Regular Article

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Optimization of railway entry and exit transition curves

https://doi.org/10.1515/eng-2022-0454
received February 15, 2023; accepted May 02, 2023

Abstract: This work concerns the optimization of the shapes of railway entry and exit transition curves (TCs). In this work, mathematical optimization and simulation methods are used. The applied computer simulation concerned the behaviour of the dynamical model of a two-axle rail vehicle. A polynomial of degree n, where $n = 9$ and 11, was adopted as a TC in the performed analyses. Three radii of the circular arc were used in the work – 600, 1,200, and 2,000 m. The aim of the research was to find optimum TC shapes taking into account the adopted criteria and to compare them with each other.

Keywords: railway transition curves, rail vehicle, dynamical assessment, computer simulation

1 Introduction

The author of the current work in many previous works [1,2] showed that for railway polynomial transition curves (TCs) of odd degrees – 5th, 7th, 9th, and 11th – the best dynamical properties (represented by the smallest values of the applied objective function FC – integral from the lateral acceleration of the centre of mass of the rail vehicle body) had the curves with the maximum number of polynomial terms. The mentioned numbers of terms of the polynomial were for the curves: 5th degree: 3, 7th degree: 5, 9th degree: 7, and 11th degree: 9. It should be added that the curvatures of the obtained TCs, however, did not have a continuity of G_1 type at the extreme – initial and final – points of the TC.

Improvement in the dynamical properties of the obtained optimum curve shapes in comparison with the standard (initial) curves were confirmed by the following:

– vehicle body lateral displacements,
– vehicle body lateral accelerations,
– wear in wheel/rail contact.

Earlier work also proved that curves meeting advanced geometrical conditions (i.e. continuity of the G_1 type at the extreme – initial and final – points of the curve) and used in railway engineering practice did not improve the dynamical interactions in the vehicle track system in the curve and the adjacent part of the circular arc (CA).

It has also been shown that it is worth taking an interest in TCs of higher odd degrees – 9 and 11. However, this only makes sense if we take into account the maximum permissible (from the mathematical point of view) number of terms of the polynomial – 7 and 9, respectively.

The curves obtained in earlier works had better properties than both the third degree parabola and standard curves if we use the FC objective function. It also managed to obtain the formulas for the optimum TCs, and what is important is that similar solutions were not found in the literature.

2 Short literature survey

The increase in the number of works dealing with the problem of the proper shape of railway TC in recent years has become a fact. According to the author of this work, there is also a visible division of such works into three different groups. These three mentioned groups of works can be presented by exemplary works [3–25].

In the first group, these are considered as theoretical works, focusing on the TCs only as the mathematical object as reported by Ahmad and Ali [3]. The authors of mentioned work focused only on the mathematical properties of the curves, like their curvatures, the existence of both inflexion points in the middle of the curve, and tangency of G_0, G_1, and G_2 types in the first and last point of the curve.

The works from the second group [4–6,7,10,11,14,15,18,20,23] examined both general and geodetic properties of railway TCs, and showed the influence of their shape on the vehicle dynamics using rather a simple vehicle model.

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The third group indicates that there are a certain number of works in which the approach adopted to the problem of the evaluation and shaping of the TCs is similar to that used by the current author [7,8,12,13,16,17,19,21,22,24,25]. An example of work belonging to the third group (using advanced vehicle dynamics) is the work by Long et al. [16]. They used the advanced rail vehicle model and appropriate simulation software to evaluate the dynamical properties of the TCs. They compared six different shapes of curves used in practice. Certain criteria were used for its implementation. These criteria were as follows:

- vehicle body lateral displacement,
- vehicle body lateral acceleration,
- vertical and lateral forces in wheel–rail contact,
- derailment coefficient,
- wear reduction in wheel–rail contact.

As a result, the authors proposed a new (non-classical) approach in which the curve function is not a scaled cant function.

The general conclusion from the analysis of the literature is as follows: there are no works in which the full dynamics of the track-vehicle system and optimization methods are considered together. Most authors use the classical approach to track-vehicle interactions when optimizing the shape of the curves. They are only interested in a simple model of the vehicle and the requirement that the physical quantities describing the vehicle's action on passengers, such as maximum unbalanced lateral acceleration and its change, are not exceeded.

Novelty, therefore, are the works, which aim at an approach in which the optimization of the TC shape uses an advanced model of the rail vehicle and the entire vehicle-track system, as well as the mathematically understood optimization methods.

3 Method

The aim of the conducted research was to evaluate the dynamical properties and optimize the shape of railway TCs. As already mentioned, the author's earlier works contributed to the planned research. The dynamical properties of railway TCs were analysed based on the passage of a two-axle rail vehicle (freight wagon) through a route consisting of:

- straight track (ST),
- TC,
- circular arc (CA).

For the purposes of the work, the wagon travelled along the route consisting of:

- ST (50 m),
- TC (l_0),
- CA (100 m),
- TC (l_0),
- ST (50 m).

The difference, therefore, was that the dynamic effects were tested on an extended route, additionally containing an exit TC and a fragment of a straight track. The study assumed that the shape of the exit TC would be a mirror image of the entry TC. The value of the curvature in last point of the entry TC was equal to $1/R$. This value was assumed as the initial point of the exit TC. The considerations are generally concerning double-track railway line. For the single-track railway lines, such considerations have no sense.

The scientific aim of the work was, therefore, to evaluate the dynamic properties and optimization of the shape of railway TCs, taking into account vehicle dynamics when passing through ST, TC, CA, TC, and ST, including the impact on a passenger. The research carried out for the purposes of this work was always in the form of planned numerical tests. They were made using a program for simulating the movement of a two-axle rail vehicle, combined into one with the library optimization procedure.

In the current work, one model of rail vehicle was applied. This model has a two-axle structure. Like every real wagon, it possesses a body connected with two wheelsets with spring-damping elements. Both the structure of the mentioned model and its parameters correspond to the real British wagon. In this work, it was used to investigate the dynamical properties and optimize the shape of the railway TCs. The nominal model of this vehicle is shown in Figure 1c. The vehicle model is supplemented with the model of the track as shown in Figure 1a and b. The whole track-vehicle system is presented in detail, e.g., in ref. [2]. The model parameters of this system are also shown in ref. [2].

4 The railway model

The approach to the TC shape modelling was widely defined in the study by Zboinski and Woznica [2]. Dynamics of relative motion is applied in this method. Definition of railway vehicle dynamics is relative to the track-based moving reference frame. The geometrical track model was generally analysed in three dimensions.

Polynomial transition curves of odd degrees – 9 and 11 – were used in the analysis. In all studies, three values for the radius of the CA were adopted: R = 600, 1,200 and 2,000 m.

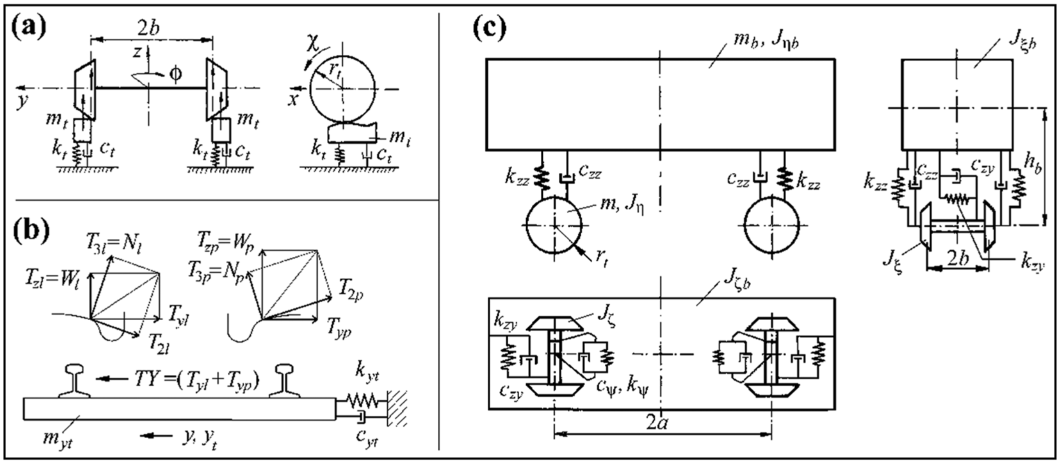

Figure 1: Two-axle vehicle model. (a) Track vertical model, (b) track lateral model, and (c) vehicle.

The following parameters were also assumed in the study:

– superelevation (cant) H ranged from 20 to 150 mm,

– wheel vertical rise along superelevation ramp f_{dop} = 56 mm/s (abrupt change in cant [26]),

– maximum change in lateral acceleration ψ_{dop} = 1 m/s^3 (abrupt change in cant deficiency is equal to 153 mm [26]).

The values f_{dop} and ψ_{dop} seem to be overestimated, but it can allow to shorten the lengths of TCs.

In the author' earlier works, one radius of a CA – 600 m – was widely studied. For this radius, the optimum TCs obtained had curvature bends at the extreme – initial and final – points of the curve. Therefore, this work was aimed at increasing the range of investigated radii of a CA, as well as the analysis of new shapes.

Generally, two speeds of the rail vehicle were adopted in the work:

– lower – corresponded to lateral acceleration in the plane of track a_{lim} equal to 0 m/s^2,

– higher – corresponded to lateral acceleration a_{lim} > 0 m/s^2.

The following TC evaluation criteria were used in the work:

(1) value of integral of the lateral acceleration of the vehicle body along the route:

$$\text{FC}_1 = L_\text{C}^{-1}\int_0^{L_\text{C}} |\ddot{y}_\text{b}|\,\text{d}l, \tag{1}$$

(2) value of integral of the change in lateral acceleration (jerk) of the vehicle body along the route:

$$\text{FC}_2 = L_\text{C}^{-1}\int_0^{L_\text{C}} |\dddot{y}_\text{b}|\,\text{d}l, \tag{2}$$

where, L_C is the total length of the route, $\ddot{y}$ is the vehicle body lateral acceleration, $\dddot{y}$ is the change in the vehicle body lateral acceleration.

The problem formulated by the author of the work is based on simulation studies. They were made, as mentioned, using an advanced program for simulating the dynamics of rail vehicles, taking into account vehicle-track and vehicle-passenger interactions available to the author. This program has been combined into one with the optimization procedure. In this way, the results of the vehicle dynamics simulation in the TCs can be and are the basis for calculating the value of the objective function in the process of optimizing the shape of the TCs.

The software scheme is also presented by Zboinski and Woznica [2]. It shows two iteration loops. The first is the equation integration (simulation) loop. It was interrupted when the length l_{lim}, being the current length of the route, reached the assumed value. The second is the optimization process loop. It was interrupted when the number of iterations reached the value i_{lim}. This value meant that the number of simulations must be performed for the optimization process to be completed. If an optimum solution was found earlier ($i < i_{\text{lim}}$), then the optimization process was automatically terminated.

5 Results

The purpose of this study was to present the results of the optimization of the shape of the railway TCs using the described vehicle model and the vehicle dynamics criteria. Every single simulation included in the optimization process consisted of the passage of the railway vehicle along the assumed route.

Traditionally, the results from individual processes of optimizing the shape of the TCs consisted of the following:

– values of optimum polynomials,
– values of quality functions,
– types of the curvatures of optimum TCs,
– vehicle body lateral displacements and accelerations.

As the initial curves in the optimization process, standard polynomial curves of degrees 9 and 11 were used. These curve are as follows [1,2]:

$$y_9 = \frac{1}{R}\left(-\frac{5}{18}\frac{l^9}{l_o^7} + \frac{5}{4}\frac{l^8}{l_o^6} - 2\frac{l^7}{l_o^5} + \frac{7}{6}\frac{l^6}{l_o^4}\right), \tag{3}$$

$$y_{11} = \frac{1}{R}\left(\frac{7}{11}\frac{l^{11}}{l_0^9} - \frac{7}{2}\frac{l^{10}}{l_0^8} + \frac{15}{2}\frac{l^9}{l_0^7} - \frac{15}{2}\frac{l^8}{l_0^6} + 3\frac{l^7}{l_0^5}\right). \tag{4}$$

In each optimization process, a constant (unchangeable) TC length was assumed, which was determined for the reference TC according to the method shown in the literature on shaping TC. These minimum lengths l_0 of the curves adopted in the work resulted from two parameters whose maximum values should not be exceeded.

– the wheel vertical rise along superelevation ramp f_{dop} (abrupt change in cant),

– the value of change in lateral acceleration ψ_{dop} (abrupt change in cant deficiency).

In general, each entry TC obtained in the work had a curvature (superelevation ramp), which could be classified into one of five groups. These five groups (types) are as follows:

1) Type 1 curvature is the curvature of standard TCs of 9th and 11th degrees,
2) Type 2 curvature is something between the curvature of standard TC of 9th or 11th degree and 3rd degree parabola,
3) Type 3 is a linear curvature (strictly speaking, only the TC in the form of a clothoid has a linear curvature),
4) Type 4 curvature has a convex character. It has slope subtype (4a) or G_1 continuity subtype (4b) at the beginning of TC, and it has a slope at the end,
5) Type 5 curvature (superelevation ramp) has a concave character.

Curvatures of all types (for l_0 = 142.15 m and R = 600 m) are shown in Figure 2.

Table 1 presents the results of the optimization of the shape of the TCs obtained in the tests.

– types of curvatures (superelevation ramps) of optimum TCs,

– values of quality functions FC_1 and FC_2.

It is shown for:

– curve radius R (in m),
– unbalanced lateral acceleration a_{lim} (m/s^2),
– vehicle velocity v (m/s),
– cant H (mm),
– polynomial degree n,
– transition length l_0 (m).

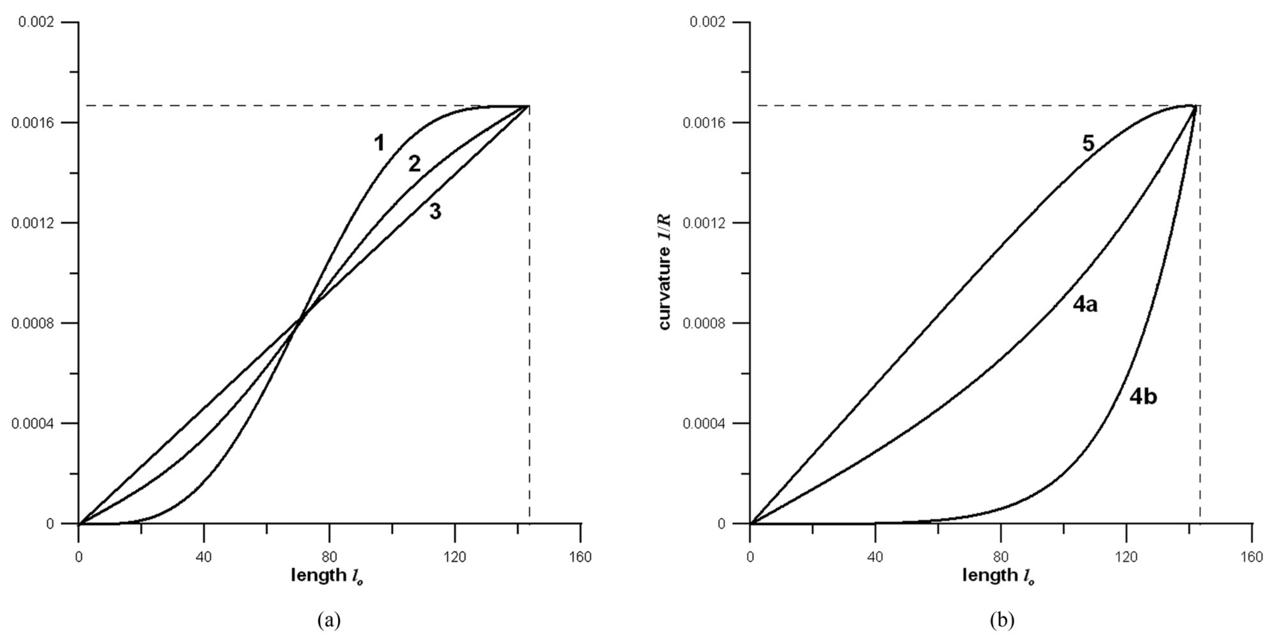

Figure 2: The curvatures: (a) Types 1, 2, and 3 and (b) Types 4 and 5.

Table 1: Optimization results

R	a_{lim}	v	H	n	l_0	FC_1	FC_2
600	0	24.26	150	9	142.15	4b/0.13512	4b/1.7210
				11	159.86	4b/0.12653	4b/1.5903
600	0.6	30.79	150	9	180.46	1/0.33925	1/3.3045
				11	202.94	2/0.29300	2/3.0047
1,200	0	24.26	75	9	71.07	4a/0.08877	4a/1.3026
				11	79.93	4a/0.08262	4a/1.2503
1,200	0.6	36.17	75	9	105.98	2/0.52012	2/4.4816
				11	119.18	2/0.51170	2/4.3048
2,000	0	24.26	45	9	42.64	4a/0.07700	4a/0.86993
				11	47.95	4a/0.06072	4a/0.97132
2,000	0.3	34.47	45	9	60.60	4a/0.20146	4a/2.8982
				11	68.15	4a/0.20634	4a/2.6075

The author of the current work is aware of the fact that such mutual relations between R, a_{lim}, v, H, and l_0 can be treated by the engineer-practitioner as not practical, but engineering formula as follows:

$$a_{\text{lim}} = v^2/R - gH/s, \tag{5}$$

where s is a track gauge, and g is the gravity, which is, however, fulfilled.

Also, the vehicle velocity is expressed in m/s instead of in km/h.

Analysing the results of the work in Table 1, it can be seen that

(a) the most curvatures of the optimum TCs – 16 – had the 4th type of the curvature,

(b) curvatures of types 3 (linear curvature) and 5 did not appear even once,

(c) curvatures of types 1 and 2 appeared a total of 8 times, and these were cases for long TCs – over 100 m,

(d) with the increase in the length of the TC, a change in the types of TCs is observed,

(e) in all analysed cases, for a specific TC length and specific simulation conditions, the optimization procedure found the same type of curvature both for FC_1 and FC_2 quality functions.

Figure 3 shows the types of curvatures of the optimum TCs as a function of the curve length for the FC_1 and FC_2 criteria.

From Figure 3, it can be seen that a certain evolution of curvature types for two FCs exists. For the longest curves, the program found only curves with curvatures having an inflexion point in their middle part (types 1 and 2).

In this study, selected test results in the form of dynamical characteristics for one optimization process of the 9th degree TCs from Table 1 are presented. For this case: arc radius R = 600 m, velocity v = 24.26 m/s, curve length l_0 = 142.15 m, and objective function 1. Figures 4 and 5 show the dynamical courses of lateral displacements and accelerations of the vehicle body mass centres, vertical displacements of the vehicle body mass centre, and angular displacements of the vehicle body around the x-axis.

In general, in all optimization processes, the optimum TCs found had better dynamic behaviours of the vehicle body than the initial TC. For the presented case, this is confirmed by the courses – lateral displacements and accelerations, vertical displacements, and angular displacements of the body seen in Figures 4 and 5. The optimum TC found by the optimization procedure in the

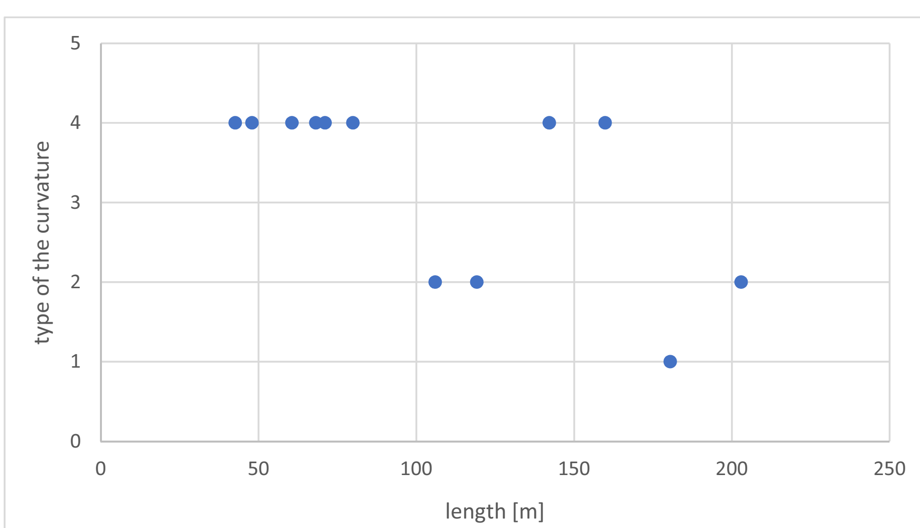

Figure 3: The types of curvatures vs the curve length for FC_1 and FC_2.

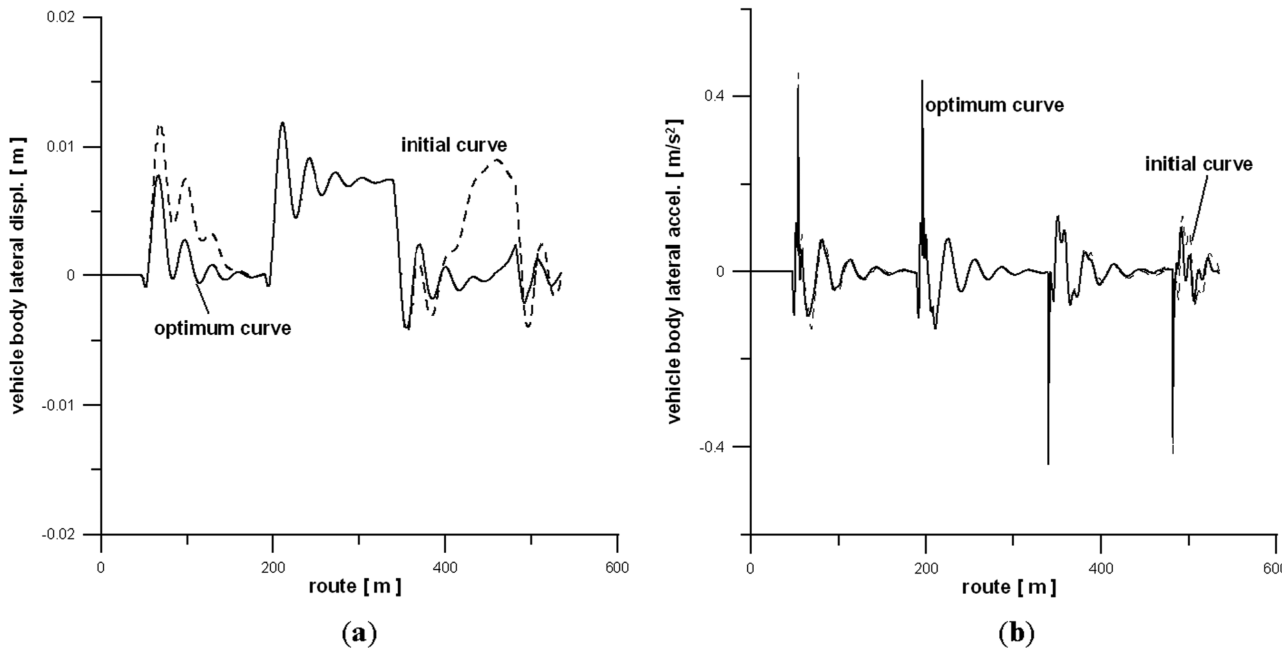

Figure 4: Vehicle body lateral displacements (a) and accelerations (b).

optimization process is a curve with curvature type 4. This curve was found in 290 steps ($i_{\lim} = 290$).

The ratio of the objective function values – the integral of the angular acceleration of the vehicle body around the *x*-axis – for the optimum curve to the objective function value for the initial curve 9 was 0.89 = (0.13512/0.15177).

6 Discussion

The achievements of this work include building, testing, and demonstrating high reliability of software in the work to optimize the shape of the spatial (3-dimensional) TC. This task can be considered non-trivial, not only due

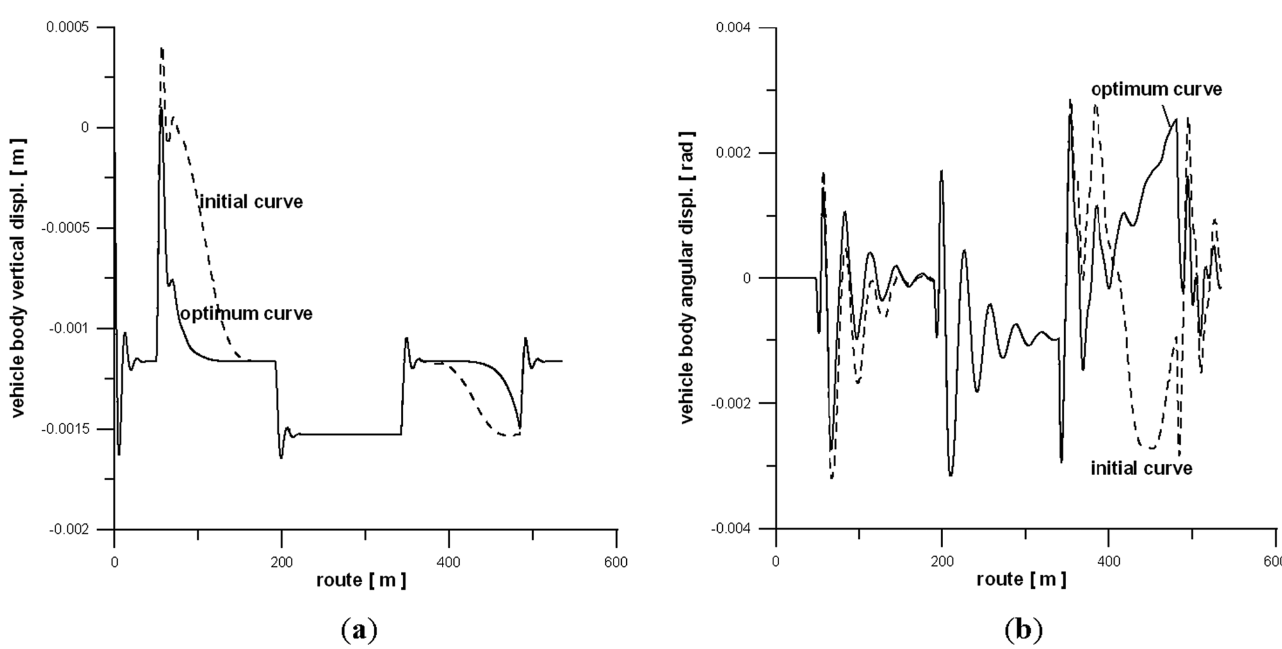

Figure 5: Vehicle body: (a) vertical displacements and (b) angular displacements around *x*-axis.

to the complexity of the calculations but also due to the preparation of a post-processor enabling the recording, archiving, and interpretation of a very large number of results of a diverse nature.

The more scientific elements include those concerning the modelling of the vehicle-track system related to the kinematics of vehicle motion along a polynomial TC of any degree and with any number of terms. It means here the determination of analytical formulas useful in the numerical determination of the components of the velocity and angular acceleration of the so-called transportation motion. On the one hand, these formulas were necessary to solve the main task of the work. On the other hand, they were not found anywhere in the literature. The author used them by modifying the simulation software for a two-axle wagon. In this way, the new dynamical model of this vehicle was obtained, capable of driving along routes containing polynomial TCs.

The author of the work also believes that he has managed to demonstrate that the methods of vehicle dynamics simulation (vehicle-track system) combined with the work and mathematical optimization methods can be a useful tool in the study and in the future, perhaps in the practical adoption of shapes and TCs. These methods, combined in the work as one whole, have never been used to optimize the shape of railway TCs. It is, therefore, an original achievement of the work. There is rather no doubt that this type of procedure is more individualized, and, thus, better adapted to the traffic prevailing on a given route compared to what is ensured by the methods used so far. Such dedicated routes are becoming more and more common.

The achievements of the work include showing that the method of shaping the TC based on the combined action of motion simulation and optimization can be effective in the case of odd degree curves as well as entry and exit transitions. It has also been shown that in the case of exit curves, shape optimization is not only possible but also that the obtained shape may be different than for the entry curve with analogous input data.

7 Conclusion

The main goal of the work was achieved. It has been shown that it is possible to find new shapes of TCs taking into account various dynamic criteria using an advanced vehicle model and mathematical optimization methods.

This study shows that the application of the criteria for evaluating the shape of railway TCs for lateral vehicle dynamics for extended routes generally results in different shapes of TCs. The adoption of two criteria for evaluating the shape of the curve, as well as a larger number of radii of the CA in the process of shape optimization, allowed more broadly than before (e.g. [1]), to look at the TCs of higher – 9th and 11th – degrees in the context of their properties affecting vehicle dynamics. As already mentioned, in all optimization processes, the optimum entry and exit TCs found had better dynamic behaviours of the vehicle body than the initial entry and exit curves.

The general conclusions drawn from the conducted research are as follows:

– For the shortest TCs (maximum length l_0 = 100 m) in all cases, TCs had the curvature of type 4,

– For very long TCs (l_0 > 180 m) in all cases, TCs had the curvatures of types 1 and 2,

– For the intermediate lengths (l_0 > 100 m and l_0 < 180 m), optimum TCs had the curvatures of types 2 and 4.

The obtained new shapes indicate the need for further study of TCs and set new research directions. Note that although the types of curvatures of the obtained optimum curves have been defined, the obtained results can be viewed more broadly. The obtained shapes indicate that it is worth taking a closer look at the TCs combining the features of a third-degree parabola and the known polynomial curves of higher degrees. Note that these curves do not necessarily have to be polynomials. Perhaps it would be advantageous to use curve types that are even more flexible from a shape-taking point of view than polynomials. Cubic splines (third degree splines) come to mind in this context. The supplementation of the optimization process with the length l_0 of the TC, treated as an additional decision variable, is considered to be interesting.

Conflict of interest: The author states no conflict of interest.

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