidently could not have reached agreement on the issue of incommensurability. If, however, we specify the type of revolution, the question of incommensurability can be exactly formulated and relatively unambiguously answered. In such discussions therefore, the question is not which conception of the development of science is true. Both conceptions are true in a sense. The question is what do they speak about.

An approach to philosophical discourse about the development of science which has been opened up here, can be elucidated by means of a metaphor of Viktor

Frankl (Frankl 1982). Let us imagine a cylinder placed on a plane. If someone is looking at it from above, he maintains that it is a circle. On the other hand anyone who sees it from the side, says it is a rectangle. The question is not – which description is true, because both are true in a sense. Our assignment is not to settle the dispute between the parties but to construct a body for which both descriptions are true. Similarly, it is not the task of the philosophy of science to find out which philosophical school is right but to construct an image of science where particular philosophical conceptions can be its true projections. The classification of scientific revolutions offers instructions for such a construction.

2. Kuhn's theory in the light of the history of mathematics

New impulses for the development of a theory often arise when the conceptual apparatus of the theory is transferred to an area, for the description of which it was not originally intended. In the new domain, shifts of meanings of particular terms take place and this throws new light on its starting points and opens up new prospects for its development. Such a situation arose in the case of Kuhn's theory of the development of science, when, in the mid-seventies, some historians tried to apply Kuhn's conceptual apparatus to the description of the development of mathematics. Kuhn definitely did not think about mathematics when formulating his theory; therefore the question whether Kuhn's theory of scientific revolutions could be used for the interpretation of the history of mathematics raised a lively discussion among renowned historians of mathematics – Michael Crowe, Joseph Dauben, and Herbert Mehrtens.

During the conference on the history of modern mathematics held in Boston in August 1974, Crowe formulated his famous thesis that "Revolutions never occur in mathematics" (Crowe 1975). Some three months later at the meeting of the History of Science Society in Norwalk, Dauben said that "revolutions do occur in mathematics and the Greek discovery of incommensurable magnitudes or Georg Cantor's creation of transfinite set theory are examples of such revolutionary transformations" (Dauben 1984). A compromise standpoint between these extreme positions was taken by Mehrtens who said that "some of Kuhn's terms (scientific community, anomaly, normal science) are of explanatory value and provide a device for a historical exploration of mathematics whereas others (revolution, crisis, incommensurability) are without explanatory value in mathematics and lead the discussion into unproductive disputes" (Mehrtens 1976). The discussion was published in the collection of papers *Revolutions in Mathematics* (Gillies 1992).

In the introduction to the collection, its editor Donald Gillies sees the reason for the disputes between Crowe and Dauben in different understandings of the concept of scientific revolution. Crowe understands revolutions very narrowly as changes during which "an entity (be it king, constitution, or theory) is overthrown and irrevocably discarded" (Gillies 1992, p. 19). On the other hand, Dauben understands revolutions in a broader sense, as changes during which the particular entity is not necessarily irrevocably discarded but is "relegated to a significantly lesser position" (Gillies 1992, p. 52). According to Gillies, both approaches to the concept of revolution are justified because they describe really existing differences. They can even be illustrated using the examples of revolutions given by Kuhn himself. Copernican revolution is an example of the revolution of the first type, because during its existence Aristotelian physics was overthrown and irrevocably discarded from the professional training of scientists. If today a student meets Aristotelian physics at all, it is in courses on the history of science. On the other hand, according to Gillies, Einstein's revolution is a revolution of the second type because Newtonian physics has not disappeared from the curricula as its consequence. It is still taught and used in a variety of cases. It was merely relegated from the position of the fundamental theory of the universe to the significantly lesser position of an approximation.

Importantly, it should be realized that the differences between the complete rejection of Aristotelian physics in the case of the Copernican revolution and the only partial shift of the Newtonian mechanics aside in the case of Einstein's revolution are differences in the behaviour of the scientific community. That is they are sociological facts which should be accepted by every supporter of Kuhn's theory. In the chapter on Frege's revolution in logic (Gillies 1992, pp. 265–305), Donald Gillies tries to apply his differentiation between the two aproaches to the concept of scientific revolution in the analysis of Frege's contribution to logic. Frege's revolution, consisting in the transition from Aristotelian syllogistic logic to the predicate calculus, does not fulfil either Crowe's or Dauben's definition. Crowe's definition is not fulfilled because Aristotelian logic is still considered to be valid, with some limitations (as in the case of Newtonian mechanics), whereas Crowe's definition requires its irrevocable overthrow. On the other hand, it does not satisfy Dauben's definition because Aristotelian logic is shifted aside more principally than in the case of Newtonian mechanics. That is, although Aristotelian logic is still considered to be valid, nobody argues in syllogisms today. On the other hand, architects or engineers use Newtonian mechanics in their calculations even today. Frege's revolution in logic appears to be a third type of scientific revolution, where the old theory is "irrevocably relegated". Evidently, if we want to specify these three, and maybe many other types of revolutions more precisely, we need a more precise conceptual apparatus than the sociological terms "overthrow" or "relegate". We think that the notion of epistemic rupture helps to achieve the necessary specification of the character of the transitions between the old and the new theory in individual types of revolution.

3. Scientific revolutions and epistemic ruptures

Kuhn's thesis that during the development of science we witness a variation of paradigms which represent isolated, closed and incommensurable worlds, polarized the community of epistemologists. One camp accepted Kuhn's attitude, seeing there a way how to free themselves from the arrogance of scientific rationality; the second camp rejected it because the price required by Kuhn, namely the loss of the possibility of rationally reconstructing the history of science, seemed to them too high. Our aim is not to take a side in this dispute. We do not want to argue either in favour of or against Kuhn's standpoint. Our aim is to make the language used in the analysis of the development of science more precise.

Our basic idea consists in introducing a distinction between "scientific revolution" and "epistemic rupture". **Epistemic rupture** means a discontinuity in the language of scientific theory. It is a fact that can be determined by analysis of the scientific texts themselves, independently of the role of these texts in the life of a scientific community (whether they are paradigmatic or marginal texts, professional or didactic texts). Thus an epistemic rupture represents a formal aspect of every scientific revolution. **Scientific revolution** means a change in the attitude of a scientific community. Thus a scientific revolution is a sociological fact explored by sociological methods, while epistemic ruptures can be investigated by the methods of the formal reconstruction of scientific texts. Both scientific revolutions and epistemic ruptures are objective facts. We do not see any sense in the polemic whether scientific revolutions exist or not. Simply, this question should be formulated in sociological terms and sociologists should find out. Sociology has methods able to clearly identify continuity or discontinuity in a particular cultural tradition or a social practice. The role of epistemology is not to decide on these issues instead of sociology. In order to avoid useless disputes between the sociology of science and epistemology, the questions and problems of the development of sciences that are of a sociological character should be clearly defined and isolated from those of epistemology. Our aim is not to replace one method of analysis with another, but to supplement each method with the prospects of the other discipline.

If we should briefly describe the relation between epistemic ruptures and scientific revolutions, it is the relation between variation and selection. The theory of epistemic ruptures describes possible variations of a particular theory which are admitted by the language of science (that is "mutations of the theory"). Only some ruptures have been selected by scientific community as promising trends for further development and particular "mutants" have been accepted as paradigms. Evidently, both components are needed: the theory of epistemic ruptures as well as the theory of scientific revolutions. Their balance creates the basis of evolution. Kuhn described the process of selection but did not elucidate where alternative theories, aspiring to become a paradigm during crisis, come from. He simply assumed their existence. It's time to try to give a theoretical account of the generation of new theories.

One of the benefits of the distinction between scientific revolution and epistemic rupture consists in the fact that epistemic ruptures can be mutually compared and classified. It turns out that epistemic ruptures can be divided into four types – idealizations, re-presentations, objectivizations, and re-formulations. It should be emphasized that not all epistemic ruptures are bound to a parallel scientific revolution. There are ruptures without any corresponding revolution. Ruptures therefore provide a much more complete and balanced image of changes in the development of science.

Another benefit from the differentiation between scientific revolution and epistemic rupture lies in the fact that we can explore how the scientific community reacts to particular ruptures. It appears that reactions to ruptures of different types are different. The classification of epistemic ruptures thus becomes the starting point for the classification of scientific revolutions. In contrast to epistemic ruptures, there are three types of scientific revolutions – idealizations, re-presentations, and objectivizations. No scientific revolution corresponds to the fourth type of epistemic ruptures, namely re-formulations. Re-formulations represent a classical instrument of normal science and are of a cumulative character. If a new planet of the solar system, a new chemical element or a new animal species, which fits in the existent system is discovered, there is no need to hold discussions about the issue. Textbooks and manuals should simply be re-written to place the new facts next to the old ones as if they had been there all the time. Their introduction into a theory is thus reduced to a re-formulation of the standard texts. No reconstruction of the conceptual scheme is required and, therefore, it does not stimulate any response in the scientific community. From the purely formal point of view it is a discontinuity – an epistemic rupture, because the number of the elements of the theory has changed. Nevertheless this rupture is not accompanied by a revolution. This is why our classification of scientific revolutions contains only three types of revolution.

4. Some consequences of making Kuhn's theory more precise

a – the requirements of consistency

The requirement for the consistency of considerations on scientific revolutions can be formulated as the first consequence of making Kuhn's theory more precise. Michael Crowe's views will be used as illustrations. Crowe expressed the thesis "Revolutions never occur in mathematics" (Crowe 1975). Crowe's justification of his view was that a necessary condition of a revolution is that an entity (king, constitution, or theory) should be overthrown and irrevocably discarded, while in the development of mathematics "formational discoveries" are at issue where a new branch of mathematics is formed without rejecting the older doctrines.

Looking at this justification of Crowe's thesis from the perspective of our classification, we see that it defines re-presentations, characterized by the formulation of a new branch of mathematics, with high precision (Kvasz 1999). However, it is inadequate in the case of idealizations. The Pythagorean rupture consisting in the transition from Egyptian mathematics based on calculative recipes to Greek mathematics based on the proof, satisfies what Crowe demands from revolutions. In its course, Egyptian recipes were overthrown and irrevocably discarded from mathematics. The Greeks did not regard them as part of mathematics but they included them with contempt into merchant calculations which they called logistics. They had good reasons for that: several Egyptian recipes were incorrect. Their results were often not even near to what they claimed to have been calculating. The calculation of the area of a quadrilateral as a product of arithmetic averages of the opposite sides (Edwards 1979, p. 2) is nonsense. The number thus obtained has nothing to do with the area of the particular object. Even if some Egyptian recipes give correct result it is a mere coincidence. The Egyptians could not know it at all because they did not have the idea of a proof. That is why Egyptian mathematics was rejected after the Pythagorean rupture. A question can be posed why Crowe does not consider this a revolution? The Pythagorean rupture based on the discovery of proof fulfils his requirement for revolution and thus clearly contradicts his thesis.

b – the requirement of unambiguity

When reading the works of philosophers of science (Kuhn, Popper, Lakatos, Piaget, Feyerabend, Polanyi) or historians (Koyré, Crowe, Dauben, Mehrtens), the distinction of scientific revolutions of different types enables us to raise the question which type of scientific revolution the particular author bears in mind. It turns out that they usually base their theory on one type of revolution, the structure of which they promote to a universal rule. A number of classical controversies in philosophy or in the history of science are based on, or at least largely influenced by, the fact that the revolutions discussed by the participants in these discussions are of different types but since they formulate their arguments in a general way, they enter into contradictions.

The controversy between Kuhn's conception of scientific revolutions and Lakatos' conception of scientific research programmes is explicitly of this type. Kuhn illustrates his theory by examples, which are in most cases re-presentations. By contrast, Lakatos' examples are re-formulations. Naturally, they come to entirely opposite conclusions concerning the nature of the development of science. Our analysis shows that this controversy is merely illusory and in fact both describe various aspects of the development of science truthfully and correctly. Similarly the controversy between Crowe and Dauben whether scientific revolutions can occur in mathematics follows from the use of the concept of scientific revolution in terms of its different types. While Crowe uses the concept of scientific revolution in terms of idealization, Dauben uses the same concept in terms of re-presentation (at least both his examples, the discovery of incommensurability and the following shift of mathematics from arithmetic to the geometrical basis as well as the discovery of the theory of sets and the following shift of mathematics to a set theoretical basis are examples of re-presentations, see Kvasz 1999).

The classification of scientific revolutions enables a mitigation of many controversies, and a better understanding of what the particular parties are actually talking about. Thus, instead of the task of settling the apparently controversial opinions, we may start the task of using the positive content of the views of the particular parties for the construction of a general picture of the development of science. This picture should be so complex enough to include places for the majority of philosophical approaches. To incorporate them into this picture, the elucidation of what they talk about is necessary in advance. Our classification opens a possibility of throwing light on this question.

c – the fine structure of scientific revolutions

In his study *The "fine structure" of mathematical revolutions: metaphysics, legitimacy, and rigour* (Giorello 1992), Giulio Giorello suggests conducting an investigation into the fine structure of scientific revolutions. In his opinion, a scientific revolution is not an event of one moment but it is rather a piecemeal and slow process consisting of different phases. The classification of epistemic ruptures provides a device for describing such a fine structure. Let us take for example the revolution associated with the birth of quantum mechanics. This revolution began with Planck's works on the black body radiation in which the idea of quanta appeared as a purely formal trick, that is as re-formulation. The next step was Einstein's theory of the photoeffect in 1905, when Einstein started to work with the radiation of quanta as if they really existed. The revolution described then deepens and assumes the character of objectivization. The overall "old" quantum mechanics is developed on this basis. The deepening of the revolution up to the level of re-presentation, which was its successful completion, was brought by de Broglie in 1923, who said that all particles and not only light are of dual wave-corpuscular character. We see that quantum re-presentation was not born in one moment but it was a process of approximately thirty years. The terms re-formulation, objectivization, and re-presentation seem to describe relatively truthfully the sequence of ruptures and thus also the dynamics of this process.

A finer analysis of the history of quantum theory would probably reveal several re-formulations and several objectivizations. This would enable a more precise account of the "fine structure" of this re-presentation just as Giorello described the "fine structure" of another re-presentation – the discovery of differential and integral calculus. For instance, transition from de Broglie's understanding of the wave function as the waves of matter to the probabilistic interpretation according to which the wave function describes only the distribution of the probability of the particle and not directly the distribution of matter was an important objectivization. This means that objectivizations also take place after the re-presentation which determines the character of the whole revolution.

Thus a scientific revolution represents a complex process which might contain several ruptures of a lower order than the largest rupture determining the character of the revolution as a whole. We believe that the sequence of these additional ruptures determining "the fine structure" of the particular revolution is not random but can be described in more detail.

d – confrontation of ruptures and revolutions

While each scientific revolution necessarily has a particular epistemic rupture, representing its formal side, not every rupture necessarily turns into a scientific revolution. The development of synthetic geometry can serve as evidence. In the paper History of Geometry and the Development of the Form of its Language (Kvasz 1998b) we described ten epistemic ruptures in the development of synthetic geometry. However, in the literature on the history of geometry only one of them is described as revolution, namely the discovery of non-Euclidian geometry. The differentiation between the concept of epistemic rupture and that of scientific revolution enables us to raise a new question: Which epistemic ruptures will turn into scientific revolutions? Strangely enough, out of ten discoveries, which are equal from the formal point of view, the scientific community chose only one as a revolutionary change, and the other nine were assigned the character of immanent development. The question is why the evaluation of the discovery of non-Euclidian geometry differs so much from other ruptures completely equivalent from the formal point of view?

The fact that the formal sides of the ruptures considered are the same shows that it is not the theory itself but rather the extra-scientific reasons that decide on the assignment of the revolutionary character to a particular rupture. We can imagine what are these reasons by recollecting the role of geometry in philosophy immediately before the discovery of non-Euclidian geometry, for instance with Kant. Kant considered the arguments of geometry to be an example of a priori synthetic judgements. As a priori judgements they represented the absolutely certain knowledge which could not be disputed by any a posteriori experience. On the other hand, since they were synthetic at the same time, they represented real knowledge in contrast to analytic judgements, which can elucidate a concept at the very most, but do not enrich our knowledge. On the basis of this account by Kant, the role of an ideal of knowledge was assigned to geometry. What does Kant need this model for? To criticize metaphysics! Metaphysics does not dispose of the synthetic a priori, that means it is either a priori and analytic but then it is not real knowledge, but it represents rather a set of linguistic explanations, or metaphysics is synthetic but a posteriori and therefore, as the knowledge based on experience, it must constantly be verified by experiences. We see that in the ideology of the Enlightenment, geometry plays a more significant role than a mere teaching about spatial forms. We think that the revolutionary character of the discovery of non-Euclidian geometry is given precisely by the fact that it breaks this myth of the Enlightenment. If there are several geometries at available, it is not possible to decide a priori which of them tells the truth about our world. That means, geometry is not synthetic a priori.

If it is really so, then examination of which epistemic ruptures turned into scientific revolutions can serve as a detector of illusions which society associated with science. The question of which epistemic ruptures turned into scientific revolutions, that is the question which of a number of equal changes attracted attention, might become a way towards a better understanding of how science actually operates in culture, what are the different illusions that society associates with science and how society tries to preserve these illusions about science. This could also explain a stormy reaction to Kuhn's book. It probably was not stimulated by the interest in science itself but rather by the interests of different parts of the scientific community, which commonly used science for their legitimization. The separation of the formal aspect from the sociological one in the development of science and the following confrontation of the two aspects enables us to uncover a lot of interesting issues in the development of science. We think that when a similar separation was made by positivists at the beginning of this century (the separation of the context of discovery and the context of verification) they never tried to compare the two contexts, and so they deprived themselves of the most productive results that could have been gained by such a separation. It is the confrontation of the formal and the sociological (or logical and psychological) that enables us to raise the most interesting questions.

e – refining the concept of paradigm

We have already said that the primary objection to Kuhn's theory lay in the fact that it does not distinguish between different types of scientific revolution. Kuhn's concept of scientific revolution actually covers three types of changes in science. They are idealizations, using the example of the scientific revolution in the 17th century, re-presentations including the example of the creation of the electromagnetic theory, and objectivizations exemplified by Einstein's revolution. The outcome is that the definition of the basic categories of his theory, like paradigm, anomaly, crisis, and revolution obtained by the analysis of such a heterogeneous material, is only approximate and has an ambiguous content. Each of Kuhn's categories actually incorporates three different concepts. Paradigm is probably something else in the case of idealization, something else in the case of re-presentation and again something else in the case of objectivization.

We propose to proceed from the differentiation of the three types of scientific revolution to a differentiation of the basic categories of Kuhn's theory. This will enable us to distinguish the paradigm of idealization, the paradigm of representation, and the paradigm of objectivization. The incommensurability between paradigms as described by Kuhn is probably of different character depending on whether it is the incommensurability of the paradigms of idealization, representation or objectivization. Possibly, the scientists separated by idealization live indeed, as Kuhn puts it, in different worlds and cannot understand one another; the scientists using different paradigms of objectivization understand each other and they merely differ in their evaluation of facts which are, however, the same for both. The relativization of the concept of incommensurability of paradigms would enable us to grasp the phenomenon, which Kuhn describes almost exclusively in psychological terms, epistemologically.

The particular types of scientific revolution consist in the change of the particular type of paradigm. Thus the idealization changes the paradigm of idealization, representation changes the paradigm of representation and objectivization changes the paradigm of objectivization. However, during idealization, there are usually also changes of the paradigms of representation and objectivization just as in the course of re-presentation the paradigm of objectivization is changing. This permits a more precise description of the structure of scientific revolution of a particular type. The revolution is not a single act, it is rather a gradual process of changes containing, in addition to the main rupture determining the type of the particular revolution, several ruptures of lower orders. In each of the three types of revolutions, anomalies have different characters and the particular crisis has a different depth. The analysis of all these aspects enables us to describe what Giulio Giorello named the "fine structure" of scientific revolutions. The hierarchical interconnections of the paradigms of different orders and the fine structure of revolutions inferred from it are phenomena representing the specific that is epistemological aspects of scientific revolutions, which Kuhn, concentrating on the sociological side of the problem, left unnoticed.

5. Kuhn and contemporary philosophy

The influence of Kuhn's ideas on contemporary philosophy surprised not only philosophers but also Kuhn himself. Only rarely does a book dealing with specific questions of the philosophy of science meet with such a response. By evaluating the impact of Kuhn's opinions it is therefore necessary to distinguish three areas. The first area is the philosophy of natural science. Kuhn's influence is here undeniable. Together with the philosophers like Polanyi, Popper, Lakatos, Toulmin or Feyerabend, Kuhn initiated a basic change in the way of formulating problems in this area, the way of seeking answers, and ideas about what is actually the subject and the role of philosophy of natural science. The idea that in the development of science we are facing progress based on accumulation of scientific knowledge and that the mission of the philosophy of science is to articulate norms which science obeys, is probably a matter of the past. This positivistic vision was replaced by the idea of revolutions and ruptures separating particular epochs and paradigms.

The second area, where Kuhn's ideas met with a large response, was the philosophy of the social sciences. In this area, the idea of particular importance was the idea that it is a paradigm and not a method that characterizes science. Therefore a social science will not become scientific by mechanically adopting the methods of the natural sciences. It is a mistake of positivism to think that science was born when data started to be accumulated and hypotheses began to be tested. According to Kuhn, science was born when a paradigm emerged that was accepted by the whole scientific community. Such a paradigm can only be born inside the discipline itself. No copying of natural science will help. Understandably, such views must have resonated in the majority of social sciences because they offered arguments against scientistically orientated schools. Freudian psychoanalysis, Marxist economy or Weber's sociology can be declared to be paradigms and they can stop caring about the criticism of their positivistically tuned colleagues. We believe that our more precise version of Kuhn's theory will shed light on the situation in some of these disciplines. By precisely defining the concept of paradigm we can elucidate which schools use this term legitimately and which only use Kuhn's original imprecision in the definition of the concept of paradigm and try to present something that has nothing to do with science as a scientific discipline.

The third area where Kuhn's ideas met with considerable response, is the current of philosophy sometimes called postmodernism. Several postmodern thinkers refer to Kuhn's idea of the incommensurability of paradigms and to his thesis that scientists defending various paradigms live in different worlds and one cannot choose among them on the basis of rational arguments. It is probably not necessary to emphasize that Kuhn was not pleased with such a reception of his ideas. We think that the differentiation of the paradigm of idealization, representation and objectivization will help to characterize the measure of incommensurability between particular paradigms. Then it will be possible to decide in every particular cases when it is legitimate to speak about different worlds and when not. While the measure of incommensurability is great between the two paradigms of idealization, and the metaphor of life in different worlds is entirely legitimate, two paradigms of objectivization do not justify the use of such a metaphor. Naturally, it would be naive to think that this more precise version will exert an influence on the postmodern discourse. However, it will enable us to decide to what extent the references of postmodern philosophers to Kuhn are legitimate.

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