High Temp. Mater. Proc. 2013; 32(2): 133–138 DOI 10.1515/htmp-2012-0041

New Phenomenological Model of Hot Mean Flow Stress

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Abstract. Based on the measurement of roll forces during the laboratory hot rolling of flat samples graded in thickness, the new phenomenological type of mean flow stress model was developed and applied on plain-carbon and HSLA steels. The obtained models describe with a very good accuracy the hot deformation resistance characteristics in the temperature range 1123 to 1463 K, large effective strain, and strain rate in the useful range of approximately 10 to 100 s^{-1}. Difficulty in the mathematical description of the influence of temperature on mean flow stress in the wide range of temperature by a single equation was solved by introducing another constant in the temperature member of the conventional model. The newly proposed model solves by phenomenological means a frequent problem of heteroscedasticity of relative deviations between the calculated and experimental values of mean flow stress values depending on temperature. It becomes more reliable from the viewpoint of the operational application, e.g. fast prediction of mean flow stress values and power/force parameters necessary in the steering systems of hot rolling mills.

Keywords. Hot forming, HSLA steel, mean flow stress, mathematical model.

PACS®(2010). 81.05.Bx, 81.20.Hy, 81.40.Jj.

1 Introduction

The ability to reliably predict power/force forming parameters, based on the knowledge of models of hot deformation resistance of various materials, is very useful for designers and users of forming equipment. Interest in these models is stable, as it is reflected in the publications from the last few years. Flow stresses are tested in trials on plastometers, most often by compression or tension tests [1, 2]. Stress curves are described by constitutive models [3, 4], or dislocation-based models [5]. Attention is paid also to the influence of phase composition on deformation behaviour of steels [6, 7].

Majority of published works concerns description of flow stress, however, simple models of mean flow stress (MFS), enabling quicker prediction of rolling forces are more advantageous, particularly for implementation into rolling mills control systems. Based on measurement of forces in the laboratory rolling of flat samples graded in thickness, the effective methods of description of the MFS values were developed [8] and applied to many steel grades, some iron aluminides, magnesium alloy AZ31 etc. – see for example [9–13]. For the most part, it is not difficult to describe mathematically with a good accuracy the influence of deformation or strain rate on MFS, but it is more questionable to describe the functional dependence of MFS on temperature, in particular in the case of its wide range. Phase transformations and other physico-metallurgical processes often do not enable to describe this relation by a single equation in the whole interval of the applied deformation temperatures. That's why the deformation behaviour of the material must be described by special equations for separated temperature subareas, which is obviously disadvantageous, e.g. in case of quick prediction of the forming forces by the control system of the rolling mill. Therefore, the aim of the works was to propose a new phenomenological equation for the description of deformation resistance in such temperature range where conventional procedures do not lead to sufficiently exact results.

2 Experimental Works

Two continuously cast HSLA steels of type C-Mn-Nb-V and similar C-Mn steel without microalloying elements were used – see Table 1.

Flat samples with thickness graded in size (total length 120 mm, width 25 mm, thickness 4.8 / 5.5 / 6.7 mm in individual steps) were manufactured. Forming of each sample provided the data on roll forces for three various reduc-

Steel	C	Mn	Si	Al	V	Nb
C-Mn	0.25	0.76	0.36	0.019	–	–
0.03Nb	0.21	1.38	0.38	0.025	0.10	0.03
0.05Nb	0.18	1.31	0.36	0.030	0.09	0.05

Table 1. Chemical composition of the tested steels in wt. %.

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Received: June 04, 2012. Accepted: June 16, 2012.

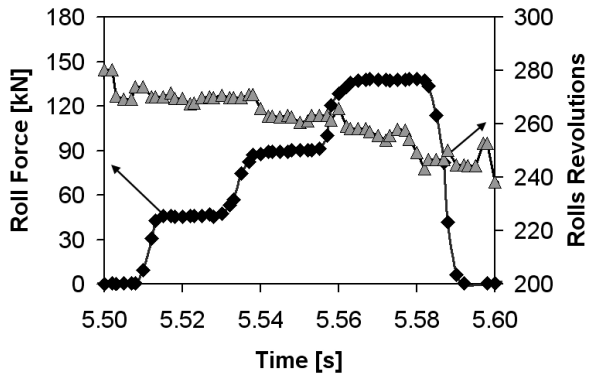

Figure 1. Experimental data registered during rolling of a sample made from steel 0.05Nb (temperature 1173 K, nominal rolls revolutions 280 rpm).

tions, thus the experiment could have been made more efficiently in this way. Each sample was measured and then heated in the furnace to the austenitizing temperature of 1473 K within 30 minutes. After partial cooling, controlled by the optical thermometer, the sample was inserted for 2 minutes into another electric furnace heated to the forming temperature. The heated sample was rolled immediately after discharging from the furnace in the stand A of the laboratory mill TANDEM (the working rolls had diameter 158 mm) [14]. During rolling of each sample the temperature was changed (from 1123 to 1463 K), together with roll gap adjustment (and thus the total strain of individual grades of the sample) and nominal revolutions of rolls varied in the range of 40–330 rpm (and hence the strain rate values). Roll forces and the actual speed of roll rotation were computer-registered – see Figure 1 for example. The model for mean flow stress σ_m [MPa] was developed afterwards, in dependence on the temperature T [K], the mean equivalent strain rate γ [s^{-1}] and the equivalent strain ε defined as [7]:

$$\varepsilon = \frac{2}{\sqrt{3}} \cdot \ln\left(\frac{H_0}{H_1}\right), \tag{1}$$

where H_0, or H_1 [mm] is the initial or exit thickness of the rolling stock. Equation (1) was derived for the calculation of equivalent strain in rolling of flat specimens with such ratio of height to width so that it could be possible to suppose the plane strain with a sufficient accuracy. In this case the zero spread value and equality of the height and longitudinal strain, expressed in absolute values, is really considered. Then equivalent strain is calculated in a simplified form from the knowledge of true (logarithmic) height deformation $\ln(H_0/H_1)$ using the recalculating coefficient $2/\sqrt{3} = 1.155$.

The MFS values were calculated from roll forces based on the knowledge of the forming factor for a particular mill stand. For each step of the given sample, roll force values were automatically determined. After cooling down the rolled stock, the width and thickness of the individual steps were also measured. All variables stated above were put down in the Excel table and recalculated by macros on values of the equivalent strain ε and mean equivalent strain rate γ (according to [15]):

$$\gamma = \frac{v_r}{l_d} \cdot \varepsilon, \tag{2}$$

where v_r [mm·s^{-1}] is the real circumferential speed of rolls with radius R [mm], $l_d = \sqrt{R \cdot (H_0 - H_1)}$ is the roll bite length [mm]. This simple Krejndlin's formula gives results that are identical with the classic formula according to Sims [16].

The mean flow stress σ_m [MPa] was calculated according to the modified equation [17]:

$$\sigma_m = \frac{F}{Q_{FR} \cdot l_d \cdot B_m}, \tag{3}$$

where F [N] is roll force, Q_{FR} is a geometric forming factor corresponding to the particular mill stand, l_d [mm] is the roll bite length and B_m [mm] is the mean width in the given place of the rolling stock (the average of widths before and after rolling). The reliability of the calculation of σ_m value is mostly influenced by the accuracy of the forming factor estimation, which actually transfers specific values of the deformation resistance to the values of equivalent stress – see [18] for example. Values of Q_{FR} for both stands of the laboratory mill Tandem were acquired and described by [8] in relation to geometric factor l_d/H_m by the equation of type:

$$Q_{FR} = J - K \cdot \exp\left(-L \cdot \frac{l_d}{H_m}\right) + \exp\left(M \cdot \frac{H_m}{l_d}\right), \tag{4}$$

where $J \ldots M$ are constants for the given facility, H_m [mm] is the mean thickness of the rolling stock in the given place (the average of thicknesses of the given step before and after rolling). These specially developed models, based on personal laboratory experiments, have been proved in terms of the accuracy of the results much better than universal, earlier designed, models (e.g. the model of Ford and Alexander [19]).

3 Mathematical Processing and Discussion of the Experimental Results

Based on the previous own experience a simple model for description of hot MFS of the investigated steels was chosen:

$$\sigma_m = A \cdot \varepsilon^B \cdot \exp(-C \cdot \varepsilon) \cdot \gamma^D \cdot \exp(-F \cdot T), \tag{5}$$

where $A \ldots F$ are material constants, obtained by multiple non-linear regression of the experimental data in statistical

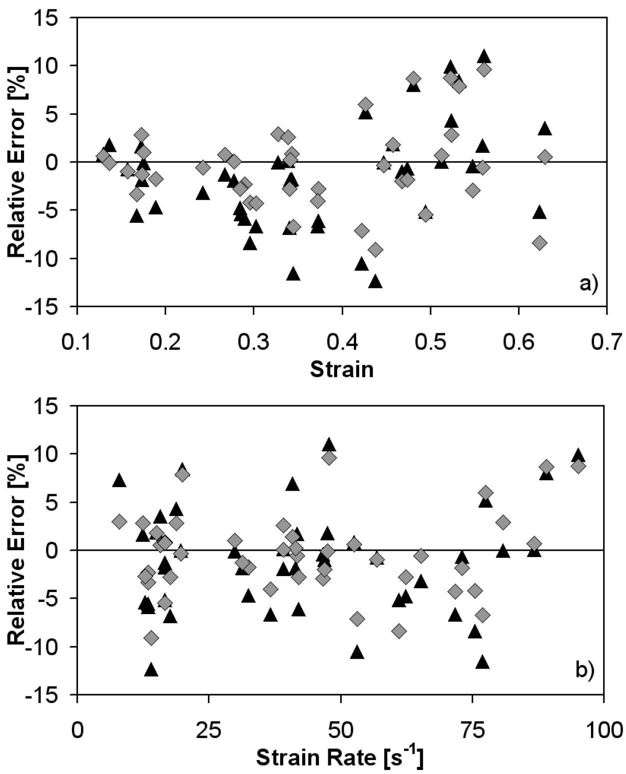

Figure 2. Relative errors of the models according to Equation (5) for steel 0.03Nb
a) depending on strain
b) depending on strain rate
(black triangles – complex model; gray diamonds – simplified model).

package UNISTAT 5.6. This model takes into consideration the hardening as well as the dynamic softening. The entire experimental data sets corresponded to the strains 0.09–0.63 and to the strain rates 8–99 s^{-1}. By their statistical processing and applying methods of the non-linear regression several mathematical models were obtained.

Due to the fact that dependence of MFS on deformation has in principle flatter course than the corresponding flow stress curve, the possibility of efficient simplification of the relation of the Equation (5) type was confirmed again, which will be documented on an example of the steel 0.03Nb. The constants determined by regression with use of the Equation (5) were the following: $A = 2293$, $B = 0.007$; $C = 0.66$, $D = 0.08$, $F = 0.0021$ at coefficient of determination $R^2 = 0.966$. If we simplify this equation by neglecting the term representing dynamic softening (it means $C = 0$), the computed constants are the following: $A = 1620$, $B = -0.18$, $D = 0.08$, $F = 0.0022$ with better accuracy of description ($R^2 = 0.982$). Negative value of the constant B in the strengthening term naturally lacks physical meaning. This problem arises at purely phenomenological description of experimental data in the case that some variables are not mutually entirely independent, although they act in this way in the relevant relation – in this case this concerns strain rate, which is directly proportional to strain (see Equation (2)). Strain term and strain rate term then mutually compensate each other, which may at purely regression processing of data lead to negative value of the constant B.

The coefficients of determination of Equation (5) in both versions (complex or simplified) for the steel 0.03Nb seem to be relatively very high. However, these parameters themselves do not implicate the situation, when an unfavourable trend of deviations between the measured and calculated values of MFS exists, in relation to some of the independent variables. This is a problem with the so called heteroscedasticity, which does not cause ordinary least squares coefficient estimates to be biased. In statistics, a sequence of random variables is heteroscedastic, if the random variables have different variances. So the plots of such relative deviations or errors [%] (defined as a proportional difference between the experimentally measured and calculated MFS value, divided by the measured MFS value) can give a useful view on the problem of accuracy and fitness of the developed models. Diagrams in Figure 2 demonstrate distribution of these relative errors in dependence on the strain and strain rate. It is evident, that complex model of the Equation (5) type shows lower accuracy and namely bigger problems with heteroscedasticity of relative deviations of predicted MFS values from experimental data. Next procedure was therefore preferentially focused on elimination of problems with heteroscedasticity of developed models.

The search of possibilities for an optimal modifications of relations of the Equation (5) type finally resulted in proposal for simplification of this equation by neglecting the term representing dynamic softening, and conversely to that, by complicated modification of the thermal term into final form:

$$\sigma_m = A \cdot \varepsilon^B \cdot \gamma^D \cdot \exp(-F \cdot T^H), \qquad (6)$$

where $A \ldots H$ are material constants, obtained by multiple non-linear regression of the experimental data. Material constants for the steel 0.03Nb calculated by regression analysis with use of Equation (6) were the following: $A = 219$, $B = -0.17$, $D = 0.08$, $F = 9.7 \cdot 10^{-13}$, $H = 3.82$. Although the obtained coefficient of determination $R^2 = 0.983$ is slightly higher than in the case of the above described simplified model, in this manner the problems with heteroscedasticity, which occurred particularly at higher temperatures, were fundamentally eliminated, as it is evident from the diagram shown in Figure 3.

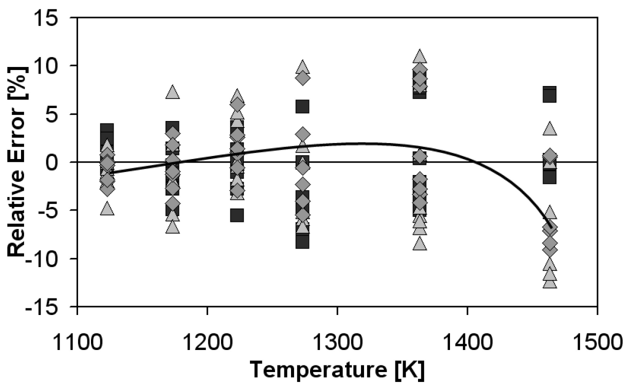

Figure 3. Relative errors of the models according to Equations (5) and (6) for steel 0.03Nb, depending on temperature (gray triangles with a trend represented by black curve – complex model; gray diamonds – simplified model; black squares – Equation (6) with modified temperature member).

Material constants were determined analogically by Equation (5) – simplified model, and by Equation (6) – model with modified temperature member for other investigated steels. The results are summarised in Table 2.

Diagrams in Figures 4 and 5 compare graphically relative errors of individual models for the steels CMn and 0.05Nb. Model newly proposed according to the Equation (6) with modified temperature member is in all cases more accurate than the models according to the Equation (5), which is evident also from the values of the coefficient of determination R^2 in Table 2. In case of the steel C-Mn without microalloying elements, the advantages of the model according to the Equation (6) are generally small, because even the simplified model according to the Equation (5) shows low heteroscedasticity (with bigger deviations only at the highest deformation temperature). The advantages of the new model with modified temperature member manifested themselves in case of the steel 0.05Nb, namely by overall reduction of relative deviations (up to 10%), and also by elimination of distinctive heteroscedasticity in dependence on the forming temperature.

4 Summary

Based on the measurement of roll forces during the laboratory rolling of flat samples graded in thickness, new MFS models of the HSLA steel of the type C-Mn-Nb-V, as well as of plain-carbon C-Mn steel, were developed. These models describe with a very good accuracy the hot deformation resistance in the temperature range from 1123 to 1463 K, equivalent strain up to about 0.6 and strain rates in the range approximately 10 to 100 s^{-1}. Difficulty in the mathematical description of the influence of temperature on MFS in the wide range of temperature by a single equation was solved by introducing another constant in the temperature member of the conventional equation $\sigma_m = f(\varepsilon, \gamma, T)$. Relative deviations of the MFS values calculated according to the Equation (6) from the experimentally determined values do not exceed 10%, which, together with coefficients of determination that always exceed 0.967, may be assessed as very good accuracy.

The possibility of making the $\sigma_m = f(\varepsilon, \gamma, T)$ models simpler in description of the combined effect of strain and strain rate was confirmed (compare with [9, 12] for example).

The newly proposed model of the Equation (6) type solves a frequent problem of heteroscedasticity of relative deviations between the calculated and experimental values of MFS in the case of the temperature effect. It is, however, necessary to draw attention to distinctly phenomenological character of the newly proposed model, i.e. that although construction of its temperature member improves the results of regression analysis of experimental data, strictly speaking it lacks physical-metallurgical basis. On the other hand, the new model is simple enough and becomes more convenient and reliable from the viewpoint of the operational application – very fast prediction of MFS and power/force parameters of hot rolling for example.

Executed works were methodologically oriented and they were not aimed yet at comparison itself of deformation behaviour of investigated steels and on explanation of influence of specific chemical composition on deformation behaviour of plain-carbon, or HSLA steels.

Steel	Model	A	B	C	D	F	H	R^2
C-Mn	Equation (5)	3771	0.12	0	0.07	0.0024	–	0.967
C-Mn	Equation (6)	948	0.12	–	0.08	$3.1 \cdot 10^{-6}$	1.85	0.969
0.03Nb	Equation (5)	1620	−0.18	0	0.08	0.0022	–	0.982
0.03Nb	Equation (6)	219	−0.17	–	0.08	$9.6 \cdot 10^{-13}$	3.82	0.983
0.05Nb	Equation (5)	1830	0.10	0	0.07	0.0022	–	0.972
0.05Nb	Equation (6)	357	0.09	–	0.07	$6.4 \cdot 10^{-13}$	3.98	0.988

Table 2. Material constants in individual models.

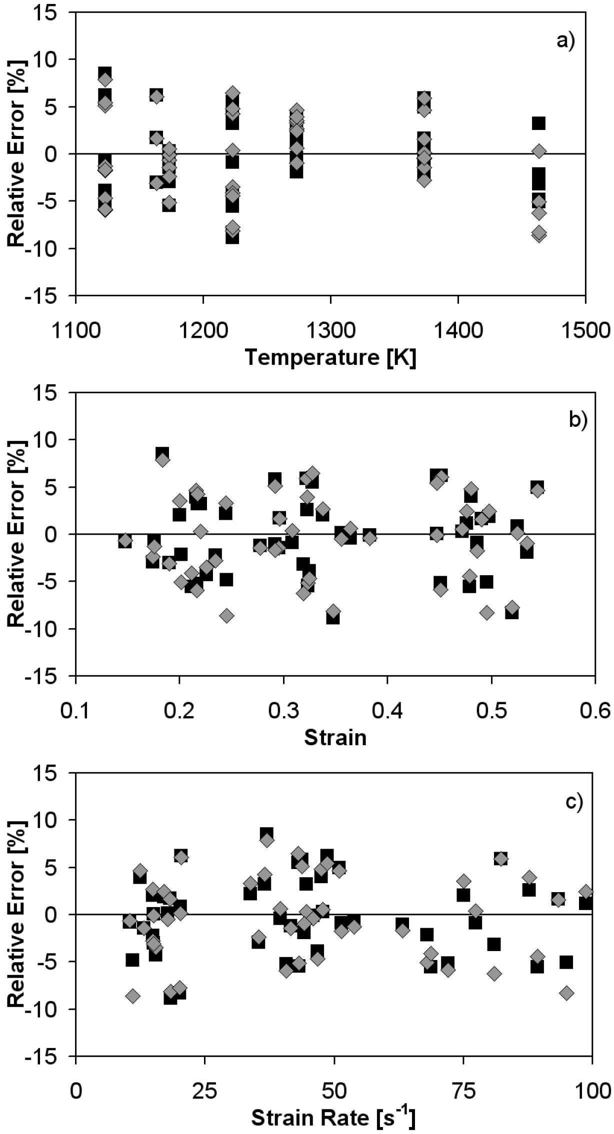

Figure 4. Relative errors of the models according to Equations (5) and (6) for steel C-Mn; a) depending on temperature; b) depending on strain; c) depending on strain rate (gray diamonds – simplified model according to Equation (5); black squares – Equation (6)).

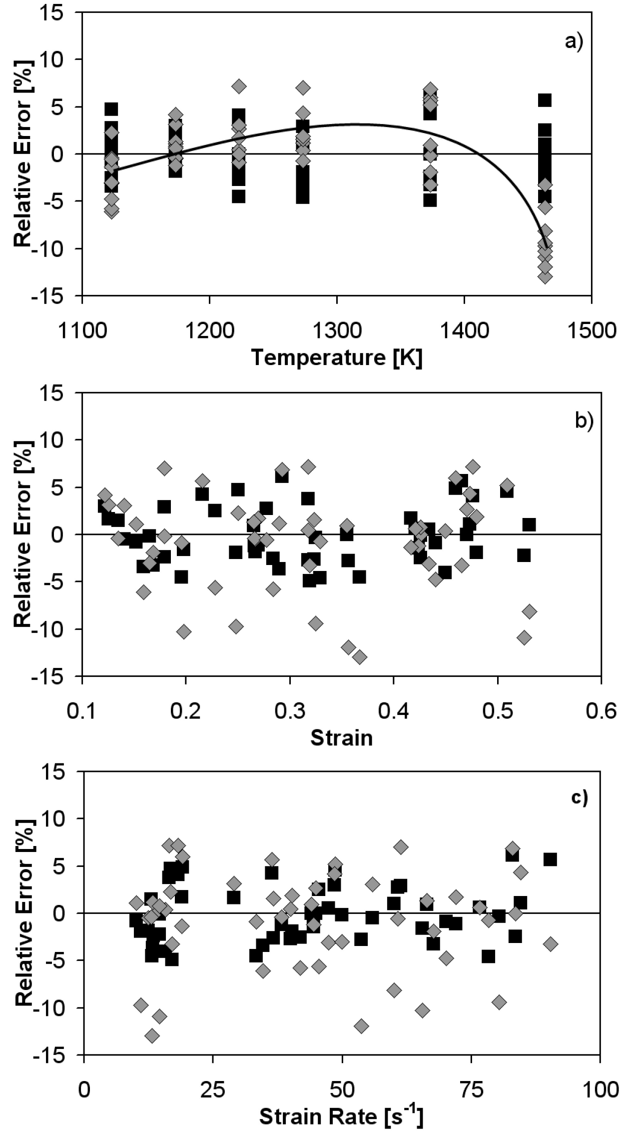

Figure 5. Relative errors of the models according to Equations (5) and (6) for steel 0.05Nb; a) depending on temperature; b) depending on strain; c) depending on strain rate (gray diamonds with a temperature trend represented by black curve – simplified model according to Equation (5); black squares – Equation (6)).

Acknowledgments

This work was realized within the projects CZ.1.05/2.1.00/01.0040 "Regional Materials Science and Technology Centre" (within the frame of the operation programme "Research and Development for Innovations" financed by the Structural Funds and from the state budget of the Czech Republic) and TA01010838 (Technology Agency of the Czech Republic).

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