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Complex Flow Stress Model for a Magnesium Alloy AZ31 at Hot Forming

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Abstract. Compression tests of magnesium alloy Mg-3Al-1Zn (AZ31) at different temperatures and strain rate were made on plastometer Gleeble 3800. Deformation behaviour and particularly shape of stress-strain curves of the alloy AZ31 differ significantly at low and high values of Zener–Hollomon parameter Z. The border between these areas was determined mathematically as $Z = 2.9\text{E}+13\ \text{s}^{-1}$. While the calculated activation energy Q was for both these areas practically identical (157 or 155 kJ mol^{-1}), mathematical description of coordinates of the peak stress differs considerably. Regression and statistical analysis of experimental data have confirmed unequivocally, that it was impossible to describe by a uniform equation the whole set of data (i.e. traditional stress-strain curves, as well as those with atypical initial stage, given by the massive twinning). That's why two mathematical models were developed enabling prediction of the flow stress of investigated magnesium alloy in dependence on temperature, strain and strain rate, with inclusion of the influence of dynamic re-crystallisation.

Keywords. magnesium alloy AZ31, stress-strain curves, activation energy, hot flow stress model.

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1 Introduction

The objective of the works consisted in obtaining a complex phenomenological model of resistance to hot deformation for the alloy AZ31, which is one of the most frequently used magnesium alloys designated for forming. The situation is remarkably complicated by its different deformation behaviour at low and high values of Zener–Hollomon parameter, expressing the temperature compensated strain rate.

Authors Kuc and Pietrzyk [1] chose a manner of obtaining the mathematical flow stress model for the alloy AZ31, which is at the first sight mathematically highly sophisticated, but also very complicated. Authors Beer and Barnett [2] investigated metallographic analysis and explanation of key features of deformation behaviour in relation to the initial grain size, structure, twinning and dynamic re-crystallisation. Authors McQueen, Leo and Cerri made very complex analysis of activation energy and necessary material constants in their work [3].

2 Experimental Procedures

Axially symmetrical samples from magnesium alloy AZ31 with chemical composition 2.82 Al–0.80 Zn–0.37 Mn (mass %) were used for obtaining the necessary data from the simulator Gleeble 3800. They were subjected to compression tests and then cooled in water in order to gain information about dynamic structure-forming processes [4]. The tests were realised at temperatures $T = 523, 573, 623, 673$ or 723 K (after uniform preheating at the temperature of 723 K) and at the strain rates $\gamma = 0.01, 0.1, 1$ or $10\ \text{s}^{-1}$. Figure 1 gives examples of the selected obtained stress-strain curves and it documents influence of deformation conditions (or of values of Zener–Hollomon parameter Z, calcu-

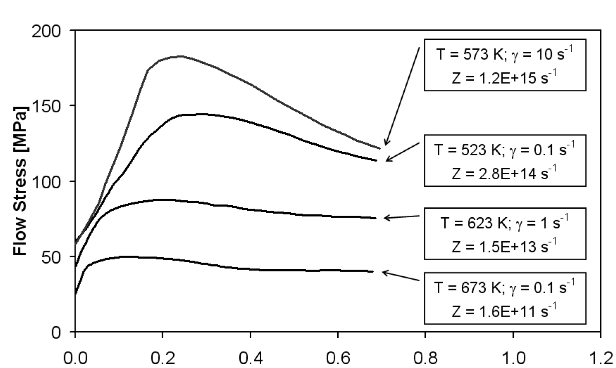

Figure 1. Influence of temperature and strain rate on the shape of stress-strain curves.

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lated by the manner described below) on their shape, which differs fundamentally in the initial stage of the curve.

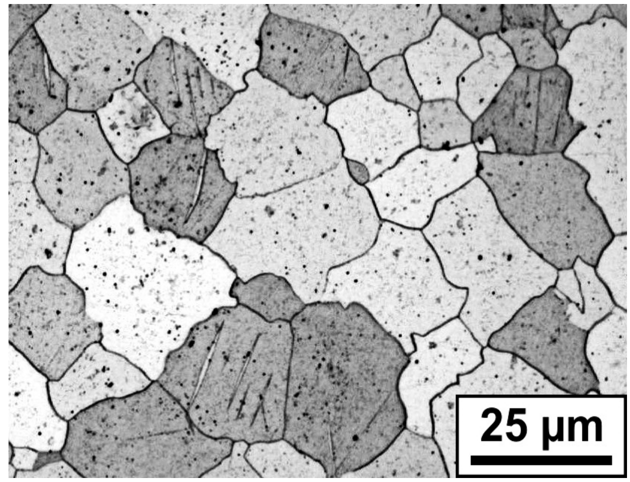
Prior to mathematical processing of the obtained data it was necessary to explain this different character of stress-strain curves by development of structure. That's why additional plastometric tests were interrupted at preselected strains ε and the current structure was always fixed by rapid cooling of the sample. Appropriate deformations were chosen in such a manner that it would be possible to obtain by metallographic analyses information on development of structure in broad range of deformations (i.e. strains), this means also with inclusion of dynamic restoration.

3 Evaluation of Experimental Results

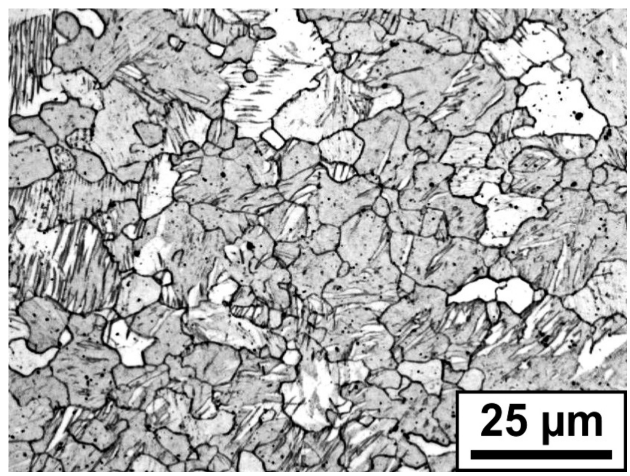
3.1 Dynamic Structure-Forming Processes

Photos in Figure 2(a) show in principle the initial structure of material after heating, since very small deformation $\varepsilon = 0.05$ did not lead to change of microstructure characterised by comparatively coarse equiaxed grains. Signs of twinning can be observed only in small extent in some grains. Other photos in Figure 2 then document development of structure during forming by low strain rate at comparatively high temperature (i.e. at low value of the parameter Z). Before achievement of the peak stress certain twinning occurred, but principal role was evidently played by the start of dynamic recrystallisation (Figure 2(b)). The grains are after achievement of the peak stress periodically refined by recrystallisation, which is manifested by their equiaxed character (Figure 2(c)).

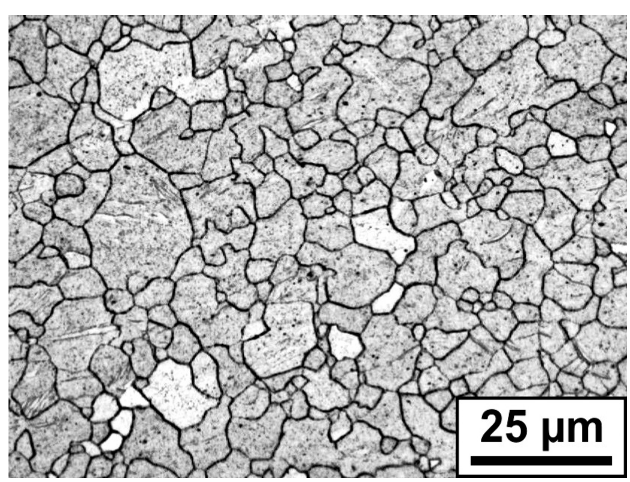
On the other hand at initial phases of forming the twinning becomes the dominant deformation mechanism due to the increase of the parameter Z – see Figure 3(a). Some grains in vicinity of the peak stress disintegrate to smaller ones (Figure 3(b)), but this phenomenon is probably not connected with traditionally understood dynamic recrystallisation, since its manifestations at high deformations are absolutely different. As it can be seen in Figure 3(c), recrystallisation manifests itself by a massive nucleation of new grains preferentially on twins and on boundaries of original grains, but growth of these nuclei is decelerated. It is therefore obvious that under different deformation conditions different deformation mechanisms prevail in investigated material, which principally influence the shape of stress-strain curves. Before description of the hot flow stress it is therefore necessary to determine the boundary conditions of these phenomena.



(a) $\varepsilon = 0.05$



(b) $\varepsilon = 0.15$

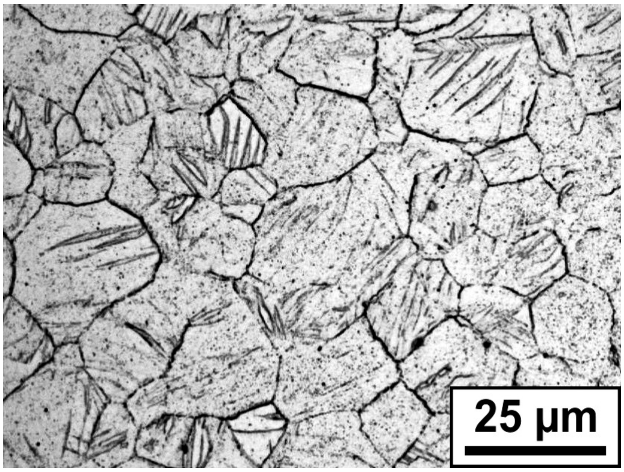


(c) $\varepsilon = 0.40$

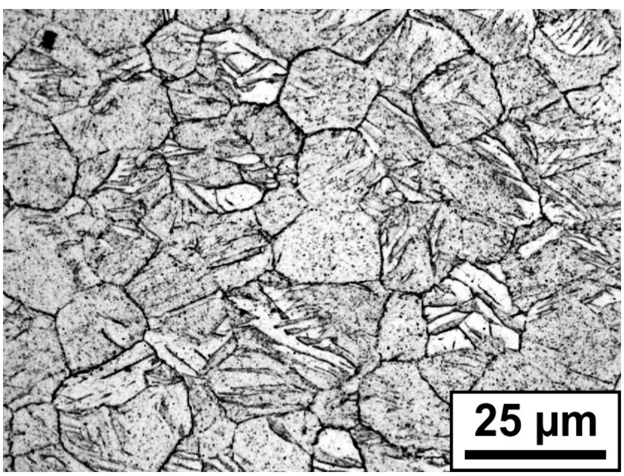
Figure 2. Microstructures of the samples analysed at $T = 573$ K, $\gamma = 0.01$ s^{-1} and at various strains (peak stress was achieved at $\varepsilon = 0.16$).

3.2 Activation Energy and Kinetics of Dynamic Recrystallisation

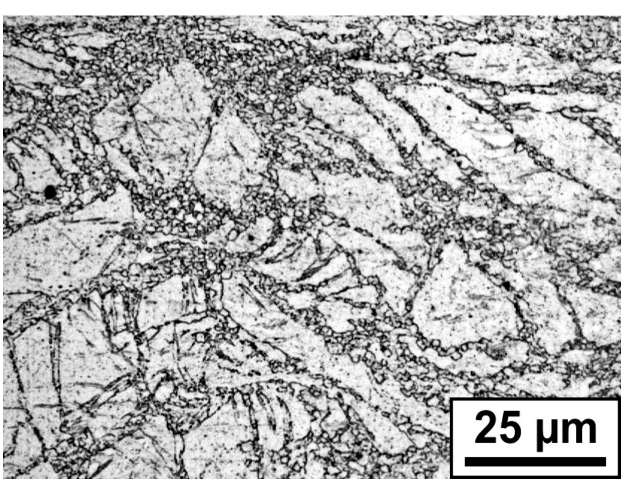
It was possible to calculate the Zener–Hollomon parameter Z [s^{-1}], given at individual curves in Figure 1, after deter-



(a) $\varepsilon = 0.15$



(b) $\varepsilon = 0.25$



(c) $\varepsilon = 0.50$

Figure 3. Microstructure of the samples analysed at T $=$ 523 K, $\gamma = 1\ \mathrm{s}^{-1}$ and at different strains (peak stress was achieved at $\varepsilon = 0.28$).

mination of the value of activation energy Q [kJ mol^{-1}] at hot forming. For this purpose an original interactive software ENERGY [5, 6] was applied. It was designated for regression quantification of constants in the relation determined originally for empirical description of the steady-state of the stress-strain curve at hot forming, which is, however, identical also in the case of description of the peak (maximum) stress $\sigma_{\max}$ [MPa] [7]:

$$\dot{e} = C\ \exp\left[\frac{-Q}{RT}\right][\sinh(\alpha\,\sigma_{\max})]^n \qquad (1)$$

where C [s^{-1}], α [MPa^{-1}] and n [$-$] are material constants, $R = 8.314$ J mol^{-1} K^{-1} and T [K] is temperature of deformation.

Strain ε_p[$-$] corresponding to the peak stress can be described in dependence on the parameter Z by the relation [8]:

$$\varepsilon_p = U \cdot Z^W \qquad (2)$$

where U [$-$] and W [$-$] are material constants and Zener-Hollomon parameter can be expressed by the relation:

$$Z = \gamma\ \exp\left[\frac{-Q}{RT}\right]. \qquad (3)$$

However, at processing of experimental data (i.e. corresponding values of $\sigma_{\max}$ and ε_p) with use of the program ENERGY it was found out, that they cannot be described by equations (1) and (2) in the full range of deformation conditions [9]. It was necessary to split the data into two groups depending on the shape of the relevant stress-strain curves – with normal or with atypical initial stage. It is evident from the diagram in Figure 4, how this different deformation behaviour manifests itself in the values of peak stress and calculation of activation energy. The calculated values of activation energy and of other material constants after appropriate separation of experimental data are summarised in Table 1. In this manner it is possible to describe both coordinates of the peak stress, to predict the maximum flow stress and start of dynamic recrystallisation of investigated alloy in dependence on temperature and strain rate.

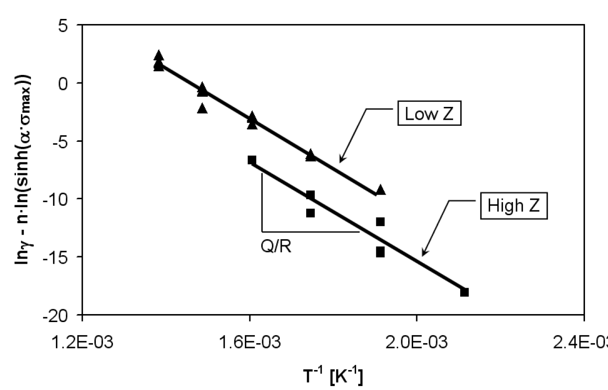


Figure 4. Final regression at calculation of activation energy from the relation (1).

	High Z	Low Z
Q [J mol^{-1}]	154830	157420
n [−]	1.6765	6.9385
α [MPa^{-1}]	0.0459	0.0034
C [s^{-1}]	6.07E+09	5.28E+16
U [−]	0.20563	0.00208
W [−]	0.005	0.153

Table 1. Material constants in the equations (1) and (2).

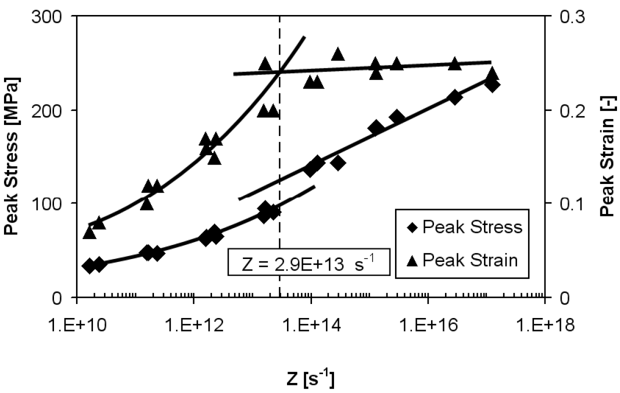


Figure 5. Influence of parameter Z on both coordinates of peak flow stress.

As it can be seen in Figure 5, the value of $Z = 2.9\text{E}+13$ s^{-1} is a purely mathematically determined border between "High Z" and "Low Z" in Table 1. The fitted curves represent calculations in accordance with the relations (1) and (2) with the constants given in Table 1.

The necessity of description of coordinates of the peak flow stress in the case of hot forming of one material by several equations was noticed for several times already, however, the appropriate border was always given by the specific temperature, or by phase composition of material – see for example [5, 10].

3.3 Flow Stress – Traditional Curves

Regression and statistical analysis of experimental data confirmed unequivocally, that it was impossible to describe the whole set of data (i.e. traditional and atypical stress-strain curves) by a universal equation. That's why in the first stage only the curves of traditional shape were used, i.e. for the values Z $\leq$ 2.9E+13 s^{-1}. On the basis of previous experience [11] the following model was used for description of the flow stress of investigated material in dependence on temperature, strain, strain rate and deformation ε_p representing the influence of dynamic recrystallisation:

$$\sigma_c = A \cdot \varepsilon^B \cdot \exp\left(-B \cdot \frac{\varepsilon}{\varepsilon_p}\right) \cdot \gamma^C \cdot \exp(-D \cdot (T - 273)) \quad (4)$$

Parameter	A	B	C	D
Value	743	0.23	0.095	0.0045

Table 2. Parameters of the function (3), determined for an extensive range of thermomechanical conditions.

where σ_c [MPa] is the predicted flow stress (calculated on the basis of the developed model), deformation ε_p is described by the equation of the type (2), and A, B, C, D are material constants. These constants are determined by methods of multiple non-linear regression, namely with use of statistical package Unistat 5.5 [4]. The equation (3) with calculated constants (see Table 2) describes experimental data in a broad interval of deformation conditions, i.e. even behind the peak stress, but not anymore in the area of steady-state.

3.4 Flow Stress – Atypical Curves

Next procedure was aimed at obtaining of mathematical description of the flow stress of the alloy AZ31 for $Z > 2.9\text{E}+13$ s^{-1}, when simple model of the type (3) is unusable due to massive twinning causing atypical development of flow stress in its initial stage (concave shape of the curve). Such a description is far from being simple, which is manifested by absence of usable results of other authors. It is obvious that it should be a combined mathematical function, but at the same time it is necessary to respect the physical principle of the given dependence.

The first proposals were directed to an additive function, which would respect different course of dependence till achievement of the inflection point in the initial stage of the curve and after passing this point. An additive function was proposed, the first part of which corresponded to exponential dependence (till achievement of the inflection point), and the second part of which corresponded to the function of the type (3) (description of the course after achievement of the inflection point). The proposed function described experimental data very well, which was confirmed by the correlation coefficient achieving the value of 0.999. However, disadvantage for its practical use consisted in the fact that it was first necessary to determine the position of the inflection point. The distinct fault on the curve occurring at very small changes of the values of calculated stress in the very proximity of this point was another drawback.

Other types of combined functions were proposed and tested in the next stage in order to eliminate these drawbacks. The function of the following type appeared to be the most suitable for description of stress-strain curves with atypical course in the initial stage:

$$\sigma_c = F + \frac{G}{1 + \exp(1 + H \cdot \exp(-J \cdot \varepsilon))} \cdot \exp(-K \cdot \varepsilon) \quad (5)$$

Parameter	F	G	H	J	K	L	M
Value	205.4	2799	4.00	13.4	1.28	0.07	0.0052

Table 3. Parameters of the function (6), determined for an extensive range of thermomechanical conditions.

where F, G, H, J, K are material constants, or function parameters.

This function was used for description of atypical courses of stress-strain curves at various strain rates and temperatures of deformation. In all the cases a very good agreement between the measured and backward calculated values was found. This was confirmed also by the obtained values of correlation coefficient, which were not lower than 0.998 for any of nine curves.

For development of a complex mathematical model of flow stress dependence on strain, strain rate and temperature of deformation, the function (5) was completed and modified to the form:

$$\sigma_c = F + \frac{G}{1 + \exp(1 + H \cdot \exp(-J \cdot \varepsilon))} \cdot \exp(-K \cdot \varepsilon) \cdot \gamma^L \cdot \exp(-M \cdot (T - 273)) \tag{6}$$

where L, M are material constants, or function parameters.

The function modified in this manner was used for creation of the model on the basis of altogether 262 experimentally determined values of stress in an extensive range of deformations even behind the peak stress (but not anymore in the area of steady-state), strain rate (within the range from 0.01 s^1 to 10 s^{-1}) and temperature (within the range from 473 K to 673 K). Non-linear regression analysis with use of statistical programs Statgraphics Plus 5.0 and Unistat was used for evaluation of parameters of this function. The determined parameters of the equation (6) are given in Table 3.

It was determined by overall F-tests that the proposed mathematical model is statistically significant at the significance level $\alpha = 0.05$ and therefore it is suitable for description of the dependence of flow stress on appropriate explaining variables. This fact was confirmed also by the high value of the correlation coefficient, which was 0.980. Testing of individual parameters of the model also lead to the conclusion that their values were statistically significant at the significance level $\alpha = 0.05$. The proposed model can be therefore in the proposed version used for prediction of the course of stress-strain curves under various conditions.

Figure 6 gives a comparison of the course of stress-strain curves at various temperatures and strain rates determined on the basis of the proposed model with experimentally determined values.

It is obvious that the found function describes experimental data very accurately. The square root of the mean square error for the analysed relation (6) was 6.89 and value of the determination index was $R^2 = 0.974$. Deviations of flow stress values determined on the basis of plastometric experiments σ [MPa] from predicted (calculated on the basis of the model) values σ_c [MPa] were backward calculated for a more detailed evaluation of quality of flow stress description by the developed model. These calculations were made for both developed models – equations (3) and (6). A relative deviation Δ [%] was defined for these purposes:

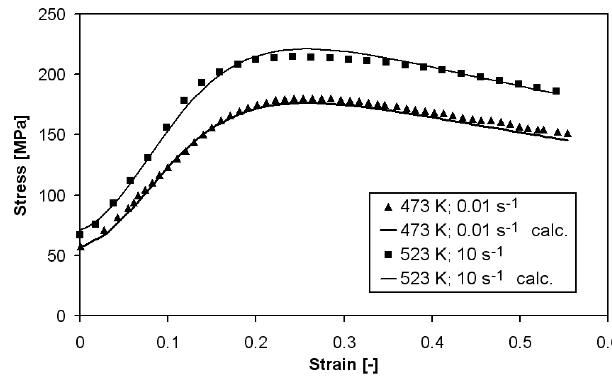


Figure 6. Comparison of flow stress of experimentally determined (points) and backward calculated (curves – calc.) in accordance with the model (6) at selected values of strain rate and deformation temperature.

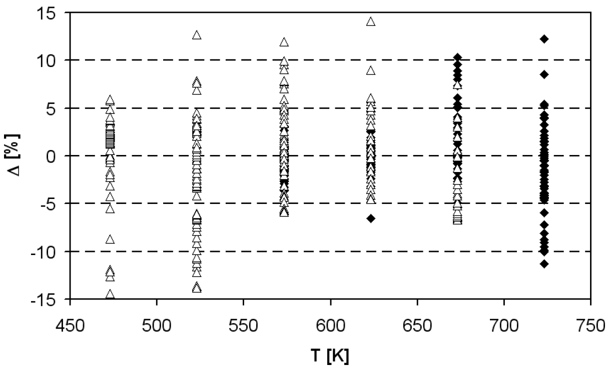
$$\Delta = \frac{\sigma - \sigma_c}{\sigma} \cdot 100 \tag{7}$$

which can be expressed as the share of residuum and experimentally determined flow stress values, multiplied by one hundred for conversion to percents. Diagrams in Figure 7 show relative deviations Δ in dependence on selected parameters of forming. It is evident that accuracy of calculation of flow stress with use of this equation is satisfactory. Scatter of deviations of experimental values and the values backward calculated with use of equations (3) and (6) is on the whole uniform in full range, without evident trends, and relative deviations in both cases do not exceed ± 15 %. The developed models therefore appear to be suitable for description of flow stress of investigated magnesium alloy.

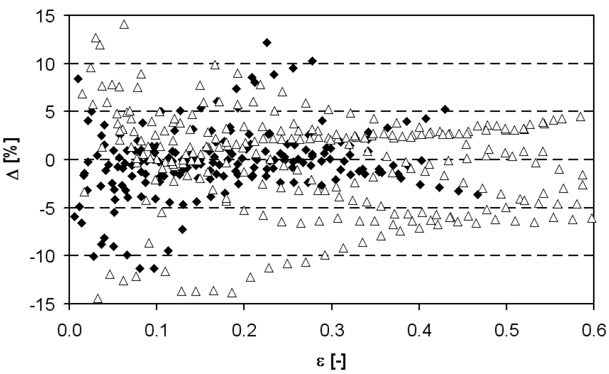
4 Discussion of Results

The obtained values of activation energy of the alloy AZ31 at hot forming are very close to the values determined by other authors – see [3] for example. This concerns also other material constants in the equation of the type (1).

Description of hot flow stress of investigated alloy is the item of interest of many researchers (see [12] for ex-



(a) dependence on temperature



(b) dependence on strain

Figure 7. Relative deviations of the values of flow stress backward calculated with use of developed models (3) ◇ and (6) Δ from experimentally determined values

ample), however, procedures leading to phenomenological description were not yet too successful, or they were too complicated or they did not reflect various deformation mechanisms. Approach to this issue based on mathematical modelling, namely on internal variable method, is different. Such a proposition of a new material model, accounting for both twinning and slip, is described in [1]. It is also possible to apply the inverse analysis for solution of similar problems [13], but none of these sophisticated procedures leads to obtaining of sufficiently simple relations describing deformation behaviour of magnesium alloy in the broad range of hot forming conditions.

The investigated alloy AZ31 was already previously subjected to experimental rolling [14] leading to obtaining of mathematical model of the mean flow stress (MFS). Table 4 represents an attempt of comparison of MFS values calculated on the basis of rolling, or of plastometric experiments. Agreement of the results is comparatively very close. In this case it is necessary to take into account the fact that model from rolling was obtained for relatively high strain rates $\gamma = 10–80\ \mathrm{s}^{-1}$ and it corresponds rather to the conditions for atypical course of stress-strain curves. Contrary to that the model based on the data from plastometer was designed

T [K]	**γ [s^{-1}]**	**ε [-]**	**MFS plastometer [MPa]**	**MFS rolling [MPa]**
573	30	0.5	163	157
573	70	0.5	179	157
673	30	0.5	91	125
673	70	0.5	102	125

Table 4. Example of comparative calculations of MFS.

for relatively low values of strain rate $\gamma = 0.001–10\ \mathrm{s}^{-1}$, which in some extent complicates comparison of results and their agreement. The model based on results of plastometric test works with flow stress. Program MATHCAD version 12 was used for recalculation to the MFS values by numeric integration in various ranges of deformation.

In spite of this and also in spite of the fact that very simple model based on data from rolling does not reflect changes of strain rate, it is possible to declare on the basis of comparative calculations a satisfactory compatibility of both models. Plastometric model is naturally more accurate for low strain rates. However, in the case of real strain rates at rolling the previously determined simple model for description of MFS, which was developed for this purpose, is sufficiently suitable for real rolling conditions.

5 Conclusions

Deformation behaviour and particularly the shape of stress-strain curves of the alloy AZ31 differ significantly at low and high values of the Zener–Hollomon parameter Z. As it was proven by microstructural analyses, the reasons for these changes of deformation behaviour do not consist in phase transformations, but in strong effect of twinning at initial stages of forming in the case of high values of temperature compensated strain rate. On the other hand at low values of the parameter Z the influence of dynamic recrystallization is dominant.

The border between these areas was determined mathematically as $Z = 2.9\mathrm{E}{+}13\ \mathrm{s}^{-1}$. While activation energy Q was for both these areas practically identical (157 or 155 kJ mol^{-1}), mathematical description of coordinates of peak stress differs considerably. This is valid particularly for the values of deformation to the peak, the dependence of which $\varepsilon_p = f(Z)$ sharply breaks in proximity of this border and at high values of Z it shows only very little slope of the curve in question.

Regression and statistical analysis of experimental data confirmed unequivocally that it was impossible to describe by a universal equation the whole set of data (i.e. traditional stress-strain curves and also those with atypical concave initial stage). For this reason two mathematical models were developed enabling prediction of flow stress of investigated magnesium alloy in dependence on temperature, strain and strain rate, with inclusion of the influence of dynamic softening (but not for the area of steady-state).

Accuracy of both models can be assessed as very good, since square root of the mean quadratic error was 1.55 or 6.89, and the value R^2 was 0.993 or 0.974 (see equations (3) and (6)). Basic advantage of the developed original models consists in their relative simplicity and suitability for a rapid application in an extensive range of deformation conditions.

Acknowledgments

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