

Research Article

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Atmospheric refractivity estimation from AIS signal power using the quantum-behaved particle swarm optimization algorithm

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Abstract: This paper proposes a new refractivity profile estimation method based on the use of AIS signal power and quantum-behaved particle swarm optimization (QPSO) algorithm to solve the inverse problem. Automatic identification system (AIS) is a maritime navigation safety communication system that operates in the very high frequency mobile band and was developed primarily for collision avoidance. Since AIS is a one-way communication system which does not need to consider the target echo signal, it can estimate the atmospheric refractivity profile more accurately. Estimating atmospheric refractivity profiles from AIS signal power is a complex nonlinear optimization problem, the QPSO algorithm is adopted to search for the optimal solution from various refractivity parameters, and the inversion results are compared with those of the particle swarm optimization algorithm to validate the superiority of the QPSO algorithm. In order to test the anti-noise ability of the QPSO algorithm, the synthetic AIS signal power with different Gaussian noise levels is utilized to invert the surface-based duct. Simulation results indicate that the QPSO algorithm can invert the surface-based duct using AIS signal power accurately, which verify the feasibility of the new atmospheric refractivity estimation method based on the automatic identification system.

Keywords: refractivity estimation, automatic identification system, surface-based duct, quantum-behaved particle swarm optimization algorithm

Abbreviations

AIS	Automatic identification system;
IMO:	International Maritime Organization;
PSO:	Particle swarm optimization;
QPSO	Quantum-behaved particle swarm optimization;
RFC:	Refractivity from clutter;
SOLAS:	Safety Of Life At Sea;
SPE:	Standard parabolic equation
SSFT:	Split-step Fourier transform;

1 Introduction

Atmospheric ducting is an abnormal propagation phenomenon resulting from the varying refractivity of air, which can cause anomalous propagation of electromagnetic waves. In the marine environment, there is a high probability of ducts occurring at any time and in any sea area, which has a significant impact on the performance of radar and communication systems [1]. Therefore, it is important to estimate the atmospheric refractivity profile for the performance evaluation and prediction of maritime radar and communication systems.

Currently, the method of remote sensing detection is mainly used to estimate atmospheric refractivity profile, and refractivity from clutter (RFC) technique has been an active field of research. RFC estimate refractivity profile of the atmosphere from the sea surface reflected radar clutter signal [2–6]. This method holds the characteristics of remote, indirect, real-time, cheap and convenient. Real-time detection can be achieved without increasing any additional equipment by RFC, because it depends on radar measurements only. In recent years, RFC has become a hot research method of estimating atmospheric refractivity profile. Although RFC has certain advantages, it actually has the following limitations [7]: In order to estimate the atmospheric refractivity profile, radar need to transmit

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high-power signals for active detection, which easily interferes with the normal operation of electronic equipment in the relevant area. In addition, the uncertainty of the currently utilized normalized radar cross section model of the sea surface will severely limit the accuracy of the inversion. When sea surface and weather (volume) clutter is hard to separate such as in precipitation, the shortcoming of the current RFC approaches is evident.

However, if we use the automatic identification system (AIS) for refractivity profile inversion, these problems do not occur. AIS system is a one-way communication system that depends on radio wave propagation for the transmission of AIS signals and the propagation path of AIS signals will be influenced by the atmospheric conditions [8–10]. Referring to the idea of RFC, this paper proposes a new refractivity profile estimation method based on the AIS signal power to improve the accuracy of estimating the refractivity profile. Using existing shipboard and shore-based AIS equipment and AIS networks, no additional equipment is required, the cost is lower, and it is convenient to operate. It can be accurately and efficiently invert the distribution of atmospheric ducts over the entire sea surface.

Obviously, atmospheric refractivity profile estimation is an inverse problem, and the powerful and efficient quantum-behaved particle swarm optimization (QPSO) algorithm is presented to estimate the surface-based duct. QPSO algorithm [11–13] is a type of particle swarm optimization (PSO) algorithm with quantum behaviour that proposed on the basis of classical particle swarm optimization algorithm, which is simple, effective and converges rapidly. Because particles in quantum space satisfy unique properties of aggregation state, there is no definite trajectory when particles move, which enables particles to search for global optimal solutions to the entire feasible solution space. Therefore, the global search performance of the QPSO algorithm is much better than that of classical PSO algorithm.

The remainder of this article is organized as follows. In Section 2, the automatic identification system is introduced. In Section 3, the forward propagation model used to calculate the AIS signal power under maritime atmospheric duct conditions is provided. In Section 4, a new method to invert the atmospheric refractivity profile using the AIS signal power is proposed. Numerical results are analyzed and discussed in Section 5, while the paper is concluded in Section 6.

2 Automatic identification system

In 2000, as a part of the Safety Of Life At Sea (SOLAS) regulations [14], the International Maritime Organization (IMO) require AIS to be fitted aboard all ships of 300 gross tonnage and upwards engaged on international voyages, cargo ships of 500 gross tonnage and upwards not engaged on international voyages and all passenger ships irrespective of size. It came into full force on December 31, 2004, and this system is known as Class A AIS, which can automatically provides vessel information, including the vessel's identity, type, position, course, speed, navigation status and other safety-related to other ships and to shore stations in its surroundings. It also receives such information from similarly fitted ships and exchanges data with shore-based facilities automatically. In 2007, Class B AIS was introduced for small vessels, including pleasure boats. Class B messages generally contain less information than Class A messages. However, they all provide essential safety information.

Two international channels are allocated for AIS and both frequencies are in the very high frequency band. They are 161.975 MHz and 162.025 MHz. As Class A AIS system is mandatory for all ships specified by the IMO, we only consider this system in this paper.

3 Forward propagation modelling

3.1 Atmospheric ducts

Since meteorological elements such as temperature, humidity and pressure in the atmospheric environment have vertical stratification unevenness characteristics, the atmospheric refractivity also has vertical stratification uneven characteristics. Therefore, electromagnetic waves propagating in the atmosphere are affected by atmospheric refraction. Atmospheric refractivity is defined by [15]

$$N = (n - 1) \times 10^6 \quad (1)$$

where n is the refractive index.

In order to take the influence of the curvature of the earth into consideration, the modified refractivity M is introduced and defined as

$$M = N + \left(\frac{z}{R_0} \right) \times 10^6 = N + 0.157z \quad (2)$$

where $R_0 = 6370$ km is the radius of the earth, and z is the height above sea level in m.

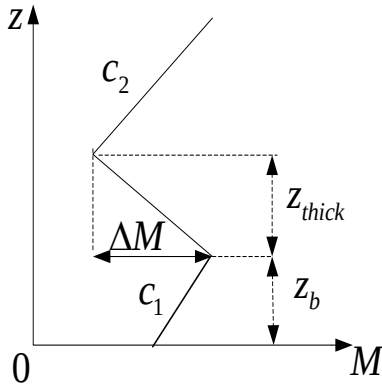


Figure 1: Atmospheric modified refractivity profile of surface-based ducts

The propagation characteristics of electromagnetic waves in the clear-air troposphere mainly depend on the modified refractivity gradient dM/dz . When the modified refractivity gradient dM/dz is less than zero, the curvature of the electromagnetic wave propagation path will exceed the curvature of the surface of the earth, that is, there is an atmospheric duct phenomenon. Since evaporation ducts have little effect on AIS transmission, this paper only considers the surface-based duct, which is illustrated in Figure 1.

As in Figure 1, the atmospheric modified refractivity profile of the surface-based duct can be modelled with a tri-linear curve:

$$M(z) = M_0 + \begin{cases} c_1 z & z \leq z_b \\ c_1 z_b - \Delta M \frac{z - z_b}{z_{thick}} & z_b < z < z_b + z_{thick} \\ c_1 z_b - \Delta M & z \geq z_b + z_{thick} \\ + c_2 (z - z_b - z_{thick}) & \end{cases} \quad (3)$$

where $M(z)$ is the modified refractivity, M_0 is the value of modified refractivity at the sea level surface, z is the vertical height, z_b is the base height, z_{thick} is the duct layer thickness, and ΔM is the duct strength, c_1 is the slope of the duct bottom layer and $c_2 = 0.118$ is the slope of the duct top layer. Obviously, the refractivity profile of the surface-based duct can be described by the four-parameter vector $m = (c_1, z_b, z_{thick}, \Delta M)$.

3.2 Parabolic equation method

The propagation of electromagnetic waves under atmospheric ducting conditions depends on many factors: antenna height, duct height, duct strength, carrier frequency, polarization and sea surface conditions, and the parabolic equation method can take all of these factors into considerations [16]. The parabolic equation method is a for-

ward full wave analysis method with the ability to handle complex boundary conditions and horizontal inhomogeneous atmospheric environment, and it also has excellent stability and accuracy. The solutions of the parabolic equation method can be achieved by the split-step Fourier transform (SSFT) technique and implemented on a personal computer in seconds for propagation over a sea surface, so it is widely applied to the wave propagation problems under atmospheric ducting conditions [17]. For the purpose of analyzing the propagation of AIS signals under atmospheric ducting conditions, the parabolic equation method may be the best choice. Therefore, we utilize the parabolic equation method to calculate the propagation loss of AIS signals in ducting channel.

In the troposphere electromagnetic wave propagation, forward narrow angle parabolic equations are usually used. Ignoring backscattering effect of electromagnetic waves, the standard parabolic equation (SPE) is defined as [18]

$$\frac{\partial u(x, z)}{\partial x} = \frac{ik_0}{2} \left[\frac{1}{k_0^2} \frac{\partial^2}{\partial z^2} + m^2(x, z) - 1 \right] u(x, z) \quad (4)$$

where $u(x, z)$ is the reduced function, x is the horizontal range and z is the height, $k_0 = 2\pi/\lambda$ is the free-space wave number, λ is the wavelength, $m(x, z) = 1 + M \times 10^{-6}$ is the range and height dependent modified atmospheric refractive index.

3.3 AIS propagation model

Since AIS is a one-way communication system, it does not need to consider the target echo signal, thus the AIS radar range equation is [19]

$$P_r = \frac{P_t G_t G_r \lambda^2}{(4\pi R)^2} F \quad (5)$$

where P_t is the transmitted power, G_t is the gain of the transmitter antenna, G_r is the gain of the receiver antenna, λ is the wavelength, R is the path length. The propagation factor F is the field relative to free space expressed in dB, which is defined as

$$F = 20 \log(|u(x, z)|) + 10 \log(x) + 10 \log(\lambda) \quad (6)$$

Therefore, the one-way propagation loss of AIS signals is defined as

$$PL(x, m) = 20 \log(4\pi) + 10 \log(x) - 30 \log(\lambda) - 20 \log(|u(x, z)|) \quad (7)$$

where x is the range from the transmitter to the receiver, m is the unknown refractivity profile parameter vector, λ is

the wavelength of AIS signals, $u(x, z)$ is the reduced function that can be solved by the parabolic equation split-step Fourier method.

The received AIS signal power can be modelled as

$$P_r(x, m) = -PL(x, m) + C \quad (8)$$

where C is the constant terms in (5).

4 A new inversion method based on the AIS signal power

4.1 The inversion step

Although RFC techniques have certain advantages, some problems exist. For example, currently used normalized sea surface radar cross section is not accurate and in some cases it is difficult to separate the sea surface and weather (volume) clutter, which will severely limit the accuracy of the inversion. AIS system is a one-way technique that does not need to consider the reflection of the signal on a target, and thus it is more appropriate for inverting the atmospheric refractivity profile. To improve the accuracy of the estimation of the refractivity profile, a new inversion method based on the AIS signal power is proposed. The inversion step is as follows.

- Step 1.** Obtain the observed AIS signal power $P_r^{obs}(x, m)$.
- Step 2.** Select the appropriate atmospheric refractivity profile model.
- Step 3.** Use the forward propagation model to calculate the AIS signal power $P_r(x, m)$.
- Step 4.** Construct the objective function $f = \min |P_r(x, m) - P_r^{obs}(x, m)|$.
- Step 5.** Optimize the objective function f determined in step 4 using a global optimization algorithm to get the unknown refractivity profile parameter vector m .

4.2 Quantum-behaved particle swarm optimization algorithm

The quantum-behaved particle swarm optimization algorithm [20] is used to optimize the objective function for obtaining the best atmospheric modified refractivity profile. The classical particle swarm optimization algorithm [21] is a random search algorithm based on swarm intelligence, which has the ability of global approximation, but due to its limited search space, it is easy to fall into the lo-

cal extreme value. From the perspective of quantum mechanics, a new particle swarm optimization algorithm, quantum-behaved particle swarm optimization algorithm is proposed. Quantum computing is a new computing technology, and its fusion with swarm intelligence algorithm has a broad application prospect. QPSO algorithm combine quantum computing method and PSO algorithm and become a more efficient algorithm.

In the PSO algorithm, each particle represents a feasible solution to the optimization problem. The pros and cons of the solution is determined by the fitness function, which depends on the actual optimization problem. For the i^{th} particle ($1 \leq i \leq M$, M is the number of particles in the population), the current position of the particle in the search space is denoted by $X_i = (x_{i1}(t), x_{i2}(t), \dots, x_{id}(t), \dots, x_{iD}(t))$, t is the current number of iterations of the algorithm and D is the dimension of the particle. According to the analysis of the particle trajectory in the PSO algorithm, to ensure the convergence of the PSO algorithm, each particle must converge to its local attractor $p_i = (p_{i1}(t), p_{i2}(t), \dots, p_{id}(t), \dots, p_{iD}(t))$, which is defined as [22]

$$p_{id}(t) = \varphi_d(t)P_{id}(t) + (1 - \varphi_d(t))P_{gd}(t) \quad (9)$$

where $\varphi_d(t) = c_1 r_{1d}(t) / (c_1 r_{1d}(t) + c_2 r_{2d}(t))$, $r_{1d}(t)$ and $r_{2d}(t)$ are two random number uniformly distributed within $(0, 1)$. c_1 and c_2 are the learning factors. During each iteration of the PSO algorithm, $P_i = (P_{i1}(t), P_{i2}(t), \dots, P_{id}(t), \dots, P_{iD}(t))$ is the current optimal position of the i^{th} particle, and $P_g = (P_{g1}(t), P_{g2}(t), \dots, P_{gd}(t), \dots, P_{gD}(t))$ is the global optimal position of the population.

Assuming that the PSO system is a quantum system, the velocity and position of the particles in the quantum space cannot be determined simultaneously. The state of each particle is determined by the wave function ψ , and $|\psi|^2$ is the probability density function of the particle position. By establishing a Delta potential well model in each dimension of p_i to prevent particle divergence, the corresponding Schrödinger equation can be solved to obtain the probability density function of the position of each dimension of the particle in the search space, which is defined as

$$F[x_{id}(t+1)] = \exp \left[\frac{-2|x_{id}(t+1) - p_{id}(t)|}{L_{id}(t)} \right] \quad (10)$$

where $L_{id}(t)$ is the characteristic length of the Delta well, which determines the search range of the particle.

Monte Carlo simulation method is used to obtain the position of the d^{th} dimension of the i^{th} particle at the

$(t + 1)^{th}$ iteration:

$$x_{id}(t + 1) = p_{id}(t) \pm \frac{L_{id}(t)}{2} \ln(1/u) \quad (11)$$

where u is a random number uniformly distributed over $(0, 1)$.

$$L_{id}(t) = 2\alpha \times |m_d(t) - x_{id}(t)| \quad (12)$$

where α is the contraction-expansion coefficient. $m_d(t)$ is the average optimal position, which is the centre point of the optimal position of all particles themselves and is defined as

$$\begin{aligned} m(t) &= (m_1(t), m_2(t), \dots, m_d(t), \dots, m_D(t)) \quad (13) \\ &= \left(\frac{1}{M} \sum_{i=1}^M P_{i1}(t), \frac{1}{M} \sum_{i=1}^M P_{i2}(t), \dots, \frac{1}{M} \sum_{i=1}^M P_{id}(t), \right. \\ &\quad \left. \dots, \frac{1}{M} \sum_{i=1}^M P_{iD}(t) \right) \end{aligned}$$

Therefore, the position update equation of the particle is

$$x_{id}(t + 1) = p_{id}(t) \pm \alpha |m_d(t) - x_{id}(t)| \times \ln(1/u) \quad (14)$$

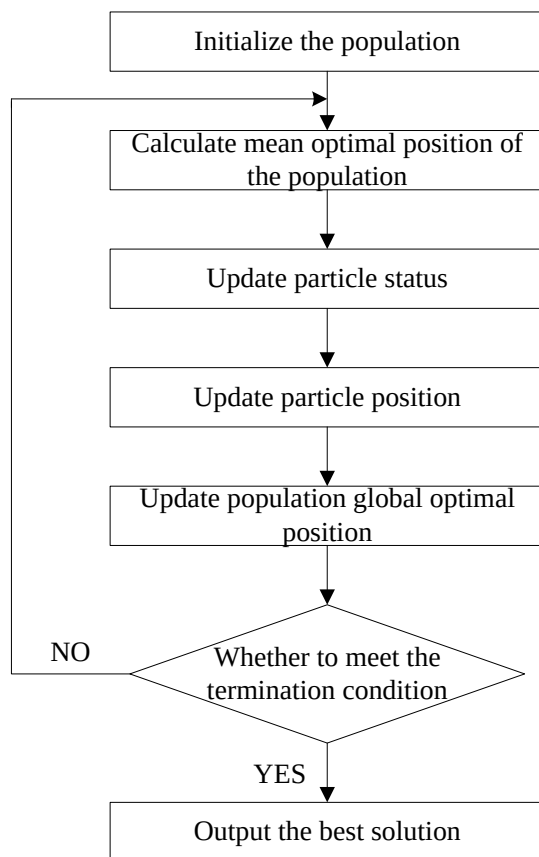


Figure 2: The flowchart of the QPSO algorithm

The PSO algorithm using the Equation (14) as the particle position update equation is called the quantum-behaved particle swarm optimization (QPSO) algorithm [23].

The flowchart of the QPSO algorithm is shown in Figure 2.

5 Results and discussion

5.1 Inversion of surface-based ducts

In this section, inversion of surface-based ducts using the AIS signal power is investigated by the QPSO algorithm via the simulation study. For the surface-based duct, there is four-parameter $m = (c_1, z_b, z_{thick}, \Delta M)$ that need to be estimated. The AIS system parameters are chosen as: the frequency is 162 MHz, the antenna is an omnidirectional antenna with a height of 20 m, and the vertical polarization is employed. The AIS signal power simulated by the profile parameter vector $m = (0.13, 40, 20, 50)$ using the AIS propagation model in Section 3.3 is treated as the observed AIS signal power.

The control parameters of the QPSO algorithm for the surface-based duct inversion are given as follows: the number of maximum iteration is 100, the population size is 60, the learning factor is 2, and the inertial weight is reduced from the initial 1 to the final 0.5. Figure 3 gives the comparison of the simulated atmospheric modified refractivity profile and the inverted profile of the surface-based duct obtained by the proposed method in the case of without noise. In order to reveal the superiority of the QPSO algorithm, the inversion results of the QPSO algorithm are compared with that of the PSO algorithm.

From Figure 3, we can see that the refractivity profile inverted by the QPSO algorithm is in excellent agreement with the simulated one, which indicates that the inversion method based on the AIS signal power can be utilized to invert the surface-based duct in a maritime environment. It can also be seen from Figure 3 that the refractivity profile inverted by the PSO algorithm is obviously less accurate than those inverted by the QPSO algorithm. The inversion results show that the QPSO algorithm is more suitable for inverting atmospheric ducts.

Error analysis of the AIS based atmospheric refractivity inversion method in term of the Root Mean Square Error (RMSE) is performed to quantitatively analyze the optimization performance of the QPSO and the PSO algorithm and is illustrated in Table 1.

Table 1: The error analyse of the QPSO and the PSO algorithm

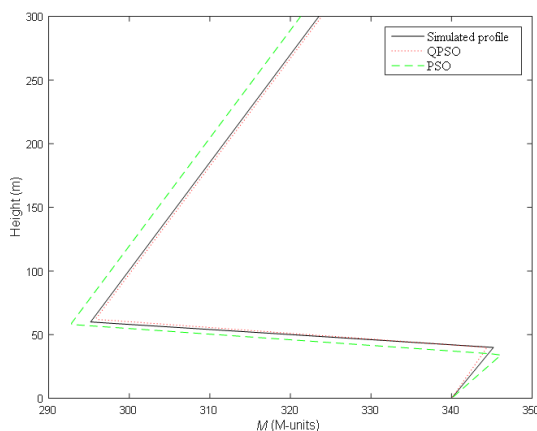
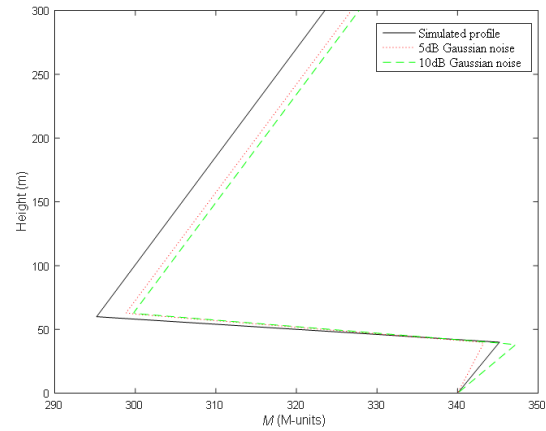
Parameter	True	QPSO	PSO
c_1	0.13	0.11	0.18
z_b	40	39.97	34.43
z_{thick}	20	22.15	23.63
ΔM	50	48.59	53.48
RMSE	0	1.29	3.75

As showed in Table 1, the RMSE of the PSO algorithm is larger than those of the QPSO algorithm, which illustrates the QPSO algorithm is better than that of the PSO algorithm for the surface-based duct inversion.

5.2 Inversion with Gaussian noise

To further analyze the anti-noise capability of the QPSO algorithm for estimating refractivity profile, the Gaussian noise with zero mean and different noise level is added to the synthetic AIS signal power. Simulated AIS signal power with 5 dB and 10 dB noise levels is taken as the observed AIS signal power to analyze the performance of the QPSO algorithm. Inversion results for different Gaussian noise levels are shown in Figure 4.

As can be seen from Figure 4, the QPSO algorithm with the noise level of 5 dB and 10 dB can still make a good estimation of atmospheric refractivity profile. Although the inversion error is slightly larger than that of the noiseless case, the overall inversion accuracy is acceptable. Therefore, the QPSO algorithm has certain anti-noise capability and good robustness for the surface-based duct estimation.

**Figure 3:** The simulated and inverted atmospheric refractivity profile without noise**Figure 4:** The simulated and inverted atmospheric refractivity profile with different Gaussian noise levels

The above analysis is based on receiving a single ship AIS signals, when multi-ship AIS signals are received (in the same direction or in different directions), the atmospheric refractivity profile over the sea in the entire region can be inverted simultaneously by the method proposed in this paper. Not only can shipboard AIS equipment be used, but also existing AIS equipment installed on other ships and shore-based equipment can be used for inversion, enabling ships to obtain current atmospheric refractivity distribution in near-real-time.

6 Conclusion

In this work, a new inversion method based on the AIS signal power has been proposed to invert the surface-based duct and the QPSO algorithm is used to search for the optimal solution from various refractivity parameters. Simulation results show that this new inversion method is feasible for the atmospheric refractivity profile estimation and the QPSO algorithm has good robustness. And it also has near-real-time performance, which provides strong support for the performance prediction of AIS system and other radio systems. It is of great value to use AIS system for realizing near-real-time, large-scale and continuous detection of atmospheric duct distribution in the maritime environment. However, an important topic that needs to be addressed more extensively is the validation of results obtained with this study for refractivity profile estimation, which will be investigated in the near future.

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