



## The study on effective thermal conductivity of carbon foam based on fractal theory

Yong Wang,<sup>1\*</sup> Qingqing An,<sup>1</sup> Lingling Cao,<sup>1</sup> Xiaojuan Si,<sup>1</sup> Donghui Liu,<sup>1</sup> Jianjun Chen,<sup>1</sup> Yimin Wang,<sup>1</sup> Kin Tak Lau<sup>2</sup>

<sup>1</sup>College of Material Science and Engineering, DongHua University, Shanghai 201620, 0086-21-67792844; email: wangyong9240@126.com.

<sup>2</sup>Department of Mechanical Engineering, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong; email: mmktlau@polyu.edu.hk.

(Received: 21 May, 2007; published: 29 August, 2007)

**Abstract:** Novel carbon foam with high thermal conductivity is prepared by thermal treating of mesophase pitch under certain temperature and pressure condition. With fractal theory, the thermal conducting property of this novel porous material is discussed. Then we deduce the area fractal dimension of carbon foam. A thermal conductivity model of carbon foam is proposed. The relationship formula of effective thermal conductivity is presented by using thermal resistance method. Through computation, the effective thermal conductivity of carbon foam is acquired. The value of model forecast is consistent with that of the actual observed for carbon foam. This method has provided the theoretical basis for better using its fine heat conduction performance.

**Key words:** carbon foam, fractal dimension, heat conduction, thermal conductivity.

### Introduction

Mesophase pitch-based graphitic carbon foams with high thermal conductivity (80–200 W/m K) was first prepared by Oak Ridge National Laboratory in 1998 [1] and attracted great interest in thermal management systems to improve single and multiphase heat transfer. Since internal structure of porous material is very complex, heterogeneity and indeterminacy will appear in a certain scope along with the irregular internal structure in actual porous medium. It is therefore difficult to evaluate thermal parameter accurately by traditional methods. Fractal is a word firstly coined by Mandelbrot [2], and has been widely used in non-linear and micro-scale science. It can well describe the disorder and stochastic performance of porous media, however it was seldom used in effective description of thermal conductivity of carbon foam.

A thermal-conductivity model was proposed in this paper to investigate the heat transfer of porous carbon foam. Images of several foam samples by scanning electron microscope (SEM) were analyzed using the fractal theory, and the area fractal dimension of carbon foam was deduced. The relationship formula of effective thermal conductivity is obtained by effective thermal conductivity of carbon foam. The results by this model are consistent with the actual observation from carbon foam and the available literature data.

### Fractal model and effective thermal conductivity

Raw foams are prepared from mesophase pitch in a stainless steel mold by heating to

appropriate temperature under adequate pressure. The raw foams are further carbonized and graphitized; the carbon foams with three-dimensional reticulate pore structure are shown in Fig. 1. The foam has a high effective conductivity and its average thermal conductivity can reach as high as 180 W/m K [3]. The main purpose of this paper is to establish a fractal model for calculating the effective thermal conductivity of carbon foam by fractal theory.

Before using the fractal theory, it is necessary to introduce some definitions of fractal indexes. The fractal dimension  $d$  is the most important parameter for describing the fractals. A two-dimensional object, such as the SEM figures of carbon foam we obtained previously, can be divided into  $N(\delta)$  self-similar smaller squares, each of which is scaled down by the length of the side. Therefore, the fractal dimension  $d$  can be defined and calculated by following equation:

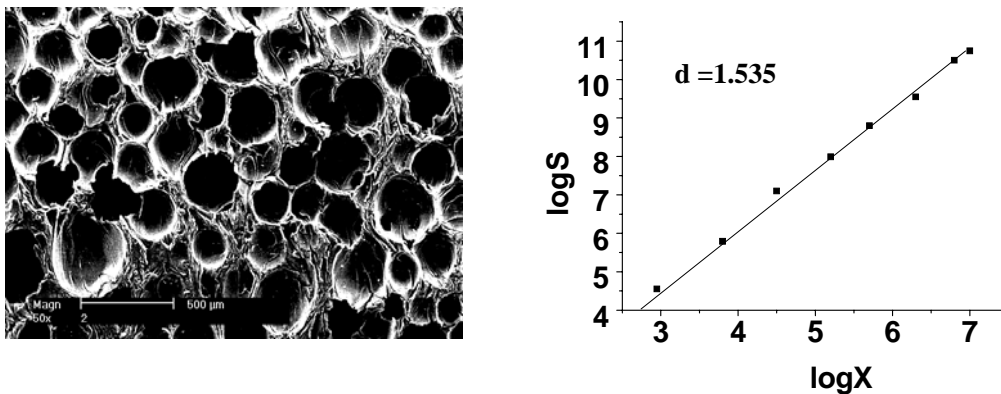
$$N(\delta) \sim \delta^d,$$

where  $N$  is the occupied space plot (line, surface or volume) of fractal objects,  $\delta$  is scale of measurement,  $d$  is the fractal dimension [4].

Fractal dimension  $d$  determined by the slope of straight line  $N(\delta)$  verses  $\delta$  in the plotting of the logarithm S-X. Carbon foam material has a network structure with the three dimensional honeycomb skeleton struts mutually connected and each honeycomb unit structure is different (Fig. 1). However looking from comparatively wide range, the honeycomb unit has a similar characteristic and carbon foam is therefore a material having fractal structure by so-called the box-counting method definition [5]. We can calculate the dimensions of these SEM images. Regarding the carbon foam sample, cell area mean value  $S$  and the measure criterion  $X$  assumes the linear relations on S-X logarithmic coordinate chart. In other words, it satisfies the following linear relation:

$$\log(S) = \log C + d \log(X),$$

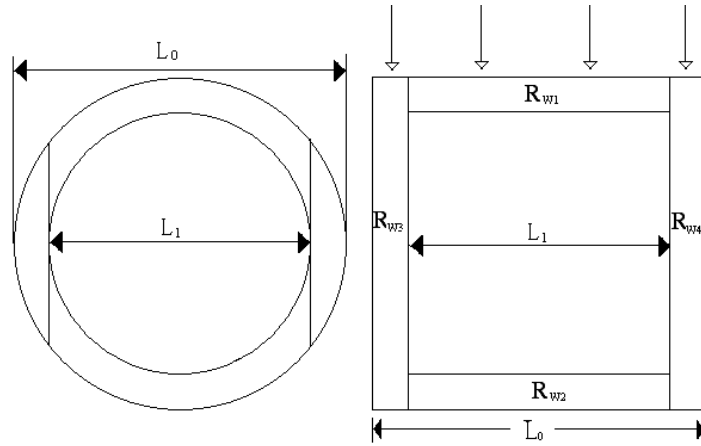
The fractal dimensions of the foam are calculated and shown in Fig. 1 where the slopes of this line are the fractal dimensions.



**Fig. 1.** The cell area fractal dimension in carbon foam.

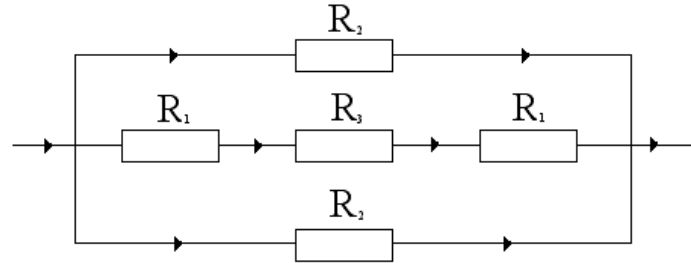
The simplified actual irregular cell rules for a regular circular cell, so that the original profile and the actual profile of the gap have the same size. As a result, the foam along the vertical direction and the direction of parallel have the same area fractal dimension  $d$ , and consequently same effective conductivity characteristics. Such a

unit cell of carbon foam is schematically shown in Fig. 2a.



**Fig. 2.** (a) Simplified heat conduction. (b) Thermal conductivity model of unit cell.

In Figure 2a,  $L_0$  is the characteristic length for whole unit cell and  $L_1$  is the characteristic length for the cavity feature length of the cell. Thus, for unit fractal cell of carbon foam, its simplified model of the thermal conductivity can be described by Figure 2b. The heat conduction process of units fractal foam can be analyzed by thermal resistant network as shown in Fig. 4. We try to use this method in this paper to establish the relation between the average porosity and the effective thermal conductivities of carbon foam. The heat conduction of thermal resistant network can be represented in Fig. 4. Paths of heat flux through a cell represented by electrical circuits.



**Fig. 4.** The thermal resistance network.

If, moreover, we assume that the thickness of the cell at the direction perpendicular to this paper is the unit length 1, the thermal resistance of every part of the cell can be given as

$$R_1 = \frac{L_0}{\left(\frac{L_0 - L_1}{2}\right) \cdot \lambda w} \quad R_2 = \frac{\left(\frac{L_0 - L_1}{2}\right)}{L_1 \cdot \lambda w} \quad R_3 = \frac{L_1}{L_1 \cdot \lambda g} = \frac{1}{\lambda g} \quad (1)$$

Similar to the calculation methods of circuit parameters, the whole thermal resistance of the cell is

$$R = \left( \frac{1}{R_2} + \frac{1}{2R_1 + R_3} + \frac{1}{R_2} \right)^{-1} \quad (2)$$

where  $R$  is the effective thermal conductivity of carbon foam. According to the paths of the thermal resistance network, the whole thermal resistance can also be written as:

$$R = \frac{L_0}{L_0 \bullet \lambda_e} = \frac{1}{\lambda_e} \quad (3)$$

Substituting Eq. (1) and (2) into Eq.(3), the equation can be simplified as:

$$\lambda_e = \frac{2}{R_2} + \frac{1}{2R_1 + R_3} = \frac{(L_0 - L_1)\lambda_w}{L_0} + \frac{L_1\lambda_w\lambda_g}{(L_0 - L_1)\lambda_g + L_1\lambda_w} \quad (4)$$

On one hand, in accordance with the previous six assumptions, the porosity of carbon foam can be given as:

$$\phi = \frac{L_1^2}{L_0^2} \quad (5)$$

where  $L_1$  is the cavity diameter of the cell, and  $L_0$  is the diameter of the cell.

Substituting Eq. (5) into Eq.(4), the correlation between the thermal conductivities and the porosity can be obtained.

$$\lambda_e = (1 - \sqrt{\phi})\lambda_w + \frac{\sqrt{\phi}\lambda_w\lambda_g}{(1 - \sqrt{\phi})\lambda_g + \sqrt{\phi}\lambda_w} \quad (6)$$

According to another definition of fractal dimension [2], the relationship between the porosity and the fractal dimension can also be presented as:

$$\phi = \kappa A^{d-1} \quad (7)$$

where  $\kappa$  is a constant.

Moreover, as shown in Fig. 1, it is apparently seen that the scale of different kinds of cell is in the range from 3 to 5  $\mu\text{m}$ . In order to build an all-purposed model for each kind of foam, we use the average scale that we have investigated previously. Then, it is assumed that  $A$  is  $(4.5 \times 4.5) \mu\text{m}^2$ . Substituting this value into Eq. (7) and combining with the average fractal dimension which is approximately equal to 1.535, we can obtain the value of constant  $\kappa$ , 0.161, and Eq. (7) can be rewritten as

$$\phi = 0.161 \times 20.25^{d-1} \quad (8)$$

Finally, after Eq.(8) has been substituted into Eq.(6), the formulas that describe the relationship between the fractal dimension and effective thermal conductivities of carbon foam are obtained.

$$\lambda_e = \left( 1 - \sqrt{0.161 \times 20.25^{d-1}} \right) \lambda_w + \frac{\sqrt{0.161 \times 20.25^{d-1}} \lambda_w \lambda_g}{\left( 1 - \sqrt{0.161 \times 20.25^{d-1}} \right) \lambda_g + \sqrt{0.161 \times 20.25^{d-1}} \lambda_w}$$

Where  $\lambda_w$  is the thermal conductivity of solid wall in carbon foam,  $\lambda_g$  is the thermal conductivity of air.  $\lambda_w$  between 1300 W/ W/m K and 1700W/ W/m K ,  $\lambda_g=0.03$  W/m K [6] respectively. By calculating, the effective thermal conductivity of carbon foam is

148 W/m K-179 W/m K. The area fractal dimensions of vertical direction to foam cell profile and that along the direction are almost the same, thus the effective thermal conductivity difference of carbon foam between the vertical and along the foaming direction is not big. The average effective thermal conductivity of carbon foam is 148 W/m K-179 W/m K. The effective thermal conductivity by calculation is extremely close to the experimental results and the deviation of the calculated and measured value is less than 5%. The fractal methods and thermal resistant network can therefore be used to well predict heat conduction in porous media. The effective thermal conductivity of carbon foam is closely related to its each component thermal conductivity and the porosity as well as fractal dimension, and has nothing to do with other factors.

## Conclusions

In this paper, based on fractal theory, the fractal dimensions of carbon in their cross-section can be determined from the SEM images using the box-counting method. The relation between the fractal dimension and the porosity of carbon foam has been established. We also succeed in establishing a fractal model for calculating the effective thermal conductivities of carbon foam. The value of model forecast is consistent with that from actual observed carbon foam and that from literature data. This method could predict the effective thermal conductivity of porous materials with different structure.

## References

- [1] Klett, J.W.; Burchell, T.D. *High Thermal Conductivity Mesophase Pitch Derived Carbon Foam* [A]. The 43<sup>rd</sup> International SAMPE Symposium [C]. Anaheim, CA, **1998** May 3- June 4.
- [2] Mandelbrot, B.B. *The Fractal Geometry of Nature*. New York: W H Freeman, **1983**.
- [3] Klett, J.; Ott, R.; McMillan, A. *Heat Exchangers for Heavy Vehicles Utilizing High Thermal Conductivity Graphite Foams* [A]. Sae Technical Paper Series [C]. **2000**-01-2207.
- [4] Yu, B. M.; Li, J. H. *Some fractal characters of porous media Fractal. Porous Media*, **2001**, 9(3): 365—37.
- [5] Adler, P.M.; Thovert, J.F. *Fractal porous media, Transport Porous Med* 13, **1993**, 41—78.
- [6] Klett, J.W.; McMillan, A.D.; Gallego, N.C.; Walls, C.A. *Journal of materials science*, **2004**, 39, 3659— 3676.