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# On the Capability of Logarithmic-Power Model for Prediction of Hot Deformation Behavior of Alloy 800H at High Strain Rates

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**Abstract:** In this study, the logarithmic-power model has been used to predict hot deformation behavior of alloy 800H at high temperatures. This is for the first time that the logarithmic-power model is examined to model the flow stress curves with negligible flow softening at high strain rates. To this end, flow stress curves of alloy 800H obtained at deformation temperatures from 850°C to 1050°C and at strain rates of 5 and 10 S<sup>-1</sup> were employed. The Johnson–Cook model and Shafiei constitutive equation were also used to prove the accuracy of the logarithmic-power model in prediction of flow stress curves of alloy 800H. Evaluation of mean error of flow stress at different deformation conditions showed that the logarithmic-power model can give a more precise estimation of flow stress curves than Johnson–Cook model. Furthermore, it was found out that the accuracy of the Logarithmic-power model and Shafiei constitutive equation was roughly the same in terms of maximum errors obtained in prediction of flow stress curves. Accordingly, it can be concluded that the logarithmic-power model can be employed as a comprehensive model for a wide range of deformation conditions.

**Keywords:** high strain rate, hot deformation, flow stress, mathematical modeling, logarithmic-power model, alloy 800H

## Introduction

Since the computer simulation of metal-forming processes is used increasingly in the industry, an accurate flow stress estimation is the preliminary requirement [1]. This is especially the case for hot forming processes,

which are usually more complicated in terms of occurrence of restoration mechanisms such as dynamic recrystallization (DRX) and dynamic recovery (DRV). The understanding of these phenomena can help one characterizing the hot deformation behavior of the metals and alloys. Considerable researches have been carried out to present constitutive equations for prediction of flow stress curves at high temperatures. However, most of the presented models developed to study material behavior at low strain rates (low Zener–Holloman parameters) are more appropriate for laboratory studies and consequently do not represent the material behavior at industrially relevant strain rates (higher than 1 S<sup>-1</sup>) [2–7]. Considering the importance of flow stress estimation in products' dimensional accuracy, it is inevitable to propose or modify a constitutive equation to predict material's behavior at high strain rates.

Recently, the authors of the present study have presented a novel constitutive equation (known as logarithmic-power model) for prediction of stress–strain curves with significant softening at low strain rates [8]. Moreover, this model has the capability to predict critical strain for initiation of DRX, transient strain associated with maximum softening rate in Meta-DRX, and peak strain simultaneously. Accordingly, it could be deduced that the logarithmic-power model can be employed as a comprehensive model for a wide range of deformation conditions with different predominant restoration mechanisms provided that this model gets approved for the high strain rate conditions.

To this end, our aim is to examine the ability of logarithmic-power model for prediction of stress–strain curves of alloy 800H at industrial deformation conditions.

## Mathematical modeling

For metals with DRV, the flow stress curves increase with strain in the initial stage of deformation and reach constant values in consequence of attaining the balance between work hardening and DRV (saturation stress,  $\sigma_s$ ).

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In addition, it is expected that at high strain rates, the concurrent deformation weakens the effect of softening on flow stress curves. Accordingly, the peak point becomes blur, and consequently, the flow stress curve is similar to the case of DRV. The derivative of the DRV stress–strain curve is positive, and then, phenomenologically the slope of stress–strain curve approaches zero.

The following nonlinear estimation of strain hardening rate vs. strain was taken into account to obtain logarithmic-power model:

$$\theta = d\sigma/d\varepsilon = 2b\varepsilon \ln(\varepsilon) + b\varepsilon + c/\varepsilon \quad (1)$$

where  $b$  and  $c$  are constants. For DRV-type stress–strain curves, using the boundary condition  $\theta = 0$  at  $\varepsilon = \varepsilon_s$ , eq. (1) can be rewritten in the following form:

$$\theta = d\sigma/d\varepsilon = b[2\varepsilon \ln(\varepsilon) + \varepsilon - A/\varepsilon] \quad (2)$$

where constant  $A$  equals to  $(2\ln(\varepsilon_s) + 1)/e$  and  $e$  is the Euler's number. It is noteworthy to mention that the value of constant  $A$  should be calculated at each deformation condition. Solution of differential eq. (2) with respect to  $\varepsilon$  within  $0 < \varepsilon \leq \varepsilon_s$  is:

$$\sigma = a + b[\varepsilon^2 \ln(\varepsilon) - A \ln(\varepsilon)] \quad (3)$$

where  $a$  and  $b$  are constants. Since constants  $a$  and  $b$  are stress variants, their values should be obtained at different temperatures and strain rates utilizing a least square fit of flow data.

## Results and discussion

In the present study, published stress–strain curves obtained from hot compression tests of alloy 800H at various deformation conditions were employed [9]. The curves were sampled at varying strain intervals (more frequently at locations of higher curvature) directly on

the published stress–strain curves. Chemical composition of the hot compression specimens of alloy 800H is presented in Table 1.

Stress–strain curves of alloy 800H obtained at various deformation temperatures from 850°C to 1050°C and at constant strain rate of 5 and 10 S<sup>−1</sup> were presented in Figure 1. It is notable to mention that the effect of adiabatic heating during high strain rate deformation has been taken into account, and consequently, the corrected flow stress curves were applied to verify the logarithmic-power model.

All flow stress curves exhibit a rapid initial increase to a saturation stress as a result of attaining the balance between strain hardening and predominant restoration mechanisms. The drop in flow stress with increasing deformation temperature may be attributed to the increase in the rate of restoration mechanisms and simultaneously decrease in the strain hardening rate, Figure 1. However, the increase in the flow stress with strain rate can be ascribed to the decrease in the time for restoration mechanisms to occur and subsequent increase in the work hardening rate.

The stepping stone to construct the flow stress curves using logarithmic-power model is to establish the dependence of the parameters  $a$  and  $b$  on temperature and strain rate for alloy 800H. In this regard, Figures 2 and 3 show the variations of these two constants with absolute temperature and strain rate, respectively. It is obvious that both constants of  $a$  and  $b$  vary linearly with absolute temperature. Linearity of these changes makes the interpolation of these constants possible at any intermediate temperatures. However, more data points are needed to establish a mathematical relation for the variations of constants  $a$  and  $b$  with strain rate for the alloy of this study. Also, it is obvious that both constants decrease with increasing deformation temperature and decreasing strain rate. As a result, it may be deduced that any factor resulting in Zener–Hollomon parameter increase would also lead to these two constants' increase.

As shown in Figures 4 and 5, the stress–strain curves predicted using logarithmic-power model are in a good agreement with experimentally measured ones for the alloy of this study. As it could be seen, at strain rates

**Table 1:** Chemical compositions (wt%) for the alloy of this study.

Element	C	Mn	Si	Ni	Cr	Al	Ti	Cu	N	Fe
Wt%	0.068	0.08	0.31	30.5	20.2	0.36	0.34	0.02	0.013	Bal.

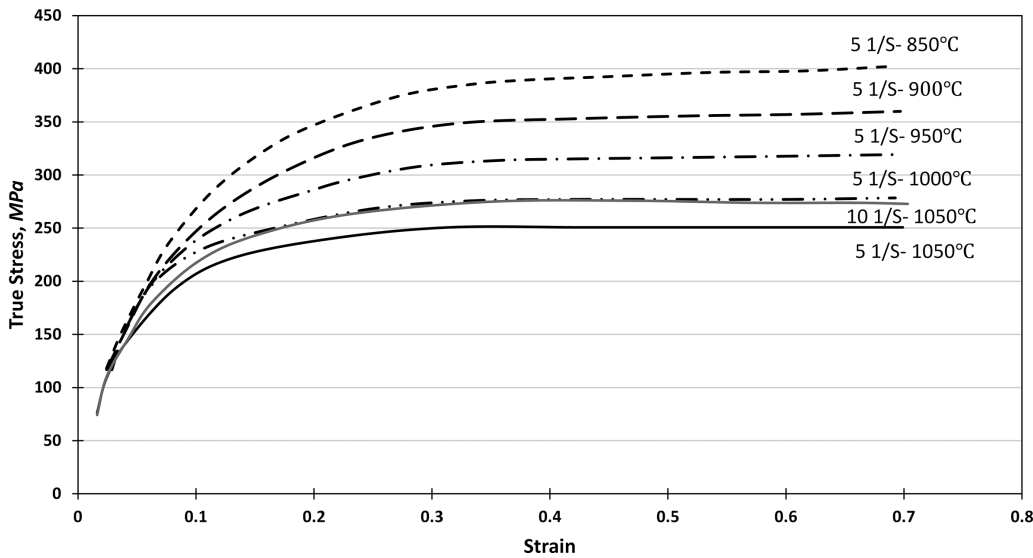


Figure 1: Flow stress curves of alloy 800H at different deformation conditions.

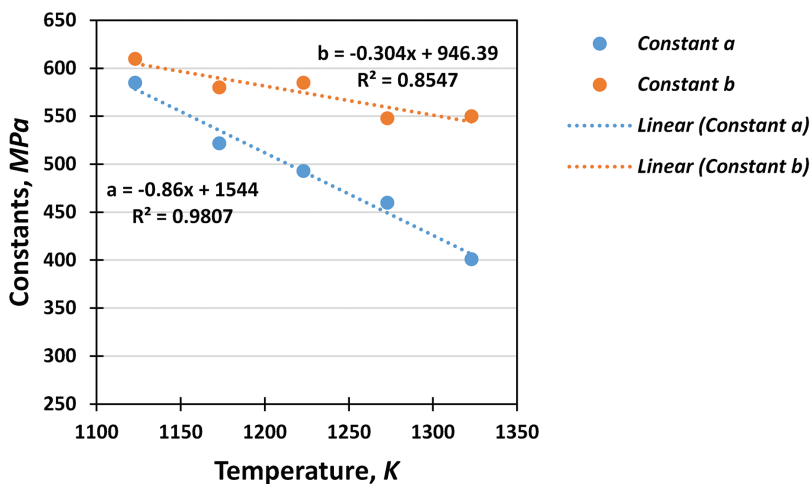


Figure 2: Dependency of logarithmic-power model constants on absolute temperature.

lower than 0.6, the logarithmic-power model predictions have a good accuracy although, at strain rates greater than 0.6 ( $0.6 < \dot{\varepsilon} < 0.7$ ), deviation from the experimental curves increases. In order to evaluate the precision of the logarithmic-power model, the predicted versus measured values of flow stress were plotted in Figure 6. The low error value, the slope of fitted line ( $\approx 1$ ), and the high value of correlation factor specify a high accordance between predicted and measured stress-strain curves.

To draw a comparison between the predictions of logarithmic-power model with previous models pertaining modeling of flow stress curves, Johnson-Cook model [10], as one of the most cited models in this field, was used. The full form of Johnson-Cook model can be expressed as follows:

$$\sigma = (A + B\dot{\varepsilon}^n) \left( 1 - \left( \frac{(T - T_r)}{(T_m - T_r)} \right)^m \right) \left( 1 + C \ln \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right) \right) \quad (4)$$

where  $A$  is yield stress of the material under reference deformation conditions (MPa),  $B$  is strain hardening constant (MPa),  $n$  is strain hardening coefficient, and  $m$  is thermal softening coefficient. In this experiment, the reference temperature  $T_r$  and strain rate  $\dot{\varepsilon}_0$  were selected as 1,123 K and  $5 \text{ S}^{-1}$ , respectively. The yield stress  $A$  is 36.63 MPa under such deformation conditions and the melting point  $T_m$  of alloy 800H is 1,633 K [8]. The Johnson-Cook equation can be re-written for the alloy of this study as follows:

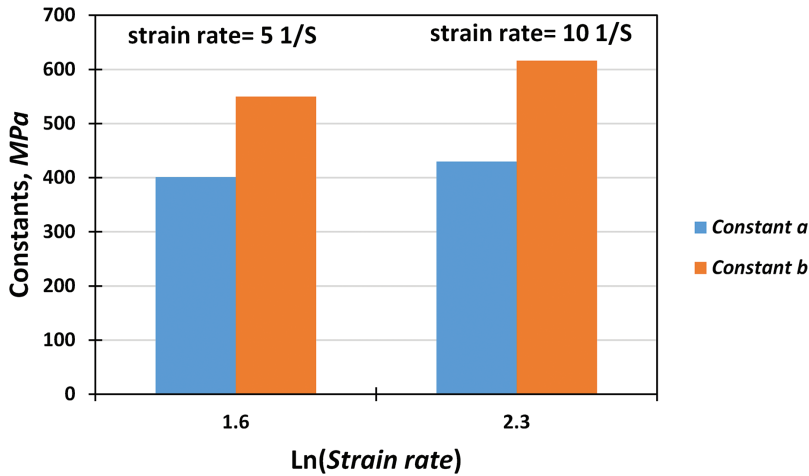


Figure 3: Dependency of logarithmic-power model constants on strain rate.

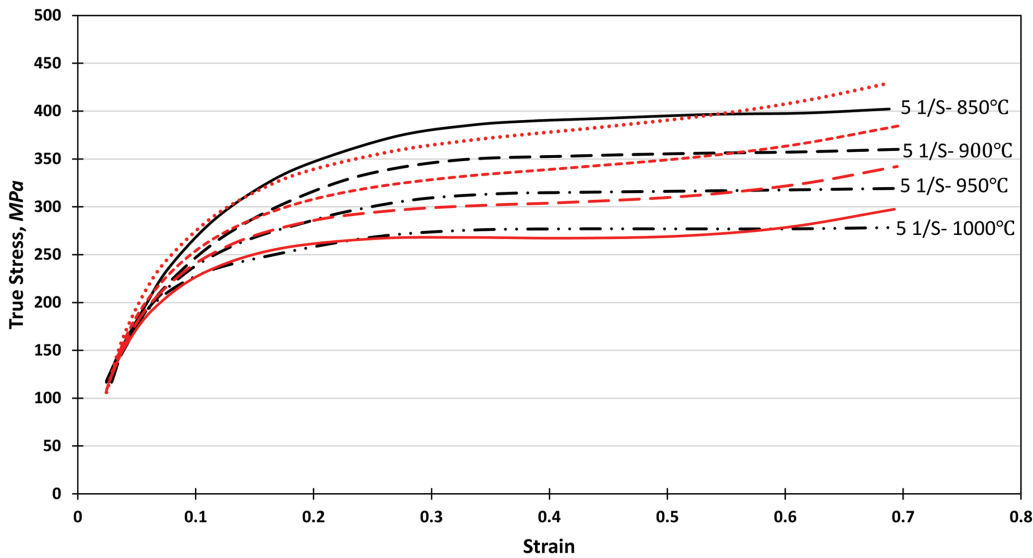
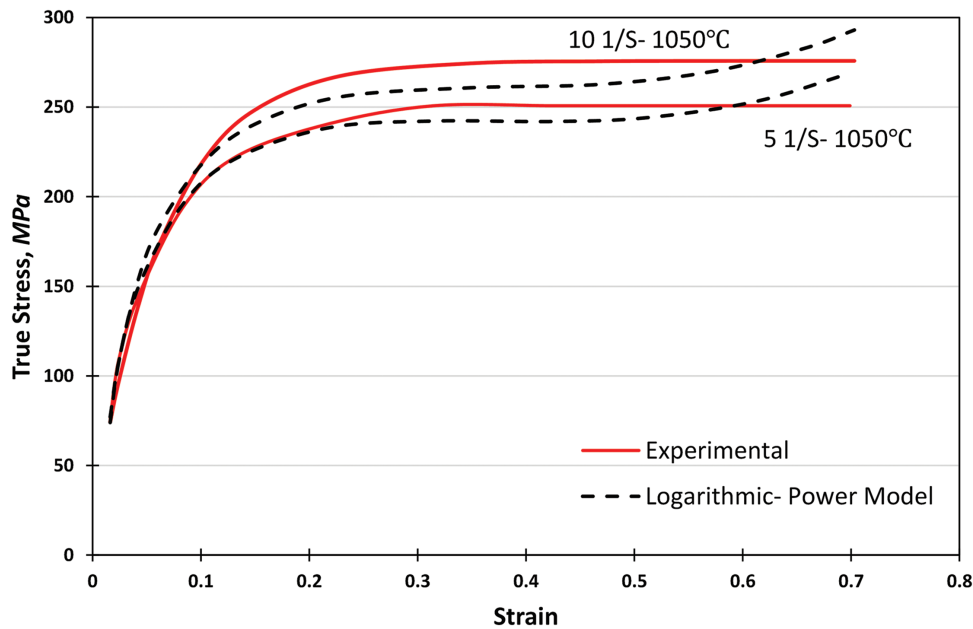


Figure 4: Comparison between predicted and experimentally measured flow stress curves obtained at different deformation temperatures and constant strain rate of  $5 \text{ S}^{-1}$ .

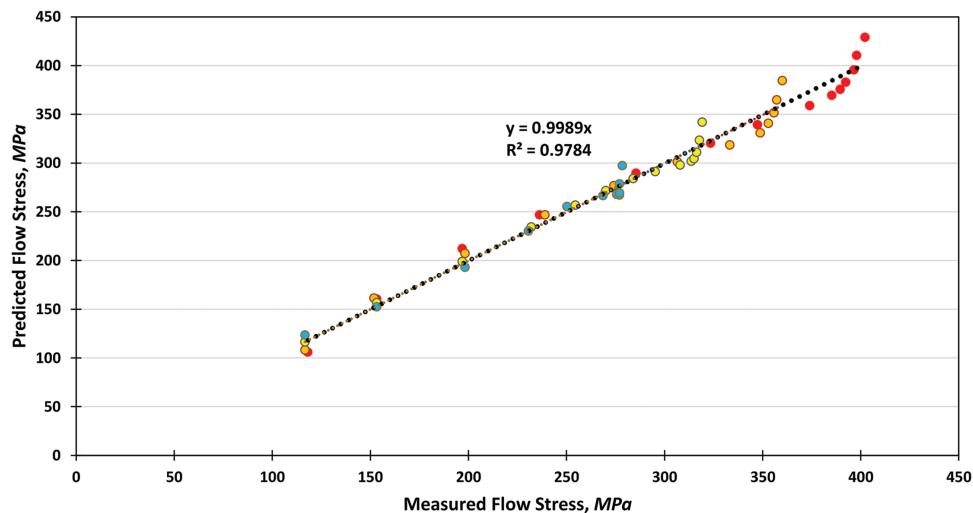
$$\sigma = (36.63 + 414\varepsilon^{0.203}) \left( 1 - \left( \frac{(T - T_r)}{(T_m - T_r)} \right)^{1.1} \right) \left( 1 + 0.093 \ln \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right) \right) \quad (5)$$

The procedures used to obtain the constants of eq. (8) can be found in full detail in Ref. [8]. The mean error in prediction of flow stress for both logarithmic-power and Johnson–Cook models was calculated to evaluate the accuracy of the predictions. Accordingly, Figures 7 and 8 show the predictions of these two models and their corresponding mean error analysis compared with

experimental flow stress curves at different deformation conditions. As can be seen, at low strains in which the strain hardening rate is high, the logarithmic-power function give a more precise estimation of flow stress curves for both deformation conditions. However, at strains greater than 0.6, the Johnson–Cook model is relatively more accurate. The reason behind the decrease in the accuracy of the logarithmic-power model at higher strains can be attributed to its initial assumptions for estimation of strain hardening rate variations with strain. Accordingly, Figure 9 shows the variations of strain hardening rate versus strain at different deformation conditions obtained using first derivative of eq. (3). As shown, the strain hardening



**Figure 5:** Comparison between predicted and experimentally measured flow stress curves obtained at different strain rates of  $5 \text{ S}^{-1}$  and  $10 \text{ S}^{-1}$  and at constant temperature of  $1,050^\circ\text{C}$ .

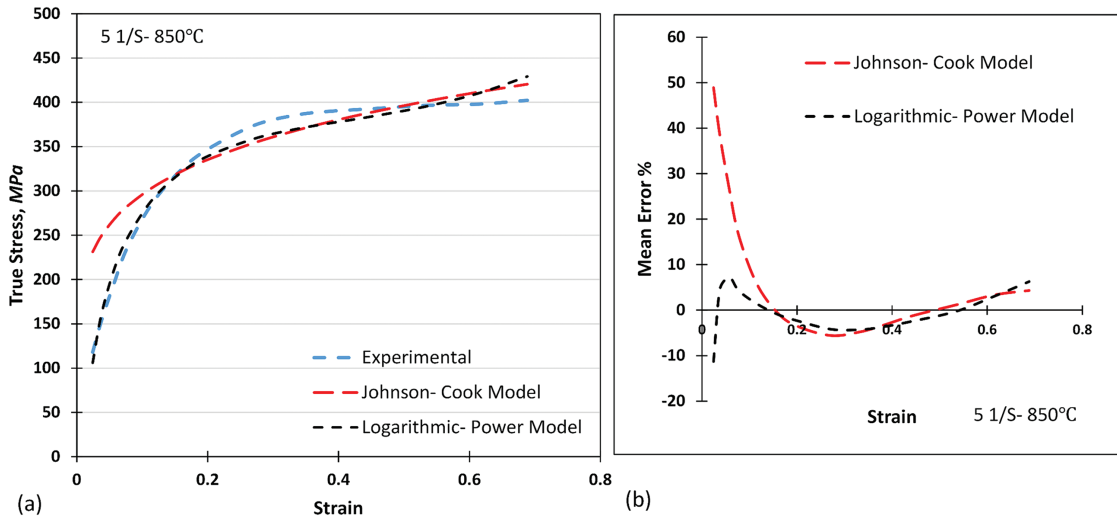


**Figure 6:** Predicted flow stress versus experimentally measured ones at different deformation conditions.

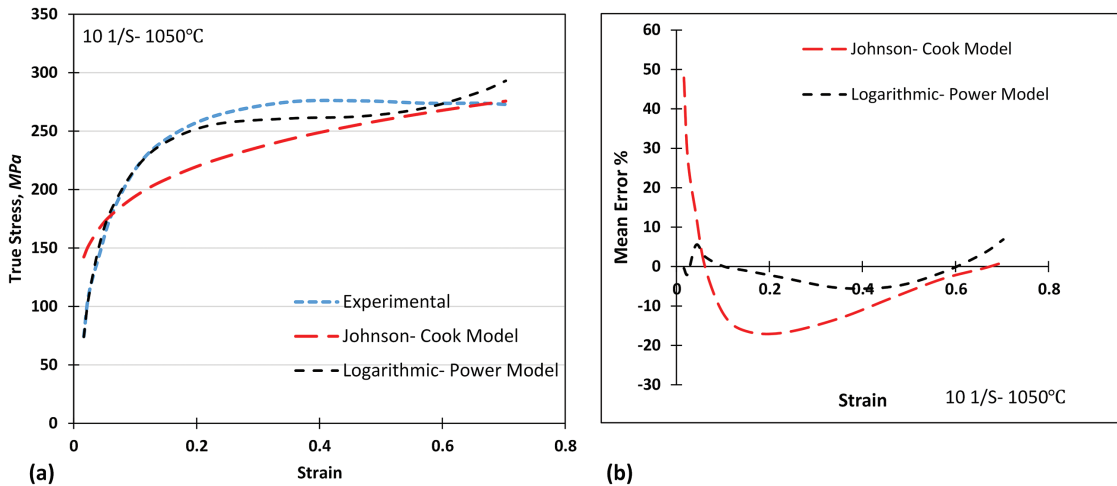
rate decreases with strain down to a minimum point representing the onset of steady state region of flow stress curves, followed by a gradual increase to a maximum point. This increase in strain hardening rate is responsible for a decrease in the accuracy of the predictions at higher strains. The maximum error of Johnson–Cook model is about 48% obtained at low strains, while the maximum error associated with the predictions of logarithmic-power model is about 11%. Hence, it could be concluded that the logarithmic-power model has a good ability to predict flow stress

curves at high strain rates with negligible softening although this model was presented to predict stress–strain curves with extended softening at low strain rates.

Recently, Shafiei [5] has proposed a constitutive equation based on linear estimation of strain hardening rate versus stress curves to predict stress–strain curves of alloy 800H at high temperatures and strain rates. Accordingly, it was concluded that, in comparison with the results obtained using Johnson–Cook model, Shafiei constitutive equation can give a more precise



**Figure 7:** Comparison between the predictions of logarithmic-power model and Johnson–Cook model at temperature of 850°C and strain rate of  $5 \text{ S}^{-1}$ : (A) flow stress curves and (B) mean error in prediction of flow stress.



**Figure 8:** Comparison between the predictions of logarithmic-power model and Johnson–Cook model at temperature of 1050°C and strain rate of  $10 \text{ S}^{-1}$ : (A) flow stress curves and (B) mean error in prediction of flow stress.

estimation of flow stress curves at different deformation conditions for alloy 800H. According to Shafiei constitutive equation, stress–strain curves can be modeled using the following expression:

$$\sigma = \sigma_s - (\sigma_s - \sigma_0) \exp(m(\varepsilon_0 - \varepsilon)) \quad (6)$$

where  $\sigma_s$ ,  $\sigma_0$ , and  $\varepsilon_0$  are saturation stress, initial stress, and initial strain, respectively. In addition, the values of constant  $m$  can be estimated using  $\ln((\sigma_s - \sigma)/(\sigma_s - \sigma_0))$  versus  $\varepsilon_0 - \varepsilon$  plots at different deformation conditions. Considering deformation temperature of 850°C and strain

rate of  $5 \text{ S}^{-1}$ , Shafiei constitutive equation can be rewritten as follows for the studied alloy:

$$\sigma = 402.1 - 283.9 \exp(11.74(\varepsilon_0 - \varepsilon)) \quad (7)$$

Accordingly, the flow stress curves predicted using the logarithmic-power model and Shafiei constitutive equation for deformation temperature of 850°C and strain rate of  $5 \text{ S}^{-1}$  were shown in Figure 10(a). As can be seen, both models underestimate the flow stress at very low strains, and consequently, maximum error in prediction of flow stress curves was obtained at this region for both models (11% for logarithmic-power model and 10.2% for Shafiei

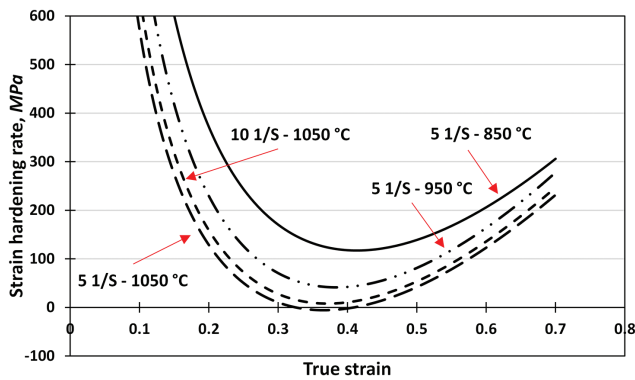


Figure 9: Variations of strain hardening rate versus strain at different deformation conditions obtained using logarithmic-power model.

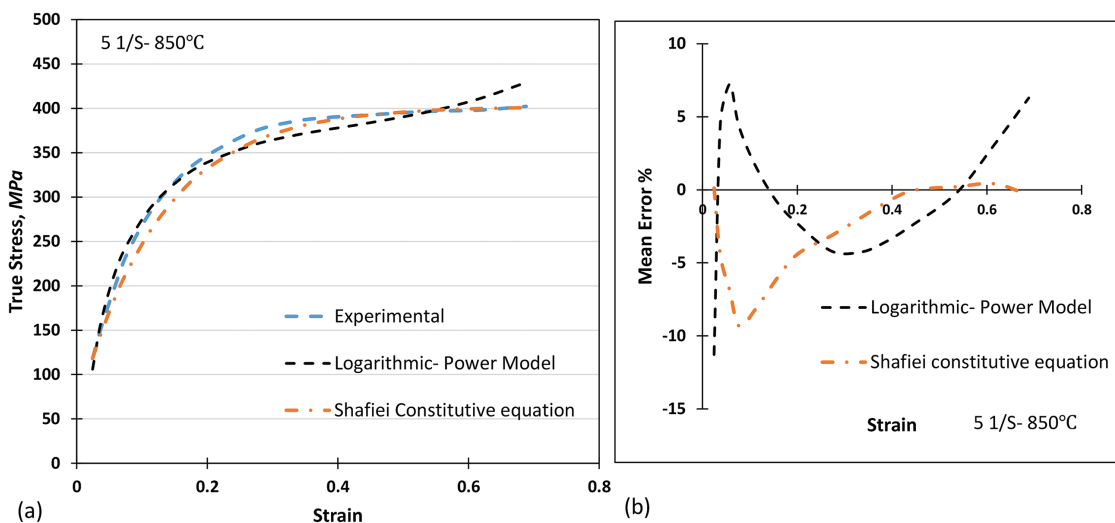


Figure 10: Comparison between the predictions of logarithmic-power model and Shafiei constitutive equation at temperature of 850°C and strain rate of  $5 \text{ S}^{-1}$ : (A) flow stress curves and (B) mean error in prediction of flow stress.

constitutive equation). Consequently, it is obvious that both models can provide a roughly the same estimation of flow stress curves in terms of maximum error values. In addition, considering the average error values for both models, Shafiei constitutive equation can give a somewhat more precise prediction of flow stress curves, Figure 10(b). On this basis, it can be concluded that, although Shafiei constitutive equation can provide a slightly better estimation of stress–strain curves than the logarithmic-power model for the alloy of this study, it can only be employed to predict the flow stress curves with negligible softening due to its linear estimation of strain hardening plots, and consequently, it is not appropriate to be used for a wide range of deformation conditions at which significant flow softening may occur.

## Conclusions

- It was found out that the logarithmic-power model can also provide a precise estimation of flow stress curves at high strain rates with negligible flow softening.
- The reason behind the decrease in the accuracy of the logarithmic-power model at higher strains can be attributed to its initial assumptions for estimation of strain hardening rate variations with strain.
- In comparison with Johnson–Cook model, the logarithmic-power model can give a more accurate estimation of flow stress curves at the studied deformation conditions.
- The error analysis showed that Shafiei constitutive equation provides a slightly better estimation of flow



stress curves than logarithmic-power model; however, its application is limited to the conditions with negligible flow softening.

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